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Set Domination in Neutrosophic Graphs

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***Abstract:** Set domination is a fundamental concept in graph theory, crucial for understanding and analyzing network structures. In this paper, we extend the notion of set domination to Neutrosophic graphs, which accommodate uncertain and indeterminate information. We introduce the concept of neutrosophic set domination, which aims to identify subsets of vertices in a neutrosophic graph that either dominate the entire graph or are adjacent to dominating vertices. We define neutrosophic domination sets and investigate their properties in neutrosophic graphs. Furthermore, we explore algorithms for computing neutrosophic set domination and discuss their computational complexity. Finally, we provide numerical examples to illustrate the application of neutrosophic set domination in real-world scenarios, demonstrating its effectiveness in analyzing and optimizing uncertain networks.*
***Keywords:** Neutrosophic graph, set domination, set domination in neutrosophic graph, Cardinality, minimal Set domination.*

I. INTRODUCTION

Graph theory is the study of graphs that involves with the relationship between edges and vertices in the fields of mathematics and computer science. With applications in computer science, information technology, biosciences, mathematics, and linguistics, to mention a few, it is a widely recognized subject.

The similarities to equivalence relations can be obtained via the premise of similarity relation, which was first proposed by L. A. Zadeh [2]. Furthermore, the resolution identity occurs rather readily from this derivation. This chapter examines fuzzy graphs from the perspective of connections and shows how the findings are applied to information network modeling and clustering analysis 1965. In 1973[3], Kauffman designed the concept of fuzzy graph notation. However, the fundamental papers by Yeh and Bang [5] and Rosenfeld [4] in 1975 [4] are associated with the creation of fuzzy graph theory. Intuitionistic fuzzy sets, which are a generalization of Zadeh's fuzzy sets and are characterized by membership and non-membership functions, were first presented by Antonassov in 1984 [6]. The combination of the membership degree plus a non-membership degree, in accordance with Atanassov, does not equal one. Through the combination of non-standard analysis, Smarandache [7] developed the concept of neutrosophic sets.

The membership value in a neutrosophic set corresponds to three components: in accuracy, indeterminacy, and truth memberships; every membership value reflects a real standard or nonstandard subset of the nonstandard unit interval $]0^-,1^+[$ and there is no restriction on their sum. A method of analysis for real-world problems such as incorrect, inconsistent, and imprecise data is the neutrosophic set. Neutrosophic set theory, as a generalization of classical set theory, fuzzy set theory and intuitionistic fuzzy set theory, is applied in a variety of fields, including control theory, decision making problems, topology, medicines and in many more real-life problems. In order to simplify the application of neutrosophic sets in practical situations, Wang et al. [8] proposed the notion of single-valued neutrosophic sets. Three independent com- ponents in single-valued neutrosophic sets have values derived from the standard unit interval [0, 1]. The investigation of single valued neutrosophic sets has become very popular recently, and many authors have taken on this problem.

II. PRELIMINARIES

A. Definition 2.1. Fuzzy Set

If X is a universe of discourse and x be any specific element of X, then a fuzzy set $\tilde{A}$ on a X can be represented as a gathering of ordered pairs $\tilde{A}=\left\{\left(x,\mu_{\tilde{A}}(x)\right): x\in X\right\}$, where each pair $\left\{x,\mu_{\tilde{A}}(x)\right\}$ is referred as a singleton. Where $\mu_{\tilde{A}}(x)$ refers to as the membership function of element x in $\tilde{A}$. Membership function is defined as a function from X to a membership space M, where M is $[0,1]$.

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B. Definition 2.2. Neutrosophic set [9]

Let v be a given set. A neutrosophic set A in v is a characterized by truth membership function $T_A(x)$,, an indeterminate membership function $I_A(x)$ and false membership function $F_A(x)$. The function $T_A(x), I_A(x)$ and $F_A(x)$ are fuzzy sets on V.

i.e., $T_A(x): V \to [0,1]; I_A(x): V \to [0,1] \;\&\; F_A(x): V \to [0,1].$

$$0 \le T_A(x) + I_A(x) + F_A(x) \le 3$$

C. Definition 2.3. Neutrosophic graph [1]

Let V be a set, also assume E be a given set, with respect to v. A neutrosophic graph is an order pair of $G = (A, B)$ when $A: V \to [0,1]$is a neutrosophic set in V and $B: E \to [0,1]$ is a neutrosophic set in E such that

$$T_B(xy) \le min\{T_A(x),\ T_A(y)\}$$
$$I_B(xy) \le min\{I_A(x),\ I_A(y)\}$$
$$F_B(xy) \le min\{F_A(x),\ F_A(y)\}$$

for all $(x, y) \in E, V$ is called a vertex set of G and E is called edge set of G respectively.

D. Definition 2.4.

[11] Let $G = (A, B)$ be a single valued neutrosophic graph on V, then the neutrosophic vertex cardinality of G,

$$|V| = \sum_{(u,v)\in V} \frac{1 + T_A(u,v) + I_A(u,v) - F_A(u,v)}{2}$$

E. Definition 2.5.

Let G = (A, B) be a single valued neutrosophic graph on V, then the neutro- sophic edge cardinality of G,

$$|E| = \sum_{(u,v)\in E} \frac{1 + T_B(u,v) + I_B(u,v) - F_B(u,v)}{2}$$

F. Definition 2.6. Set domination

Let $G = (A, B)$ be a connected graph. A set $B \subset V$ is a set dominating set (SD set) if for every set $S \subset V - B$, there exist a non-empty set $T \subset B$ such that the subgraph $< T \cup S >$ induced by $T \cup S$ is connected. The set dominating number $\gamma_{NS}(G)$ is minimum cardinality of a set dominating neutrosophic set.

III. STUDY OF NEUTROSOPHIC GRAPH IN SET DOMINATIONS

Theorem 3.1. Every neutrosophic connected graph G of order n ≥ 4 has a set dominating set D whose complement T $\subset$ V $-$ D is also a set-dominating set G^*

Proof. Let G^* be any spanning tree of *G*, and let *u* be any vertex in *V*. Then the vertices in G^* fall into two disjoint sets *D* and D^* consisting, respectively, of the vertices with an even and odd distance from *u* in G^*. Clearly both *D* and $D^* = V - D$ are set-dominating set of *G*.

Theorem 3.2. If G is a neutrosophic connected graph with no isolated vertices, then the complement T $\subset$ V $-$D of every minimal set dominating set D $\subset$ V is a set dominating set.

Proof. Let *D* be any minimal set dominating set of *G*. Assume vertex $u \in D$ is not det dominating set by any vertex in $V - D$. Since, *G* has no isolated vertices *u* must be set dominated by atleast one vertex in $D - u$, that is $D - u$ is a set dominating set. Contradicting the minimality of *D*. Thus, every vertex in D is set dominated by atleast one vertex in $V - D$, and $V - D$ is a set dominating set.

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Theorem 3.3. Every neutrosophic Hamiltonian graph G of order $n \geq 4$ has a set dominating set B whose complement $V - B$ is also a set-dominating set G^*

Proof.

Let G^* be any spanning tree of G, and let u be any vertex in V. Then the vertices in G^* fall into two disjoint sets B and B^* consisting, respectively, of the vertices with an even and odd distance from u in G^*. Clearly both B and $B^* = V - B$ are set-dominating set of G.

Theorem 3.4. If a Neutrosophic graph NG has an independent sd-set then diam $NG \leq 4$.

Proof.

Suppose D^N is an independent sd-set of NG. We consider three different cases.

case 1: Let $u, v \in V - D$. Since D is an independent sd-set, both u ad v are adjacent to a common vertex in D, or u and v are adjacent. In either case, $d(u,v) \leq 2$.

case 2: Let $u, v \in D$. Since NG is connected and D is independent, there exist vertices u_1, v_1 in $V - D$ such that u_1u and v_1v are edges. Hence, by case 1, we have $d(u,v) \leq 2 + d(u_1, v_1) \leq 4$

case 3: Let $u \in D$ and $v \in V - D$. Then, there exists $u_1 \in V - D$ such that u_1 is adjacent to u and $d(u,v) \leq 1 + d(u_1, v) \leq 1 + 2$, by case 1.

Thus, for all $u, v \in V(NG); d(u,v) \leq 4$ and hence diam $G \leq 4$.

IV. NUMERICAL EXAMPLES

A. Problem: 4.1

Consider a Neutrosophic graph with four vertices, v_1, v_2, v_3, v_4, and the following neutrosophic attributes:

$$v_1 : (0.4, 0.3, 0.2); v_2 : (0.5, 0.3, 0.3)$$

$$v_3 : (0.5, 0.5, 0.2); v_4 : (0.5, 0.3, 0.2)$$

Here, each vertex v_i is associated with three values representing its truth, indeterminacy, and falsity attributes, respectively. Using MATLAB code to analyze the Neutrosophic graph is set domination

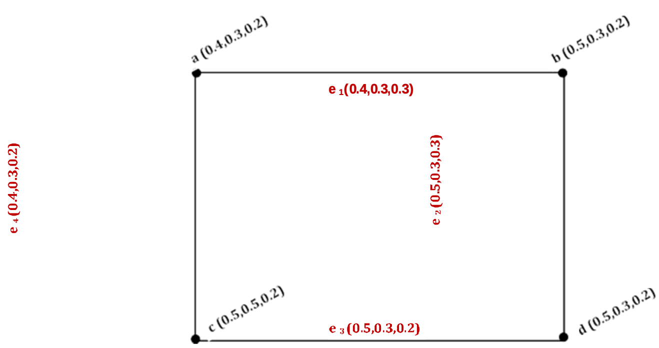


Figure 1: NG-SD

Solution:

Dominating Sets:

Vertex 1: 2 3 4
Vertex 2: 3 4
Vertex 3: 4
Vertex 4:

Minimal SD-Sets:

Vertex 1: 2 3 4
Vertex 2: 3 4
Vertex 3: 4
Vertex 4:

Domination Number: 3
SD-Dominating Sets:

Vertex 1: 2 3 4
Vertex 2: 3 4
Vertex 3: 4
Vertex 4:

B. Problem: 4.2

Consider a neutrosophic graph with four vertices, v_1, v_2, v_3, v_4, and the following neutrosophic attributes:

$$v_1 : (0.4, 0.3, 0.2); v_2 : (0.5, 0.3, 0.3)$$
$$v_3 : (0.5, 0.5, 0.2); v_4 : (0.5, 0.3, 0.2)$$

Here, each vertex v_i is associated with three values representing its truth, indeterminacy, and falsity membership function, respectively. Write MATLAB code to analyze the neutrosophic attributes of the graph and determine the set domination set, which consists of vertices that dominate all other vertices in the graph. Additionally, you should plot the truth, indeterminacy, and falsity attributes for each vertex.

Solution:

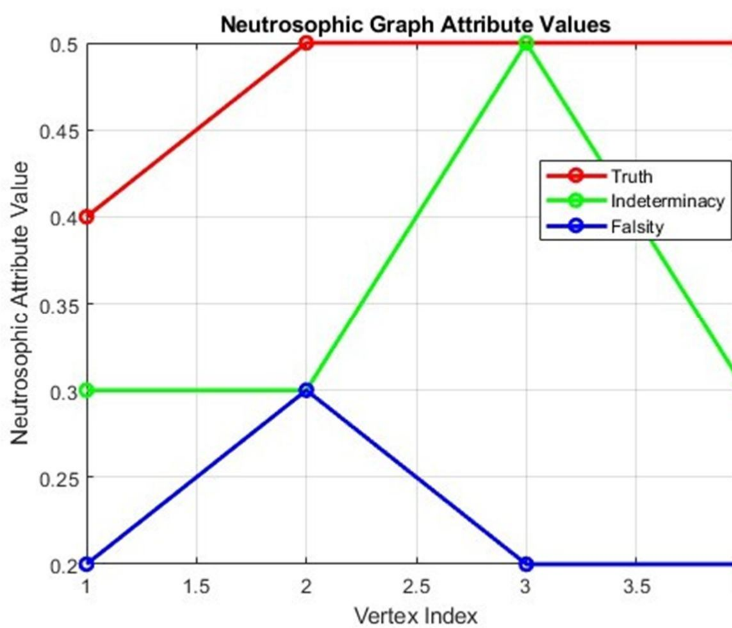


Figure 2: Neutrosophic Set Domination Graph

C. Problem: 4.3

Consider a neutrosophic graph with four vertices, v_1, v_2, v_3, v_4, and the following neutrosophic attributes:

$$v_1 : (0.4, 0.3, 0.2); v_2 : (0.5, 0.3, 0.3)$$
$$v_3 : (0.5, 0.5, 0.2); v_4 : (0.5, 0.3, 0.2)$$

Here, each vertex v_i is associated with three values representing its truth, indeterminacy, and falsity membership function, respectively. Write MATLAB code to analyze the neutrosophic attributes of the graph and determine the minimal-set domination set, which consists of vertices that dominate all other vertices in the graph. Additionally, you should plot the truth, indeter- minacy, and falsity attributes for each vertex.

Solution:

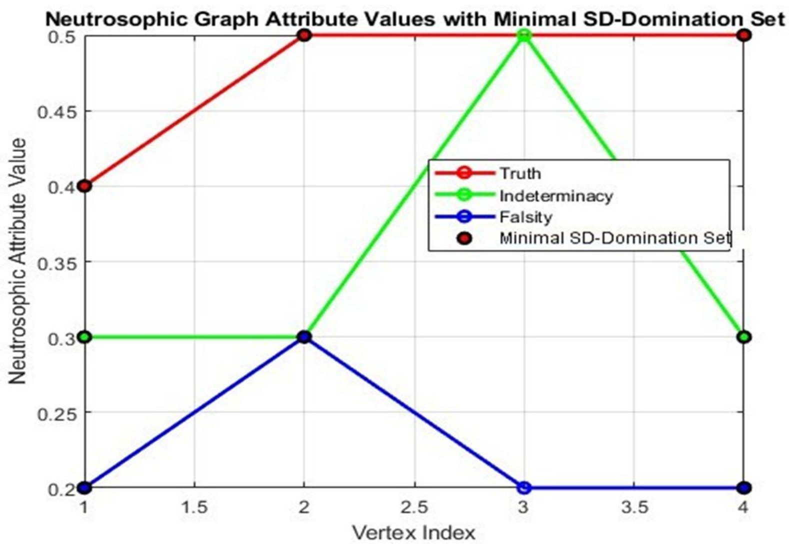


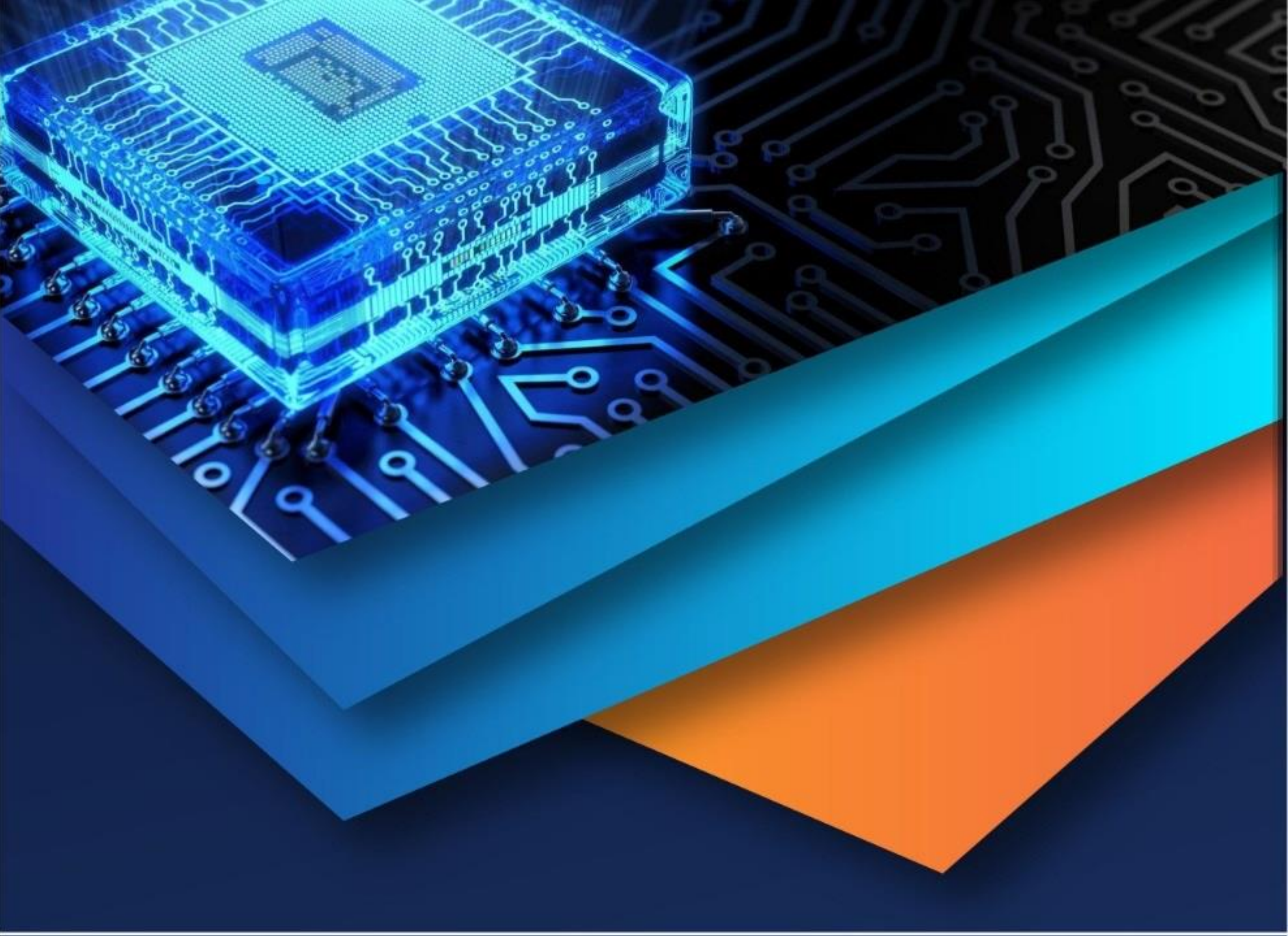
Figure 3: Neutrosophic Minimal Set Domination Graph

V. CONCLUSION

Fuzzy graph and intuitionistic graph techniques fail in situations where there is indeterminacy. The neutrosophic graph performs well in these circumstances. This study develops the concepts of set domination set in a neutrosophic graph. Here, appropriate examples are used to construct the definitions of set dominating sets, minimal SD-sets, independent set dominating sets, and set domination numbers in neutrosophic graphs. Finally, the set dominance set theorems in the neutrosophic graph are derived and Numerical examples solved in MATLAB. Future developments will expand and apply the idea of the set domination set in neutrosophic graphs to real-world problems.

REFERENCES

[1] Akram, M., Shahzadi, S.: Neutrosophic soft graphs with application. J. Intell. Fuzzy Syst. 32(1), 841-858 (2017)
[2] Zadeh, L.A.: Fuzzy sets. Information and Control, Vol 8, No 3, 338 - 353 (1965)
[3] Kaufmann, A.: Introduction to the Theory of Fuzzy Sets. Academic Press Inc., Orlando, Florida (1973)
[4] Rosenfeld, A. Fuzzy graphs. In fuzzy sets and their application to cognitive and decision processes.
[5] Zadeh, L.A., Fu, K.S., Shimura, M. (eds.) pp. 77 - 95. Academic Press, New York (1975)
[6] R. Yeh, S. Bang : Fuzzy relations, Fuzzy graphs and their applications to clustering anal- ysis,Computer Science, Mathematics University of Texas, Austin, Texas U.S.A. (1975) DOI:10.1016/B978-0-12-775260-0.50010-4
[7] K. Atanassov, Intuitionistic fuzzy relations, in L.Antonov,Ed., III International School "Au- tomation" and Scientific Instrumentation", Varna (1984) 56-57.
[8] Smarandache, F.: Neutrosophic set a generalization of the intuitionistic fuzzy set, Granular computing. In: 2006 IEEE International Conference, pp. 38-42. doi:10.1109/GRC.2006.1635754
[9] Wang, H., Smarandache, F., Zhang, Y.Q., Sunderraman, R.: Single-valued neutrosophic sets. Multisspace Multistruct. 4, 410 - 413 (2010)
[10] Smarandache, F.: Neutrosophy: A Unifying Field in LogicsâĂŤNeutrosophic Logic. Neu- trosophy, Neutrosophic Set, Neutrosophic Probability. American Research Press, Santa Fe (1999)
[11] Florentin Smarandache, W. B. Vasantha Kandasamy., Ilanthenral Kan- dasamy.:Neutrosophic Graphs: A New Dimension to Graph THeory,January 2015, DOI: 10.6084/M9.FIGSHARE.1574172.
[12] Muhammad Akram, Saba Siddique.,Bijan Davvaz.: New concepts in neutrosophic graphs with application,J. Appl. Math. Comput. (2017) DOI 10.1007/s12190-017-1106-3
[13] S. Satham Hussain , R. Jahir Hussain ., Florentin Smarandache.: Domination Number in Neutrosophic Soft Graphs, Neutrosophic Sets and Systems, Vol. 28, 2019.



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