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Stability Analysis of Two-Dimensional Prey-Predator model on Fuzzy Uncertainty

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Abstract- ***This paper presents a simple prey–predator model under a fuzzy set environment. Fuzzy mathematical model enables us to use mathematical tools to verify the system both quantitatively and qualitatively. The classical Lotka–Volterra model is extended by incorporating fuzzy parameters to handle uncertainty in ecological interactions. The model is converted into an α-cut representation to obtain a system of interval differential equations. In the mean time we calculate Jacobian matrix around equilibrium point and we analyse stability of the system on fuzzy set. The implementation of the models in various fields is discussed. A numerical example using triangular fuzzy numbers is also discussed.***
Keywords: Fuzzy set, Lotka-Volterra model, α-cut, Numerical simulation, triangular number.

I. INTRODUCTION

Ecology is one of the enlarged scientific subject. The word Ecology was invented by German ecologist E. Haeckel. Ecology is the culture of relation between prey, predator and their surrounding environment [1]. A mathematical model is a kind of tools which applies mathematical ideas and language. Mathematical models utilise different fields like engineering, natural science, operation research etc. A. Lotka and V. Volterra was first construct Prey-predator model in 1925. It is identified by oscillations in both prey and predator population size. Mathematical models of prey–predator interactions are widely used in ecology. However, real-world systems often involve uncertainty due to environmental fluctuations and imprecise data. Fuzzy set theory provides a useful framework to incorporate such uncertainty [2]. There are many mathematical components such as Dynamical system, Statistical models, Differential equations, Game theory. Dynamical system analysis is an important technique to examine nonlinear mathematical systems. Fuzzy set theory was invented by L. Zadeh in 1965, he developed a general method for extending mathematical thoughts to deal with fuzzy quantities. Many real time events are not crisp or precise but they are vague or uncertain in nature as the variables or parameters including some mathematical models of computer systems, engineering, biological science, physics are not crisp and this tends to uncertainties of the results, therefore it becomes important to illustrate such ambiguities in linguistic in nature such situations can be achieve through fuzzy numbers. To the ecological based modelling or handling vague information in an useful avenue for fuzzy numbers which plays an significant role and therefore fuzzy differential equations with primary situation with fuzzy sets can be used for modelling a subjective ideas where boundaries are fuzzy and it can be analyse different areas because of its wide area of applications. Fuzzy differential equations are increasingly being applied in this model for modelling in Ecology. Lotka-Volterra's research overlapped in predator's debate-prey engagement. This approach is called as principle of extension. In several fields of science and technology where data is ambiguous or vague, fuzzy set concept can apply. Mathematical model on fuzzy environment is the techniques to study its behaviour in both quantitatively and qualitatively possibly [3][4].

Sl. No.	Crisp set theory	Fuzzy set theory
1	In this theory object and assumptions are clearly mention.	In this class object and assumptions are uncertain i.e vague.
2	There is exact boundaries location of the set and are defined by a simple statement.	A fuzzy set is defined by its ambiguous boundaries about the location of the set.
3	Widely applied in modern digital system.	It is used in fuzzy controllers.

4	This theory is the classical set theory.	Fuzzy sets theory is an extension of crisp set theory.
5	An object in a set is only member or element.	Elements have varying degree of membership, a logic based on two truth values.

Table-1: Comparisons between crisp and fuzzy set

In the view of research efforts, here we discussed some preliminaries definition and proposition related to fuzzy number.

Definition : Fuzzy set [5]:

Let Y be a non-empty set. An IFS $\tilde{T}$ in Y is considered by a set $\tilde{T}$= {(y,M(y)):y⊑ Y } . The mapping M(y):Y→ [0,1] is the membership function and M(y) is the membership value of y in Y of the fuzzy set $\tilde{T}$.

Definition 1.2 Fuzzy number [5]:

A fuzzy set is defining a fuzzy interval in the real number R is applied to appropriate fuzzy numbers. Some of fuzzy numbers are Triangular fuzzy number, Trapezodial fuzzy number, Penagonal fuzzy number, Hexagonal fuzzy number, Symmetric hexagonal fuzzy number.

Definition : α-cut of a fuzzy number [5]:

A classical set define from fuzzy set with each element having membership value greater or equal for the define value $\alpha \subset [0,1]$ in the following way: A α-cut of fuzzy number $\tilde{M}$ in Y is denoted by M_{α} and is defined as the following crisp set

$M_{\alpha} = \{y | u_{\tilde{M}}(y) \geq \alpha\}$ where $\alpha \sqsubseteq [0,1]$. M_{α} is a non-empty closed bounded set in Y and it can represented by, $M_{\alpha}=[M_{LT}(\alpha), M_{RT}(\alpha)]$, where $M_{LT}(\alpha)$ and $M_{RT}(\alpha)$ are the lower and upper bounds of the intervals respectively.

The α-cut of triangular fuzzy number $\tilde{M}=(a_1, a_2, a_3)$ can be expressed as $=[M_{LT}(\alpha), M_{RT}(\alpha)]$ where, $M_{LT}(\alpha) = a_1 + \alpha(a_2 - a_1), M_{RT}(\alpha) = a_3 - \alpha(a_3 - a_2) \; \forall \; \alpha \in [0,1]$.

Definition: Triangular fuzzy number [5]:

A triangular fuzzy number (TFN) represented by three points like as $\tilde{M} - (a_1, a_2, a_3)$ and represented of membership function as follows

$$\mu_{\tilde{M}}(y) = \begin{cases} \frac{y - a_1}{a_2 - a_1}, a_1 \leq y \leq a_2 \\ 1, \quad y - a_2 \\ \frac{a_3 - y}{a_3 - a_2}, a_2 \leq y \leq a_3 \\ 0, otherwise \end{cases}$$

Definition: Interval Arithmetic's [5]:

Let A=$[a_1, a_2]$ and B=$[b_1, b_2]$ are two any closed intervals define on real number R such that $a_1 \leq a_2$ and $b_1 \leq b_2$ then addition and subtraction are get as follows:

(i) Addition : A+B=$[(a_1 + b_1), (a_2 + b_2)]$

(ii) Subtraction: A-B==$[(a_1 - b_1), (a_2 - b_2)]$

II. PRINCIPLE OF BIO-MATH ON FUZZY SET

A. *Classical Model*

The prey-predator equations of Lotka-Volterra are both non-linear first order ordinary differential equations. It's used to predict two differential retails structure. Over time, the population changes according to the system of equations [7]-[8]:

$\frac{du}{dt}$=ru-αuv

$\frac{dv}{dt}$=**β**uv-dv

where **u**(t) and **v**(t) represent prey and predator populations respectively, **r** and **α** are the prey and predator growth rate respectively. **d** is the death rate of predator and **β** be the predator reproduction rate.

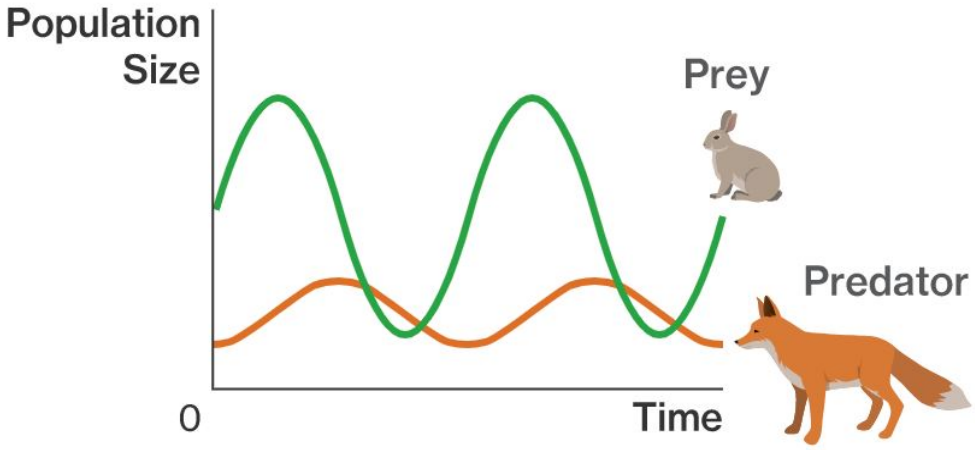

Fig.1. Prey and Predator competition in environment.

B. Fuzzy Model Formulation

In ecological model any of the three cases arise either initial condition are fuzzy number, parameters of the model are fuzzy numbers or both condition i.e initial conditions and parameters are fuzzy numbers. In the fuzzy framework, parameters **r, α, β, d** are replaced by fuzzy numbers. The system becomes:

$$\frac{\tilde{du}}{dt} = \tilde{r}\,\tilde{u} - \tilde{\alpha}\tilde{u}\,\tilde{v}$$
$$\frac{\tilde{dv}}{dt} = -\tilde{d}\tilde{v} + \tilde{\beta}\,\tilde{u}\,\tilde{v}$$

α-cut Representation

Using α-cuts, fuzzy numbers are transformed into interval values:

$\tilde{u}^{\alpha} = [u_L^{\alpha}, u_U^{\alpha}]$
$\tilde{v}^{\alpha} - [v_L^{\alpha}, v_U^{\alpha}]$

This leads to a system of lower and upper bound differential equations [9]. Fuzzy critical points for each **α**-level are $E^{\alpha} = (\frac{d^{\alpha}}{\beta^{\alpha}}, \frac{r^{\alpha}}{\alpha^{\alpha}})$. Now calculate linearized Jacobian matrix for **α**-cut system is $J = \begin{pmatrix} r - \alpha v & -\alpha u \\ \beta v & -d + \beta v \end{pmatrix}$.

The eigenvalues at this equilibrium point are - $i\sqrt{r^{\alpha}d^{\alpha}}$ and +i $\sqrt{r^{\alpha}d^{\alpha}}$, these shows that all eigenvalues are purely imaginary for all **α**-level. Therefore each **α**-system is a center i.e the system is unstable.

III. MOTIVATION AND NOVELTY

Our observation impact on scientific landscape. In recent time many researchers typically apply fuzzy number and Triangular fuzzy number in two dimensional prey-predator models. Fuzzy differential systems serve as an established method of modelling the distribution of ambiguity of information in a dynamics arena [10, 11].

Fuzzy ecological models in prey-predator population modelling in the paper. In this work, we study on basic concept of fuzzy prey-predator model, where we consider parameters are fuzzy.

Dealing with this parameters is assumed more realistic and logically sound. This alteration from the norm contributes a new and distinctive point of view to understand the ecological system more accurately.

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IV. CHALLENGES AND LIMITATIONS

Despite its advantages, fuzzy uncertainty has certain limitations:

- Most works focus on triangular fuzzy set, which limited the scope of advanced fuzzy environment.
- Many models consider isolated effect but unified framework combining multiple uncertainties are still limited,
- Sensitivity analysis is difficult to explore on this framework.
- Theoretical results are very difficult and many works rely mostly on numerical simulations.
- Limited for higher-order fuzzy systems, such as Pythagorean fuzzy sets, remains largely unexplored.

Some studies indicate that long-term plant diversity may not significantly differ from conventional methods [12].

Prey-predator mathematical model is the formation of relation between different organisms in environment. Using differential equations within an ecosystem we develop model to do this we need different assumptions such as listed below:

- The prey populations get enough time to search sufficient food.
- Predator species totally rely on the prey species for food.
- Prey population's decreases due to either natural death or predation by predator.
- The predator number diminish due to natural death.
- The population's size change rate is proportional to its recent size.

Systems of ordinary differential equations illustrating the prey-predator inter-relation and various ecological models exists in nature..

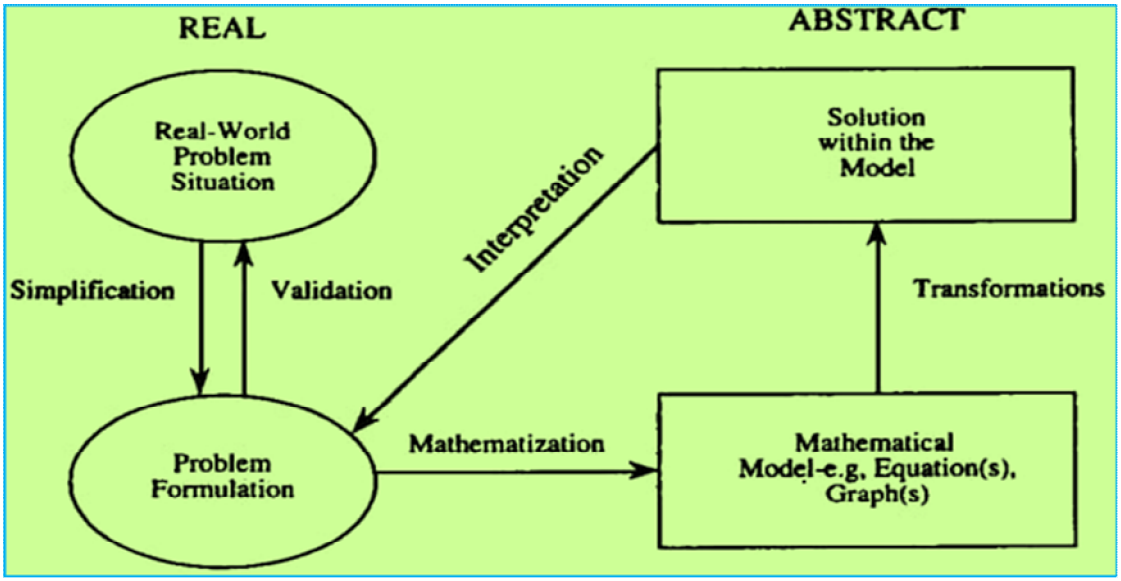

Fig.2. Comprehensive set of population based model

V. FUTURE PROSPECTS

Mainly in our work we focused on predator-prey model which illustrates the fuzzy behaviour. A very prominent method for demonstrating the ambiguity of different real world scenario in dynamic ecosystems is the fuzzy se theory. While we working with imprecise or vague uncertainty data, a fuzzy method can be quite helpful. We analysis the system of equations under different fuzzy uncertainty such as fuzzy initial condition using Laplace transformation. To manage this fuzzy uncertainty one can take fuzzy model to get better outcomes. But since it is not restricted, it can be advised for further researchers such as explore the ecological model with bifurcation under fuzziness. Fuzzy with the delay terms can also be enhance.

VI. CONCLUSIONS

In recent times, several forms of biological modelling have relied heavily on mathematical modelling. Mathematical modelling now has been included in several branches of research in particular in the field of ecology. In this study, we presented a prey-predator model that was tested in fuzzy environment to discover the fuzzy solution of the suggested model. The mathematical modelling of dynamical system with two species with fuzzy framework is the main work we examined this in our paper. The fuzzy prey–predator model provides a more realistic representation of ecological systems under uncertainty. Finally we simplify the real world problem into a real model, then we calculate into a mathematical model and we analyse and interpret a model to a real situation in life. It can be further extended to advanced fuzzy frameworks such as Pythagorean fuzzy sets and fractional models.

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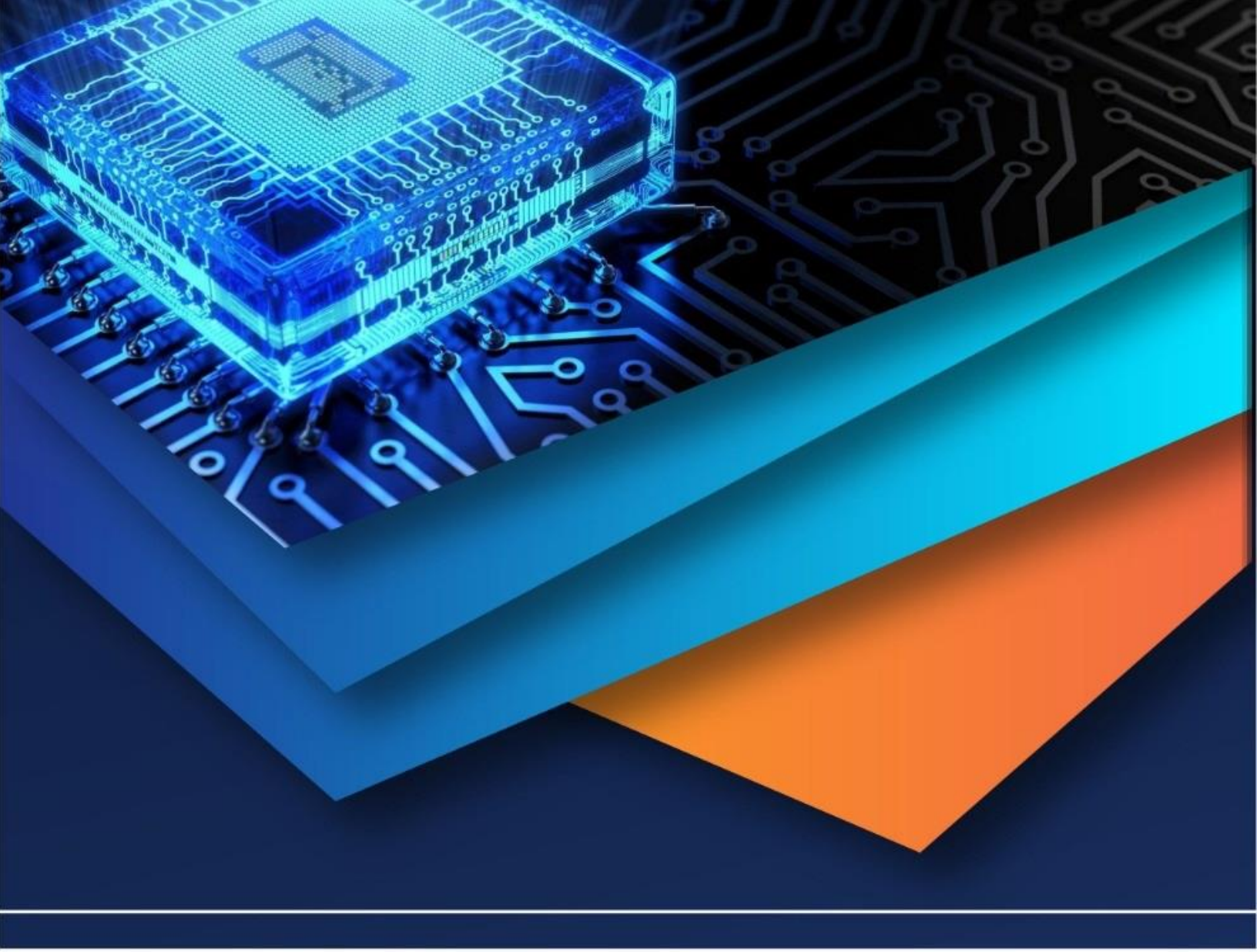

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