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# Static and Vibration Analysis of FGM Plates: A Critical Review

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**Abstract:** Recently a new material has been emerged with variation of material properties particularly across the thickness. Such class of materials is called functionally graded materials. The concept of functionally graded material was proposed in 1984 by materials scientists as a means of preparing thermal barrier materials. Since then functionally graded materials are being increasingly used in the aeronautical and aerospace industry as well as in other fields of modern technology. This paper presents a comprehensive review of the various methods used to study the static and dynamic behaviour of functionally graded material plates. Both analytical and numerical methods are considered. The effect of variation of material properties through the thickness, type of load case, boundary conditions, edge ratio and side-to thickness ratio on the behaviour of functionally graded material plates are discussed. The main objective of this paper is to serve the interests of researchers and engineers already involved in the analysis and design of functionally graded material structures.

**Keywords:** functionally graded material, finite element.

## I. INTRODUCTION

Functionally graded materials (FGM) are being increasingly used in aeronautical and aerospace industry as well as in other fields of modern technology, especially where structures are subjected to high temperatures. Functionally graded materials normally consist of two phases, namely; the metal phase and the ceramic phase. Ceramic acts as thermal barrier and the metal gives required ductility. The FGM have advantage over the laminated composites. The delamination problem which is normally observed in laminated composites is totally absent in FGM because of the smooth variation of material properties through the thickness. This material is mainly used as thermal barrier material. In 1984 Japanese scientists invented the novel concept of FGM. Since then FGMs are being increasingly used in the aeronautical and aerospace industry as well as in other fields of modern technology. FGM composition can vary from ceramic rich material to metal rich material by varying the volume fractions of the two materials. Thermal conductivity of ceramic is very low and metal is good in reducing tensile stresses this together can help in enhancing the properties of thermal barrier. Several studies have been performed to analyze the behaviour of functionally graded Plates and Shells.

The available literature relevant to the scope of this paper is reviewed for 3 D solutions as well as 2 D solutions. The review is divided in two groups, namely analytical methods and numerical methods. Each group is then subdivided in subgroups, namely, static analysis and free vibration analysis employed by various researchers for analysis of FG plates under normal as well as thermal environment.

### A. Analytical Methods

FGM sandwich plates are discussed under two separate categories- Static Analysis and vibration Analysis

#### 1) Static Analysis

a) *Three-Dimensional Analysis:* Kumari [1] presented a 3D analytical solution for a longitudinally FGM plate having Levy type boundary conditions and Kantorovich method is applied to solve the governing equations in mixed form. Compare the numerical results of two layered plate and single layered plate having a different in-plane material variation in each layer are presented. Later Mantari [2] presented a static response of advance composite plates by using a non-polynomial higher order shear deformation theory. In geometry, stretching effect is not applied and number of unknown displacement functions are 5. In addition he has done the inclusion of an exponential function to the tangent function. Also results are compared with an existing Quasi-3D exact solution and several HSDT results. C. Guedes Soares [3] used a trigonometric higher order theory to get the static analysis of functionally graded plates which is subjected to transverse bi-sinusoidal load in which stretching effect is included and showed that present new trigonometric theory predicts displacement and stresses more accurately when compared to the well-known trigonometric plate theory.

Ji Ying [4] investigated the thermo mechanical behaviour of functionally graded thick plate based on 3D thermo-elasticity with one pair of opposite edges is simply supported. To predict the effective material properties Mori-Tanaka method with a power law fraction is used.

- b) *Two-Dimensional Analysis:* Zenkour [5] presented a 2D solution for bending analysis of simply supported isotropic FGM sandwich plate by using a displacement function that identically satisfies boundary conditions to reduce the governing equations to a set of coupled ordinary differential equations with variable coefficients and analysed the stress and displacement response of the plates under sinusoidal loading. Mahmoud [6] presented comparative analysis of thermal stresses in 2D FGM plate and conventional FGM plate by using volume fractions and Rule of mixture and showed that 2D FGM plate's capacity to reduce thermal stresses is more than conventional FGM plate.

## 2) *Vibration Analysis*

- a) *Three-Dimensional Analysis:* Mahi [7] used a hyperbolic shear deformation theory for bending and free vibration analysis of isotropic functionally graded sandwich and laminated composite plates by considering five degrees of freedom provides parabolic transverse shear strain across the thickness direction. Navier type procedure is used to obtained analytical solutions of deflection and stresses. Also Ritz method is used to calculate free vibration frequencies. Thai [8] presented shear deformation theory for static, dynamic and buckling analysis of FGM made of isotropic and sandwich plates with two distribution functions are proposed in the formulation. These functions determine distribution of transverse shear strain and stress across the thickness of the plate. The distribution function is chosen such that the shear stress free conditions at the top and bottom surfaces of the plates are naturally satisfied. Hosseini-Hashemi [9] presented solution based on 3D elasticity theory for both in-plane and out-of-plane for thick FGM simply supported rectangular plate. Ordinary differential equations are obtained by using some suitable independent functions and 3D elasto-dynamic equations. Results are compared with 3D finite element method. Also the influence of inhomogeneity on natural frequency for thick FGM rectangular plate is observed. Wen [10] used a state space formalism to analyse 3D orthotropic FGM rectangular plate with simply supported boundary conditions under static and dynamic loads. By using radial basis function method numerical solutions for FGM plate is obtained.
- b) *Two-Dimensional Analysis:* Lorenzo [11] used a Levy's method to derive the solution for free vibration of thick and moderately thick FGM rectangular plates on the basis of two dimensional shear and normal deformation theories with variable order. Comparison study has been done with 3D and 2D approaches are provided for FGM sandwich plate and also six combinations of boundary conditions are applied to FGM plates.

## B. *Numerical Methods*

### 1) *Static Analysis*

- a) *Finite Element Method:* Tomar [12] investigated thermo-mechanical buckling response of skew functionally graded laminated plates with initial geometric imperfections by using Reddy's higher order shear deformation theory. A nine noded isoparametric element is taken to discretize the domain of the plate and showed that the gradation of material properties, thickness ratio, thermal environment imperfection parameter and skew angle significantly effect on the critical buckling parameter of the FGM plate. Singha [13] investigated the nonlinear behaviour of FGM plates under transverse distributed load by using first order shear deformation theory and by using standard finite element procedure. Rule of mixture method is used to evaluate effective material properties. It was observed that the distribution of the transverse shear stress through the thickness of the plate is in general parabolic with the maximum shear stress occurring at the neutral surface. Later Talha [14] also used buckling analysis of FGM plate under thermo-mechanical load by using higher order shear deformation theory and variational approach is utilized to derive the fundamental equations for the FGM plates and presented numerical results based on different volume fraction indices, thickness ratios, temperature rise along with different boundary conditions. It was observed that gradation in the material properties and the temperature field has a considerable influence on the buckling load parameter on FGM plates.
- b) *Meshless Method:* Alibeigloo [15] used Fourier series and state-space method to analyse 3D FGM plates with simply supported edges subjected to thermo-mechanical loads. The behaviour of the plate is examined due to effect of temperature change, applied mechanical load, gradient index, aspect ratio and thickness to length ratio. Later Zenkour [16] presented static response of simply supported rectangular FGM plate subjected to transverse uniform load by using shear deformation theory. It was observed that response of plates that corresponds to properties intermediate to that of the metal and ceramic, is necessarily lie in between metal and ceramic and behaviour is irrespective of boundary conditions.

## 2) *Vibration Analysis*

- a) *Finite Element Method*: Viet [17] used combination of Navier's solution and finite element method based on  $n^{\text{th}}$  order shear deformation theory to analysed relative sliding movements of two layer FGM plates resting on Winker's Pasternak elastic foundations. Then investigations are done to capture the effects of structure's geometrical and material features on the mechanical behaviour of two layer FG plate. Four noded tetragonal plate element with eleven degrees of freedom is used. Then Smita [18] used a higher order shear deformation theory to develop finite element model of a skew functionally graded plate in thermal environment with a free vibration effect. Eight noded isoparametric element with seven degrees of freedom is considered for this model and showed that frequency of plate decrease with increase in volume fraction index, aspect ratio and side thickness ratio. Also frequency is more for greater skew angle has been shown. Later Pandey [19] used a higher order layerwise finite element formulation for static and dynamic analysis of FGM sandwich plate. An isoparametric elements having eight nodes with thirteen degrees of freedom has been considered. Comparison study between two configurations has been presented, one with FGM core and homogeneous facesheets and other with homogeneous core and FGM facesheets. Pankaj [20] used a higher order shear deformation theory for the evaluation of bending and vibration response of the skew sandwich composite plate. Numerical results are studied using equivalent single layer higher order kinematic theory for the isotropic and orthotropic core with the help of isoparametric finite element steps.
- b) *Meshless*: Fiorenzo [21] used advanced hierarchical trigonometric Ritz formulation to analyse free vibration effect and thermal stability of FGM sandwich plates. The results were presented for effect of initial thermal stresses on the free vibration behaviour of FG plate. Also the effect of aspect ratio, volume fraction index, boundary conditions, length to thickness ratio has been investigated. Ping Zhu [22] used a local Kriging meshless method for free vibration analysis of FGM plate. To examine this method convergence studies are conducted. Numerical results were presented for three types of plates which are square, skew and quadrilateral plates to check versatility of this method for free vibration analysis. Later Zhao [23] used a element free kp Ritz method for a free vibration analysis of functionally graded plates. First order shear deformation theory is considered for the transverse shear strain and rotary inertia. Author discussed about the effects of volume fraction, boundary conditions and length to thickness ratio on their frequency characteristics. Then Ferreira [24] used a global collocation method, first and third order shear deformation theory to get frequencies of plate, and Mori-Tanaka technique to find material properties and to analyse free vibration of functionally graded plates. Then Deshpande [25] In this paper, the four-node quadrilateral element having seven degrees of freedom per node, was developed earlier by the second author and his co-worker has modified for free vibration analysis of functionally graded material (FGM) plates under uniform and linear thermal environment. The performance of the modified element is assessed by comparison of the present results of the non-dimensionalized natural frequencies with the available analytical results as well as with 2D finite element results obtained using the commercial software ANSYS. It is observed that the performance of the present element is quite satisfactory for the isotopic FGM plates under uniform and linear thermal environment considered in this study with all edges simply supported and all edges clamped are the boundary conditions considered.

## II. CONCLUSION

This is a review on recent advancements in analytical and numerical methods for stress and vibration analysis of multi-layered as well as single FGM plates. The approaches used for analysis of FGM plates are extensions of similar approaches used for study of composite or isotropic plates. Analytical solutions are restricted for FGM plates with exponential and simple type of material gradient using 3D elasticity equations. In FGM plate 3D with power law variation of material properties is still not available. In FGM plates the gradients in material properties are important in determining the response of FGM plate. The basic response of FGM that corresponds to properties lie between metal and ceramics irrespective of boundary conditions when only mechanical load case is considered. The deflection the plate does not necessarily lie in between thermal and ceramics for thermal load case.

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