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Some Properties of Fuzzy Finite Tree Automata

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Abstract: This paper examines fuzzy tree automata's deterministic, reduced, and homomorphic properties. It is demonstrated through an example that there is a deterministic fuzzy tree automaton for the nondeterministic one. For every given fuzzy tree automaton, an equivalent reduced fuzzy tree automaton is displayed. A theorem that fuzzy tree languages are preserved by tree homomorphism is derived.

Keywords: Fuzzy tree automaton, Deterministic fuzzy tree atuomaton, Fuzzy tree homomorphism.

I. INTRODUCTION

Automata on infinite words, weighted automata, fuzzy automata on finite and infinite words, tree automata, and weighted tree automata are some of the ways that finite state automata have been expanded. We refer to [8] for a general treatment of fuzzy automata and languages. Previous research on fuzzy tree automata has been done in [2, 5]. The deterministic fuzzy tree automaton Ad exhibits the same behavior as any fuzzy tree automaton A, as demonstrated in this paper by the proof of L(A) = L(Ad). The membership value in A and that in Ad, however, do not have to match for any tree t. An example is provided to demonstrate the existence. Additionally, it is demonstrated that there is an equivalent reduced fuzzy tree automaton for every fuzzy tree automaton. Also, it is demonstrated that h(L) is recognized by a fuzzy tree automaton if h is a linear tree homomorphism and L is recognized by a fuzzy tree automaton. This can be further extended in the field of fuzzy hyper graph and Plus Weighted Linear Grammar as cited in [10, 11,12,13,14,15,16].

II. BASIC DEFINITIONS

For the definition of standard notions, [2,3,5] are mentioned.

1) *Definition II.1.*

A fuzzy finite tree automaton (NFFTA) is a system $A = (Q, \Sigma, \delta, \Gamma)$ where,

- Q is a set of state symbols.
- Σ is a set of ranked alphabets called input symbols.
- $\delta = \{\delta_\sigma : Q^n \times Q \times \Sigma_n \to [0, 1] \mid \sigma \in \Sigma_n, n \geq 0\}$

  A transition rule may be defined by

  $\delta_\sigma : Q^n \times \Sigma_n \times [0, 1] \to Q$
- $\Gamma \subseteq$ Fuzzy(Q) is a set of fuzzy final states.

A fuzzy final state $\gamma \in \Gamma$ may be defined by $\gamma : Q \to [0, 1]$

An NFFTA is called finite if Q is finite and if $\delta_\sigma(q_1, q_2, \ldots q, \sigma) = 0$ for all but a finite number of $\delta_\sigma \in \sigma$ and if $\Gamma(q) = 0$ for all but a finite number of states $q \in Q$.

For $t \in T_\Sigma$, a fuzzy set $\rho(t) \subseteq$ Fuzzy(Q) is defined by induction on structure of t.

2) *Definition II.2.*

When $t = \sigma \in \Sigma_0$, then $\rho(t)(q) = \delta(q, \sigma)$, $\forall q \in Q$. Assume that $t = \sigma(t_1, t_2, \ldots t_n)$ for some $\sigma \in \Sigma_n$ and $t_1, t_2, \ldots t_n \in T_\Sigma$.

$$\rho(t)(q) = \bigvee_{q_1,q_2,\ldots q_n \in Q}\left(\delta(q_1,q_2,\ldots q_n,q,\sigma) \wedge \left(\bigwedge_{i=1}^{n} \rho(t_i)(q_i)\right)\right)$$

The behavior of an NFFTA A is a fuzzy set |A| on a set of trees $t \in T_\Sigma$ is defined by

$$|A|(t) = \bigvee_{q\in Q}\left(\rho(t)(q) \wedge \Gamma(q)\right)$$

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3) *Definition II.3.*

A tree $t \in T_\Sigma$ is accepted by NFFTA $A = (Q, \Sigma, \delta, \Gamma)$ if $|A|(t) > 0$ and $|A|(t) = \mu_{L(A)}(t)$.

4) *Definition II.4.*

The fuzzy tree language L(A) recognized by NFFTA A is the set of all ground terms accepted by A. A set L of ground terms is recognizable if L = L(A) for some A.

III. DETERMINISTIC FUZZY FINITE TREE AUTOMATON

This section introduces the definition of deterministic fuzzy finite tree automaton and some related theorems are proved.

1) *Definition III.1.*

A deterministic fuzzy finite tree automaton (DFFTA) is an FFTA $A_d = (Q_d, \Sigma, \delta_d, \Gamma_d)$ such that for each $\sigma \in \Sigma_n$ and $q_1, q_2, \ldots q_n \in Q_d$ there exists at most one $q \in Q_d$ such that $\delta_d(\sigma(q_1, q_2, \ldots q_n) \to q) > 0$ or

$\sigma(q_1, q_2, \ldots q_n) \xrightarrow{m} q$; $m > 0$.

2) *Theorem III.2*

Let L be a recognizable set of ground terms. Then there exists a DFFTA that accepts L.

Proof : Let $A = (Q, \Sigma, \delta, \Gamma)$ be a NFFTA. Define a DFFTA $A_d = (Q_d, \Sigma, \delta_d, \Gamma_d)$. The states of Q_d are all the subsets of Q. That is $Q_d = 2^Q$. We denote by S a state of Q_d, $S = \{q_1, q_2, \ldots q_n\}$ for some states $q_1, q_2, \ldots q_n \in Q$. δ_d and Γ_d are constructed from the following algorithm.

input : NFFTA $A = (Q, \Sigma, \delta, \Gamma)$

begin

/* a state S of the DFFTA is in 2^Q */

Set $\delta_d = \phi$;

repeat

if $f \in \Sigma_n$ and $S_1, S_2, \ldots S_n \in Q_d$ then

begin

$S = \{q \in Q \mid \exists\, q_1 \in S_1, q_2 \in S_2, \ldots, q_n \in S_n,$

$f(q_1, q_2, \ldots q_n) \xrightarrow{m} q \in \delta\}$

$m = \vee\{m_i \mid f(q_1, q_2, \ldots q_n) \xrightarrow{m_i} q,$

$\exists\, q_1 \in S_1, q_2 \in S_2, \ldots, q_n \in S_n\}$

end

Set $\delta_d = \delta_d \cup \{f(S_1, S_2, \ldots S_n) \xrightarrow{m} S\}$;

until no new rule is added to δ_d

$\Gamma_d(S) = \vee\{\Gamma(q) \mid q \in S\}\ \forall\, S \in Q_d$,

output : DFFTA $A_d = A_d = (Q_d, \Sigma, \delta_d, \Gamma_d)$.

Clearly A_d is a DFFTA. Let us prove that $L(A) = L(A_d)$, that is to prove $t \xrightarrow[A_d]{*} S$, $\rho(t)(S) > 0$ if and only if

$S = \{q \in Q \mid t \xrightarrow[A]{*} q\ ,\ \rho(t)(q) > 0\}$

Suppose that $t \xrightarrow[A_d]{*} S$, $\rho(t)(S) > 0$,

we prove $t \xrightarrow[A]{*} q$, $\rho(t)(q) > 0$, by induction on the structure of terms.

Base case : Let us consider $t = a \in \Sigma_0$. Then there is only one rule.

$a \xrightarrow{m} S \in \delta_d \because \rho(t)(S) = m > 0$, where

$m = \vee\{a \xrightarrow{m_i} q \mid q \in Q\}\ [\because\ m > 0]$

$S = \{q \in Q \mid t \xrightarrow[A]{*} q,\ \rho(t)(q) > 0\}$

Induction step : Let us consider a term $t = f(t_1, t_2, \ldots t_n)$

Assume that the derivation if of the form

$t \xrightarrow[A_d]{*} f(S_1, S_2, \ldots S_n) \xrightarrow[A_d]{} S$, where $t_i \xrightarrow[A_d]{*} S_i$,

$\rho(t_i)(S_i) > 0$ $i = 1, 2, \ldots, n$. By induction hypothesis, for each $i = 1, 2, \ldots, n$.

$$S_i = \{q \in Q \mid t_i \xrightarrow[A]{*} q,\ \rho(t_i)(q) > 0\} \qquad (1)$$

Since S_i's are in Q_d, $f(S_1, S_2, \ldots S_n) \to S$ is a rule in δ_d with membership value $m_1 > 0$. From the algorithm,

$S = \{q \in Q \mid \exists\, q_1 \in S_1,\ q_2 \in S_2, \ldots, q_n \in S_n,\ f(q_1, q_2, \ldots q_n) \xrightarrow{m_1} q \in \delta\}$ and $m_1 = \vee_{i=1}^{n} \{m_i\}$ implies, $\exists$ $m_i > 0$ such that $f(q_1, q_2, \ldots q_n) \xrightarrow{m_i} q$.

From (1) $\exists\, q_i \in Q$ such that $t_i \xrightarrow[A]{*} q_i$, $\rho(t_i)(q_i) > 0$

$i = 1, 2, \ldots, n$. Since there exists $q_1 \in S_1$, $q_2 \in S_2$, $\ldots$, $q_n \in S_n$, and $f(q_1, q_2, \ldots q_n) \xrightarrow{m_i} q$,

we have $t \xrightarrow[A]{*} q$, $\rho(t)(q) > 0$

Conversely suppose that $t \xrightarrow[A]{*} q$, $\rho(t)(q) > 0$, we prove

$t \xrightarrow[A_d]{*} S$, $\rho(t)(S) > 0$ by induction on the structure of terms.

Base case : Let us consider $t = a \in \Sigma_0$.

Then there is a rule $a \xrightarrow{m} q \in \delta$, $\rho(t)(q) = m > 0$.

Suppose $S = \{q \in Q \mid a \xrightarrow{m_2} q \in \delta\}$

$m_2 = \vee\{m_i \mid a \xrightarrow{m_i} q \in \delta\}$, implies that $m_2 > 0$.

Therefore $a \xrightarrow{m_2} S$, $\rho(t)(S) > 0$.

Induction step : Let us consider a term $t = f(t_1, t_2, \ldots t_n)$

Assume that the derivation is of the form

$t \xrightarrow[A]{*} f(q_1, q_2, \ldots q_n) \xrightarrow[A]{} q$, where $t \xrightarrow[A]{*} q$, $\rho(t)(q) > 0$, by induction, for each $i = 1, 2, \ldots, n$. $t_i \xrightarrow[A_d]{*} S_i$,

$\rho(t_i)(S_i) = m_i > 0$ and the state S_i is defined by

$$S_i = \{q \in Q \mid t_i \xrightarrow[A]{*} q \in \delta\} \qquad (2)$$

Since q_i's are in Q, $f(q_1, q_2, \ldots q_n) \to q$ is a rule in δ with membership value $m > 0$.

Thus $S = \{q \in Q \mid \exists\, q_1 \in S_1,\ q_2 \in S_2, \ldots, q_n \in S_n,\ f(q_1, q_2, \ldots q_n) \xrightarrow{m'} q \in \delta\}$ and

$m' = \vee\{m_i \mid f(q_1, q_2, \ldots q_n) \xrightarrow{m_i} q \in \delta,\ \exists\, q_1 \in S_1,\ q_2 \in S_2, \ldots, q_n \in S_n\}$

Implies that $m' > 0$, therefore $f(S_1, S_2, \ldots S_n) \xrightarrow{m'} S$ is a rule in δ_d. From equation (2)

$\exists\, S_i \in Q_d$ such that $t_i \xrightarrow[A_d]{*} S_i$, $\rho(t_i)(S_i) = m_i > 0$ for each $i = 1, 2, \ldots, n$ and also

$f(S_1, S_2, \ldots S_n) \xrightarrow[A_d]{m'} S$ is a rule in δ,.

Therefore $t \xrightarrow[A_d]{*} S$ and $\rho(t)(S) = m$,

$m = \wedge\{m_1, m_2, \ldots m_n, m'\} > 0$.

Example III.3.

Consider the NFFTA, $A = (Q, \Sigma, \delta, \Gamma)$, where $Q = \{q, q_g, q_f\}$, $\Sigma_0 = \{a\}$, $\Sigma_1 = \{g\}$, $\Sigma_2 = \{f\}$, $\Gamma(q) = 0.5$, $\Gamma(q_g) = 0.6$, $\Gamma(q_f) = 0.9$ and δ is the following set of transition rules :

$$\{a \xrightarrow{0.8} q,\ g(q) \xrightarrow{0.3} q,\ g(q) \xrightarrow{0.5} q_g,$$
$$g(q_g) \xrightarrow{0.7} q_f,\ f(q,q) \xrightarrow{0.6} q\}$$

Consider the ground term $t = f(g(a), g(a))$ and its sequence of reduction is as follows :

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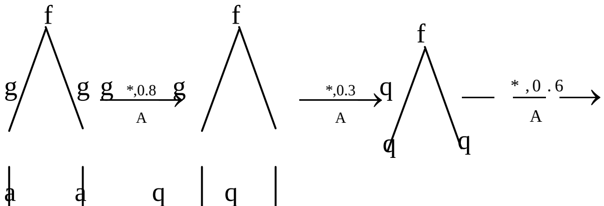


$\rho(t)(q) = 0.8 \wedge 0.3 \wedge 0.6 = 0.3$ and

$\mu_{L(A)}(t) = 0.3 \wedge 0.5 = 0.3 > 0$

The language accepted by the NFFTA is

$$\mu_{L(A)}(t) = \begin{cases} 0.5, & \text{if } t \in \{g(g(t')) \mid t' \in T_\Sigma\} \\ 0.3, & \text{if } t \in \{f(g^i(a), g^j(a),) \mid i, j \geq 1\} \\ 0.5, & \text{if } t \in \{f^i(a, a) \mid i, j \geq 1\} \end{cases}$$

Using the theorem (III.2), we get the DFFTA

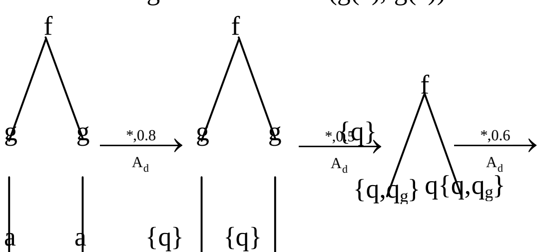
$A = (Q_d, \Sigma, \delta_d, \Gamma_d)$, where

$Q_d = \{\{q\}, \{q, q_g\}, \{q, q_g, q_f\}\}$, $\Sigma_0 = \{a\}$,

$\Sigma_1 = \{g\}$, $\Sigma_2 = \{f\}$, $\Gamma_d(\{q\}) = 0.5$, $\Gamma_d(\{q, q_g\}) = 0.6$, $\Gamma_d(\{q, q_g, q_f\}) = 0.9$ and δ_d is the following set of transition rules :

$$\left\{a \xrightarrow{0.8} \{q\}, g\{q\} \xrightarrow{0.5} \{q, q_g\}, g\left(\{q, q_g\}\right) \xrightarrow{0.7} \{q, q_g, q_f\}, \quad f(S_1, S_2) \xrightarrow{0.6} \{q\}, \text{ where } S_1, S_2 \in Q_d\right\}$$

Consider the ground term t = f(g(a), g(a)) and its sequence of reduction is as follows :



$\rho(t)(\{q\}) = 0.8 \wedge 0.5 \wedge 0.6 = 0.5$ and

$\mu_{L(A_d)}(t) = 0.5 \wedge 0.5 = 0.5 > 0$

The language accepted by the DFFTA is

$$\mu_{L(A)}(t) = \begin{cases} 0.5, & \text{if } t \in \{g(g(t')) \mid t' \in T_\Sigma\} \\ 0.5, & \text{if } t \in \{f(g^i(a), g^j(a),) \mid i, j \geq 1\} \\ 0.5, & \text{if } t \in \{f^i(a, a) \mid i \geq 1\} \end{cases}$$

The language accepted by A is equal to the language accepted by A_d, but the membership is not retained.

IV. REDUCED FUZZY FINITE TREE AUTOMATON

Reduced Fuzzy Finite Tree Automaton is discussed in this section. It is demonstrated that there is an equivalent reduced fuzzy tree automaton for every fuzzy tree automaton.

Definition IV.1.

A state Q is accessible if there exists a term t such that $\rho(t)(q) > 0$. An NFFTA is said to be reduced if all its states are accessible.

Theorem IV.2.

Let L be a recognizable set of ground terms. Then there exists a reduced FFTA that accepts L.

Proof : Let $A = (Q, \Sigma, \delta, \Gamma)$, be an FFTA. If all the states of Q are accessible. ie. $\rho(t)(q) > 0$ $\forall q \in Q$, then A is reduced. Suppose that A is irreduced and L be the language accepted by it. Let $Q^r = Q - \{s|s$ is not reacheable$\}$.

Define $A^r = (Q^r, \Sigma, \delta^r, \Gamma^r)$, where

$\delta^r : {Q_n}^r \times \Sigma_n \times Q^r \to [0, 1]$ is defined by

$\forall q_1, q_2, \ldots, q_n, q \in Q$ and $f \in \Sigma_n$,

$$\delta^r(q_1,q_2,\ldots,q_n,f,q) = \begin{cases} \delta(q_1,q_2,\ldots,q_n,f,q), & \text{if } q \text{ is accessible.} \\ 0, & \text{if } q \text{ is not accessible.} \end{cases}$$

$$\Gamma^r(q) = \begin{cases} \Gamma(q), & \text{if } q \text{ is accessible.} \\ 0, & \text{if } q \text{ is not accessible.} \end{cases}$$

Clearly A^r is a reduced FFTA. Now we prove that A and A^r are equivalent.

Let L_1 be the fuzzy tree language accepted by A^r.

If $\mu_{L(A)}(t) = 0$ then $\rho(t)(q) = 0$ or $\Gamma(q) = 0 \forall q \in Q$, hence $\mu_{L_1(A)}(t) = 0$.

If $\mu_{L(A)}(t) > 0$ then $\rho(t)(q) > 0$ and $\Gamma(q) > 0$, implies that

$$\Gamma(q) = \Gamma^r(q) \forall q \in Q \quad (3)$$

Therefore q is accessible in A^r and in the sequence of moves $\rho^r(t)(q) = \rho(t)(q)$ and hence

$$\rho(t)(q) \wedge \Gamma(q) = \rho^r(t)(q) \wedge \Gamma^r(q) \quad (4)$$

Equation (4) is true for every $q \in Q$ which satisfies equation (3). Implies that

$\bigvee_{q\in Q}\{\rho(t)(q) \wedge \Gamma(q)\} = \bigvee_{q\in Q^r}\{\rho^r(t)(q) \wedge \Gamma^r(q)\}$. Therefore $\mu_{L(A)}(t) = \mu_{L_1(A)}(t)$.

V. TREE HOMOMORPHISM

This section deals with Tree Homomorphism. It is demonstrated that h(L) is recognized by a fuzzy tree automaton if h is a linear tree homomorphism and L is recognized by a fuzzy tree automaton.

Definition V.1.

Let Σ and Σ' be two sets of function of symbols, possibly not disjoint. For each $n > 0$, Σ contains a symbol f of arity n, we define a set of variables $\chi_n = \{ x_1,x_2,\ldots x_n \}$ disjoint from Σ and Σ'. Let h_Σ be a mapping which with $f \in \Sigma$ of arity n, associate as term $t_f \in T(\Sigma', \chi_n)$. The tree homomorphism $h : T(\Sigma) \to T(\Sigma')$ determined by h_Σ is defined as follows :

- $h(a) = t_a \in T(\Sigma')$ for each $a \in \Sigma$ of arity 0.
- $h(f(t_1,t_2,\ldots t_n)) = t_f(x_1 \leftarrow h(t_1), x_2 \leftarrow h(t_2), \ldots, x_n \leftarrow h(t_n))$, where

$t_f(x_1 \leftarrow h(t_1), x_2 \leftarrow h(t_2), \ldots, x_n \leftarrow h(t_n))$ is the result applying the substitution

$\{x_1 \leftarrow h(t_1), x_2 \leftarrow h(t_2), \ldots, x_n \leftarrow h(t_n)\}$

Definition V.2.

A tree homomorphism is linear if for each $f \in \Sigma$ of arity n, $h_\Sigma(f) = t_f$ is a linear term in $T(\Sigma', \chi_n)$.

Theorem V.3.

Let h be a linear tree homomorphism and L be recognized by fuzzy tree automaton. Then h(L) is recognized by a fuzzy tree automaton.

Proof: Let $A = (Q, \Sigma, \delta, \Gamma)$ be reduced DFFTA such that L(A)=L and h be a linear fuzzy tree homomorphism from $T(\Sigma) \to T(\Sigma')$. determined by a mapping h_Σ. We construct an NFFTA $A' = (Q', \Sigma', \delta', \Gamma')$ as follows. Let us consider a ruler $r = f(q_1,q_2,\ldots q_n) \xrightarrow{m} q \in \delta$, a term $t_f = h_\Sigma(f) \in T(\Sigma', X_n)$ and the set of positions $pos(t_f)$. We define a set of states $Q^r = \{q_p{}^r \mid p \in pos(t_f)$ and a set of rules δ_r as follows for all positions

$p \in pos(t_f)$.

If $t_f(p) = g \in \Sigma'$ then $g\left(q^r_{p_1},q^r_{p_2},\ldots q^r_{p_n}\right) \xrightarrow{m} q^r_p \in \delta_r$

If $t_f(p) = x_i$ then $q_i \xrightarrow{m} q^r_p \in \delta_r$

$q^r_e \xrightarrow{m} q \in \delta_r$

The preceding construction is made for each rule in δ. We assume that all the states set Q^r are disjoint and are disjoint from Q. Define $A' = (Q', \Sigma', \delta', \Gamma')$ by

$$Q' = Q \bigcup \bigcup_{r\in\delta} Q^r$$

$\Gamma': Q' \to [0,1]$ such that $\Gamma'(P)=\Gamma(p)\ \forall p \in Q$ and $\Gamma'(P)=\Gamma(q)\ \forall p \in Q^r$,

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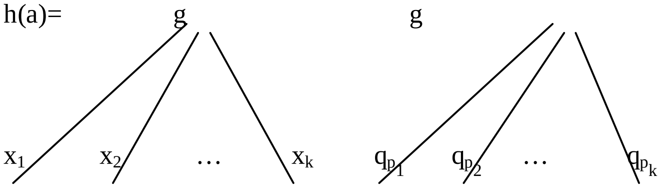
$\delta' = \bigcup_{r \in \delta} \delta_r$

Now we prove h(L) = L(A′). To prove h(L) $\subseteq$ L(A′), we prove that if t $\xrightarrow[A]{*m}$ q, ρ(t)(q)=m>0 then

h(t) $\xrightarrow[A]{*m}$ q, ρ(t)(q) = m > 0 by induction on the length of the reduction of the ground term $t \in T(\Sigma)$ by automaton A.

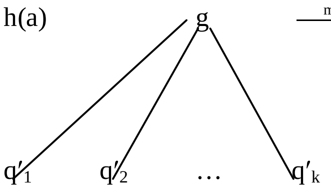
Base case: Suppose that $t \xrightarrow{m} q$, ρ(t)(q) = m > 0, t= a $\in \Sigma_0$ and a $\xrightarrow{m}$ q $\in$ δ. Let h(a) = g $\in \Sigma'_k$ and assume that

h(a)=



g((q₁)′, (q₂)′, . . . , (q_k)′) $\xrightarrow{m}$ (q_ε)′, $q_{p_1} \xrightarrow{m}$ (q₁)′, $q_{p_2} \xrightarrow{m}$ (q₂)′, . . . , $q_{p_k} \xrightarrow{m}$ (q_k)′, $q_\varepsilon \xrightarrow{m}$ (q)′ now

h(a)



$\xrightarrow{m} q_\varepsilon{}' \xrightarrow{m}$ q

Induction step : suppose that $t_f(x_1 \leftarrow q_1, x_2 \leftarrow q_2, \ldots, x_n \leftarrow q_n) = g(q_1, q_2, \ldots q_k)$

$t_f(q_1, q_2, \ldots q_n) = g(q_1^r, q_2^r, \ldots q_k^r) \xrightarrow{m_1} q_e^r$, $q_1 \xrightarrow{m_2} q_1^r$, $q_2 \xrightarrow{m_2} q_2^r$, . . . , $q_k \xrightarrow{m_2} q_k^r$, $(q_\varepsilon)^r \xrightarrow{m_2}$ q, h(t) $\xrightarrow{*m_1}$ g(q_1^r, q_2^r, . . . , q_k^r) $\xrightarrow{m_2} q_e^r \xrightarrow{m_2}$ q

Thus $t_f(x_1 \leftarrow q_1, x_1 \leftarrow q_1, \ldots, x_1 \leftarrow q_1) \xrightarrow{*m_2} q$ and hence h(t) $\xrightarrow[A']{*m}$ q , where m = $m_1 \wedge m_2$ > 0, implies that h(L) $\subseteq$ L(A′).

To prove L(A′) $\subseteq$ h(L), we prove that t′ $\xrightarrow[A']{*m}$ $\in$ Q then t′ = h(t) with t $\xrightarrow{*m} q$ for some t $\in$ T(Σ) by induction on the number of states in Q occurring along the sequence of reduction t′ $\xrightarrow{*m} q$, ρ(t)(q) = m > 0.

Base case : Suppose that t′ $\xrightarrow[A']{*m}$ q$\in$ Q, ρ(t)(q) = m > 0 and no states apart from q occurs in the reduction. The state set Q^r are disjoint, only some rules of δ^r can be used in the reduction. Let t′ = $h_\Sigma(f)$ for some f $\in$ Σ and the rule r = $f(q_1, q_2, \ldots q_n) \xrightarrow{m}$ q. Since the automaton is reduced there exists some ground term t with head(t) = f, implies that t $\xrightarrow[A]{m}$ q.

Induction step : Suppose that t′ $\xrightarrow[A']{*m_1}$ v{ $x'_1 \leftarrow q_1$, $x'_2 \leftarrow q_2$, . . . , $x'_m \leftarrow q_m$} $\xrightarrow[A']{*m_2}$ q, ρ(t′)(q) = m, m = $m_1 \wedge m_2$ where v is a linear term in T(Σ′, { x'_1, x'_2, . . . , x'_m }),

t′ = v{ $x'_1 \leftarrow u'_1$, $x'_2 \leftarrow u'_2$, . . . , $x'_m \leftarrow u'_m$ }, $u'_i \xrightarrow[A']{*m_i} q_i$ $\in$Q and no states in Q apart from q occurs in the reduction of v($x'_1 \leftarrow q_1$, $x'_2 \leftarrow q_2$, . . . , $x'_m \leftarrow q_m$} to q. The state set Q^r are disjoint, only some rules of δ^r can be used in the reduction v($x'_1 \leftarrow q_1$, $x'_2 \leftarrow q_2$, . . . , $x'_m \leftarrow q_m$} to q. Thus there exists a linear term t_f such that v($x'_1 \leftarrow q_1$, $x'_2 \leftarrow q_2$, . . . , $x'_m \leftarrow q_m$} = v($x_1 \leftarrow q_1$, $x_2 \leftarrow q_2$, . . . , $x_n \leftarrow q_n$}for some symbol f $\in \Sigma_n$ and the rule for r = $f(q_1, q_2, \ldots q_n) \xrightarrow{m_2}$ q $\in$ δ. By induction hypothesis there exists terms $u_1, u_2, \ldots u_m$ in L such that

$u_i' = h(u_i)$ and $u_i \xrightarrow{*\, m_i} q_i$ for $i \in \{1, 2, \ldots, m\}$. Consider the term $t = f(v_1, v_2, \ldots v_m)$, where $v_i = u_i$ if x_i occurs in t_f, then $h(t)=$

$t_f\{x_1 \longleftarrow h(v_1), x_2 \longleftarrow h(v_2) \ldots\ldots x_n \longleftarrow h(v_n)\}$ implies that $h(t) = V\{x_1' \leftarrow h(u_1),\ x_2' \leftarrow h(u_2),\ \ldots, x_m \leftarrow h(u_m)\}$.

By induction hypothesis and by definition of v_i, we have $t \xrightarrow[A]{*\, m} q$.

Hence $L(A') \subseteq h(L)$.

Example V.4.

Let A = $A = (Q, \Sigma, \delta, \Gamma)$ be a FFTA such that $Q = \{q_0, q_1\}$, $\Sigma_0 = \{0, 1\}$, $\Sigma_1 = \{not\}$, $\Sigma_2 = \{or, and\}$, $\Gamma(q_0) = 0.5$, $\Gamma(q_1) = 0.6$ and δ is defined by

$$0 \xrightarrow{1} q_0,\ 1 \xrightarrow{1} q_1,\ not(q_0) \xrightarrow{0.3} q_1,\ not(q_1) \xrightarrow{0.5} q_0,\ and(q_0, q_0) \xrightarrow{0.4} q_0,\ and(q_0, q_1) \xrightarrow{0.6} q_0$$

$$\text{and } (q_1, q_0) \xrightarrow{0.7} q_0,\ \text{and}(q_1, q_1) \xrightarrow{0.8} q_1,\ \text{or}(q_0, q_0) \xrightarrow{0.7} q_0,$$

$$\text{or } (q_0, q_1) \xrightarrow{0.9} q_1,\ \text{or}(q_1, q_0) \xrightarrow{0.6} q_1,\ \text{or}(q_1, q_1) \xrightarrow{0.5} q_1$$

Let the mapping $h : T(\Sigma) \to T(\Sigma')$ be defined by

$h_\Sigma(0) = 0$, $h_\Sigma(1) = 1$, $h_\Sigma(or) = or$,

$h_\Sigma(not) = not$, $h_\Sigma(and) = not(or(not(x_1), not(x_2))$

Now we define $A' = (Q', \Sigma', \delta', \Gamma')$.

For rule 1 : $0 \xrightarrow{1} q_0 \in \delta$, we have rules in δ_1 are $0 \xrightarrow{1} (q_0)_e^1$, $(q_0)_e^1 \xrightarrow{1} q_0$, $Q^1 = \{(q_0)_e^1\}$. By considering all the 12 rules, we get $\delta' = \bigcup_{i=1}^{12} \delta_i, Q' = \bigcup_{i=1}^{12} Q^i \cup Q$,

$\Gamma'(p) = \Gamma(p) \forall p \in Q$, $\Gamma'(p) = \Gamma(q) \forall p \in Q^i$.

Clearly $|A|(t) = |A'|h(t)$

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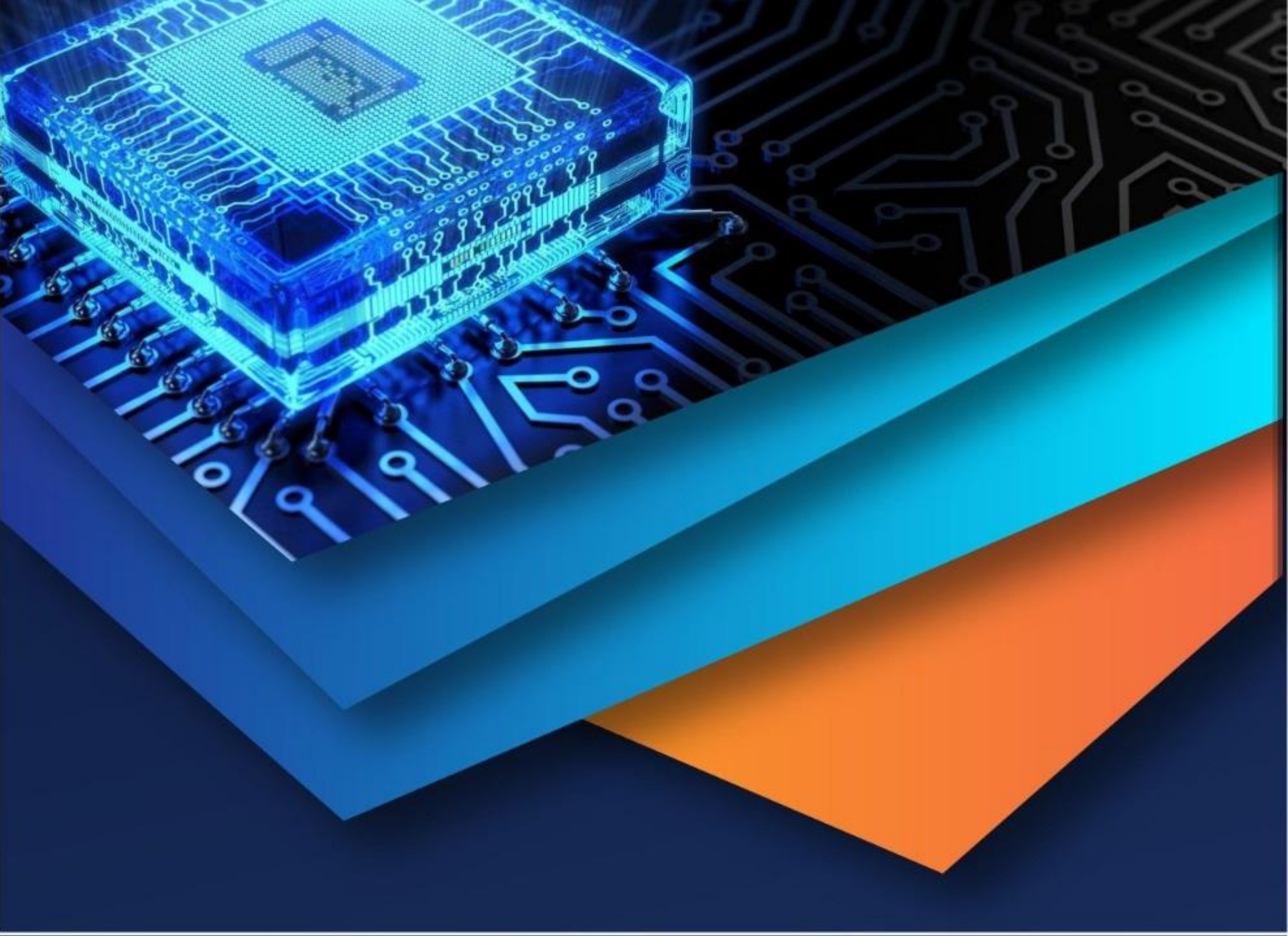

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