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Some New Topological Indices of Friendship Graph

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Abstract: In this paper we compute the first and second Kulli-Basava indices, hyper Kulli-Basava indices, some connectivity Kulli-Basava indices, geometric-arithmetic Kulli-Basava indices and reciprocal Kulli-Basava indices of the friendship graph.
Keywords: Topological Index, Kulli-Basava Index, hyper Kulli-Basava index, Kulli-Basava connectivity indices, Kulli-Basava geometric-arithmetic Kulli-Basava index.

I. INTRODUCTION

A topological index is a numerical quantity derived from a graph structure. There are several classes of topological indices such as degree based, distance based, counting based etc. The Kulli- Basava indices are degree based topological indices.
Let G be a finite simple connected graph with vertex set $V(G)$ and edge set $E(G)$. Let $\mathcal{S}_e(v)$ denote the sum of the degrees of all edges incident to a vertex v in G. The first and second Kulli-Basava indices of graph G were introduced in [1], defined as

$$\mathcal{KB}_1(G) = \sum_{uv\in E(G)} [\mathcal{S}_e(u) + \mathcal{S}_e(v)]$$

$$\text{and} \quad \mathcal{KB}_2(G) = \sum_{uv\in E(G)} \mathcal{S}_e(u)\mathcal{S}_e(v).$$

The first and second hyper Kulli-Basava indices were introduced by Kulli in [2], defined as

$$\mathcal{HKB}_1(G) = \sum_{uv\in E(G)} [\mathcal{S}_e(u) + \mathcal{S}_e(v)]^2$$

$$\text{and } \mathcal{HKB}_2(G) = \sum_{uv\in E(G)} [\mathcal{S}_e(u)\mathcal{S}_e(v)]^2$$

The sum connectivity Kulli-Basava index, product connectivity Kulli-Basava index, atom bond connectivity Kulli-Basava index, geometric-arithmetic Kulli-Basava index and reciprocal Kulli-Basava index of a graph were introduced by Kulli in [3], defined as

$$\mathcal{SKB}(G) = \sum_{uv\in(G)} \frac{1}{\sqrt{\mathcal{S}_e(u)+\mathcal{S}_e(v)}}$$

$$\mathcal{PKB}(G) = \sum_{uv\in E(G)} \frac{1}{\sqrt{\mathcal{S}_e(u)\mathcal{S}_e(v)}}$$

$$\mathcal{ABCKB}(G) = \sum_{uv\in E(G)} \sqrt{\frac{\mathcal{S}_e(u)+\mathcal{S}_e(v)-2}{\mathcal{S}_e(u)\mathcal{S}_e(v)}}$$

$$\mathcal{GAKB}(G) = \sum_{uv\in E(G)} \frac{2\sqrt{\mathcal{S}_e(u)\mathcal{S}_e(v)}}{\mathcal{S}_e(u)+\mathcal{S}_e(v)}$$

$$\text{and} \quad \mathcal{RKB}(G) = \sum_{uv\in E(G)} \sqrt{\mathcal{S}_e(u)\mathcal{S}_e(v)}\,.$$

In this paper, we compute the above indices for the friendship graph $\mathcal{F}_n$.In [4], Kulli introduced the first and second Kulli-Basava polynomials and the first and second hyper Kulli-Basava polynomials, defined as

$$\mathcal{KB}_1(G,x) = \sum_{uv\in E(G)} x^{\mathcal{S}_e(u)+\mathcal{S}_e(v)}$$

$$\mathcal{KB}_2(G,x) = \sum_{uv\in E(G)} x^{\mathcal{S}_e(u)\mathcal{S}_e(v)}$$

$$\mathcal{HKB}_1(G,x) = \sum_{uv\in E(G)} x^{[\mathcal{S}_e(u)+\mathcal{S}_e(v)]^2}$$

$$\text{and } \mathcal{HKB}_2(G,x) = \sum_{uv\in E(G)} x^{[\mathcal{S}_e(u)\mathcal{S}_e(v)]^2}.$$

The polynomials of the first four Kulli-Basava indices are also obtained.

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II. KULLI-BASAVA INDICES OF FRIENDSHIP GRAPH

Friendship graph $\mathcal{F}_n$ is obtained by joining n copies of cycle graph C_3 with a common vertex. Let the common vertex be denoted by v_0 and the other vertices of the cycles be denoted by $v_1, v_2, \dots, v_{2n}$. Note that $|V(\mathcal{F}_n)| = 2n+1$ and $|E(\mathcal{F}_n)| = 3n$. We can see that, $\mathcal{S}_e(v_0) = 4n^2$ and $\mathcal{S}_e(v_i) = n+2$, for $i = 1,2, \dots, 2n$.

The Friendship graph $\mathcal{F}_n$ has two types of edges:

$$E_1(\mathcal{F}_n) = \{uv \in E(\mathcal{F}_n): \mathcal{S}_e(u) = 2n^2 \text{ and } \mathcal{S}_e(v) = 2(n+1)\}$$

and $$E_2(\mathcal{F}_n) = \{uv \in E(\mathcal{F}_n): \mathcal{S}_e(u) = 2(n+1) \text{ and } \mathcal{S}_e(v) = 2(n+1)\}.$$

Also, $|E_1(\mathcal{F}_n)| = 2n$ and $|E_2(\mathcal{F}_n)| = n$.

1) Theorem 1

The first and second Kulli- Basava indices of the Friendship graph $\mathcal{F}_n$ are:

$$\mathcal{KB}_1(\mathcal{F}_n) = 4n^3 + 8n^2 + 8n$$

and $$\mathcal{KB}_2(\mathcal{F}_n) = 8n^4 + 12n^3 + 8n^2 + 4n.$$

Proof:

The first Kulli-Basava index, $\mathcal{KB}_1(\mathcal{F}_n) = \sum_{uv\in E(\mathcal{F}_n)} [\mathcal{S}_e(u) + \mathcal{S}_e(v)]$

$$= 4n(n^2 + n + 1) + 4n(n+1)$$
$$= 4n^3 + 8n^2 + 8n.$$

The second Kulli- Basava index, $\mathcal{KB}_2(\mathcal{F}_n) = \sum_{uv\in E(\mathcal{F}_n)} [\mathcal{S}_e(u)\mathcal{S}_e(v)]$

$$= [2n \times 4n^2(n+1)] + 4n(n+1)^2$$
$$= 8n^4 + 12n^3 + 8n^2 + 4n.$$

2) Theorem 2

The first and second hyper Kulli- Basava indices of the Friendship graph $\mathcal{F}_n$ are:

$$\mathcal{HKB}_1(\mathcal{F}_n) = 8n^5 + 16n^4 + 10n^3 + 48n^2 + 24n$$

and $$\mathcal{HKB}_2(\mathcal{F}_n) = 32n^7 + 64n^6 + 48n^5 + 64n^4 + 96n^3 + 64n^2 + 16n.$$

Proof:

The first hyper Kulli-Basava index, $\mathcal{HKB}_1(\mathcal{F}_n) = \sum_{uv\in E(\mathcal{F}_n)} [\mathcal{S}_e(u) + \mathcal{S}_e(v)]^2$

$$= 8n(n^2 + n + 1)^2 + 16n(n+1)^2$$
$$= 8n^5 + 16n^4 + 10n^3 + 48n^2 + 24n.$$

The second hyper Kulli- Basava index, $\mathcal{HKB}_2(\mathcal{F}_n) = \sum_{uv\in E(\mathcal{F}_n)} [\mathcal{S}_e(u)\mathcal{S}_e(v)]^2$

$$= 2n\left(4n^2(n+1)\right)^2 + n(4(n+1)^2)^2$$
$$= 32n^7 + 64n^6 + 48n^5 + 64n^4 + 96n^3 + 64n^2 + 16n.$$

3) Theorem 3

The sum connectivity Kulli-Basava index, product connectivity Kulli-Basava index and atom bond connectivity Kulli-Basava index of the Friendship graph $\mathcal{F}_n$ are:

$$\mathcal{SKB}(\mathcal{F}_n) = \frac{2n}{\sqrt{2(n^2+n+1)}} + \frac{n}{2\sqrt{(n+1)}}$$

$$\mathcal{PKB}(\mathcal{F}_n) = \frac{1}{\sqrt{n+1}} + \frac{n}{2(n+1)}$$

and $$\mathcal{ABCKB}(\mathcal{F}_n) = \sqrt{2n} + \frac{\sqrt{4n+2}}{2(n+1)}.$$

Proof:

The sum connectivity Kulli-Basava index, $\mathcal{SKB}(\mathcal{F}_n) = \sum_{uv\in E(\mathcal{F}_n)} \frac{1}{\sqrt{\mathcal{S}_e(u)+\mathcal{S}_e(v)}} = \frac{2n}{\sqrt{2(n^2+n+1)}} + \frac{n}{2\sqrt{(n+1)}}$.

The product connectivity Kulli-Basava index, $\mathcal{PKB}(\mathcal{F}_n) = \sum_{uv\in E(\mathcal{F}_n)} \frac{1}{\sqrt{\mathcal{S}_e(u)\mathcal{S}_e(v)}} = \frac{1}{\sqrt{n+1}} + \frac{n}{2(n+1)}$.

The atom bond connectivity Kulli-Basava index, $\mathcal{ABCKB}(G) = \sum_{uv\in E(\mathcal{F}_n)} \sqrt{\frac{\mathcal{S}_e(u)+\mathcal{S}_e(v)-2}{\mathcal{S}_e(u)\mathcal{S}_e(v)}} = \sqrt{2n} + \frac{\sqrt{4n+2}}{2(n+1)}$.

4) Theorem 4

The geometric-arithmetic Kulli-Basava index and reciprocal Kulli-Basava index of the Friendship graph $\mathcal{F}_n$ are:

$$\mathcal{GAKB}(\mathcal{F}_n) = \frac{4n^2\sqrt{n+1}}{n^2+n+1} + n$$

$$\text{and } \mathcal{RKB}(\mathcal{F}_n) = 4n^2\sqrt{n+1} + 2n(n+1).$$

Proof:

The geometric-arithmetic Kulli-Basava index, $\mathcal{GAKB}(\mathcal{F}_n) = \sum_{uv\in E(\mathcal{F}_n)} \frac{2\sqrt{\mathcal{S}_e(u)\mathcal{S}_e(v)}}{\mathcal{S}_e(u)+\mathcal{S}_e(v)} = \frac{4n^2\sqrt{n+1}}{n^2+n+1} + n.$

The reciprocal Kulli-Basava index, $\mathcal{RKB}(\mathcal{F}_n) = \sum_{uv\in E(\mathcal{F}_n)} \sqrt{\mathcal{S}_e(u)\mathcal{S}_e(v)} = 4n^2\sqrt{n+1} + 2n(n+1).$

5) Theorem 5

The first and second Kulli-Basava polynomials of the friendship graph $\mathcal{F}_n$ are:

$$\mathcal{KB}_1(\mathcal{F}_n, x) = 2n\,x^{2(n^2+n+1)} + nx^{4(n+1)}$$

$$\text{and } \mathcal{KB}_2(\mathcal{F}_n, x) = 2n\,x^{4n^2(n+1)} + nx^{4(n+1)^2}$$

Proof:

(i) $\mathcal{KB}_1(\mathcal{F}_n, x) = \sum_{uv\in E(\mathcal{F}_n)} x^{\mathcal{S}_e(u)+\mathcal{S}_e(v)} = 2n\,x^{2(n^2+n+1)} + nx^{4(n+1)}$

(ii) $\mathcal{KB}_2(\mathcal{F}_n, x) = \sum_{uv\in E(\mathcal{F}_n)} x^{\mathcal{S}_e(u)\mathcal{S}_e(v)} = 2n\,x^{4n^2(n+1)} + nx^{4(n+1)^2}$

6) Theorem 6

The first and second hyper Kulli-Basava polynomials of the friendship graph $\mathcal{F}_n$ are:

$$\mathcal{HKB}_1(\mathcal{F}_n, x) = 2n\,x^{4(n^2+n+1)^2} + nx^{16(n+1)^2)}$$

$$\text{and } \mathcal{HKB}_2(\mathcal{F}_n, x) = 2n\,x^{16n^4(n+1)^2} + nx^{16(n+1)^4}$$

Proof:

(i) $\mathcal{HKB}_1(\mathcal{F}_n, x) = \sum_{uv\in E(\mathcal{F}_n)} x^{[\mathcal{S}_e(u)+\mathcal{S}_e(v)]^2} = 2n\,x^{4(n^2+n+1)^2} + nx^{16(n+1)^2)}$

(ii) $\mathcal{HKB}_2(\mathcal{F}_n, x) = \sum_{uv\in E(\mathcal{F}_n)} x^{[\mathcal{S}_e(u)\mathcal{S}_e(v)]^2} = 2n\,x^{16n^4(n+1)^2} + nx^{16(n+1)^4}$

III. CONCLUSIONS

In this paper the first and second Kulli-Basava indices, the first and second hyper Kulli-Basava indices, the sum connectivity Kulli-Basava index, product connectivity Kulli-Basava index, atom bond connectivity Kulli-Basava index and geometric-arithmetic Kulli-Basava index of the friendship graph are computed. We also obtained the first and second Kulli-Basava index polynomials and the first and second hyper Kulli-Basava index polynomials of the friendship graph.

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