iJRASET
International Journal For Research in Applied Science and Engineering Technology

INTERNATIONAL JOURNAL FOR RESEARCH

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY

Volume: 14 **Issue:** VIII **Month of publication:** August 2026

DOI: https://doi.org/10.22214/ijraset.2026.84495

www.ijraset.com

Call: 08813907089 | E-mail ID: ijraset@gmail.com

International Journal for Research in Applied Science & Engineering Technology (IJRASET)
ISSN: 2321-9653; IC Value: 45.98; SJ Impact Factor: 7.538
Volume 14 Issue VIII Aug 2026- Available at www.ijraset.com

Shayou Permutation Labelling for Some Families of Graphs

P. Shalini[1], K. Youvamathi [2]
[1]*Associate Professor in Mathematics,* [2]*II-M.Sc. PG & Research Department of Mathematics, Cauvery College For Women (Autonomous), Tiruchirappalli-18*

Abstract: ***This paper introduces a new graph labeling called Shayou Permutation Labeling. A formula is proposed to define the edge labels from the vertex labels. A function π is said to be a Shayou Permutation Labeling of a graph G if there exists a mapping from the vertex set of G to the set {1 , 2 ,…, p } such that each edge uv is assigned the label*** $\pi(uv) = \frac{(\pi(u)+\pi(v))!}{(\pi(u)+\pi(v)-1)!}$ ***,and the resulting edge labels are distinct numbers. The proposed labeling is based on the additive relationship between the labels of the end vertices of each edge. This labeling provides a systematic approach for assigning edge labels using factorial expressions derived from vertex labels. The concept contributes to the study of graph labeling by introducing a new labeling technique that can be extended to other classes of graphs. The existence of this labeling is studied for some classes of graphs, namely, the path graph and star graph. The results demonstrate that these graph families admit Shayou Permutation Labeling under the proposal labeling rule. Graph labelling can be classified into different types which is cited as [9,13,14,15,16,17]. Graph labeling concept can be extended to automata theory which is cited as [10, 11, 12]. Graph labeling is also extended to domination [1, 6, 7, 8].***

Keywords: Graph Labeling, Shayou Permutation Labeling, Factorial Labeling, Permutation Labeling.

I. INTRODUCTION

Graph theory is a significant branch of applied mathematics that deals with the study of graphs, which consist of a set of vertices and a set of edges connecting pairs of vertices. Owing to its simple structure and wide applicability, graph theory has become an active area of research with applications in computer science, communication networks, chemistry, biology, physics, operations research, engineering, and social sciences. Graphs provide an efficient mathematical model for representing and analyzing relationships among real-world objects and systems.

Throughout this paper, all graphs are assumed to be finite, simple, and undirected. Let G = (V , E) be a graph, where V(G) denotes the vertex set and E(G) denotes the edge set. The order of G, denoted by *p*, is the number of vertices, while the size of G, denoted by *q*, is the number of edges. Graph labeling is an important area of graph theory in which integers are assigned to the vertices, edges, or both according to specified conditions. Numerous labeling schemes, including graceful labeling, additive labeling, multiplicative labeling, strongly multiplicative labeling, square labeling, and cube labeling, have been introduced and extensively studied because of their theoretical significance and practical applications.

Motivated by these existing labeling techniques, this paper introduces a new concept called Shayou Permutation Labeling. In this labeling, the vertices are assigned distinct labels through a permutation, and each edge receives a label according to the proposed additive permutation formula.

The objective is to obtain distinct edge labels while preserving the required labeling conditions. The properties of this labeling are investigated for selected families of graphs, thereby extending the scope of graph labeling and providing new directions for future research.

II. BASIC DEFINITIONS

1) Definition 2.1

A linear graph (or simply a graph) G = (V, E) consists of set of objects $V = \{v_1 , v_2 ,\ldots\ldots, v_n\}$ called vertices and another set $E = \{e_1 , e_2 ,\ldots\ldots, e_k\}$, whose elements are called edges, such that each edge e_k is identified with an unordered pair (v_i , v_j) of vertices.

2) Definition 2.2

A graph G is said to be connected if there is at least one path between every pair of vertices in G, otherwise the graph is dis-connected.

III. MAIN RESULT

Definition 3.1l

Let G = (V, E) be a finite, simple, and undirected graph with p vertices. A Shayou Permutation Labeling of G is a bijection $\pi : V(G) \rightarrow \{1,2,\dots,p\}$,which assigns distinct labels to the vertices of G. For every edge uv in E(G), the induced edge label is defined by $\pi(uv) = \frac{(\pi(u)+\pi(v))!}{(\pi(u)+\pi(v)-1)!}$, where π(u) and π(v) are the labels of the end vertices u and v respectively.

Definition 3.2

A graph G is called a Shayou Permutation Graph if it admits a Shayou Permutation Labeling.

Theorem 3.1

The Path P_n is a Shayou Permutation Graphs for $n \geq 2$.

Proof

Let G be the path graph P_n

Let $V(P_n) = \{v_1, v_2, .., v_n\}$ be the vertex set and $E(P_n) = \{e_1, e_2, \dots, e_{n-1}\}$ be the edge set, where $e_i = v_i v_{i+1}$, $1 \leq i \leq n-1$ which are denoted as in figure 3.1

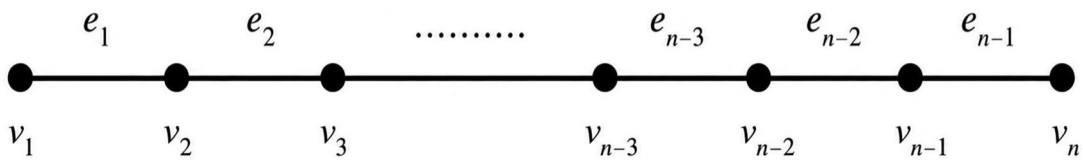


Figure 3.1. Path P_n with standard labeling

The Path P_n consists of n vertices and n-1 edges.

Assign the vertex labels by the bijection

$$\pi : V(G) \rightarrow \{1,2,\dots,n\}$$

defined as $\pi(v_i) = i$, $1 \leq i \leq n$.

Then, by the Shayou Permutation Labeling,

$$\pi(e_i) = 2i + 1 , 1 \leq i \leq n-1$$

The edges of the path are assigned distinct numbers.

Hence, the path P_n ($n \geq 2$) is a Shayou Permutation Graph.

Example : 3.1

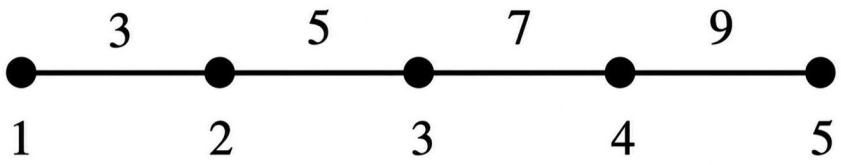


Figure 3.2. Path P_5

Example : 3.2

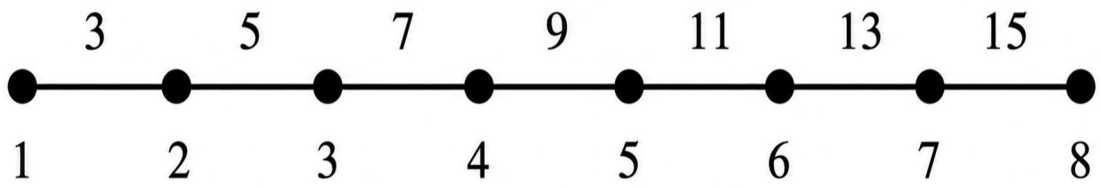


Figure 3.3. Path P_8

Theorem 3.2

The star $K_{1,n}$ is a Shayou Permutation Graphs for $n \geq 2$.

Proof

Let G be a star graph $K_{1,n}$

Let v_0 be the centre vertex and $\{ v_1, v_2, \ldots, v_n \}$ be the pendant vertices of $K_{1,n}$,and let $\{ e_1, e_2, \ldots, e_n \}$ be the edges of $K_{1,n}$, where $e_i = v_0 v_i$, $1 \leq i \leq n-1$.

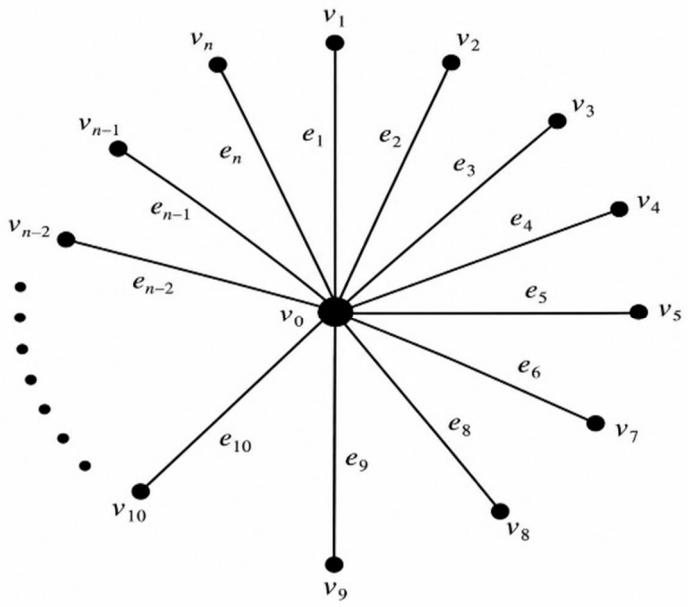


Figure 3.4. Star $K_{1,n}$ with standard labeling

The star graph $K_{1,n}$ consists of n + 1 vertices and n edges.

Define the bijection $\pi : V(G) \rightarrow \{ 1,2,\ldots, n+1\}$ by

$\pi(v_0) = 1$,

$\pi(v_i) = i + 1$, $1 \leq i \leq n$.

Then the induced edge labels are

$\pi (e_i) = i + 2$, $1 \leq i \leq n$.

The edges of the star are assigned distinct numbers .

Hence, the star $K_{1,n}$ $(n \geq 2)$ is a Shayou Permutation Graph.

Example : 3.3

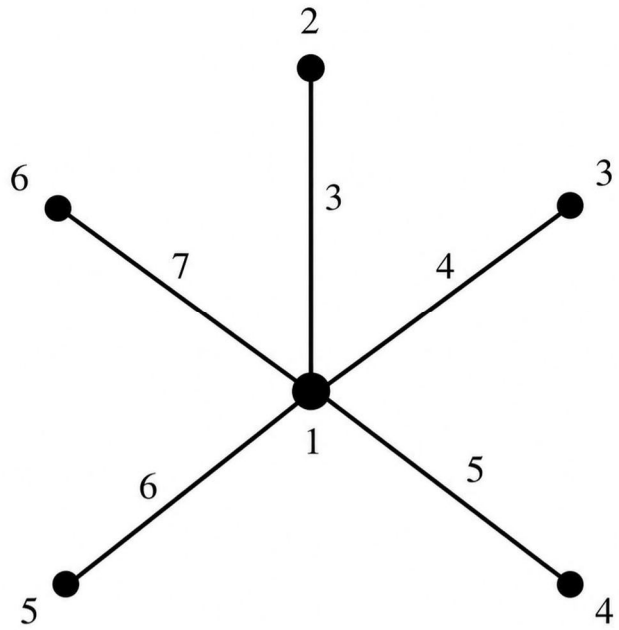


Figure 3.5. Star $K_{1,5}$

Example : 3.4

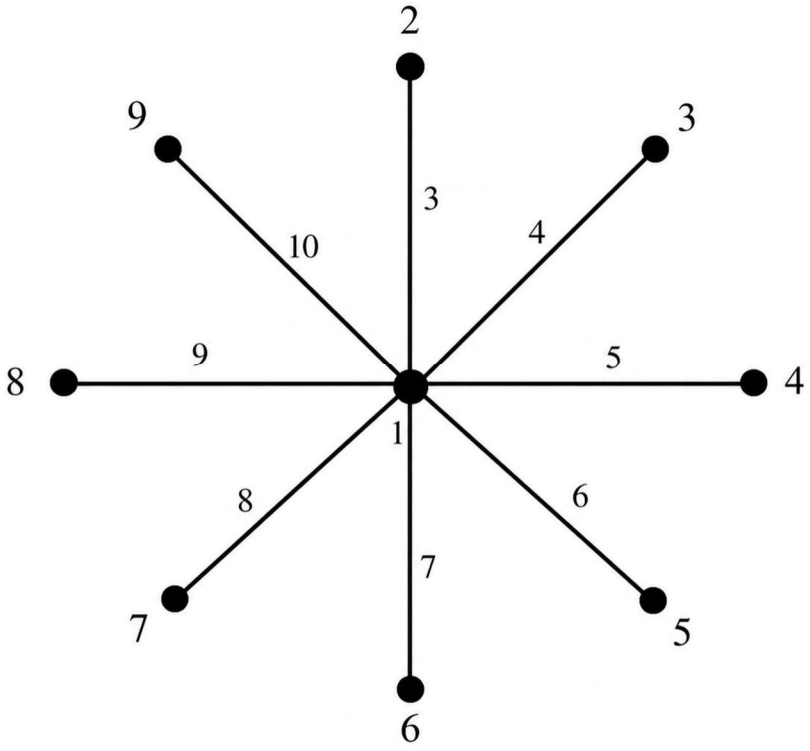


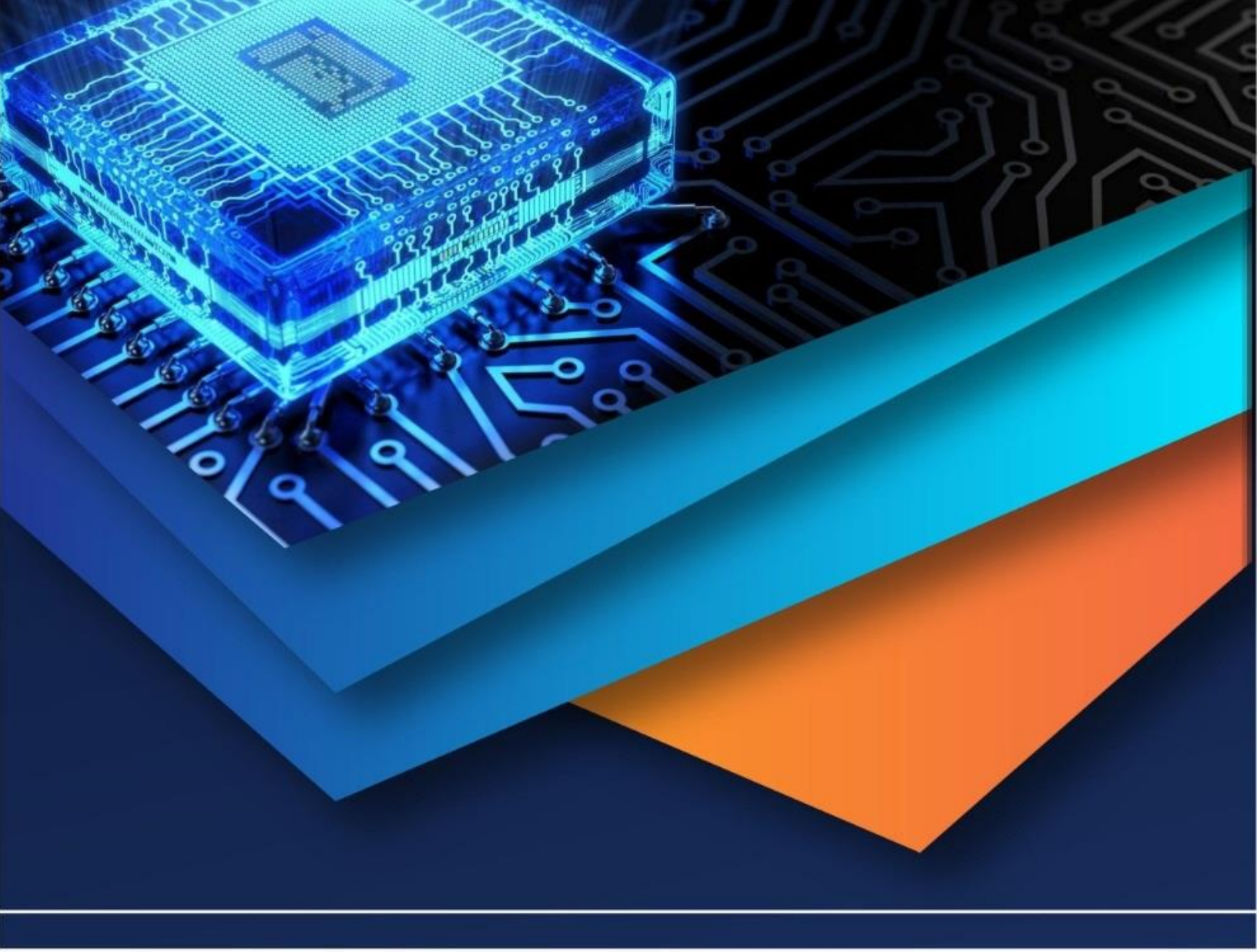
Figure 3.6. Star $K_{1,8}$

IV. CONCLUSION

This paper introduces a new graph labeling scheme called Shayou Permutation Labeling. The proposed labeling assigns distinct edge labels using a factorial-based formula and a bijective vertex labeling. The existence of this labeling has been established for the path graph P_n and the star graph $K_{1,n}$. This work contributes a new direction to graph labeling theory and provides a foundation for extending the concept to other classes of graphs in future research.

REFERENCES

[1] Amalorpava Jerline, J., Litta, E., Dhanalakshmi, K., Benedict Michael Raj, L., "F – Index of Generalized Mycielskian Graphs", International Journal of Recent Scientific Research, 10(6),33076 – 33081.
[2] Bodendick, R. and Walther, G., On number theoretical methods in graph labelingsRes.Exp.Maths (2,/1995) 3-25.
[3] Bloom, D.F. Hsu, On graceful directed graphs, SIAMJ, Alg. Discrete Math.,6(1985),519-536.
[4] Gallian, M.A., "A Dynamic survey of graph labelings" Electronic journal, 2000 (Volume-23).
[5] Harary, F., Graph Theory, New Delhi: Narosa Publishing House, 2001.
[6] Litta, E., Narmadha, S, "Proper Colourings in r – Regular Inverse sum Indeg Index Graph", Research and Applications Towards Mathematics and Computer Science, Volume 9, February 2024, Pg. No. 66-76, B P International, Print ISBN: 978-81-970187-8-7 eBook ISBN: 978-81-970187-9-4.
[7] Litta, E., Narmadha, S.,"Augmented Zagreb Index of corona Product of Some Special Graphs", International Journal for Research in Applied Science & Engineering Technology, Volume 12, March 2024 Pg. No. 580 – 585.
[8] Litta, E., Jayavarshini, N.,, "Proper Colourings in r – Regular Randic Index Graph", , International Journal for Research in Applied Science & Engineering Technology, Volume 12, February 2024 Pg. No. 1603– 1607.
[9] [9] Prakash, V., Gopi, R., Shalini, P., Anti Skolem Mean Labeling of Quadrilateral Snake Related Graphs, Tuijin Jishu/Journal of Propulsion Technology, Vol. 44, No. 5(2023), 495 – 500.
[10] Saridha. S, Haridha Banu. S"An innovative Ideas on Plus Weighted Linear Grammar",Volume 15, Issue 2, Aryabhatta journal of mathematics & Informatics (AJMI), June-Dcc 2023, Page No: 173-178, ISSN: 2394-9309.
[11] Saridha. S, Vidhya. R, "Application of Fuzzy Hypergraph in Medical Diagnosis", Aryabhatta Journal of Mathematics & Informatics,Volume 12, Issue I, Page no: 872-875, ISSN 2321-9653, Jan 2024, Impact factor: 7.538.
[12] Saridha, S., Shalini, P., "An overview of plus weighted DiGraph and its Applications", Aryabhatta Journal of Mathematics and Informatics, Volume 18, Issue 1, Jan -June 2026. Pg. No. 27-32.
[13] Shalini, P. Paul Dhayabaran, D, Minimization of Multiplicative Graphs, International Journal of Current Research, Vol. 7, Issue, 08, pp. 19511 – 19518, August, 2015.
[14] Shalini, P., Gowri, R., Paul Dhayabaran, D., "An Absolute Differences of Cubic and Square Difference Labeling For Some Families Of Graphs", The International journal of analytical and experimental modal analysis, Volume XI, Issue 10, October 2019, Page no: 538-544.
[15] Shalini, P., Priyadarshini, K., SP Mean Difference Labeling for Some Families of Graphs",Research and Applications Towards Mathematics and Computer Science, Volume 9, February 2024, Pg. No. 1-12, B P International, Print ISBN: 978-81-970187-8-7, eBook ISBN: 978-81-970187-9-4.
[16] Shalini, P., "Skolem Odd Vertex Graceful Signed Graphs for Path", Aryabhatta Journal of Mathematics and Informatics, Volume 15, Issue 1, Jan -June 2023. Pg. No. 121-126.
[17] Shalini, P., Saridha, S.,"A Study on Vertex Sum Cube Labeling for Path and Star Graphs", Aryabhatta Journal of Mathematics and Informatics, Volume 18, Issue 1, Jan -June 2026. Pg. No. 63-68.



10.22214/IJRASET

45.98

IMPACT FACTOR: 7.129

IMPACT FACTOR: 7.429

INTERNATIONAL JOURNAL FOR RESEARCH

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY