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# Some New Topological Indices of Friendship Graph

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**Abstract:** In this paper we compute the first and second Kulli-Basava indices, hyper Kulli-Basava indices, some connectivity Kulli-Basava indices, geometric-arithmetic Kulli-Basava indices and reciprocal Kulli-Basava indices of the friendship graph.

**Keywords:** Topological Index, Kulli-Basava Index, hyper Kulli-Basava index, Kulli-Basava connectivity indices, Kulli-Basava geometric-arithmetic Kulli-Basava index.

## I. INTRODUCTION

A topological index is a numerical quantity derived from a graph structure. There are several classes of topological indices such as degree based, distance based, counting based etc. The Kulli-Basava indices are degree based topological indices.

Let  $G$  be a finite simple connected graph with vertex set  $V(G)$  and edge set  $E(G)$ . Let  $S_e(v)$  denote the sum of the degrees of all edges incident to a vertex  $v$  in  $G$ . The first and second Kulli-Basava indices of graph  $G$  were introduced in [1], defined as

$$KB_1(G) = \sum_{uv \in E(G)} [S_e(u) + S_e(v)]$$

$$\text{and } KB_2(G) = \sum_{uv \in E(G)} S_e(u)S_e(v).$$

The first and second hyper Kulli-Basava indices were introduced by Kulli in [2], defined as

$$HKB_1(G) = \sum_{uv \in E(G)} [S_e(u) + S_e(v)]^2$$

$$\text{and } HKB_2(G) = \sum_{uv \in E(G)} [S_e(u)S_e(v)]^2$$

The sum connectivity Kulli-Basava index, product connectivity Kulli-Basava index, atom bond connectivity Kulli-Basava index, geometric-arithmetic Kulli-Basava index and reciprocal Kulli-Basava index of a graph were introduced by Kulli in [3], defined as

$$SKB(G) = \sum_{uv \in E(G)} \frac{1}{\sqrt{S_e(u) + S_e(v)}}$$

$$PKB(G) = \sum_{uv \in E(G)} \frac{1}{\sqrt{S_e(u)S_e(v)}}$$

$$ABCKB(G) = \sum_{uv \in E(G)} \sqrt{\frac{S_e(u) + S_e(v) - 2}{S_e(u)S_e(v)}}$$

$$GAKB(G) = \sum_{uv \in E(G)} \frac{2\sqrt{S_e(u)S_e(v)}}{S_e(u) + S_e(v)}$$

$$\text{and } RKB(G) = \sum_{uv \in E(G)} \sqrt{S_e(u)S_e(v)}.$$

In this paper, we compute the above indices for the friendship graph  $F_n$ . In [4], Kulli introduced the first and second Kulli-Basava polynomials and the first and second hyper Kulli-Basava polynomials, defined as

$$KB_1(G, x) = \sum_{uv \in E(G)} x^{S_e(u) + S_e(v)}$$

$$KB_2(G, x) = \sum_{uv \in E(G)} x^{S_e(u)S_e(v)}$$

$$HKB_1(G, x) = \sum_{uv \in E(G)} x^{[S_e(u) + S_e(v)]^2}$$

$$\text{and } HKB_2(G, x) = \sum_{uv \in E(G)} x^{[S_e(u)S_e(v)]^2}.$$

The polynomials of the first four Kulli-Basava indices are also obtained.

## II. KULLI-BASAVA INDICES OF FRIENDSHIP GRAPH

Friendship graph  $\mathcal{F}_n$  is obtained by joining  $n$  copies of cycle graph  $\mathcal{C}_3$  with a common vertex. Let the common vertex be denoted by  $v_0$  and the other vertices of the cycles be denoted by  $v_1, v_2, \dots, v_{2n}$ . Note that  $|V(\mathcal{F}_n)| = 2n + 1$  and  $|E(\mathcal{F}_n)| = 3n$ . We can see that,  $\mathcal{S}_e(v_0) = 4n^2$  and  $\mathcal{S}_e(v_i) = n + 2$ , for  $i = 1, 2, \dots, 2n$ .

The Friendship graph  $\mathcal{F}_n$  has two types of edges:

$$E_1(\mathcal{F}_n) = \{uv \in E(\mathcal{F}_n) : \mathcal{S}_e(u) = 2n^2 \text{ and } \mathcal{S}_e(v) = 2(n+1)\}$$

$$\text{and } E_2(\mathcal{F}_n) = \{uv \in E(\mathcal{F}_n) : \mathcal{S}_e(u) = 2(n+1) \text{ and } \mathcal{S}_e(v) = 2(n+1)\}.$$

Also,  $|E_1(\mathcal{F}_n)| = 2n$  and  $|E_2(\mathcal{F}_n)| = n$ .

### 1) Theorem 1

The first and second Kulli- Basava indices of the Friendship graph  $\mathcal{F}_n$  are:

$$\mathcal{KB}_1(\mathcal{F}_n) = 4n^3 + 8n^2 + 8n$$

$$\text{and } \mathcal{KB}_2(\mathcal{F}_n) = 8n^4 + 12n^3 + 8n^2 + 4n.$$

Proof:

$$\begin{aligned} \text{The first Kulli-Basava index, } \mathcal{KB}_1(\mathcal{F}_n) &= \sum_{uv \in E(\mathcal{F}_n)} [\mathcal{S}_e(u) + \mathcal{S}_e(v)] \\ &= 4n(n^2 + n + 1) + 4n(n + 1) \\ &= 4n^3 + 8n^2 + 8n. \end{aligned}$$

$$\begin{aligned} \text{The second Kulli- Basava index, } \mathcal{KB}_2(\mathcal{F}_n) &= \sum_{uv \in E(\mathcal{F}_n)} [\mathcal{S}_e(u)\mathcal{S}_e(v)] \\ &= [2n \times 4n^2(n + 1)] + 4n(n + 1)^2 \\ &= 8n^4 + 12n^3 + 8n^2 + 4n. \end{aligned}$$

### 2) Theorem 2

The first and second hyper Kulli- Basava indices of the Friendship graph  $\mathcal{F}_n$  are:

$$\mathcal{HKB}_1(\mathcal{F}_n) = 8n^5 + 16n^4 + 10n^3 + 48n^2 + 24n$$

$$\text{and } \mathcal{HKB}_2(\mathcal{F}_n) = 32n^7 + 64n^6 + 48n^5 + 64n^4 + 96n^3 + 64n^2 + 16n.$$

Proof:

$$\begin{aligned} \text{The first hyper Kulli-Basava index, } \mathcal{HKB}_1(\mathcal{F}_n) &= \sum_{uv \in E(\mathcal{F}_n)} [\mathcal{S}_e(u) + \mathcal{S}_e(v)]^2 \\ &= 8n(n^2 + n + 1)^2 + 16n(n + 1)^2 \\ &= 8n^5 + 16n^4 + 10n^3 + 48n^2 + 24n. \end{aligned}$$

$$\begin{aligned} \text{The second hyper Kulli- Basava index, } \mathcal{HKB}_2(\mathcal{F}_n) &= \sum_{uv \in E(\mathcal{F}_n)} [\mathcal{S}_e(u)\mathcal{S}_e(v)]^2 \\ &= 2n(4n^2(n + 1))^2 + n(4(n + 1)^2)^2 \\ &= 32n^7 + 64n^6 + 48n^5 + 64n^4 + 96n^3 + 64n^2 + 16n. \end{aligned}$$

### 3) Theorem 3

The sum connectivity Kulli-Basava index, product connectivity Kulli-Basava index and atom bond connectivity Kulli-Basava index of the Friendship graph  $\mathcal{F}_n$  are:

$$\mathcal{SKB}(\mathcal{F}_n) = \frac{2n}{\sqrt{2(n^2+n+1)}} + \frac{n}{2\sqrt{(n+1)}}$$

$$\mathcal{PKB}(\mathcal{F}_n) = \frac{1}{\sqrt{n+1}} + \frac{n}{2(n+1)}$$

$$\text{and } \mathcal{ABCKB}(\mathcal{F}_n) = \sqrt{2n} + \frac{\sqrt{4n+2}}{2(n+1)}.$$

Proof:

$$\text{The sum connectivity Kulli-Basava index, } \mathcal{SKB}(\mathcal{F}_n) = \sum_{uv \in E(\mathcal{F}_n)} \frac{1}{\sqrt{\mathcal{S}_e(u) + \mathcal{S}_e(v)}} = \frac{2n}{\sqrt{2(n^2+n+1)}} + \frac{n}{2\sqrt{(n+1)}}.$$

$$\text{The product connectivity Kulli-Basava index, } \mathcal{PKB}(\mathcal{F}_n) = \sum_{uv \in E(\mathcal{F}_n)} \frac{1}{\sqrt{\mathcal{S}_e(u)\mathcal{S}_e(v)}} = \frac{1}{\sqrt{n+1}} + \frac{n}{2(n+1)}.$$

$$\text{The atom bond connectivity Kulli-Basava index, } \mathcal{ABCKB}(G) = \sum_{uv \in E(\mathcal{F}_n)} \sqrt{\frac{\mathcal{S}_e(u) + \mathcal{S}_e(v) - 2}{\mathcal{S}_e(u)\mathcal{S}_e(v)}} = \sqrt{2n} + \frac{\sqrt{4n+2}}{2(n+1)}.$$

4) *Theorem 4*

The geometric-arithmetic Kulli-Basava index and reciprocal Kulli-Basava index of the Friendship graph  $\mathcal{F}_n$  are:

$$GAKB(\mathcal{F}_n) = \frac{4n^2\sqrt{n+1}}{n^2+n+1} + n$$

$$\text{and } \mathcal{RKB}(\mathcal{F}_n) = 4n^2\sqrt{n+1} + 2n(n+1).$$

Proof:

The geometric-arithmetic Kulli-Basava index,  $GAKB(\mathcal{F}_n) = \sum_{uv \in E(\mathcal{F}_n)} \frac{2\sqrt{s_\sigma(u)s_\sigma(v)}}{s_\sigma(u)+s_\sigma(v)} = \frac{4n^2\sqrt{n+1}}{n^2+n+1} + n$ .

The reciprocal Kulli-Basava index,  $\mathcal{RKB}(\mathcal{F}_n) = \sum_{uv \in E(\mathcal{F}_n)} \sqrt{s_\sigma(u)s_\sigma(v)} = 4n^2\sqrt{n+1} + 2n(n+1)$ .

5) *Theorem 5*

The first and second Kulli-Basava polynomials of the friendship graph  $\mathcal{F}_n$  are:

$$\mathcal{KB}_1(\mathcal{F}_n, x) = 2n x^{2(n^2+n+1)} + nx^{4(n+1)}$$

$$\text{and } \mathcal{KB}_2(\mathcal{F}_n, x) = 2n x^{4n^2(n+1)} + nx^{4(n+1)^2}$$

Proof:

(i)  $\mathcal{KB}_1(\mathcal{F}_n, x) = \sum_{uv \in E(\mathcal{F}_n)} x^{s_\sigma(u)+s_\sigma(v)} = 2n x^{2(n^2+n+1)} + nx^{4(n+1)}$

(ii)  $\mathcal{KB}_2(\mathcal{F}_n, x) = \sum_{uv \in E(\mathcal{F}_n)} x^{s_\sigma(u)s_\sigma(v)} = 2n x^{4n^2(n+1)} + nx^{4(n+1)^2}$

6) *Theorem 6*

The first and second hyper Kulli-Basava polynomials of the friendship graph  $\mathcal{F}_n$  are:

$$\mathcal{HKB}_1(\mathcal{F}_n, x) = 2n x^{4(n^2+n+1)^2} + nx^{16(n+1)^2}$$

$$\text{and } \mathcal{HKB}_2(\mathcal{F}_n, x) = 2n x^{16n^4(n+1)^2} + nx^{16(n+1)^4}$$

Proof:

(i)  $\mathcal{HKB}_1(\mathcal{F}_n, x) = \sum_{uv \in E(\mathcal{F}_n)} x^{[s_\sigma(u)+s_\sigma(v)]^2} = 2n x^{4(n^2+n+1)^2} + nx^{16(n+1)^2}$

(ii)  $\mathcal{HKB}_2(\mathcal{F}_n, x) = \sum_{uv \in E(\mathcal{F}_n)} x^{[s_\sigma(u)s_\sigma(v)]^2} = 2n x^{16n^4(n+1)^2} + nx^{16(n+1)^4}$

III. CONCLUSIONS

In this paper the first and second Kulli-Basava indices, the first and second hyper Kulli-Basava indices, the sum connectivity Kulli-Basava index, product connectivity Kulli-Basava index, atom bond connectivity Kulli-Basava index and geometric-arithmetic Kulli-Basava index of the friendship graph are computed. We also obtained the first and second Kulli-Basava index polynomials and the first and second hyper Kulli-Basava index polynomials of the friendship graph.

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