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Some Results on Radical Screen Transversal Lightlike Submanifold of an Indefinite Kaehler Norden Manifold

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Abstract: In this paper, we introduce the notion of radical screen transversal and screen transversal anti-invariant lightlike submanifolds of an indefinite Kaehler Norden manifold. We investigate the geometry of distributions involved and obtain necessary and sufficient conditions for the induced connection on radical screen transversal and screen transversal anti-invariant lightlike submanifolds to be metric connection. Further, we provide the necessary and sufficient conditions for foliations determined by above distributions to be totally geodesic on radical screen transversal lightlike submanifold of an indefinite Kaehler Norden manifold.

AMS 2020 Mathematics Subject Classification: 53C15, 53C40, 53C50.
Keywords: Semi-Riemannian manifolds, radical distribution, screen distribution, screen transversal vector bundle, lightlike transversal vector bundle, Norden metric.

I. INTRODUCTION

The theory of almost complex Norden manifolds was introduced in 1960 by A. P. Norden [9] and was further developed by Ganchev et al.[6],[7],[8] who studied Kaehler Norden manifolds. The geometry of an indefinite almost Hermitian manifold is entirely different from the geometry of an almost complex Norden manifold. The difference arises due to the behaviour of an almost complex structure $\bar{J}$ which is an isometry with respect to the semi-Riemannian metric $\tilde{\bar{g}}$ in first case and is an anti-isometry with respect to the metric $\tilde{\bar{g}}$ in the second case.

The general theory of lightlike submanifolds contrasts that of classical theory of non-degenerate submanifolds. Since in case of lightlike submanifold, the intersection of tangent bundle and the normal bundle called the radical distribution is non-trivial, therefore, there arises more difficulties in studying lightlike submanifolds rather than in the non-degenerate case.The lightlike geometry has been developed by K. L. Duggal and A. Bejancu [1][4].Many geometers have proved various important results for radical transversal and screen transversal lightlike submanifolds for indefinite Kaehler manifolds using the lightlike theory, which motivated us to study these submanifolds in the setting of indefinite Kaehler Norden manifolds.

In this paper, we introduce the notion of radical screen transversal and screen transversal anti-invariant lightlike submanifolds of an indefinite Kaehler Norden manifold. We investigate the geometry of various distributions involved and obtain necessary and sufficient conditions for the induced connection on radical screen transversal and screen transversal anti-invariant lightlike submanifolds to be metric connection.Further, we provide the necessary and sufficient conditions for foliations determined by above distributions to be totally geodesic on radical screen transversal lightlike submanifold of an indefinite Kaehler Norden manifold.

II. PRELIMINARIES

1) Lightlike submanifolds. Let $(\bar{M}, \tilde{\bar{g}})$ be a real $(m+n)$-dimensional semiRiemannian manifold of constant index q such that $m,n \geq 1$, $1 \leq q \leq m+n-1$ and $(M,\tilde{g})$ be an m-dimensional submanifold of $\bar{M}$ and $\tilde{g}$ the induced metric of $\tilde{\bar{g}}$ on M. If $\tilde{\bar{g}}$ is degenerate on the tangent bundle TM of M then M is known as a lightlike submanifold of $\bar{M}$. For a degenerate metric $\tilde{g}$ on M, $TM^{\perp}$ is a degenerate n-dimensional subspace of $T_x\bar{M}$. Thus both T_xM and $T_xM^{\perp}$ are degenerate orthogonal subspaces but no longer complementary. In this case, there exists a subspace $Rad(T_xM) = T_xM \cap T_xM^{\perp}$ which is known as radical (null) subspace. If the mapping $Rad(TM) : x \in M \longrightarrow RadT_xM$, defines a smooth distribution on M of rank $r > 0$ then the submanifold M of $\bar{M}$ is called an r-lightlike submanifold[4] and $Rad(TM)$ is called the radical distribution on M. Screen distribution $S(TM)$ is a semi-Riemannian complementary distribution of $Rad(TM)$ in TM, that is,

$$TM = Rad(TM) \perp S(TM)$$

and $S(TM^{\perp})$ is a complementary vector subbundle to $Rad(TM)$ in $TM^{\perp}$. Let $tr(TM)$ and $ltr(TM)$ be complementary (but not orthogonal) vector bundles to TM in $\bar{TM}|_M$ and to $Rad(TM)$ in $S(TM^{\perp})^{\perp}$ respectively. Then, we have

$$tr(TM) = ltr(TM) \perp S(TM^{\perp})$$

2 .

$$\bar{TM}|_M = TM \oplus tr(TM) = (Rad(TM) \oplus ltr(TM)) \perp S(TM) \perp S(TM^{\perp}).$$

For a quasi-orthonormal fields of frames of $\bar{M}$ along M, we have the following well known result.

Theorem 2.1. [4] *Let* $(M, g, \tilde{S}(TM), S(TM^{\perp}))$ *be an r-lightlike submanifold of a semi-Riemannian manifold dim* < 5 $(\bar{M}, \bar{\tilde{g}})$. *Then there exists a complementary vector bundle* $ltr(TM)$ *of* $Rad(TM)$ *in* $S(TM^{\perp})^{\perp}$ *and a basis of* $\Gamma(ltr(TM)|_U)$ *consisting of smooth section* $\{N_i\}$ *of* $S(TM^{\perp})^{\perp}|_U$, *where* U *is a coordinate neighborhood of M, such that*
$\bar{g}(N_i, \xi_j) = \delta_{ij}$, $\bar{g}(N_i, N_j) = 0$, *for any* $i,j \in \{1,2,..,r\}$,
where $\{\xi_1, \ldots, \xi_r\}$ *is a lightlike basis of* $\Gamma(Rad(TM))$.

Let $(M, \tilde{g})$ be an r-lightlike submanifold of an almost complex manifold with Norden metric $(\bar{M}, \bar{J}, \bar{g}, \bar{\tilde{g}})$. Let $\bar{\tilde{\nabla}}$ be the Levi-Civita connection of the metric $\bar{\tilde{g}}$ on $\bar{M}$ and $\tilde{\nabla}$ be the induced connection on M then the Gauss and Weingarten formulae are given by

$\bar{\tilde{\nabla}}_X Y = \tilde{\nabla}_X Y + \tilde{h}(X,Y), \bar{\tilde{\nabla}}_X V = -\tilde{A}_V X + \nabla^t_X V,$

where $\{\tilde{\nabla}_X Y, \tilde{A}_V X\}$ and $\{\tilde{h}(X,Y), \nabla^t_X V\}$ belong to $\Gamma(TM)$ and $\Gamma(tr(TM))$, respectively and $\tilde{\nabla}$ and ∇^t are linear connections on TM and $tr(TM)$, respectively. Moreover, $\tilde{\nabla}$ is torsion-free linear connection, $\tilde{h}$ is a $\Gamma(tr(TM))$-valued symmetric F(M)-bilinear form on $\Gamma(TM)$ and $\tilde{A}$ is a $\Gamma(TM)$-valued F(M)-bilinear form on $\Gamma(tr(TM)) \times \Gamma(TM)$. In general, $\tilde{\nabla}$ and ∇^t are not metric connections. Let L and S be the projection morphisms of $tr(TM)$ on $ltr(TM)$ and $S(TM^{\perp})$, respectively then

(1) $\bar{\tilde{\nabla}}_X Y = \tilde{\nabla}_X Y + \tilde{h}^l(X,Y) + \tilde{h}^s(X,Y),$

(2) $\bar{\tilde{\nabla}}_X V = -\tilde{A}_V X + D^l_X V + D^s_X V$, where

$D^l_X V = L(\nabla^t_X V)$; $D^s_X V = S(\nabla^t_X V)$.

Besides D^l and D^s do not define linear connections on $tr(TM)$ but they are Otsuki connections on $tr(TM)$ with respect to L and S, respectively. Therefore (1) and (2) can be expressed as

(3) $$\bar{\tilde{\nabla}}_X Y = \tilde{\nabla}_X Y + \tilde{h}^l(X,Y) + \tilde{h}^s(X,Y)$$

3 ,

(4) $$\bar{\tilde{\nabla}}_X N = -\tilde{A}_N X + \tilde{\nabla}^l_X N + D^s(X,N),$$

(5) $$\bar{\tilde{\nabla}}_X W = -\tilde{A}_W X + D^l(X,W) + \tilde{\nabla}^s_X W,$$

where $\tilde{\nabla}^l$ and $\tilde{\nabla}^s$ are defined by $\nabla^l_X N = D^l_X N$ and $\nabla^s_X W = D^s_X W$ are metric linear connections on $ltr(TM)$ and $S(TM^{\perp})$, respectively. D^l and D^s are defined by $D^l(X,W) = D^l_X W$ and $D^s(X,N) = D^s_X N$ are F(M)-bilinear mappings. Using (3) − (5) and taking into account the fact that $\bar{\tilde{\nabla}}$ is a metric connection, we obtain

(6) $\bar{\tilde{g}}(\tilde{h}^s(X,Y), W) + \bar{\tilde{g}}(Y, D^l(X,W)) = \bar{\tilde{g}}(\tilde{A}_W X, Y),$

$\bar{\tilde{g}}(D^s(X,N), W) = \bar{\tilde{g}}(\tilde{A}_W X, N), \bar{\tilde{g}}(\tilde{A}_N X, N') + \bar{\tilde{g}}(\tilde{A}_{N'} X, N) = 0, \bar{\tilde{g}}(\tilde{\nabla}_X Y, N) + \bar{\tilde{g}}(Y, \nabla^l_X N) = \bar{\tilde{g}}(\tilde{A}_N X, Y).$

(7) $\bar{\tilde{g}}(\tilde{h}^l(X,Y), \xi) + \bar{\tilde{g}}(Y, \tilde{h}^l(X,\xi)) + \bar{\tilde{g}}(Y, \tilde{\nabla}_X \xi)) = 0.$

Let P' be the projection morphism of TM on $S(TM)$ then new induced geometric objects on the screen distribution $S(TM)$ are given as below.

(8) $\tilde{\nabla}_X P'Y = \tilde{\nabla}^*_X P'Y + \tilde{h}^*(X, P'Y), \tilde{\nabla}_X \xi = -\tilde{A}^*_\xi X + \tilde{\nabla}^{*t}_X \xi,$

for any $X,Y \in \Gamma(TM)$ and $\xi \in \Gamma Rad(TM)$, where $\{\tilde{\nabla}^*_X P'Y, \tilde{A}^*_\xi X\}$ and $\{\tilde{h}^*(X,P'Y), \tilde{\nabla}^{*t}_X \xi\}$ belong to $\Gamma(S(TM))$ and $\Gamma(RadTM)$, respectively. $\tilde{\nabla}^*$ and $\tilde{\nabla}^{*t}$ are linear connections on complementary distributions $S(TM)$ and $Rad(TM)$, respectively. $\tilde{h}^*$ and $\tilde{A}^*$ are $\Gamma(Rad(TM))$-valued and $\Gamma(S(TM))$-valued bilinear forms and are known as the second fundamental forms of distributions $S(TM)$ and $Rad(TM)$,respectively.

Further,using (3) and (8), we obtain $\bar{\tilde{g}}(\tilde{h}^l(X,PY),\xi) = \tilde{h}(\tilde{A}^*_\xi X, PY)$ and $\bar{\tilde{g}}(\tilde{h}^*(X,PY),N) = \tilde{g}(\tilde{A}_N X, PY)$, for any $X,Y \in \Gamma(TM)$, $\xi \in \Gamma(Rad(TM))$ and $N \in \Gamma(ltr(TM))$.

From the geometry of Riemannian submanifolds and non degenerate submanifolds, it is known that the induced connection $\nabla^{\sim}$ on a non degenerate submanifold is a metric connection. Unfortunately, this is not true for a lightlike submanifold.Indeed, considering $\bar{\nabla}^{\sim}$ a metric connection, we have

$(\bar{\nabla}_X \tilde{g})(Y,Z) = \bar{\tilde{g}}(h^{\tilde{l}}(X,Y),Z) + \bar{\tilde{g}}(h^{\tilde{l}}(X,Z),Y),$

for any $X,Y,Z \in \Gamma(TM)$.

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2) Almost complex Norden manifolds. Let $(\bar{M}^{2n}, \bar{J}, \bar{g})$ be an almost complex manifold with almost complex structure $\bar{J}$ and metric $\bar{g}$ on it. The metric $\bar{g}$ is called a Norden metric on $\bar{M}$ if

(9) $\bar{g}(\bar{J}X, \bar{J}Y) = -\bar{g}(X,Y),$

for all differentiable vector fields X and Y on $\bar{M}$ and then $\bar{M}$ is called an almost complex Norden manifold. Next to it, another metric $\bar{\tilde{g}}$ is defined by

(10) $\bar{\tilde{g}}(X,Y) = \bar{g}(\bar{J}X,Y)$ and $\bar{\tilde{g}}(\bar{J}X,Y) = \bar{\tilde{g}}(X,\bar{J}Y)$.

It is clear that $\bar{\tilde{g}}$ is also a Norden metric on $\bar{M}$ and both the metrics $\bar{g}$ and $\bar{\tilde{g}}$ are indefinite of signature (n,n). The metric $\bar{\tilde{g}}$ is called an associated metric of $\bar{M}$. Let $\bar{\nabla}$ and $\bar{\nabla}^{\sim}$ be the Levi-Civita connection of $\bar{g}$ and $\bar{\tilde{g}}$ respectively then

(11) $\Phi(X,Y) = \bar{\tilde{\nabla}}_X Y - \bar{\nabla}_X Y$

is a tensor field of type (1,2) on $\bar{M}$ and $\Phi(X,Y) = \Phi(Y,X)$. A tensor field F of type (0,3) can also be defined on $\bar{M}$ by $F(X,Y,Z) = \bar{g}((\bar{\nabla}_X \bar{J})Y,Z)$ and satisfies the following symmetries

$F(X,Y,Z) = F(X,Z,Y) = F(X,\bar{J}Y,\bar{J}Z).$

In [6], eight classes of almost complex Norden manifolds are characterized by conditions for the tensor F. The relations between the tensor F and Φ are given by

$$X,Y,Z) = \frac{1}{2}[F(\bar{J}Z,X,Y) - F(X,Y,\bar{J}Z) - F(Y,\bar{J}Z,X)]$$

(12) $\Phi($ $$F(X,Y,Z) = \Phi(X,Y,\bar{J}Z) + \Phi(X,Z,\bar{J}Y),$$,

for any $X,Y,Z \in \Gamma(T\bar{M})$, where $\Phi(X,Y,Z) = \bar{g}(\Phi(X,Y),Z)$. For a Kaehler Norden manifold $\bar{M}$, the characterization condition $F(X,Y,Z) = 0$ is equivalent to $(\bar{\nabla}_X \bar{J})Y = 0$. Therefore from (12), for a Kaehler Norden manifold we have $\Phi = 0$.

Let M be an m-dimensional submanifold of an almost complex Norden manifold $(\bar{M}, \bar{J}, \bar{g}, \bar{\tilde{g}})$ and g, $\tilde{g}$ be the induced metrics on the tangent space T_xM of $\bar{g}$, $\bar{\tilde{g}}$ respectively. Hence for any $x \in M$, we have

$g(X_x,Y_x) = \bar{g}(X_x,Y_x), \qquad \tilde{g}(X_x,Y_x) = \bar{\tilde{g}}(X_x,Y_x),$

for any $X_x,Y_x \in T_xM$. We denote the tangent bundle of both the submanifolds (M,g) and $(M,\tilde{g})$ of $\bar{M}$ by TM and the normal bundle of (M,g) and $(M,\tilde{g})$ by $TM^{\perp}$ and $TM^{\tilde{\perp}}$, respectively.

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III. RADICAL SCREEN TRANSVERSAL LIGHTLIKE SUBMANIFOLDS

Definition 3.1. *Let $(M,g,\tilde{S}(TM),S(TM)^{\perp})$ be a lightlike submanifold of an indefinite Kaehler Norden manifold $(\bar{M}, \bar{J}, \bar{g}, \bar{\tilde{g}})$. Then $(M,\tilde{g})$ is known as a screen transversal(ST) lightlike submanifold of $\bar{M}$ if $\bar{J}Rad(TM) = S(TM)^{\perp}$.*

Lemma 3.2. *Let $(M,\tilde{g})$ be a lightlike submanifold of an indefinite Kaehler Norden manifold $(\bar{M}, \bar{J}, \bar{g}, \bar{\tilde{g}})$ such that $\bar{J}Rad(TM) \subset S(TM)^{\perp}$.Then $\bar{J}ltr(TM)$ is also a subvector bundle of screen transversal bundle.*

Proof. Firstly assume that $\bar{J}ltr(TM)$ is invariant with respect to $\bar{J}$. Now by virtue of the definition of a lightlike submanifold, we know the existence of vector fields $\xi \in Rad(TM)$ and $N \in ltr(TM)$ such that $\bar{\tilde{g}}(\xi,N) = 1$. Also, from 9, we have

$\bar{\tilde{g}}(\xi,N) = -\bar{\tilde{g}}(\bar{J}\xi, \bar{J}N) = 1.$

However, if $\bar{J}N \in ltr(TM)$ then by hypothesis, we find that $\bar{\tilde{g}}(\bar{J}\xi, \bar{J}N) = 0$. Thus, we have a contradiction which further implies that $\bar{J}N$ does not belong to $ltr(TM)$. Next, we suppose that $\bar{J}N \in S(TM)$, then proceeding in a similar manner as above, yields

$1 = \bar{\tilde{g}}(\xi,N) = -\bar{\tilde{g}}(\bar{J}\xi, \bar{J}N) = 0,$

since $\bar{J}\xi \in S(TM)^{\perp}$ and $\bar{J}N \in S(TM)$, therefore $\bar{J}N$ does not belong to $S(TM)$. Proceeding in the similar manner, we can also prove that $\bar{J}N$ does not belong to $Rad(TM)$. Thus, making use of the decomposition of a lightlike submanifold, we conclude that $\bar{J}N \in S(TM)^{\perp}$.

Hence proved. □

Definition 3.3. *Let $(M,\tilde{g})$ be a screen transversal lightlike submanifold of an indefinite Kaehler Norden manifold $(\bar{M}, \bar{J}, \bar{g}, \bar{\tilde{g}})$. We say that*

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(1) *$(M,\tilde{g})$ is a Radical Screen transversal(RST) lightlike submanifold of $\bar{M}$ if $\bar{J}(S(TM)) = S(TM)$.*

(2) *$(M,\tilde{g})$ is a Screen transversal(ST) anti-invariant lightlike submanifold of $\bar{M}$ if $\bar{J}(S(TM)) \subset S(TM)^{\perp}$.*

If $(M,\tilde{g})$ is a screen transversal(ST) anti-invariant lightlike submanifold of $\bar{M}$ then $S(TM)^{\perp}$ can be decomposed as

$$S(TM)^{\perp} = \bar{J}Rad(TM) \oplus \bar{J}ltr(TM) \perp \bar{J}(S(TM)) \perp D_0 \quad (6)$$

where D_0 is a non-degenerate orthogonal complementary distribution to $\bar{J}Rad(TM) \oplus \bar{J}ltr(TM) \perp \bar{J}(S(TM))$ in $S(TM)^{\perp}$.

Thus, for Radical screen transversal(RST) lightlike submanifold of $\bar{M}$, we have

$S(TM)^{\perp} = \bar{J}Rad(TM) \oplus \bar{J}ltr(TM) \perp D_0$,

where D_0 is a non-degenerate orthogonal complementary distribution to $\bar{J}Rad(TM) \oplus \bar{J}ltr(TM)$ in $S(TM)^{\perp}$.

Theorem 3.4. *Let $(M,\tilde{g})$ be a ST lightlike submanifold of $\bar{M}$. Then, the distribution D_0 is invariant with respect to $\bar{J}$.*

Proof. For any $X \in D_0$, $\xi \in Rad(TM)$, $N \in ltr(TM)$, $Z \in S(TM)$ and using 10, we have $\tilde{g}(\bar{J}X,\xi) = \tilde{g}(X,\bar{J}\xi) = 0$, which proves that $\bar{J}X$ does not belong to $ltr(TM)$. Similarly, one can easily verify that

$\tilde{g}(\bar{J}X,N) = \tilde{g}(X,\bar{J}N) = 0$, $\tilde{g}(\bar{J}X,\bar{J}\xi) = -\tilde{g}(X,\xi) = 0$,

$\tilde{g}(\bar{J}X,\bar{J}N) = -\tilde{g}(X,N) = 0$, $\tilde{g}(\bar{J}X,Z) = \tilde{g}(X,\bar{J}Z) = 0$, $\tilde{g}(\bar{J}X,\bar{J}Z) = -\tilde{g}(X,Z) = 0$.

Hence, the distribution D_0 is invariant with respect to $\bar{J}$, which completes the proof. □

Let $(M,\tilde{g})$ be ST anti-invariant lightlike submanifold of an Kaehler Norden manifold $\bar{M}$. Let P and Q be projection morphisms of S(TM) and Rad(TM), respectively. Then, for any $X \in TM$, we can write

$$X = PX + QX, \quad (13)$$

on the other hand, if we apply $\bar{J}$ to (13, we obtain

$$\bar{J}X = P_1X + P_2X,$$

where $\bar{J}PX = P_1X \in S(TM)$ and $\bar{J}QX = P_2X \in S(TM^{\perp})$. Similarly, for any $V \in tr(TM)$, we have

$$\bar{J}V = tV + fV, \quad (7)$$

where tV and fV are the tangent and transversal parts respectively. Suppose that $\bar{J}_1$, $\bar{J}_2$, $\bar{J}_3$, $\bar{J}_4$ be the projection morphisms on $\bar{J}Rad(TM)$, $\bar{J}S(TM)$, $\bar{J}ltr(TM)$ and D_0, respectively. Now for any $V \in S(TM^{\perp})$, we have

(14) $V = \bar{J}_1V + \bar{J}_2V + \bar{J}_3V + \bar{J}_4V$

Applying $\bar{J}$ to above equation yields

(15) $\bar{J}V = S_1V + S_2V + R_1V + R_2V$,

where $S_1V = \bar{J}\bar{J}_1V \in Rad(TM)$, $S_2V = \bar{J}\bar{J}_2V \in S(TM)$, $R_1V = \bar{J}\bar{J}_3V \in ltr(TM)$ and $R_2V = \bar{J}\bar{J}_4V \in D_0$. It is well known that the induced connection on a screen transversal anti-invariant lightlike submanifold of an indefinite Kaehler Norden manifold need not be a metric connection. Thus, in the next theorem, we are investigating the criteria under which the induced connection reduces to a metric connection.

Theorem 3.5. *Let $(M,\tilde{g})$ be a ST anti-invariant lightlike subamnifold of an indefinite Kaehler Norden manifold $\bar{M}$. Then the induced connection $\tilde{\nabla}$ on $(M,\tilde{g})$ is a metric connection if and only if $S_2\tilde{\nabla}^s_X\bar{J}\xi = 0$, for some $X \in TM$ and $\xi \in Rad(TM)$.*

Proof. By using the definition of an indefinite Kaehler Norden manifold, we have

$$\bar{\tilde{\nabla}}_X\bar{J}Y = \bar{J}\bar{\tilde{\nabla}}_XY.$$

Taking $Y = \xi$ in the above equation, we find

$$-\tilde{A}_{\bar{J}\xi}X + \tilde{\nabla}^s_X\bar{J}\xi + D^l(X,\bar{J}\xi) = \bar{J}(\tilde{\nabla}_X\xi + \tilde{h}^l(X,\xi) + \tilde{h}^s(X,\xi)).$$

On applying $\bar{J}$ to above equation, using (15) and equating the tangential parts gives,

$$-\tilde{\nabla}_X\xi = S_1\tilde{\nabla}^s_X\bar{J}\xi + S_2\tilde{\nabla}^s_X\bar{J}\xi.$$

Thus, the proof is completed. □

In the following theorems, we are obtaining the integrability conditions for the radical distribution and screen distribution.

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Theorem 3.6. *Let $(M,\tilde g)$ be a ST anti-invariant lightlike subamnifold of an indefinite Kaehler Norden manifold $(\bar M, \tilde{\bar g})$. Then the radical distribution is integrable if and only if*

$$\tilde\nabla^s_X \bar J Y = \tilde\nabla^s_Y \bar J X,$$

for all $X,Y \in Rad(TM)$.

By using the definition of a ST anti-invariant lightlike subamnifold, we know that radical distribution is integrable if and only if $\tilde g([X,Y],Z) = 0$ for some $X,Y \in Rad(TM)$ and $Z \in S(TM)$. Again using the fact that $(\bar M, \tilde{\bar g})$ is an indefinite Kaehler Norden manifold yields

$\tilde g([X,Y],Z) = -\tilde g(\tilde{\bar\nabla}_X \bar J Y, \bar J Z) + \tilde g(\tilde{\bar\nabla}_Y \bar J X, \bar J Z)$

using equation (5), we obtain

$$\tilde g([X,Y],Z) = \tilde g(\tilde\nabla^s_X \bar J Y - \tilde\nabla^s_Y \bar J X, \bar J Z).$$

Thus, the proof is completed. □

Theorem 3.7. *Let $(M,\tilde g)$ be a ST anti-invariant lightlike subamnifold of an indefinite Kaehler Norden manifold $(\bar M, \tilde{\bar g})$. Then the screen distribution is integrable if and only if*

$$\tilde g(\tilde\nabla^s_X \bar J Y - \tilde\nabla^s_Y \bar J X, \bar J N) = 0$$

for all $X,Y \in S(TM)$ and $N \in ltr(TM)$.

Proof. Using the fact that $\bar J N \in S(TM)^\perp$ and (5) we have

$$\tilde g([X,Y],N) = \tilde g(\tilde\nabla^s_X \bar J Y - \tilde\nabla^s_Y \bar J X, \bar J N),$$

which completes the proof. □

4. RST lightlike submanifolds of an indefinite Kaehler Norden manifold

In this section, we study Radical ST lightlike submanifold of an indefinite Kaehler Norden manifold. Initially, we investigate the integrability of various distributions involved and later, give the necessary and sufficient condition for the induced connection to be a metric connection.

Theorem 4.1. *Let $(M,\tilde g)$ be a RST lightlike submanifold of an indefinite Kaehler Norden manifold $\bar M$. Then the screen distribution is integrable if and only if*

$$\tilde h^s(X, \bar J Y) = \tilde h^s(Y, \bar J X)$$

for all $X,Y \in S(TM)$.

We know that screen distribution is integrable if and only if $\tilde g([X,Y],N) = 0$, for all $X,Y \in S(TM)$ and $N \in ltr(TM)$.

We obtain,

$$\begin{aligned} \tilde g([X,Y],N) &= \tilde g(\tilde\nabla_X Y, N) - \tilde g(\tilde\nabla_Y X, N) \\ &= -\tilde g(\tilde{\bar\nabla}_X \bar J Y, \bar J N) + \tilde g(\tilde{\bar\nabla}_Y \bar J X, \bar J N) \end{aligned}$$

using (3), we have

$$\tilde g([X,Y],N) = -[\tilde g(\tilde h^s(X,\bar J Y) - \tilde h^s(Y,\bar J X), \bar J N)].$$

Thus the proof is completed. □

Theorem 4.2. *Let $(M,\tilde g)$ be a RST lightlike submanifold of an indefinite Kaehler Norden manifold $\bar M$. Then the radical distribution is integrable if and only if*

$$\tilde A_{\bar J X} Y = \tilde A_{\bar J Y} X$$

for all $X,Y \in Rad(TM)$.

Proof. For any $X,Y \in Rad(TM)$ and $Z \in S(TM)$ and using the definition of an indefinite Kaehler Norden manifold, we have

$\tilde g([X,Y],Z) = -\tilde g(\tilde{\bar\nabla}_X \bar J Y, \bar J Z) + \tilde g(\tilde{\bar\nabla}_Y \bar J X, \bar J Z)$

using (5), we obtain

$$\tilde g([X,Y],Z) = -\tilde g(\tilde A_{\bar J X} Y - \tilde A_{\bar J Y} X, \bar J Z) = 0.$$

Thus, the proof is completed. □

Theorem 4.3. *Let* $(M,\tilde g)$ *be a RST lightlike submanifold of an indefinite Kaehler Norden manifold* $(\bar M, \bar{\tilde g})$. *Then the distribution* D_0 *is invariant with respect to* $\bar J$.

Proof. Proof is similar to those given in Theorem 3.4. □

Theorem 4.4. *Let* $(M,\tilde g)$ *be a RST lightlike submanifold of an indefinite Kaehler Norden manifold* $(\bar M, \bar{\tilde g})$. *Then the screen distribution defines a totally geodesic foliation if and only if* $\tilde h^s(X,\bar J Y)$ *has no component in* $\bar J Rad(TM)$ *for* $X,Y \in S(TM)$.

By using the definition of RST lightlike submanifold $S(TM)$ defines totally geodesic foliation if and only if $\tilde g(\tilde\nabla_X Y,N) = 0$, for some $X,Y \in S(TM)$ and $N \in ltr(TM)$. Using the definition of an indefinite Kaehler Norden manifold $\bar M$, we have

$\tilde g(\tilde\nabla_X Y,N) = -\tilde g(\tilde\nabla_X \bar J Y, \bar J N)$

and using (3), we obtain

$\tilde g(\tilde\nabla_X Y,N) = -\tilde g(\tilde h^s(X,\bar J Y), \bar J N))$.

Thus, the proof is completed. □

Theorem 4.5. *Let* $(M,\tilde g)$ *be a RST lightlike submanifold of an indefinite Kaehler Norden manifold* $(\bar M, \bar{\tilde g})$. *Then the radical distribution defines a totally geodesic foliation on* $(M,\tilde g)$ *if and only if* $\tilde h^s(X,\bar J Y)$ *has no component in* $\bar J ltr(TM)$ *for every* $X,Y \in Rad(TM)$.

Proof. We know that for every $X,Y \in Rad(TM)$ and $Z \in S(TM)$, we have

$$\tilde g(\tilde\nabla_X Y,Z) = 0.$$

using (5), we have

$$\tilde g(\tilde\nabla_X Y,Z) = \tilde g(\tilde A_{\bar J Y} X, \bar J Z)$$

and use of (6), yields

$\tilde g(\tilde\nabla_X Y,Z) = \tilde g(\tilde A_{\bar J Y} X,\bar J Z) = \tilde g(\tilde h^s(X,\bar J Z),\bar J Y)$.

hence proved the required result. □

In the next theorem, we have investigated the necessary and sufficient criterion for the induced connection $\tilde\nabla$ on $(M,\tilde g)$ to be a metric connection.

Theorem 4.6. *Let* $(M,\tilde g)$ *be a RST lightlike submanifold of an indefinite Kaehler Norden manifold* $(\bar M, \bar{\tilde g})$. *Then the induced connection* $\tilde\nabla$ *on* $(M,\tilde g)$ *is a metric connection if and only if* $\tilde h^s(Y,Z)$ *has no component in* $\bar J ltr(TM)$ *for every* $Y,Z \in S(TM)$.

Proof. Using the definition of an indefinite Kaehler Norden manifold, we have $(\bar{\tilde\nabla}_Z \bar J)X = 0$, for some $Z \in S(TM)$ and $X \in ltr(TM)$, which implies

$$\bar{\tilde\nabla}_Z \bar J X = \bar J \bar{\tilde\nabla}_Z X. \tag{16}$$

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Taking inner product of above equation with $Y \in S(TM)$, gives

$\bar{\tilde g}(\bar{\tilde\nabla}_Z \bar J X, Y) = \bar{\tilde g}(\bar J \bar{\tilde\nabla}_Z X, Y) = \bar{\tilde g}(\bar{\tilde\nabla}_Z X, \bar J Y)$.

Using equations (3), (5) and (6), we get the required result. □

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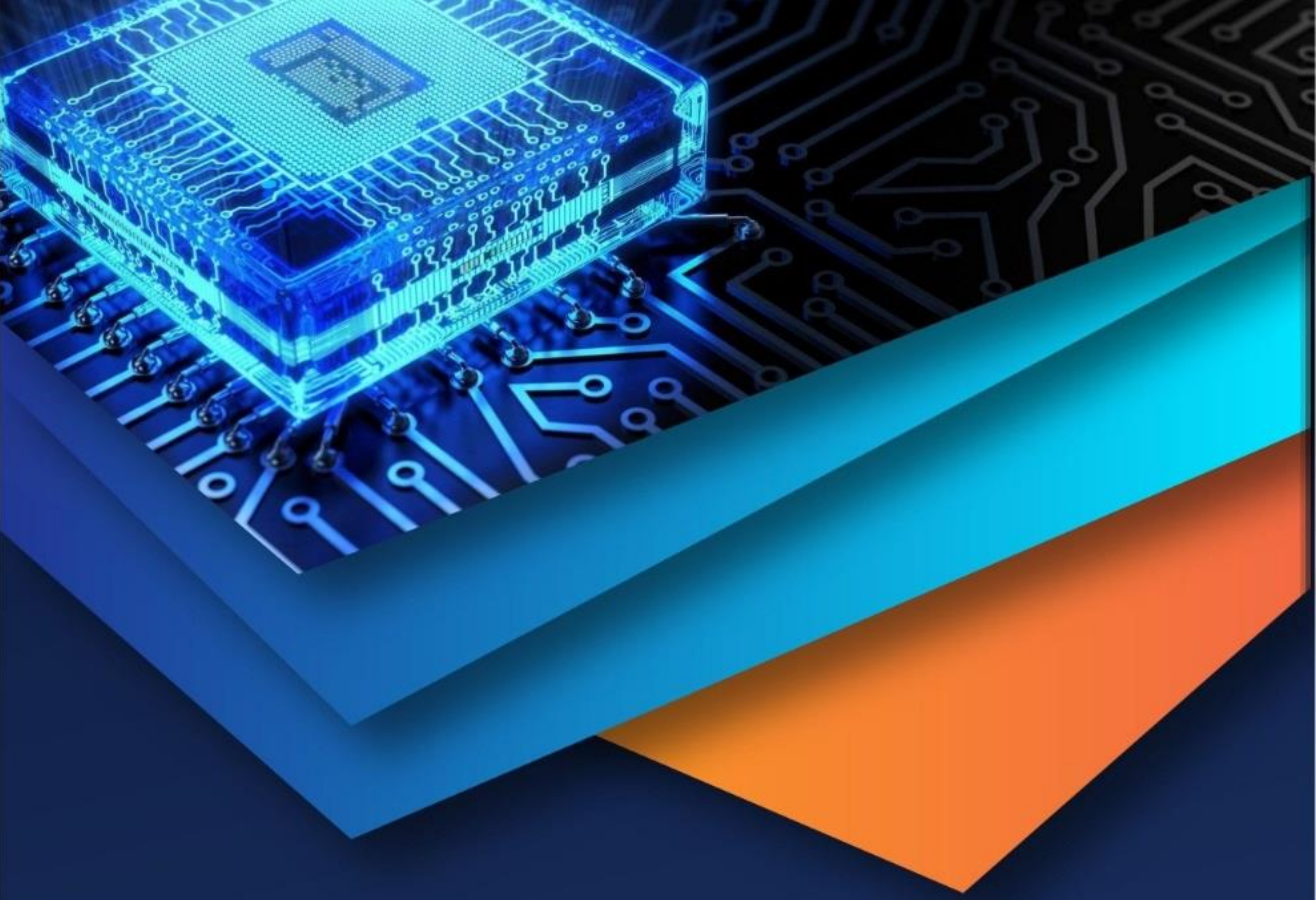

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