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# Some Properties of Fuzzy Finite Tree Automata

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**Abstract:** This paper examines fuzzy tree automata's deterministic, reduced, and homomorphic properties. It is demonstrated through an example that there is a deterministic fuzzy tree automaton for the nondeterministic one. For every given fuzzy tree automaton, an equivalent reduced fuzzy tree automaton is displayed. A theorem that fuzzy tree languages are preserved by tree homomorphism is derived.

**Keywords:** Fuzzy tree automaton, Deterministic fuzzy tree automaton, Fuzzy tree homomorphism.

## I. INTRODUCTION

Automata on infinite words, weighted automata, fuzzy automata on finite and infinite words, tree automata, and weighted tree automata are some of the ways that finite state automata have been expanded. We refer to [8] for a general treatment of fuzzy automata and languages. Previous research on fuzzy tree automata has been done in [2, 5]. The deterministic fuzzy tree automaton  $Ad$  exhibits the same behavior as any fuzzy tree automaton  $A$ , as demonstrated in this paper by the proof of  $L(A) = L(Ad)$ . The membership value in  $A$  and that in  $Ad$ , however, do not have to match for any tree  $t$ . An example is provided to demonstrate the existence. Additionally, it is demonstrated that there is an equivalent reduced fuzzy tree automaton for every fuzzy tree automaton. Also, it is demonstrated that  $h(L)$  is recognized by a fuzzy tree automaton if  $h$  is a linear tree homomorphism and  $L$  is recognized by a fuzzy tree automaton. This can be further extended in the field of fuzzy hyper graph and Plus Weighted Linear Grammar as cited in [10, 11,12,13,14,15,16].

## II. BASIC DEFINITIONS

For the definition of standard notions, [2,3,5] are mentioned.

1) *Definition II.1.*

A fuzzy finite tree automaton (NFFTA) is a system  $A = (Q, \Sigma, \delta, \Gamma)$  where,

- $Q$  is a set of state symbols.
- $\Sigma$  is a set of ranked alphabets called input symbols.
- $\delta = \{ \delta_\sigma : Q^n \times Q \times \Sigma_n \rightarrow [0, 1] \mid \sigma \in \Sigma_n, n \geq 0 \}$

A transition rule may be defined by

$$\delta_\sigma : Q^n \times \Sigma_n \times [0, 1] \rightarrow Q$$

- $\Gamma \subseteq \text{Fuzzy}(Q)$  is a set of fuzzy final states.

A fuzzy final state  $\gamma \in \Gamma$  may be defined by  $\gamma : Q \rightarrow [0, 1]$

An NFFTA is called finite if  $Q$  is finite and if  $\delta_\sigma(q_1, q_2, \dots, q_n, \sigma) = 0$  for all but a finite number of  $\delta_\sigma \in \sigma$  and if  $\Gamma(q) = 0$  for all but a finite number of states  $q \in Q$ .

For  $t \in T_\Sigma$ , a fuzzy set  $\rho(t) \subseteq \text{Fuzzy}(Q)$  is defined by induction on structure of  $t$ .

2) *Definition II.2.*

When  $t = \sigma \in \Sigma_0$ , then  $\rho(t)(q) = \delta(q, \sigma), \forall q \in Q$ . Assume that  $t = \sigma(t_1, t_2, \dots, t_n)$  for some  $\sigma \in \Sigma_n$  and  $t_1, t_2, \dots, t_n \in T_\Sigma$ .

$$\rho(t)(q) = \bigvee_{q_1, q_2, \dots, q_n \in Q} \left( \delta(q_1, q_2, \dots, q_n, \sigma) \wedge \left( \bigwedge_{i=1}^n \rho(t_i)(q_i) \right) \right)$$

The behavior of an NFFTA  $A$  is a fuzzy set  $|A|$  on a set of trees  $t \in T_\Sigma$  is defined by

$$|A|(t) = \bigvee_{q \in Q} (\rho(t)(q) \wedge \Gamma(q))$$

3) *Definition II.3.*

A tree  $t \in T_{\Sigma}$  is accepted by NFFTA  $A = (Q, \Sigma, \delta, \Gamma)$  if  $|A|(t) > 0$  and  $|A|(t) = \mu_{L(A)}(t)$ .

4) *Definition II.4.*

The fuzzy tree language  $L(A)$  recognized by NFFTA  $A$  is the set of all ground terms accepted by  $A$ . A set  $L$  of ground terms is recognizable if  $L = L(A)$  for some  $A$ .

**III. DETERMINISTIC FUZZY FINITE TREE AUTOMATON**

This section introduces the definition of deterministic fuzzy finite tree automaton and some related theorems are proved.

1) *Definition III.1.*

A deterministic fuzzy finite tree automaton (DFFTA) is an FFTA  $A_d = (Q_d, \Sigma, \delta_d, \Gamma_d)$  such that for each

$\sigma \in \Sigma_n$  and  $q_1, q_2, \dots, q_n \in Q_d$  there exists at most one  $q \in Q_d$  such that  $\delta_d(\sigma(q_1, q_2, \dots, q_n) \rightarrow q) > 0$  or

$$\sigma(q_1, q_2, \dots, q_n) \xrightarrow{m} q; m > 0.$$

2) *Theorem III.2*

Let  $L$  be a recognizable set of ground terms. Then there exists a DFFTA that accepts  $L$ .

*Proof:* Let  $A = (Q, \Sigma, \delta, \Gamma)$  be a NFFTA. Define a DFFTA  $A_d = (Q_d, \Sigma, \delta_d, \Gamma_d)$ . The states of  $Q_d$  are all the subsets of  $Q$ . That is  $Q_d = 2^Q$ . We denote by  $S$  a state of  $Q_d$ ,  $S = \{q_1, q_2, \dots, q_n\}$  for some states  $q_1, q_2, \dots, q_n \in Q$ .  $\delta_d$  and  $\Gamma_d$  are constructed from the following algorithm.

input : NFFTA  $A = (Q, \Sigma, \delta, \Gamma)$

begin

/\* a state  $S$  of the DFFTA is in  $2^Q$  \*/

Set  $\delta_d = \phi$ ;

repeat

if  $f \in \Sigma_n$  and  $S_1, S_2, \dots, S_n \in Q_d$  then

begin

$S = \{q \in Q \mid \exists q_1 \in S_1, q_2 \in S_2, \dots, q_n \in S_n,$

$$f(q_1, q_2, \dots, q_n) \xrightarrow{m} q \in \delta\}$$

$m = \vee \{m_i \mid f(q_1, q_2, \dots, q_n) \xrightarrow{m_i} q,$

$\exists q_1 \in S_1, q_2 \in S_2, \dots, q_n \in S_n\}$

end

Set  $\delta_d = \delta_d \cup \{f(S_1, S_2, \dots, S_n) \xrightarrow{m} S\}$ ;

until no new rule is added to  $\delta_d$

$\Gamma_d(S) = \vee \{\Gamma(q) \mid q \in S\} \forall S \in Q_d,$

output : DFFTA  $A_d = A_d = (Q_d, \Sigma, \delta_d, \Gamma_d)$ .

Clearly  $A_d$  is a DFFTA. Let us prove that  $L(A) = L(A_d)$ , that is to prove  $t \xrightarrow{A_d} S, \rho(t)(S) > 0$  if and only if

$$S = \{q \in Q \mid t \xrightarrow{A} q, \rho(t)(q) > 0\}$$

Suppose that  $t \xrightarrow{A_d} S, \rho(t)(S) > 0,$

we prove  $t \xrightarrow{A} q, \rho(t)(q) > 0,$  by induction on the structure of terms.

Base case : Let us consider  $t = a \in \Sigma_0$ . Then there is only one rule.

$a \xrightarrow{m} S \in \delta_d \because \rho(t)(S) = m > 0,$  where

$m = \vee \{a \xrightarrow{m_i} q \mid q \in Q\} [\because m > 0]$

$$S = \{q \in Q \mid t \xrightarrow{A} q, \rho(t)(q) > 0\}$$

Induction step : Let us consider a term  $t = f(t_1, t_2, \dots, t_n)$

Assume that the derivation if of the form

$$t \xrightarrow[A_d]{*} f(S_1, S_2, \dots S_n) \xrightarrow[A_d]{*} S, \text{ where } t_i \xrightarrow[A_d]{*} S_i,$$

$\rho(t_i)(S_i) > 0 \ i = 1, 2, \dots, n$ . By induction hypothesis, for each  $i = 1, 2, \dots, n$ .

$$S_i = \{q \in Q \mid t_i \xrightarrow[A]{*} q, \rho(t_i)(q) > 0\} \quad (1)$$

Since  $S_i$ 's are in  $Q_d$ ,  $f(S_1, S_2, \dots S_n) \rightarrow S$  is a rule in  $\delta_d$  with membership value  $m_1 > 0$ . From the algorithm,

$$S = \{q \in Q \mid \exists q_1 \in S_1, q_2 \in S_2, \dots, q_n \in S_n, f(q_1, q_2, \dots, q_n) \xrightarrow{m_1} q \in \delta\} \text{ and } m_1 = \bigvee_{i=1}^n \{m_i\} \text{ implies, } \exists m_i > 0 \text{ such that}$$

$$f(q_1, q_2, \dots, q_n) \xrightarrow{m_i} q.$$

$$\text{From (1) } \exists q_i \in Q \text{ such that } t_i \xrightarrow[A]{*} q_i, \rho(t_i)(q_i) > 0$$

$$i = 1, 2, \dots, n. \text{ Since there exists } q_1 \in S_1, q_2 \in S_2, \dots, q_n \in S_n, \text{ and } f(q_1, q_2, \dots, q_n) \xrightarrow{m_i} q,$$

$$\text{we have } t \xrightarrow[A]{*} q, \rho(t)(q) > 0$$

Conversely suppose that  $t \xrightarrow[A]{*} q, \rho(t)(q) > 0$ , we prove

$$t \xrightarrow[A_d]{*} S, \rho(t)(S) > 0 \text{ by induction on the structure of terms.}$$

Base case : Let us consider  $t = a \in \Sigma_0$ .

Then there is a rule  $a \xrightarrow{m} q \in \delta, \rho(t)(q) = m > 0$ .

$$\text{Suppose } S = \{q \in Q \mid a \xrightarrow{m_2} q \in \delta\}$$

$$m_2 = \bigvee \{m_i \mid a \xrightarrow{m_i} q \in \delta\}, \text{ implies that } m_2 > 0.$$

$$\text{Therefore } a \xrightarrow{m_2} S, \rho(t)(S) > 0.$$

Induction step : Let us consider a term  $t = f(t_1, t_2, \dots, t_n)$

Assume that the derivation is of the form

$$t \xrightarrow[A]{*} f(q_1, q_2, \dots, q_n) \xrightarrow[A]{*} q, \text{ where } t_i \xrightarrow[A]{*} q_i, \rho(t_i)(q_i) > 0, \text{ by induction, for each } i = 1, 2, \dots, n. t_i \xrightarrow[A_d]{*} S_i,$$

$\rho(t_i)(S_i) = m_i > 0$  and the state  $S_i$  is defined by

$$S_i = \{q \in Q \mid t_i \xrightarrow[A]{*} q \in \delta\} \quad (2)$$

Since  $q_i$ 's are in  $Q$ ,  $f(q_1, q_2, \dots, q_n) \rightarrow q$  is a rule in  $\delta$  with membership value  $m > 0$ .

$$\text{Thus } S = \{q \in Q \mid \exists q_1 \in S_1, q_2 \in S_2, \dots, q_n \in S_n, f(q_1, q_2, \dots, q_n) \xrightarrow{m'} q \in \delta\} \text{ and}$$

$$m' = \bigvee \{m_i \mid f(q_1, q_2, \dots, q_n) \xrightarrow{m_i} q \in \delta, \exists q_1 \in S_1, q_2 \in S_2, \dots, q_n \in S_n\}$$

Implies that  $m' > 0$ , therefore  $f(S_1, S_2, \dots S_n) \xrightarrow{m'} S$  is a rule in  $\delta_d$ . From equation (2)

$$\exists S_i \in Q_d \text{ such that } t_i \xrightarrow[A_d]{*} S_i, \rho(t_i)(S_i) = m_i > 0 \text{ for each } i = 1, 2, \dots, n \text{ and also}$$

$$f(S_1, S_2, \dots S_n) \xrightarrow[A_d]{m'} S \text{ is a rule in } \delta.$$

$$\text{Therefore } t \xrightarrow[A_d]{*} S \text{ and } \rho(t)(S) = m,$$

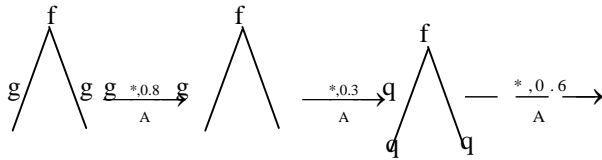
$$m = \bigwedge \{m_1, m_2, \dots, m_n, m'\} > 0.$$

**Example III.3.**

Consider the NFFTA,  $A = (Q, \Sigma, \delta, \Gamma)$ , where  $Q = \{q, q_g, q_f\}, \Sigma_0 = \{a\}, \Sigma_1 = \{g\}, \Sigma_2 = \{f\}, \Gamma(q) = 0.5, \Gamma(q_g) = 0.6, \Gamma(q_f) = 0.9$  and  $\delta$  is the following set of transition rules :

$$\left\{ a \xrightarrow{0.8} q, g(q) \xrightarrow{0.3} q, g(q) \xrightarrow{0.5} q_g, \right. \\ \left. g(q_g) \xrightarrow{0.7} q_f, f(q, q) \xrightarrow{0.6} q \right\}$$

Consider the ground term  $t = f(g(a), g(a))$  and its sequence of reduction is as follows :



$$\rho(t)(q) = 0.8 \wedge 0.3 \wedge 0.6 = 0.3 \text{ and}$$

$$\mu_{L(A)}(t) = 0.3 \wedge 0.5 = 0.3 > 0$$

The language accepted by the NFFTA is

$$\mu_{L(A)}(t) = \begin{cases} 0.5, & \text{if } t \in \{g(g(t')) \mid t' \in T_\Sigma\} \\ 0.3, & \text{if } t \in \{f(g^i(a), g^j(a)) \mid i, j \geq 1\} \\ 0.5, & \text{if } t \in \{f^i(a, a) \mid i, j \geq 1\} \end{cases}$$

Using the theorem (III.2), we get the DFFTA

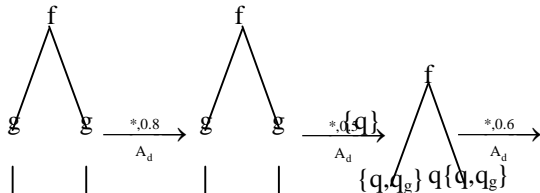
$A = (Q_d, \Sigma, \delta_d, \Gamma_d)$ , where

$Q_d = \{\{q\}, \{q, q_g\}, \{q, q_g, q_f\}\}, \Sigma_0 = \{a\},$

$\Sigma_1 = \{g\}, \Sigma_2 = \{f\}, \Gamma_d(\{q\}) = 0.5, \Gamma_d(\{q, q_g\}) = 0.6, \Gamma_d(\{q, q_g, q_f\}) = 0.9$  and  $\delta_d$  is the following set of transition rules :

$$\{a \xrightarrow{0.8} \{q\}, g\{q\} \xrightarrow{0.5} \{q, q_g\}, g\{q, q_g\} \xrightarrow{0.7} \{q, q_g, q_f\}, f(S_1, S_2) \xrightarrow{0.6} \{q\}, \text{ where } S_1, S_2 \in Q_d\}$$

Consider the ground term  $t = f(g(a), g(a))$  and its sequence of reduction is as follows :



$$\rho(t)(\{q\}) = 0.8 \wedge 0.5 \wedge 0.6 = 0.5 \text{ and}$$

$$\mu_{L(A_d)}(t) = 0.5 \wedge 0.5 = 0.5 > 0$$

The language accepted by the DFFTA is

$$\mu_{L(A)}(t) = \begin{cases} 0.5, & \text{if } t \in \{g(g(t')) \mid t' \in T_\Sigma\} \\ 0.5, & \text{if } t \in \{f(g^i(a), g^j(a)) \mid i, j \geq 1\} \\ 0.5, & \text{if } t \in \{f^i(a, a) \mid i \geq 1\} \end{cases}$$

The language accepted by A is equal to the language accepted by  $A_d$ , but the membership is not retained.

#### IV. REDUCED FUZZY FINITE TREE AUTOMATON

Reduced Fuzzy Finite Tree Automaton is discussed in this section. It is demonstrated that there is an equivalent reduced fuzzy tree automaton for every fuzzy tree automaton.

*Definition IV.1.*

A state Q is accessible if there exists a term t such that  $\rho(t)(q) > 0$ . An NFFTA is said to be reduced if all its states are accessible.

*Theorem IV.2.*

Let L be a recognizable set of ground terms. Then there exists a reduced FFTA that accepts L.

*Proof:* Let  $A = (Q, \Sigma, \delta, \Gamma)$ , be an FFTA. If all the states of Q are accessible. ie.  $\rho(t)(q) > 0 \forall q \in Q$ , then A is reduced. Suppose that A is irreduced and L be the language accepted by it. Let  $Q^f = Q - \{s \mid s \text{ is not reachable}\}$ .

Define  $A^f = (Q^f, \Sigma, \delta^f, \Gamma^f)$ , where

$\delta^f : Q_n^f \times \Sigma_n \times Q^f \rightarrow [0, 1]$  is defined by

$\forall q_1, q_2, \dots, q_n, q \in Q$  and  $f \in \Sigma_n$ ,

$$\delta^r(q_1, q_2, \dots, q_n, f, q) = \begin{cases} \delta(q_1, q_2, \dots, q_n, f, q), & \text{if } q \text{ is accessible.} \\ 0, & \text{if } q \text{ is not accessible.} \end{cases}$$

$$\Gamma^r(q) = \begin{cases} \Gamma(q), & \text{if } q \text{ is accessible.} \\ 0, & \text{if } q \text{ is not accessible.} \end{cases}$$

Clearly  $A^r$  is a reduced FFTA. Now we prove that  $A$  and  $A^r$  are equivalent.

Let  $L_1$  be the fuzzy tree language accepted by  $A^r$ .

If  $\mu_{L(A)}(t) = 0$  then  $\rho(t)(q) = 0$  or  $\Gamma(q) = 0 \forall q \in Q$ , hence  $\mu_{L_1(A)}(t) = 0$ .

If  $\mu_{L(A)}(t) > 0$  then  $\rho(t)(q) > 0$  and  $\Gamma(q) > 0$ , implies that

$$\Gamma(q) = \Gamma^r(q) \forall q \in Q \quad (3)$$

Therefore  $q$  is accessible in  $A^r$  and in the sequence of moves  $\rho^r(t)(q) = \rho(t)(q)$  and hence

$$\rho(t)(q) \wedge \Gamma(q) = \rho^r(t)(q) \wedge \Gamma^r(q) \quad (4)$$

Equation (4) is true for every  $q \in Q$  which satisfies equation (3). Implies that

$$\bigvee_{q \in Q} \{\rho(t)(q) \wedge \Gamma(q)\} = \bigvee_{q \in Q} \{\rho^r(t)(q) \wedge \Gamma^r(q)\}. \text{ Therefore } \mu_{L(A)}(t) = \mu_{L_1(A)}(t).$$

### V. TREE HOMOMORPHISM

This section deals with Tree Homomorphism. It is demonstrated that  $h(L)$  is recognized by a fuzzy tree automaton if  $h$  is a linear tree homomorphism and  $L$  is recognized by a fuzzy tree automaton.

*Definition V.1.*

Let  $\Sigma$  and  $\Sigma'$  be two sets of function of symbols, possibly not disjoint. For each  $n > 0$ ,  $\Sigma$  contains a symbol  $f$  of arity  $n$ , we define a set of variables  $\chi_n = \{x_1, x_2, \dots, x_n\}$  disjoint from  $\Sigma$  and  $\Sigma'$ . Let  $h_\Sigma$  be a mapping which with  $f \in \Sigma$  of arity  $n$ , associate as term  $t_f \in T(\Sigma', \chi_n)$ . The tree homomorphism  $h : T(\Sigma) \rightarrow T(\Sigma')$  determined by  $h_\Sigma$  is defined as follows :

- $h(a) = t_a \in T(\Sigma')$  for each  $a \in \Sigma$  of arity 0.
- $h(f(t_1, t_2, \dots, t_n)) = t_f(x_1 \leftarrow h(t_1), x_2 \leftarrow h(t_2), \dots, x_n \leftarrow h(t_n))$ , where  $t_f(x_1 \leftarrow h(t_1), x_2 \leftarrow h(t_2), \dots, x_n \leftarrow h(t_n))$  is the result applying the substitution  $\{x_1 \leftarrow h(t_1), x_2 \leftarrow h(t_2), \dots, x_n \leftarrow h(t_n)\}$

*Definition V.2.*

A tree homomorphism is linear if for each  $f \in \Sigma$  of arity  $n$ ,  $h_\Sigma(f) = t_f$  is a linear term in  $T(\Sigma', \chi_n)$ .

*Theorem V.3.*

Let  $h$  be a linear tree homomorphism and  $L$  be recognized by fuzzy tree automaton. Then  $h(L)$  is recognized by a fuzzy tree automaton.

Proof: Let  $A = (Q, \Sigma, \delta, \Gamma)$  be reduced DFFTA such that  $L(A)=L$  and  $h$  be a linear fuzzy tree homomorphism from  $T(\Sigma) \rightarrow T(\Sigma')$ . determined by a mapping  $h_\Sigma$ . We construct an NFFTA  $A' = (Q', \Sigma', \delta', \Gamma')$  as follows. Let us consider a ruler  $r = f(q_1, q_2, \dots, q_n) \xrightarrow{m} q \in \delta$ , a term  $t_f = h_\Sigma(f) \in T(\Sigma', X_n)$  and the set of positions  $\text{pos}(t_f)$ . We define a set of states  $Q^r = \{q_p^r \mid p \in \text{pos}(t_f)\}$  and a set of rules  $\delta_r$  as follows for all positions  $p \in \text{pos}(t_f)$ .

$$\text{If } t_f(p) = g \in \Sigma' \text{ then } g(q_{p_1}^r, q_{p_2}^r, \dots, q_{p_n}^r) \xrightarrow{m} q_p^r \in \delta_r$$

$$\text{If } t_f(p) = x_i \text{ then } q_i \xrightarrow{m} q_p^r \in \delta_r$$

$$q_e^r \xrightarrow{m} q \in \delta_r$$

The preceding construction is made for each rule in  $\delta$ . We assume that all the states set  $Q^r$  are disjoint and are disjoint from  $Q$ . Define  $A' = (Q', \Sigma', \delta', \Gamma')$  by

$$Q' = Q \cup \bigcup_{r \in \delta} Q^r$$

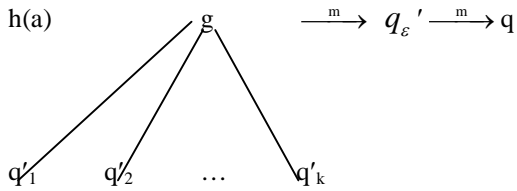
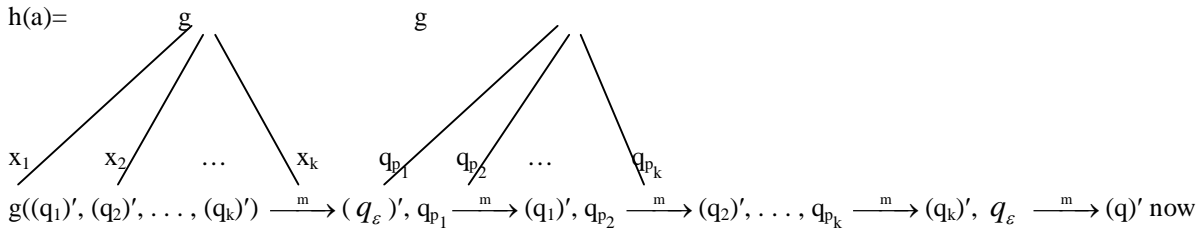
$$\Gamma' : Q' \rightarrow [0, 1] \text{ such that } \Gamma'(P) = \Gamma(p) \forall p \in Q \text{ and } \Gamma'(P) = \Gamma(q) \forall p \in Q^r,$$

$$\delta' = \bigcup_{r \in \delta} \delta_r$$

Now we prove  $h(L) = L(A')$ . To prove  $h(L) \subseteq L(A')$ , we prove that if  $t \xrightarrow[A]{*m} q$ ,  $\rho(t)(q) = m > 0$  then

$h(t) \xrightarrow[A]{*m} q$ ,  $\rho(t)(q) = m > 0$  by induction on the length of the reduction of the ground term  $t \in T(\Sigma)$  by automaton A.

Base case: Suppose that  $t \xrightarrow{m} q$ ,  $\rho(t)(q) = m > 0$ ,  $t = a \in \Sigma_0$  and  $a \xrightarrow{m} q \in \delta$ . Let  $h(a) = g \in \Sigma'_k$  and assume that



Induction step : suppose that  $t_f(x_1 \leftarrow q_1, x_2 \leftarrow q_2, \dots, x_n \leftarrow q_n) = g(q_1, q_2, \dots, q_k)$

$$t_f(q_1, q_2, \dots, q_n) = g(q_1^r, q_2^r, \dots, q_k^r) \xrightarrow{m_1} q_e^r, q_1 \xrightarrow{m_2} q_1^r, q_2 \xrightarrow{m_2} q_2^r, \dots, q_k \xrightarrow{m_2} q_k^r, (q_\epsilon^r) \xrightarrow{m_2} q, h(t) \xrightarrow{*m_1} g(q_1^r, q_2^r, \dots, q_k^r) \xrightarrow{m_2} q_e^r \xrightarrow{m_2} q$$

Thus  $t_f(x_1 \leftarrow q_1, x_1 \leftarrow q_1, \dots, x_1 \leftarrow q_1) \xrightarrow{*m_2} q$  and hence  $h(t) \xrightarrow[A]{*m} q$ , where  $m = m_1 \wedge m_2 > 0$ , implies that  $h(L) \subseteq L(A')$ .

To prove  $L(A') \subseteq h(L)$ , we prove that  $t' \xrightarrow[A]{*m} \in Q$  then  $t' = h(t)$  with  $t \xrightarrow{m} q$  for some  $t \in T(\Sigma)$  by induction on the number of states in Q occurring along the sequence of reduction  $t' \xrightarrow{m} q$ ,  $\rho(t)(q) = m > 0$ .

Base case : Suppose that  $t' \xrightarrow[A]{*m} q \in Q$ ,  $\rho(t)(q) = m > 0$  and no states apart from q occurs in the reduction. The state set  $Q^f$  are disjoint, only some rules of  $\delta^f$  can be used in the reduction. Let  $t' = h_\Sigma(f)$  for some  $f \in \Sigma$  and the rule  $r = f(q_1, q_2, \dots, q_n) \xrightarrow{m} q$ . Since the automaton is reduced there exists some ground term t with  $head(t) = f$ , implies that  $t \xrightarrow[A]{m} q$ .

Induction step : Suppose that  $t' \xrightarrow[A]{*m_1} v\{x'_1 \leftarrow q_1, x'_2 \leftarrow q_2, \dots, x'_m \leftarrow q_m\} \xrightarrow[A]{*m_2} q$ ,  $\rho(t')(q) = m$ ,  $m = m_1 \wedge m_2$  where v is a linear term in  $T(\Sigma', \{x'_1, x'_2, \dots, x'_m\})$ ,

$t' = v\{x'_1 \leftarrow u'_1, x'_2 \leftarrow u'_2, \dots, x'_m \leftarrow u'_m\}$ ,  $u'_i \xrightarrow[A]{*m_i} q_i \in Q$  and no states in Q apart from q occurs in the reduction of  $v\{x'_1 \leftarrow q_1, x'_2 \leftarrow q_2, \dots, x'_m \leftarrow q_m\}$  to q. The state set  $Q^f$  are disjoint, only some rules of  $\delta^f$  can be used in the reduction  $v\{x'_1 \leftarrow q_1, x'_2 \leftarrow q_2, \dots, x'_m \leftarrow q_m\}$  to q. Thus there exists a linear term  $t_f$  such that  $v\{x'_1 \leftarrow q_1, x'_2 \leftarrow q_2, \dots, x'_m \leftarrow q_m\} = v(x_1 \leftarrow q_1, x_2 \leftarrow q_2, \dots, x_n \leftarrow q_n)$  for some symbol  $f \in \Sigma_n$  and the rule for  $r = f(q_1, q_2, \dots, q_n) \xrightarrow{m_2} q \in \delta$ . By induction hypothesis there exists terms  $u_1, u_2, \dots, u_m$  in L such that

$u'_i = h(u_i)$  and  $u_i \xrightarrow{*m_i} q_i$  for  $i \in \{1, 2, \dots, m\}$ . Consider the term  $t = f(v_1, v_2, \dots, v_m)$ , where  $v_i = u_i$  if  $x_i$  occurs in  $t$ , then  $h(t) = t_f \{x_1 \longleftarrow h(v_1), x_2 \longleftarrow h(v_2), \dots, x_n \longleftarrow h(v_n)\}$  implies that  $h(t) = V \{x'_1 \leftarrow h(u_1), x'_2 \leftarrow h(u_2), \dots, x_m \leftarrow h(u_m)\}$ .

By induction hypothesis and by definition of  $v_i$ , we have  $t \xrightarrow{*m} q$ .

Hence  $L(A') \subseteq h(L)$ .

*Example V.4.*

Let  $A = A = (Q, \Sigma, \delta, \Gamma)$  be a FFTA such that  $Q = \{q_0, q_1\}$ ,  $\Sigma_0 = \{0, 1\}$ ,  $\Sigma_1 = \{\text{not}\}$ ,  $\Sigma_2 = \{\text{or, and}\}$ ,  $\Gamma(q_0) = 0.5$ ,  $\Gamma(q_1) = 0.6$  and  $\delta$  is defined by

$$0 \xrightarrow{1} q_0, \quad 1 \xrightarrow{1} q_1, \quad \text{not}(q_0) \xrightarrow{0.3} q_1, \quad \text{not}(q_1) \xrightarrow{0.5} q_0, \quad \text{and}(q_0, q_0) \xrightarrow{0.4} q_0, \quad \text{and}(q_0, q_1) \xrightarrow{0.6} q_0$$

$$\text{and}(q_1, q_0) \xrightarrow{0.7} q_0, \quad \text{and}(q_1, q_1) \xrightarrow{0.8} q_1, \quad \text{or}(q_0, q_0) \xrightarrow{0.7} q_0,$$

$$\text{or}(q_0, q_1) \xrightarrow{0.9} q_1, \quad \text{or}(q_1, q_0) \xrightarrow{0.6} q_1, \quad \text{or}(q_1, q_1) \xrightarrow{0.5} q_1$$

Let the mapping  $h : T(\Sigma) \rightarrow T(\Sigma')$  be defined by

$$h_\Sigma(0) = 0, \quad h_\Sigma(1) = 1, \quad h_\Sigma(\text{or}) = \text{or},$$

$$h_\Sigma(\text{not}) = \text{not}, \quad h_\Sigma(\text{and}) = \text{not}(\text{or}(\text{not}(x_1), \text{not}(x_2)))$$

Now we define  $A' = (Q', \Sigma', \delta', \Gamma')$ .

For rule 1 :  $0 \xrightarrow{1} q_0 \in \delta$ , we have rules in  $\delta_1$  are  $0 \xrightarrow{1} (q_0)_e^1, (q_0)_e^1 \xrightarrow{1} q_0, Q^1 = \{(q_0)_e^1\}$ . By considering all the 12 rules, we get  $\delta' =$

$$\bigcup_{i=1}^{12} \delta_i, Q' = \bigcup_{i=1}^{12} Q^i \cup Q,$$

$$\Gamma'(p) = \Gamma(p) \forall p \in Q, \Gamma'(p) = \Gamma(q) \forall p \in Q^i.$$

Clearly  $|A|(t) = |A'|h(t)$

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