iJRASET
International Journal For Research in
Applied Science and Engineering Technology

INTERNATIONAL JOURNAL FOR RESEARCH

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY

Volume: 14 **Issue:** III **Month of publication:** March 2026

DOI: https://doi.org/10.22214/ijraset.2026.78575

Solution of Certain Admissible Control Systems Characterised by Nonlinear Integral and Integro-Differential Equations within a Bounded Banach Space via the Laplace Adomian Decomposition Algorithm

Baiju S
Associate Professor, Department of Mathematics, Government College, Kariavattom, Thiruvananthapuram, Kerala, India - 695 581

Abstract: A system of linear/nonlinear integral and integro-differential equations that exists in a bounded Banach space is described in the paper along with the acceptable control functions involved. We obtain a dual space of the p-integrable space and the solution to its dual from the theory. The system's kernel produces an admissible control function. Considering these factors and the existence of Laplace transform and its inverse conditions stipulated, we solved some various systems of nonlinear Volterra integro-differential equations using the Laplace-Adomian Decomposition Algorithm. The Laplace-Adomian Decomposition Algorithm is used without the need of restrictive assumptions, discretization, transformation or the guess of initial term. When compared to the current/exact solution, numerical results show that the Laplace-Adomian Decomposition Algorithm is very promising and effective particularly for polynomial, exponential and differential nonlinearity. The method developed by the author with an extension, modification and new approach to initial and boundary value problem named Shooting Type Laplace Adomian Decomposition Algorithm (STLADA). The paper describes a system of linear/nonlinear integral and integro-differential equations that occur in a bounded Banach space, along with the appropriate control functions involved. From the theory, we derive a dual space of the p-integrable space and the solution to its dual. An acceptable control function is generated by the system's kernel. We used the Laplace-Adomian Decomposition Algorithm, an analytical approach to solve a number of different systems of nonlinear Volterra integro-differential equations in light of these parameters as well as the presence of the Laplace transform and its inverse conditions. Restrictive assumptions, discretisation, transformation, and initial term guesswork are not required when using the Laplace-Adomian Decomposition Algorithm. Numerical results indicate that the Laplace-Adomian Decomposition Algorithm is highly promising and efficient when compared to the current/exact solution, especially for polynomial, exponential, and differential nonlinearity. The Shooting Type Laplace Adomian Decomposition Algorithm (STLADA) is an extension, modification, and novel technique to the boundary value problems that the author created.
Keywords: System of integral and integro-differential equation, Laplace-Adomian Decomposition Algorithm, Admissible control functions, Analytic and numerical solution, p-integrable space, nonlinear equations, Adomian Polynomials.

I. PRELIMINARIES

The theory and application of integro-differential equations are essential in practical mathematics. In addition to being mathematical models for a variety of real-world situations, these equations are utilised to reformulate mathematical problems. These problems have analytical answers that are essential because they offer a physical comprehension of the problems that the numerical results cannot. Nonlinear problems can be solved analytically using a variety of methods, such as the Taylor Series Method [2], Adomian's Method [1, 6, 16], Taylor Expansion Approach [15], Laplace-Adomian Decomposition Algorithm [14], Variational Iteration Method [12, 27, 28], and Homotopy Perturbation Method [13, 18]. The numerical solution of nonlinear integro-differential equations has been thoroughly studied by several authors. Many works in recent years have focused on the development of increasingly complex and efficient methods for integro-differential equations, such as the Wavelet-Galerkin Method [4], Lagrange's Interpolation Method [26], and Tau Method [19, 20, 21, 22, 25]. A particular class of integro-differential equations is solved using most of the previously discussed methods.

In this paper, we propose the Laplace-Adomian Decomposition Algorithm (LADA) to solve a system of nonlinear Volterra integro-differential equations, inspired by the work [7, 8, 9, 10, 11, 14, 17, 24, 25, 27, 28, 29, 30, 31, 32, 33, 34]. Biazar [14] developed the Adomian Decomposition Method to solve systems of integral differential equations. The variational iteration method was used in [27, 28] to solve a system of linear integrate-differential equations. Instead of selecting the exact solution with uncertain constants, the variational iteration approach's initial iteration was selected at random. Biazar et al. [13] solved the system of integro-differential equations using He's Homotopy Perturbation Method. A New Variational Iteration Method for solving systems of integro-differential equations was recently developed by J. Sabei Nadjafi and M. Tamamgar [27] used an efficient algorithm. A set of integro-differential equations was solved by E. Yusufoglu [35].
G. Ebadi [17] developed the operational method to the Tau technique for the numerical solution of the system of nonlinear Volterra integro-differential equations with initial or boundary conditions without linearising. The operational approach to the Tau technique has lately been applied to the numerical solution of the system of linear integral and integro-differential equations [19, 21, 22, 25]. LADA may solve linear, nonlinear, integral, or integro-differential equations in any order. This approach is very straightforward, efficient, and assumes nothing about the starting term, and it produces results that are incredibly close to the exact solution. In the first section, we explain the admissible control functions which are usually in the space $L^p([0,T])$ describes a control system involved in system of integral and integro-differential equations. Following the approach explanation in Section 3, numerical results are given in Section 4.

II. CONTROL SYSTEM DESCRIBED BY A NONLINEAR INTEGRAL AND INTEGRO-DIFFERENTIAL EQUATIONS

The solution of nonlinear equations requires a control system with constrained resources. The behaviour of the control system determines the acceptable control functions, which are typically selected from the closed ball centred at the origin with radius μ and involved in the space $L^p([0,T]), p > 1$. These control functions are bounded, continuous with respect to p for each fixed μ, and satisfy the Lipschitz condition with respect to μ for each fixed p. Systems with finite energy supplies that are depleted by consumption are modelled by control functions with integral constraints on the controls. Nonlinear integral and integro-differential equations describe the behaviour of these systems. The closed ball of the space $L^p([0,T])$ contains the admissible control functions; $p > 1$ with radius μ and centred at the origin are continuous with regard to p in Hausdorff metric. In [3, 18, 19, 20], the conditions of the sets on p and μ are examined.
Consider the linear/nonlinear system of integral/integro-differential equations,

$$y_i^{(k)}(t) = \alpha_i\left(t, y_i^{(k')}(t)\right) + \lambda \int_{t_0}^{t} K_i(t, x, y_i^{(k)}(x), u_j(x))dx \qquad (1)$$

with $i = 1,2,3,\dots,n,\ \ j = 1,2,3,\dots,m; k$ and k' be the number of times its derivative, k' be restricted such that $k \neq k'$ and $k > k'$. Also $y_i(x) \in \mathbb{R}^m$ is the state vector and $u_i(x) \in \mathbb{R}^m$ is the control vector, $t \in [t_0, T]$, $\lambda \in \mathbb{R}$. Let $p > 1$ and $\mu > 0$ such that $u_i(.) \in X = \langle L^p([t_0,T]); \mathbb{R}^m\rangle$, the inner product space such that $||u_i(.)||_p \leq \mu$ is an admissible control functions, where

$$||u_i(.)||_p = \left[\int_{t_0}^{T} ||u_i(t)||^p dt\right]^{\frac{1}{p}} \qquad (2)$$

Define $u_{p,\mu} = \{u_i(.) \in L^p; \mathbb{R}^m): ||u_i(.)||_p \leq \mu\}$ is the is the set of all admissible control functions in X. All the control functions satisfies the properties of the inner product space $L^p([0,T])$. The functions $\alpha_i(t, y_i^{(k')}(t))$, $K_i(t, x, y_i^{(k)}(x), u_j(x))$, and the interval of integration $[t_0, T]$ satisfies the following conditions.

1. The function $\alpha_i(.) : [t_0, T] \times \mathbb{R}^n \to \mathbb{R}^n$ and $K_i(.) : [t_0, T] \times [t_0, T] \times \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^n$ are continuous.
2. There exists $M \in [0,1)$, $M_1, M_2, M_3 \geq 0$, $N_1, N_2, N_3 \geq 0$ such that
   (a) $||\alpha_i\left(t, y_l^{(k')}(t)\right) - \alpha_i\left(t, y_{l'}^{(k')}(t)\right)|| \leq M_0 \| y_l^{(k)} - y_{l'}^{(k)}\|$
   (b) $||K_i(t_1, x, y_l, u_1) - K_i(t_2, x, y_l', u_2)|| \leq [M_1 + N_1(||u_1|| + ||u_2||)| t_1 - t_2| + [M_2 + N_2(||u_1|| + ||u_2||)| x_1 - x_2| + [M_3 + N_3(||y_l|| + ||y_{l'}||)| u_1 - u_2|$

for every $(t_1, x, y_l, u_1), (t_2, x, y_{l'}, u_2) \in [t_0, T] \times [t_0, T] \times \mathbb{R}^n \times \mathbb{R}^m$

(c) There exists $p_0 > 1$ and $\mu_0 > 1$ such that $0 \le \lambda[M_2(T-t_0) + 2N_2(T-t_0)^{\frac{p_0-1}{p_0}}\mu_0 < 1 - M$

If $K_i(t_1, x, y_{i,} u_j) = \phi(t, x, y_i) + \psi(t, x, y_i)u_j$, where the functions $(t, x, y_i) \to \phi(t, x, y_i)$ and $(t, x, y_i) \to \psi(t, x, y_i)$ are continuous with respect to (t, x, y_i) and satisfies Lipchitz conditions. Also,

$$L(\lambda; p, \mu) = M + \lambda[M_2(T-t_0) + 2N_2(T-t_0)^{\frac{p_0-1}{p_\mu}} \quad (3)$$

Condition (c) implies $0 \le L(\lambda; p_0, \mu_0) < 1$, there exists $\eta > 0$ such that $0 \le L(\lambda; p, \mu) < 1$ for every $p \in [p_0 - \eta, p_0 + \eta]$ where $p_0 - \eta > 1$ and $\mu_0 - \eta > 0$.

Define the conjugate operator,

$$L^*(\lambda) = \max\{L(\lambda; p, \mu): p \in [p_0 - \eta, p_0 + \eta], \mu \in [\mu_0 - \eta, \mu_0 + \eta]\} \quad (4)$$

Let p and μ be fixed and $u^*(.) \in U_{p,q}$, under these conditions, there is a continuous function $u^*(.): [t_0, T] \to \mathbb{R}^n$ satisfying the system.

The above conditions produced the following theorem with the motivation obtained from [22, 23]. Further properties are also documented in the same author.

$$y_i^*(t) = \alpha_i(t, y_i^{*(k)}) + \lambda \int_{t_0}^{t} K_i(l, x, y_i^*(x), u_j^*(x))dx \quad (5)$$

for every $t \in [t_0, T]$ is the trajectory of the system (1) generated by the admissible control function $u_j^*(.) \in U_{p,q}$ yields the control function $u_j(.) \in U_{p,q}$ denoted by $y_i(.; u(.))$.

Theorem 1: Conditions(a), (b) and (c) holds. Then every admissible control function $u_j(.) \in U_{p,\mu}$ generates a unique trajectory $y_i(.; u(.))$ of the system (1), where $p \in [p_0 - \alpha, p_0 + \alpha], \mu \in [\mu_0 - \eta, \mu_0 + \mu]$ with $\mu > 0$.

From the above point of view and the conditions of stipulated in Laplace transform α_i and K_i in which the Laplace and its inverse exists are considered in the upcoming sections with some applications.

III. LAPLACE-ADOMIAN DECOMPOSITION ALGORITHM (LADA)

Consider the linear/nonlinear integro-differential equations

$$\begin{aligned} y_1^{(m_1)} = {} & F_1(x, y_2(x), y_2^{(1)}(x), \dots, y_2^{(m_2)}(x), y_3(x), y_3^{(1)}(x), \dots, y_3^{(m_3)}(x), \dots, y_n(x), \\ & y_n^{(1)}(x), \dots, y_n^{(m_l)}(x)) + \int_0^x K_1[x, t, y_1(t), y_1^{(1)}(t), \dots, y_1^{(m_1)}(t), y_2(t), y_2^{(1)}(t), \dots, y_2^{(m_2)}(t), \\ & y_3(t), y_3^{(1)}(t), \dots, y_3^{(m_3)}(t) \dots, y_n(t), y_n^{(1)}(t), \dots, y_n^{(m_l)}(t)]dt \end{aligned}$$

$$\begin{aligned} y_2^{(m_2)} = {} & F_2(x, y_1(x), y_1^{(1)}(x), \dots, y_1^{(m_1)}(x), y_3(x), y_3^{(1)}(x), \dots, y_3^{(m_3)}(x), \dots, y_n(x), \\ & y_n^{(1)}(x), \dots, y_n^{(m_l)}(x)) + \int_0^x K_2[x, t, y_1(t), y_1^{(1)}(t), \dots, y_1^{(m_1)}(t), y_2(t), y_2^{(1)}(t), \dots, y_2^{(m_2)}(t), \\ & y_3(t), y_3^{(1)}(t), \dots, y_3^{(m_3)}(t) \dots, y_n(t), y_n^{(1)}(t), \dots, y_n^{(m_l)}(t)]dt \end{aligned}$$

.

.

$$\begin{aligned} y_n^{(m_l)} = {} & F_n(x, y_1(x), y_1^{(1)}(x), \dots, y_1^{(m_1)}(x), y_2(x), y_2^{(1)}(x), \dots, y_2^{(m_2)}(x), \dots, y_{n-1}(x), \\ & y_{n-1}^{(1)}(x), \dots, y_{n-1}^{(m_l-1)}(x)) + \int_0^x K_n[x, t, y_1(t), y_1^{(1)}(t), \dots, y_1^{(m_1)}(t), y_2(t), y_2^{(1)}(t), \dots, y_2^{(m_2)}(t), \\ & y_3(t), y_3^{(1)}(t), \dots, y_3^{(m_3)}(t) \dots, y_n(t), y_n^{(1)}(t), \dots, y_n^{(m_l)}(t)]dt \end{aligned} \quad (6)$$

In this system $m_1, m_2, \dots, m_l$ are the order of derivatives, F_i and $K_i; i = 1,2,3, \dots, n$ are continuous functions of several variables, which are given and $y_1(x), y_2(x), y_3(x), \dots, y_n(x)$ are the functions to be determined. From the system, taking the first nonlinear integro-differential equation and applying Laplace transform on both sides, we obtain

$$\mathcal{L}(y_1{}^{(m_1)}) = \mathcal{L}(F_1(x, y_2(x), y_2{}^{(1)}(x), \dots, y_2{}^{(m_2)}(x), y_3(x), y_3{}^{(1)}(x), \dots, y_3{}^{(m_3)}(x), \dots, y_n(x), y_n{}^{(1)}(x), \dots, y_n{}^{(m_l)}(x))) + \frac{1}{s}\mathcal{L}[K_1[x, t, y_1(t), y_1{}^{(1)}(t), \dots, y_1{}^{(m_1)}(t), y_2(t), y_2{}^{(1)}(t), \dots, y_2{}^{(m_2)}(t), y_3(t), y_3{}^{(1)}(t), \dots, y_3{}^{(m_3)}(t) \dots, y_n(t), y_n{}^{(1)}(t), \dots, y_n{}^{(m_l)}(t)] \quad (7)$$

Take $y_1(x) = \sum_{m=0}^{\infty} y_{1m}(x)$, $y_2(x) = \sum_{m=0}^{\infty} y_{2m}(x)$, …, $y_n(x) = \sum_{m=0}^{\infty} y_{nm}(x)$, be the solution of the system. The nonlinear terms in the system is decomposed into Adomian Polynomials [1]. Substitution of these terms in (2) and using the iterative algorithm in LADA, we get a system of equations in terms of the unknown functions and Adomian polynomials. From the iterative algorithm, we get the initial term y_{10}. Next, we take the second equation from the system and proceeding as above, we get the initial term y_{20}. Continuing in this way, we get y_{n0}. Substitution of these initial terms successively in the iterative algorithm, we get the successive terms of the approximate solution. Defining the sum up to n terms of the corresponding series as $S_n(x), T_n(x), R_n(x), \dots; n = 1,2,3, \dots$, the approximate analytic solution of the system of integro-differential equation is

$$\left.\begin{aligned} S_n(x) &= y_{10}(x) + y_{11}(x) + y_{12}(x) + \cdots + y_{1n}(x) \\ T_n(x) &= y_{20}(x) + y_{21}(x) + y_{22}(x) + \cdots + y_{2n}(x) \\ R_n(x) &= y_{30}(x) + y_{31}(x) + y_{32}(x) + \cdots + y_{3n}(x) \end{aligned}\right\} \quad (8)$$

and so on.

IV. SOME APPLICATIONS

A. *Second order - Differential and Polynomial nonlinearity*

Consider the following system of nonlinear integro-differential equation [13, 28, 34]

$$u''(x) = 1 - \frac{1}{3}x^3 - \frac{1}{2}v'^2(x) + \frac{1}{2}\int_0^x [u^2(t) + v^2(t)]dt;$$

$$v''(x) = -1 + x^2 - x\,u(x) + \frac{1}{4}\int_0^x [u^2(t) - v^2(t)]dt$$

with the initial conditions $u(0) = 1, u'(0) = 2, v(0) = -1, v'(0) = 0$.

Consider the first nonlinear equation from the system and taking Laplace transform on both sides, we get

$$\mathcal{L}(u(x)) = \frac{1}{s} + \frac{2}{s^2} + \frac{1}{s^3} - \frac{2}{s^6} - \frac{1}{2s^2}\mathcal{L}\left(\left(v'(x)\right)^2\right) + \frac{1}{2s^2}\mathcal{L}(u^2(x)) + \frac{1}{2s^2}\mathcal{L}(v^2(x)) \quad (9)$$

Let $u(x) = \sum_{n=0}^{\infty} u_n(x)$ and $v(x) = \sum_{n=0}^{\infty} v_n(x)$ be the solution of the system. The nonlinear terms in (6) can be decomposed into Adomian polynomials as follows.

$u^2(x) = \sum_{n=0}^{\infty} A_n(x)$, $v^2(x) = \sum_{n=0}^{\infty} B_n(x)$, $(v')^2(x) = \sum_{n=0}^{\infty} C_n(x)$ where

$A_0 = u_0{}^2,\ A_1 = 2u_0u_1, A_2 = u_1{}^2 + 2u_0u_2, A_3 = 2u_0u_3 + 2u_1u_2, A_4 = 2u_0u_4 + 2u_1u_3 + u_2{}^2$;
$B_0 = v_0{}^2,\ B_1 = 2v_0v_1, B_2 = v_1{}^2 + 2v_0v_2, B_3 = 2v_0v_3 + 2v_1v_2, B_4 = 2v_0v_4 + 2v_1v_3 + v_2{}^2$;
$C_0 = v'_0{}^2,\ C_1 = 2v'_0v'_1, C_2 = v'_1{}^2 + 2v'_0v'_2,\ C_3 = 2v'_0v'_3 + 2v'_1v'_2,\ C_4 = 2v'_0v'_4 + 2v'_1v'_3 + v'_2{}^2$ and so on. Substituting these results into (9) and using linearity property of Laplace transforms, the iterative algorithm yields

$$\mathcal{L}(u_0(x)) = \frac{1}{s} + \frac{2}{s^2} + \frac{1}{s^3} - \frac{2}{s^6} \quad (10)$$

$$\mathcal{L}(u_{n+1}(x)) = -\frac{1}{2s^2}\mathcal{L}(C_n) + \frac{1}{2s^2}\mathcal{L}(A_n) + \frac{1}{2s^2}\mathcal{L}(B_n) \quad (11)$$

Again, consider the second nonlinear integro-differential equation from the system and proceeding as above, we get the following iterative algorithm.

$$\mathcal{L}(v_0(x)) = -\frac{1}{s} - \frac{1}{s^3} + \frac{2}{s^5} \quad (12)$$

$$\mathcal{L}(v_{n+1}(x)) = -\frac{1}{s^2}\mathcal{L}(x\,u_n(x)) + \frac{1}{4s^3}\mathcal{L}(A_n) - \frac{1}{4s^3}\mathcal{L}(B_n) \quad (13)$$

Using inverse Laplace transforms in (10) and (12), we get the initial terms $u_0(x)$ and $v_0(x)$. That is,

$$u_0(x) = 1 + 2x + \frac{1}{2}x^2 - \frac{1}{60}x^5 \qquad (14)$$

$$v_0(x) = -1 - \frac{1}{2}x^2 + \frac{1}{12}x^4 \qquad (15)$$

Substituting $u_0(x)$ and $v_0(x)$ into (11) and (13), with the use of Adomian polynomials stated above and taking inverse Laplace transforms, we get the following iterative scheme.

$$u_1(x) = \frac{1}{6}x^3 + \frac{1}{24}x^4 + \frac{1}{20}x^5 + \frac{7}{360}x^6 + \frac{1}{1260}x^7 - \frac{1}{960}x^8 - \frac{1}{6720}x^9 - \frac{1}{86400}x^{10} + \frac{1}{285120}x^{11} + \frac{1}{12355200}x^{13}$$

$$v_1(x) = -\frac{1}{6}x^3 - \frac{1}{8}x^4 - \frac{1}{120}x^5 + \frac{1}{240}x^6 + \frac{1}{5040}x^7 + \frac{11}{40320}x^8 + \frac{1}{120960}x^9 - \frac{1}{172800}x^{10} - \frac{1}{570240}x^{11} + \frac{1}{24710400}x^{13}$$

$$u_2(x) = -\frac{1}{40}x^5 - \frac{1}{72}x^6 + \frac{3}{560}x^7 + \frac{5}{1152}x^8 + \frac{11}{18144}x^9 + \frac{1}{129600}x^{10} - \frac{59}{6652800}x^{11} - \frac{41}{4838400}x^{12} - \frac{701}{622702080}x^{13} - \frac{17}{215255040}x^{14} + \frac{12461}{326918592000}x^{15} + \frac{391}{63402393600}x^{16} - \frac{977}{8468748288000}x^{17} + \frac{149}{10888390656000}x^{18} - \frac{1}{99461260800}x^{19} + \frac{1}{99461260800}x^{19} + \frac{1}{2028229632000}x^{20} - \frac{1}{5915669700000}x^{21}$$

$$v_2(x) = -\frac{1}{180}x^6 - \frac{1}{2520}x^7 - \frac{19}{26880}x^8 - \frac{137}{725760}x^9 + \frac{13}{302400}x^{10} + \frac{143}{7257600}x^{11} + \frac{151}{319334400}x^{12} - \frac{983}{2075673600}x^{13} - \frac{589}{4981616640}x^{14} - \frac{751}{108972864000}x^{15} + \frac{5329}{2324754432000}x^{16} + \frac{1297}{3387499315200}x^{17} + \frac{889}{21776781312000}x^{18} - \frac{1}{198922521600}x^{19} - \frac{1}{4056459264000}x^{20} - \frac{1}{11831339520000}x^{21}$$

$$u_3(x) = -\frac{1}{240}x^6 - \frac{1}{168}x^7 - \frac{11}{3360}x^8 - \frac{13}{60480}x^9 + \frac{683}{3628800}x^{10} + \frac{391}{7983360}x^{11} + \frac{3293}{87091200}x^{12} + \frac{13291}{1556755200}x^{13} - \frac{11449}{17435658240}x^{14} - \frac{1245401}{2615348736000}x^{15} - \frac{1482101}{20922789888000}x^{16} + \frac{79}{403273728000}x^{17} + \frac{6177803}{3201186852864000}x^{18} + \frac{8199931}{24329020081766400}x^{19} + \frac{4661513}{270322445352960000}x^{20} - \frac{101896849}{1703031405723648000}x^{21} - \frac{1180097003}{1124000727777607680000}x^{22} + \frac{3220963}{8285902800924672000}x^{23} + \frac{33743893}{492419366454951936000}x^{24} + \frac{5712829}{5275921783445913600000}x^{25} - \frac{825403}{2485035622637568000000}x^{26} + \frac{9829}{4792568700801024000000}x^{27} - \frac{1}{12880069455052800000}x^{28} + \frac{1871}{593484978112266240000}x^{29}$$

$$v_3(x) = \frac{11}{26880}x^8 + \frac{1}{8064}x^9 - \frac{17}{181440}x^{10} - \frac{5927}{159667200}x^{11} + \frac{643}{383201280}x^{12} + \frac{9437}{4981616640}x^{13} + \frac{119}{3228825600}x^{14} + \frac{120877}{1743565824000}x^{15} - \frac{101477}{8369115955200}x^{16} - \frac{158087}{27360571392000}x^{17} - \frac{11017793}{12804747411456000}x^{18} + \frac{119087}{5529322745856000}x^{19} + \frac{3491393}{108128978141184000}x^{20} + \frac{299137}{742061614694400000}x^{21} + \frac{817373}{6192841475358720000}x^{22} - \frac{332699}{5699298222858240000}x^{23} - \frac{1195759}{1036672350431477760000}x^{24} - \frac{18083293}{5275921783445913600000}x^{25} - \frac{73}{3567036778905600000}x^{26} + \frac{14827}{958513740160204800000}x^{27} - \frac{1}{33488180583137280000}x^{28} + \frac{4283}{356090986867359744000000}x^{29}$$

Hence the approximate solution of the system of nonlinear integro-differential equation is $S_n(x)$ and $T_n(x)$ defined in (8) whose convergence is shown in Tables 1 and 2 and is compared with exact solution on interval$[-1,1]$. Exact solution of the above system is $u(x) = x + e^x$ and $v(x) = x - e^x$. This problem was solved using HPM [13], VIM [34] and Modified Variation of Parameters Method (MVPM) [34]. Comparison with the exact solution, present method, MVPM and VIM are shown in Fig. 1 for $u(x)$ and Fig. 2 for $v(x)$. From this we see that $u(x)$ is same for all the three methods but LADA is very close to the exact solution than MVPM and VIM in the case of $v(x)$. From Fig. 1, these methods coincide if $|x| < 0.7$ and the error increases if $|x| > 0.7$ for MVPM and VIM.

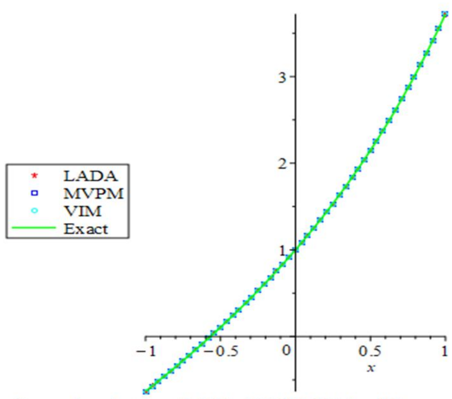
International Journal for Research in Applied Science & Engineering Technology (IJRASET)
ISSN: 2321-9653; IC Value: 45.98; SJ Impact Factor: 7.538
Volume 14 Issue III Mar 2026- Available at www.ijraset.com

x	S_2	S_4	S_6	Exact	Error
0.0	1.00000000	1.000000000	1.000000000	1.00000000	0.00000000
0.2	1.421403110	1.421402759	1.421402759	1.421402758	7.04×10^{-10}
0.4	1.891853781	1.891824697	1.891824698	1.891824698	0.00000000
0.6	2.422539334	2.422118691	2.422118801	2.422118800	4.13×10^{-10}
0.8	3.028490234	3.025538081	3.025540921	3.025540928	2.31×10^{-9}
1.0	3.732110306	3.718246637	3.718281497	3.718281828	8.90×10^{-8}

Table 1: Convergence of the sequence $S_n(x)$ and comparison with exact solution

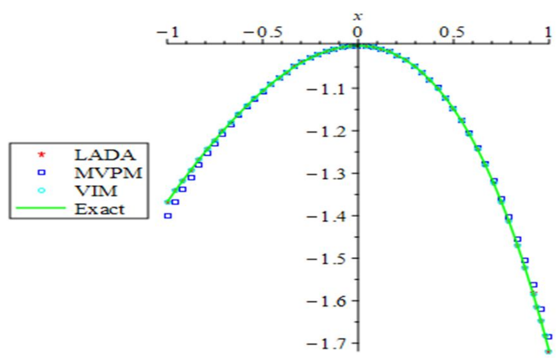
x	T_2	T_4	T_6	Exact	Error
0.0	-1.000000000	-1.000000000	-1.000000000	-1.00000000	0.00000000
0.2	-1.021402760	-1.021402759	-1.021402759	-1.021402758	9.79×10^{-10}
0.4	-1.091825007	-1.091824698	-1.091824698	-1.091824698	0.00000000
0.6	-1.222127173	-1.222118805	-1.222118801	-1.222118800	8.18×10^{-10}
0.8	-1.425627870	-1.425541073	-1.425540915	-1.425540928	9.12×10^{-9}
1.0	-1.718813428	-1.718284079	-1.718281520	-1.718281828	1.79×10^{-7}

Table 2: Convergence of the sequence $T_n(x)$ and comparison with exact solution



Comparison between LADA, MVPM, VIM and Exact

Fig.1



Comparison between LADA, MVPM, VIM and Exact

Fig.2

B. First order - Differential and Product nonlinearity

Consider the system of nonlinear integro-differential equations (convolution type) [13, 28]

$$u'(x) = 1 - \frac{1}{2}(v'(x))^2 + \int_0^x [(x-t)v(t) + u(t)v(t)]\,dt \,;\, v'(x) = 2x + \int_0^x [(x-t)u(t) - v^2(t) + u^2(t)]dt$$

with the initial conditions $u(0) = 0, v(0) = 1$.

Applying Laplace transform on both sides of the first equation in the system, we get

$$\mathcal{L}(u(x)) = \frac{1}{s^2} - \frac{1}{2s}\mathcal{L}\left((v'(x))^2\right) + \frac{1}{s^2}\mathcal{L}(u(x)v(x)) \tag{16}$$

Let $u(x) = \sum_{n=0}^{\infty} u_n(x)$ and $v(x) = \sum_{n=0}^{\infty} v_n(x)$ be the solution of the system. The nonlinear terms be decomposed into Adomian polynomials as $(v'(x))^2 = \sum_{n=0}^{\infty} A_n(x)$, $u(x)v(x) = \sum_{n=0}^{\infty} B_n(x)$ where

$A_0 = {v'_0}^2$, $A_1 = 2v'_0 v'_1$, $A_2 = {v'_1}^2 + 2v'_0 v'_2$, $A_3 = 2v'_0 v'_3 + 2v'_1 v'_2$, $A_4 = 2v'_0 v'_4 + 2v'_1 v'_3 + {v'_2}^2$;

$B_0 = u_0 v_0$, $B_1 = v_0 u_1 + u_0 v_1$, $B_2 = u_0 v_2 + v_0 u_2 + u_1 v_1$, $B_3 = u_0 v_3 + v_0 u_3 + u_1 v_2 + u_2 v_1$, $B_4 = u_0 v_4 + u_2 v_2 + u_1 v_3 + u_3 v_1 + u_4 v_0$; and so on.

International Journal for Research in Applied Science & Engineering Technology (IJRASET)
ISSN: 2321-9653; IC Value: 45.98; SJ Impact Factor: 7.538
Volume 14 Issue III Mar 2026- Available at www.ijraset.com

Substitution of the above results into (16) yields the following iterative algorithm.

$$\mathcal{L}(u_0(x)) = \frac{1}{s^2} \tag{17}$$

$$\mathcal{L}(u_{n+1}(x)) = -\frac{1}{2s}\mathcal{L}(A_n) + \frac{1}{s^2}\mathcal{L}(B_n) \tag{18}$$

The nonlinear terms in the second equation of the system can be decomposed into Adomian polynomials as $v^2(x) = \sum_{n=0}^{\infty} P_n(x)$ and $u^2(x) = \sum_{n=0}^{\infty} Q_n(x)$ where

$P_0 = {v_0}^2$, $P_1 = 2v_0v_1$, $P_2 = {v_1}^2 + 2v_0v_2$, $P_3 = 2v_0v_3 + 2v_1v_2$, $P_4 = 2v_0v_4 + 2v_1v_3 + {v_2}^2$;
$Q_0 = {u_0}^2$, $Q_1 = 2u_0u_1$, $Q_2 = {u_1}^2 + 2u_0u_2$, $Q_3 = 2u_0u_3 + 2u_1u_2$, $Q_4 = 2u_0u_4 + 2u_1u_3 + {u_2}^2$ and so on.

Substituting these equations into the second equation of the system yields the following iterative scheme

$$\mathcal{L}(v_0(x)) = \frac{1}{s} + \frac{2}{s^3} \tag{19}$$

$$\mathcal{L}(v_{n+1}(x)) = -\frac{1}{s^2}\mathcal{L}(P_n) + \frac{1}{s^2}\mathcal{L}(Q_n) \tag{20}$$

Applying inverse Laplace transform to (17) and (19), we get the initial term as $u_0(x) = x$, $v_0(x) = 1 + x^2$.
Using the initial term and Adomian polynomials to (18) and (20), we get the following iterative scheme.

$$u_1(x) = \frac{1}{3}x^3 + \frac{1}{15}x^5$$

$$v_1(x) = -\frac{1}{2}x^2 - \frac{1}{24}x^4 - \frac{1}{30}x^6$$

$$u_2(x) = \frac{2}{3}x^3 + \frac{1}{60}x^5 + \frac{25}{504}x^7 + \frac{1}{2520}x^9$$

$$v_2(x) = \frac{1}{12}x^4 + \frac{1}{90}x^6 + \frac{53}{10080}x^8 + \frac{1}{1350}x^{10}$$

$$u_3(x) = -\frac{1}{6}x^3 - \frac{2}{15}x^5 - \frac{17}{630}x^7 - \frac{1109}{90720}x^9 - \frac{1171}{453600}x^{11} - \frac{47}{7207200}x^{13}$$

$$v_3(x) = \frac{13}{360}x^6 - \frac{1}{672}x^8 - \frac{23}{302400}x^{10} - \frac{359}{4989600}x^{12} - \frac{1}{70200}x^{14}$$

$$u_4(x) = \frac{7}{120}x^5 - \frac{5}{84}x^7 + \frac{487}{36288}x^9 + \frac{292}{155925}x^{11} + \frac{1175221}{1556755200}x^{13} + \frac{220469}{2095632000}x^{15}$$

$$v_4(x) = -\frac{1}{80}x^6 - \frac{13}{1008}x^8 - \frac{31}{90720}x^{10} - \frac{229}{725700}x^{12} - \frac{95399}{5448613200}x^{14} + \frac{20989}{8072061000}x^{16} + \frac{41}{161109000}x^{18}$$

Summing up all the corresponding terms of the approximation, we get the approximate solution as $S_n(x)$ and $T_n(x)$ described in (8). Convergence of the solution is shown in Tables 3 and 4 and is compared with the exact solution on interval $[-1,1]$. Exact solution of the system is u(x) = sinhx and v(x) = coshx. This problem was solved using HPM [13] and VIM [28]. Comparison with the exact solution, present method and VIM are shown in Fig. 3 for u(x) and Fig. 4 for v(x). From Fig. 3, we see that the solution obtained using LADA is very close to the exact. But the solution obtained from VIM has error increases if $|x| > 0.8$. In Fig. 4, the solution obtained from LADA and VIM are same as the exact solution.

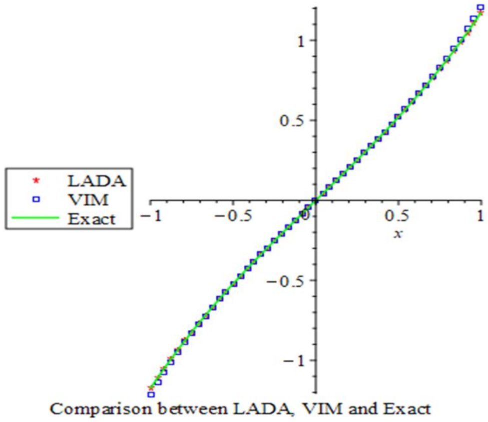


Fig. 3

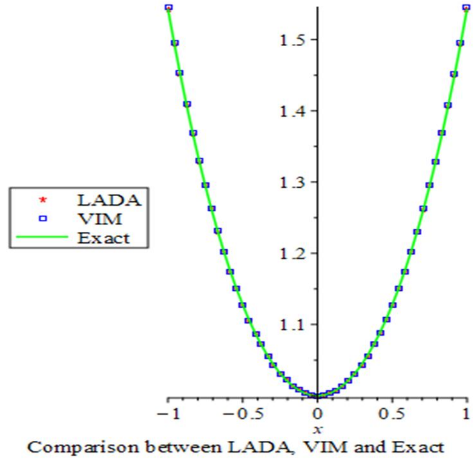


Fig. 4

x	T_7	T_8	T_9	T_{10}	Exact	Error
-1.0	-0.9580891119	-0.9751737328	-0.9786967749	-0.9823942798	-1.0	0.01760572
-0.8	-0.7899924106	-0.7943177906	-0.7948498590	-0.7953754889	-0.8	5.8×10^{-3}
-0.6	-0.5981933156	-0.5988247526	-0.5988614107	-0.5988894756	-0.6	1.85×10^{-3}
-0.4	-0.3998037323	-0.3998405604	-0.3998412303	-0.3998415751	-0.4	3.96×10^{-4}
-0.2	-0.1999944892	-0.1999947583	-0.1999947589	-0.1999947591	-0.2	2.62×10^{-5}
0.0	0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0	0.00000000
0.2	0.1999945152	0.1999947354	0.1999947349	0.1999947350	0.2	2.63×10^{-5}
0.4	0.3998109839	0.3998370203	0.3998366322	0.3998367762	0.4	4.08×10^{-4}
0.6	0.5983993502	0.5988278130	0.5988090615	0.5988219954	0.6	1.96×10^{-3}
0.8	0.7921816512	0.7954899356	0.7952201524	0.7955456715	0.8	5.57×10^{-3}
1.0	0.9701612991	0.9886264674	0.9871839227	0.9905801123	1.0	9.42×10^{-3}

Table 3: Convergence of the sequence $T_n(x)$ and comparison with exact solution

x	R_4	R_6	R_8	R_{10}	R_{12}	Exact	Error
-1.0	2.33856253	2.13910426	2.68663324	2.69504918	2.93167225	3.00	0.0227759
-0.8	1.68287271	1.60718843	1.88257670	1.87743399	1.97308863	1.92	0.0276503
-0.6	1.01966788	1.01081677	1.08868202	1.08838104	1.10155395	1.08	0.0199571
-0.4	0.47177535	0.47223828	0.48248584	0.48252789	0.48305224	0.48	6.36×10^{-3}
-0.2	0.11974142	0.11979785	0.12008985	0.12009026	0.12009206	0.12	7.67×10^{-4}
0.0	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00	0.0000000
0.2	0.12025319	0.12014517	0.11992198	0.11992274	0.11992397	0.12	6.34×10^{-4}
0.4	0.48813527	0.48379398	0.47747941	0.47772355	0.47799209	0.48	4.18×10^{-3}
0.6	1.14393756	1.09908926	1.05659276	1.06510553	1.07080077	1.08	8.52×10^{-3}
0.8	2.21086232	1.91468883	1.77948129	1.91680423	1.94129293	1.92	0.0110900
1.0	4.01297382	2.25822166	2.46244124	4.20627679	3.06121619	3.00	0.0204053

Table 4: Convergence of the sequence $R_n(x)$ and comparison with exact solution

The aforementioned numerical examples demonstrate that the results are extremely near to the appropriate exact solution and are acquired in a relatively small number of iterations. The corresponding tables evaluate the error's absolute value. Additionally, the numerical results show that compared to MVPM [34] and Variational Iteration technique [28], the current technique converges relatively quickly. Abbasbandy et al. compared these issues with HPM and ADM using VIM, and they concluded that VIM is more precise and efficient. Using MVPM [34], Mohyud-Din et al. solved the first and second problems and compared their answer to the precise solution. When compared to these techniques, LADA is the most precise and dependable approach. LADA can be used to solve a wide range of Volterra integro-differential equations, both linear and nonlinear.

C. Integral System

Consider the nonlinear integral system [5]

$$u(t) = 2t + \frac{1}{3}t^3 - \frac{1}{2}t^2 + \int_0^t v(s) + u^2(s)]\,ds\,;\, v(t) = -1 + \int_0^t [u(s) - v(s)]ds$$

Applying Laplace transform on both sides of the first equation in the system, we get

$$\mathcal{L}(u(t)) = \frac{2s^2-s+2}{s^4} - \frac{1}{s}\mathcal{L}(u^2(t)) + \frac{1}{s}\mathcal{L}(v(t)) \qquad (21)$$

Laplace transform of the second one is

$$\mathcal{L}(u(t)) = 1 + \mathcal{L}(v(t))(s+1) \qquad (22)$$

Substituting (22) into (21) yields,

$$\mathcal{L}(u(t)) = -\frac{1}{s^2+s-1} + \frac{(s+1)(2s^2-s+2)}{s^3(s^2+s-1)} - \frac{(s+1)}{s^2+s-1}\mathcal{L}(u^2(t)) \qquad (23)$$

Let $u(x) = \sum_{n=0}^{\infty} u_n(x)$ and $v(x) = \sum_{n=0}^{\infty} v_n(x)$ be the solution of the system. The nonlinear term $u^2(x) = \sum_{n=0}^{\infty} A_n(x)$ in (23) can be decomposed into Adomian polynomials as

$A_0 = {u_0}^2,\ A_1 = 2u_0u_1,\ A_2 = {u_1}^2 + 2u_0u_2,\ A_3 = 2u_0u_3 + 2u_1u_2,\ A_4 = 2u_0u_4 + 2u_1u_3 + {u_2}^2$ and so on.

Substituting these results into (23) and using linearity property of Laplace transforms, the iterative algorithm yields

$$\mathcal{L}(u_0(t)) = -\frac{1}{s^2+s-1} + \frac{(s+1)(2s^2-s+2)}{s^3(s^2+s-1)}$$

$$\mathcal{L}(u_1(t)) = -\frac{(s+1)}{s^2+s-1}\mathcal{L}(A_0)$$

$$\mathcal{L}(u_2(t)) = -\frac{(s+1)}{s^2+s-1}\mathcal{L}(A_1)$$

In general,

$$\mathcal{L}(u_{n+1}(t)) = -\frac{(s+1)}{s^2+s-1}\mathcal{L}(A_n)$$

Using inverse Laplace transform, we get the initial term and some successive terms of the solution are as follows.

$$u_0(t) = -6 - 3t - t^2 + \frac{14\sqrt{5}e^{\frac{-t}{2}}\sinh\left(\frac{t\sqrt{5}}{2}\right)}{5} + 6e^{\frac{-t}{2}}\cosh\left(\frac{t\sqrt{5}}{2}\right)$$

$$u_1(t) = 606 + t^4 + 14t^3 + 93t^2 + 348t - \frac{1172\sqrt{5}\sinh(t\sqrt{5})}{55e^t} - \frac{524\sqrt{5}\cosh(t\sqrt{5})}{11e^t} + \frac{44t^3e^{\frac{-t}{2}}\cosh\left(\frac{t\sqrt{5}}{2}\right)}{55e^t} + \frac{74t^2e^{\frac{-t}{2}}\cosh\left(\frac{t\sqrt{5}}{2}\right)}{5} + \frac{1408te^{\frac{-t}{2}}\cosh\left(\frac{t\sqrt{5}}{2}\right)}{25} - \frac{6142e^{\frac{-t}{2}}\cosh\left(\frac{t\sqrt{5}}{2}\right)}{11} + \frac{4\sqrt{5}t^3e^{\frac{-t}{2}}\sinh\left(\frac{t\sqrt{5}}{2}\right)}{3} + \frac{166\sqrt{5}t^2e^{\frac{-t}{2}}\sinh\left(\frac{t\sqrt{5}}{2}\right)}{25} + \frac{632\sqrt{5}te^{\frac{-t}{2}}\sinh\left(\frac{t\sqrt{5}}{2}\right)}{25} - \frac{343526\sqrt{5}e^{\frac{-t}{2}}\sinh\left(\frac{t\sqrt{5}}{2}\right)}{1375}$$

$$u_2(t) = -229476 - 136272t - \frac{2528t^3\cosh(t\sqrt{5})}{55e^t} - \frac{17112t^2\cosh(t\sqrt{5})}{121e^t} - \frac{5592616t\cosh(t\sqrt{5})}{6655e^t} + \frac{7543177848\sqrt{5}\sinh(t\sqrt{5})}{1830125e^t} - \frac{512}{25e^t} - \frac{16t^2}{5e^t} - \frac{48t}{25e^t} + \frac{36904e^{\frac{-3t}{2}}\cosh\left(\frac{3t\sqrt{5}}{2}\right)}{95} + \frac{344e^{\frac{-3t}{2}}\cosh\left(\frac{t\sqrt{5}}{2}\right)}{55} + \frac{61167528820e^{\frac{-t}{2}}\cosh\left(\frac{t\sqrt{5}}{2}\right)}{278179} + \frac{32t^6e^{\frac{-t}{2}}\cosh\left(\frac{t\sqrt{5}}{2}\right)}{45} + \frac{2188t^5e^{\frac{-t}{2}}\cosh\left(\frac{t\sqrt{5}}{2}\right)}{375} + \frac{74t^4e^{\frac{-t}{2}}\cosh\left(\frac{t\sqrt{5}}{2}\right)}{5} - \frac{1713548t^3e^{\frac{-t}{2}}\cosh\left(\frac{t\sqrt{5}}{2}\right)}{4125} - \frac{3749902t^2e^{\frac{-t}{2}}\cosh\left(\frac{t\sqrt{5}}{2}\right)}{1375} + \frac{16504\sqrt{5}e^{\frac{-3t}{2}}\sinh\left(\frac{3t\sqrt{5}}{2}\right)}{95} + \frac{152\sqrt{5}e^{\frac{-3t}{2}}\sinh\left(\frac{t\sqrt{5}}{2}\right)}{55} + \frac{85473096022308\sqrt{5}e^{\frac{-t}{2}}\sinh\left(\frac{t\sqrt{5}}{2}\right)}{869309375} - \frac{74805536e^{\frac{-t}{2}}\cosh\left(\frac{t\sqrt{5}}{2}\right)}{6875} + \frac{8\sqrt{5}t^6e^{\frac{-t}{2}}\sinh\left(\frac{t\sqrt{5}}{2}\right)}{25} + \frac{324\sqrt{5}t^5e^{\frac{-t}{2}}\sinh\left(\frac{t\sqrt{5}}{2}\right)}{125} + \frac{2408\sqrt{5}t^4e^{\frac{-t}{2}}\sinh\left(\frac{t\sqrt{5}}{2}\right)}{325}$$

$$-\frac{256948\sqrt{5}t^3e^{\frac{-t}{2}}\sinh\left(\frac{t\sqrt{5}}{2}\right)}{1375}-\frac{8406962\sqrt{5}t^2e^{\frac{-t}{2}}\sinh\left(\frac{t\sqrt{5}}{2}\right)}{6875}-\frac{33496696\sqrt{5}te^{\frac{-t}{2}}\sinh\left(\frac{t\sqrt{5}}{2}\right)}{6875}$$

$$-39060t^2-820t^4-6918t^3-2t^6-58t^5+\frac{3373381992\cosh(t\sqrt{5})}{366025e^t}$$

$$-\frac{3392\sqrt{5}t^3\sinh(t\sqrt{5})}{165e^t}-\frac{38264\sqrt{5}t^2\sinh(t\sqrt{5})}{605e^t}-\frac{500216\sqrt{5}t\sinh(t\sqrt{5})}{1331e^t}$$

$$v_1(t)=379+221t+62t^2+10t^3+t^4-\frac{524\sqrt{5}\sinh(t\sqrt{5})}{55e^t}-\frac{1172t^2\cosh(t\sqrt{5})}{55e^t}-\frac{8}{5e^t}+\frac{28t^3e^{\frac{-t}{2}}\cosh\left(\frac{t\sqrt{5}}{2}\right)}{15} \quad 6t^2e^{\frac{-t}{2}}\cosh\left(\frac{t\sqrt{5}}{2}\right)-$$

$$\frac{716\,te^{\frac{-t}{2}}\cosh\left(\frac{t\sqrt{5}}{2}\right)}{25}-\frac{3928\,e^{\frac{-t}{2}}\cosh\left(\frac{t\sqrt{5}}{2}\right)}{11}+\frac{4\sqrt{5}\,t^3e^{\frac{-t}{2}}\sinh\left(\frac{t\sqrt{5}}{2}\right)}{5}+ \quad \frac{62\sqrt{5}t^2\,e^{\frac{-t}{2}}\sinh\left(\frac{t\sqrt{5}}{2}\right)}{25}+12\sqrt{5}te^{\frac{-t}{2}}\sinh\left(\frac{t\sqrt{5}}{2}\right)-\frac{221352\sqrt{5}\,e^{\frac{-t}{2}}\sinh\left(\frac{t\sqrt{5}}{2}\right)}{1375}$$

Summing up all the corresponding terms of the approximation, we get the approximate solution as $S_n(t)$ and $T_n(t)$ described in(8). Convergence of the solution is shown in Tables 5 and 6 and is compared with the exact solution on interval[0,1]. Exact solution of the system is $u(t) = t$ and$v(t) = t - 1$. Comparison with the exact solution and present method are shown in Fig. 5, Fig. 6 and Fig. 7 for $u(t)$ and Fig. 8 for $v(t)$. From the graphs and tables, one can see that error increases if t increase.

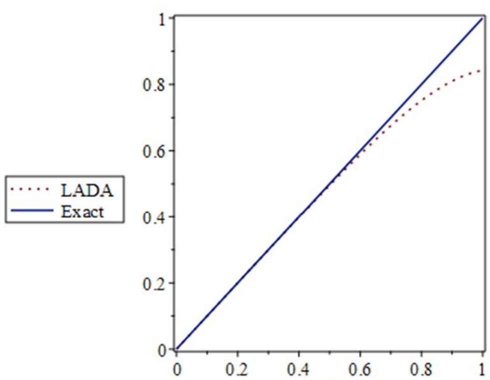


Fig.5: Comparison and convergence of LADA up to $S_2(t)$

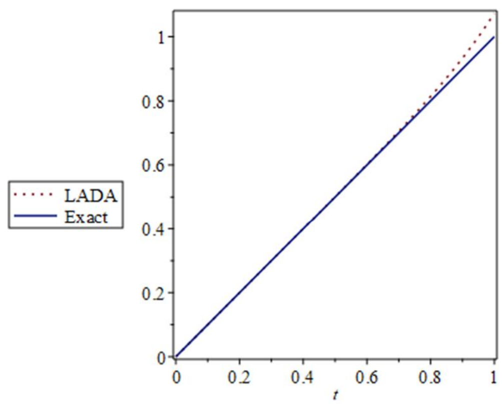


Fig.6: Comparison and convergence of LADA up to $S_3(t)$

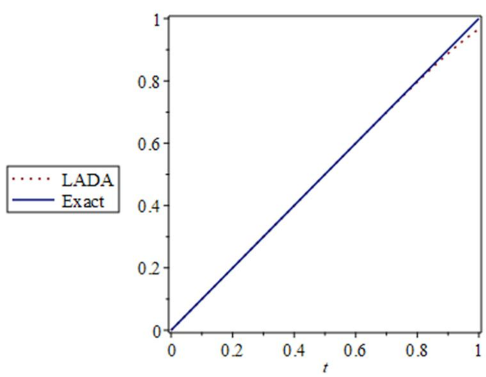


Fig.7: Comparison and convergence of LADA up to $S_4(t)$

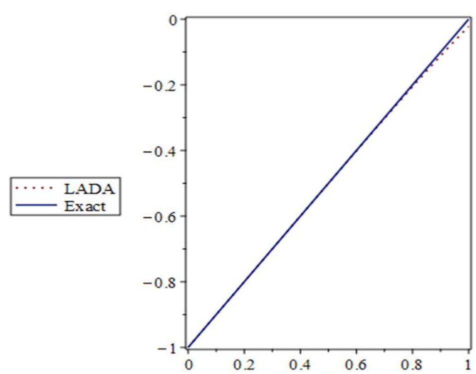


Fig.8: Comparison and convergence of LADA up to $T_2(t)$

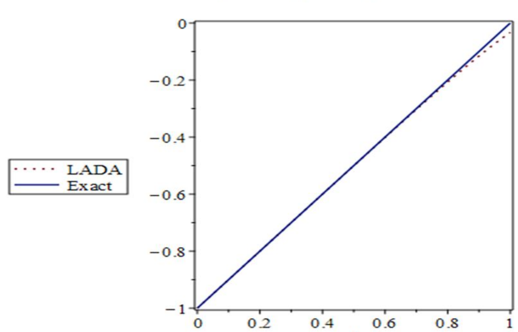


Fig.9: Comparison and convergence of LADA up to $T_3(t)$

t	S_2	S_3	Exact
0.0	0.00000000	0.0000000	0.0
0.1	0.09999885	0.0997596	0.1
0.2	0.19995744	0.1999298	0.2
0.3	0.29967114	0.2999947	0.3
0.4	0.39859589	0.4000902	0.4
0.5	0.49564854	0.5003321	0.5
0.6	0.58896705	0.6017174	0.6
0.7	0.67562691	0.7048037	0.7
0.8	0.75128131	0.8133759	0.8
0.9	0.75128131	0.9330060	0.9
1.0	0.84234542	1.0727650	1.0

Table 5: Convergence of the sequence $S_n(t)$ and comparison with exact solution

t	T_2	Exact
0.0	-1.0000000	-1.0
0.1	-0.9000000	-0.9
0.2	-0.8000010	-0.8
0.3	-0.7000156	-0.7
0.4	-0.6000881	-0.6
0.5	-0.5003346	-0.5
0.6	-0.4010010	-0.4
0.7	-0.3025329	-0.3
0.8	-0.2056798	-0.2
0.9	-0.1116193	-0.1
1.0	-0.0221173	0.0

Table 6: Convergence of the sequence $T_n(t)$ and comparison with exact solution

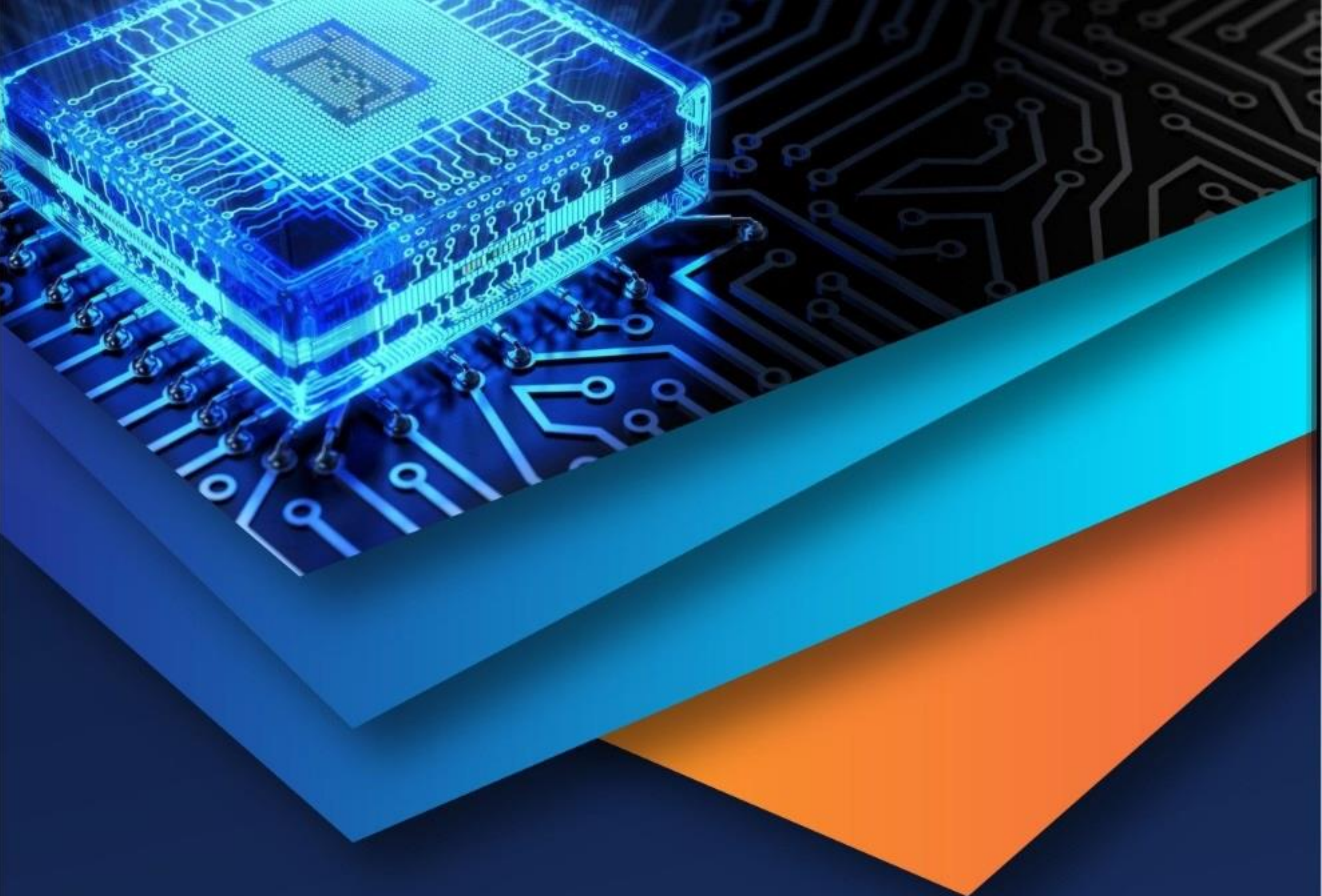
V. CONCLUSION

This work used the Laplace-Adomian Decomposition Algorithm to explore three systems of nonlinear and one system of linear Volterra integro-differential equations. This approach does not choose the initial term at random; rather, it takes it directly from the procedure. The numerical findings demonstrate how quickly LADA converges when compared to alternative techniques. It just takes a small number of iterations, and the outcome is really near to the precise answer. This approach is simple to apply and may be applied to a wide range of general form problems. Without undergoing any transformations or making any restrictive assumptions, this method is applied directly to the system of equations. A large class of linear and nonlinear initial and boundary value problems have analytic solutions that can be found using LADA, which is a highly powerful tool. Maple is used to perform the computations.

REFERENCES

[1] Adomian G, Solving Frontier Problems of Physics: The Decomposition Method, Kluwer Boston, 1994.
[2] Alan Goldfine, Taylor series methods for the solution of Volterra integral and integro-differential equations., Math.Comput., **31**, 1977, 691-707.
[3] Anar Huseyin, Nesir Huseyin, Eskisehir, Dependence on the parameters of the set of trajectories of the control systems described by a nonlinear Volterra integral equation, Appl.Math., **59**, 2014, 303-317.
[4] Avudainayagam A, Vani C, Wavelet-Galerkin method for integro-differential equations, Appl.Math.Comput., **32**, 2000, 247-254.
[5] Aytac Arikoglu, Ibrahim Ozkol, Solutions of integral and integro-differential equation systems by using differential transform method, Comp.Math.Appln., 56, 2008, 2411-2417.
[6] Babolian E, Biazar J and Vahidi A R, The Decomposition method applied to system of Fredholm integral equations of the second kind, Appl.Math., **148**, 2004, 443-452.
[7] Baiju S and Doney Kurian, Solution of nonlinear Stiff Ordinary Differential Equation Systems. Bull.Ker.Math.Assoc., 18**(1),** 2024, 71-81.
[8] Baiju S and Sivakumar T R, Laplace-Adomian Decomposition Algorithm for system of linear and nonlinear integro-differential equations, Jour.Sci.Tech.Man., **4(2),** 2011, 33-42.

[9] Baiju S and Sivakumar T R, Laplace – Adomian Decomposition Algorithm to the solution of the Brusselator System, Jour.Sci.Tech.Man., **4(2)**, 2011, 21-29.
[10] Baiju S, Reversion Method applied to Boundary Value Problems, Acad.Resear.Rev.Lett., **2(2)**, 2018, 11-19.
[11] Baiju S, Analytic Solution of Nonlinear Boundary Value Problem governing the Electrical Behaviour of cell membrane, Acad.Resear.Rev.Lett., **1(2),** 2017, 79-92.
[12] Batiha B, Noorani M S M and Hashim I, Numerical solution of the nonlinear integro-differential equations, Int.I.Open Problems.Comput.Math., **1(1),** 2008, 36-42.
[13] Biazar J, Ghazvini H, Eslami M, He's homotopy perturbation method for system of integro-differential equations, Chaos, Solitons and Fractals, **39(3)**, 2009, 1253-1258.
[14] Biazar~J, Solution of systems of integral differential equations by Adomian Decomposition method, Appl.Math.Comput., 168, 2005, 1232-1238 .
[15] Darania P and Ebadian A, Development of the Taylor Expansion Approach for nonlinear integro-differential equations, Int.J.Contemp.Math.Sciences., **1(14),** 2006, 651-664.
[16] Evans D J, Bulut H, A new approach to the Gas Dynamics equation: An application of the decomposition method, Appl.Comput.Math., **79(7),** 2002, 817-822.
[17] Ebadi G, Rahimi-Ardabili M Y and Shahmorad S, Numerical solution of the system of nonlinear Volterra integro-differential equations, Southeast Asian Bulletin of Mathematics., **33**, 2009, 835-846.
[18] El-Shahed M, Application of He's homotopy perturbation method to Volterra integro-differential equations, Int.J.Nonlinear Sci.Numer.Simul., **6(2),** 2005, 163-168.
[19] Guseinov K G, Nazlipinar A S, On the continuity properties of of the attainable sets of nonlinear control systems with integral constraints on controls, Abstr.Appl.Anal., 2008, ID.295817
[20] Guseinov K G, Nazlipinar A S, On the continuity property of L^p balls and on application, J.Math.Anal.Appl., 335, 2007, 1347-1359.
[21] Hosseini S M, Shahmorad S, Numerical solution of a class of integro-differential equations by the Tau method with an error estimation, Appl.Math.Comput., **136**, 2003, 559-570.
[22] Hosseini S M and Shahmorad S, Tau method solution of Fredholm integro-differential equations with arbitrary polynomial bases, Appl.Math.Model., 27, 2003, 145-154.
[23] Huseyin A, Huseyin N, Precompactness of the set of trajectories of the controllable system described by a nonlinear Volterra integral equation, Math.Model.Anal., **17**, 2012, 686-695.
[24] Perviz Sargolzaei, Mehdi Safinezhad and Mohammad Hadi Rostami, Solution of Fredholm integral differential equation system by Adomian Decomposition Method, J.Modern Math.Stat., **2(3),** 2008, 98-101.
[25] Pour-Mahmoud J, Rahimi-Ardabili M Y, Shahmorad~S, Numerical solution of the System of Fredholm integro-differential equations by the Tau method, Appl.Math.Comput., **168**, 2005, 465-478.
[26] Rashed M T, Lagrange interpolation to compute the numerical solution of differential and integro-differential equations, Appl.Math.Comput., 2003.
[27] Saberi-Nadjafi J, Tamamgar M, The Variational Iteration Method ; A highly premising method for solving the system of integro-differential equations, Comput.Math.Appl., **56**, 2008, 346-351.
[28] Saeid Abbasbandy, Elyas Shivanian, Application of the Variational Iteration Method for system of nonlinear Volterra integro-differential equations, Math.Comput.Appl., **14(2),** 2009, 147-158.
[29] Sivakumar T R and Baiju S, Shooting-Type Laplace – Adomian Decomposition Algorithm for nonlinear Differential equations with boundary Condition at Infinity, Appl.Math.Lett., **24**, 2011, 1702-1709.
[30] Sivakumar~T~R and Baiju~S, Shooting Type Laplace-Adomian Decomposition Algorithm and their applications, Bull.Ker.Math.Assoc., **9(2),** 2012, 259-277.
[31] Sivakumar T R, Some variants of Adomian's Decomposition Method and their applications, Ph.D Thesis, IIT, Madras, 1990.
[32] Venkataranghan S N and Sivakumar T R, A modification of Adomian's Decomposition Method for boundary value problems with homogeneous boundary conditions, J.Math.Phy.Sci., **24**, 1990, 19-24.
[33] Suheil A Khuri, A Laplace Decomposition Algorithm applied to a class of nonlinear differential equations, J.Appl.Math., **1(4),** 2001, 141-155.
[34] Syed Tauseef Mohyud-Din, Muhammad Aslam Noor and KhalidaInayat Noor, Modified variation of parameters method for second order integro-differential equations and coupled system, World.Appl.Sci., 6**(8),** 2009, 1139-1146.
[35] Yusufoglu E, An efficient algorithm for solving integro-differential equations system, Appl.Math.Compu., **192**, 2007, 151-155.



10.22214/IJRASET

INDEX COPERNICUS 45.98

IMPACT FACTOR: 7.129

IMPACT FACTOR: 7.429

INTERNATIONAL JOURNAL FOR RESEARCH

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY