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# Stationary Performance Evaluation of an M/M/2 Queue with Constant Retrials and State-Dependent Service Rate

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***Abstract:*** ***This paper investigates the stationary analysis of an M/M/2 retrial queuing system with a state dependent service rate. Customers arrive according to a Poisson process, and if both servers are busy, they enter an orbit and retry after a random time with a constant repeated attempt rate. The model is formulated as a Markov process with a two-dimensional state descriptor. Steady state equations are derived, and conditions for ergodicity are established. Numerical illustrations demonstrate how probabilities change with orbit size, providing insights into system performance.***
***Keywords: Retrial queues, M/M/2 model, state dependent service, stationary distribution, birth-death process,bivarite process.***

## I. INTRODUCTION

Queuing models play a significant role in analyzing and optimizing service systems in areas such as telecommunication networks, computer systems, and customer service environments. Among these, retrial queuing systems are particularly important when blocked customers do not leave the system permanently but instead retry for service after a random interval. Queuing theory provides a fundamental framework for analyzing service systems where resources are shared among randomly arriving customers. In many practical situations, when a service request cannot be accommodated immediately, the customer may reattempt after some delay instead of leaving permanently. Such systems are referred to as retrial queues.

The study of retrial queues and state-dependent service mechanisms has been extensively developed through earlier research in queueing theory and stochastic models. [1]Artalejo (1996) analyzed the stationary characteristics of the M/M/2 retrial queue with constant repeated attempts, providing a foundation for the present work. [2]Garg and Singh (1993) examined queue-dependent servers, highlighting the impact of varying service behavior on system dynamics. Classical texts by [3]Gross and Harris (1985) and [4]Kleinrock (1975) established the fundamental principles of queueing systems, while [5]Medhi (1991) contributed comprehensive treatments of stochastic models relevant to retrial phenomena. These works collectively provide the theoretical and methodological basis upon which the present study extends the analysis of M/M/2 retrial queues with state-dependent service rates.

The model:

M/M/2 retrial queuing system with state dependent service rate. In this model the customer arrives in a Poisson distribution with rate $\lambda>0$. The customer immediately undergone service, when anyone of the service is free. If all the servers are busy, then the customers join in the orbit, and the customers becomes a source of repeated calls to retry after a random amount of time. The retrial intensity is $\mu \geq 0$. State –transition diagram is shown in the fig 1.

When there is one customer in the orbit, the retrial intensity is $\mu$ and when there is a j customer, the retrial intensity is $j\mu$. $\mu=0$, if there is no customer in the orbit (empty). Any primary arrival or retrial can serve immediately if it finds anyone of the free servers. The successive service times are mutually independent and the service rate is $v_1$if one server is busy and $v_2$ if both the servers are busy. The service rate is 'STATE DEPENDENT'.

Assume that the primary arrivals, intervals between repeated attempts and service times are all mutually independent. At any time t, the state of the system can by described by the bivariate process{C, N}.

$$X(t) = \{(C(t), N(t) \mid t \geq 0\}$$

Where,

C (t) denotes the number of busy servers at time t and

N (t) denotes the number of customers (sources) in the orbit at time t.
The process is Markovian.

Here,

C (t) can take values 0, 1, 2.
N (t) can take values 0,1,2,3…

(i.e.) the state space is the cross product of finite set {0,1,2}and half-line,

We define the limit distribution

$P_{ij}=\lim_{t\to\infty} P\{C(t=i, N(t)=j);\ i=0,1,2$
j=0, 1, 2….

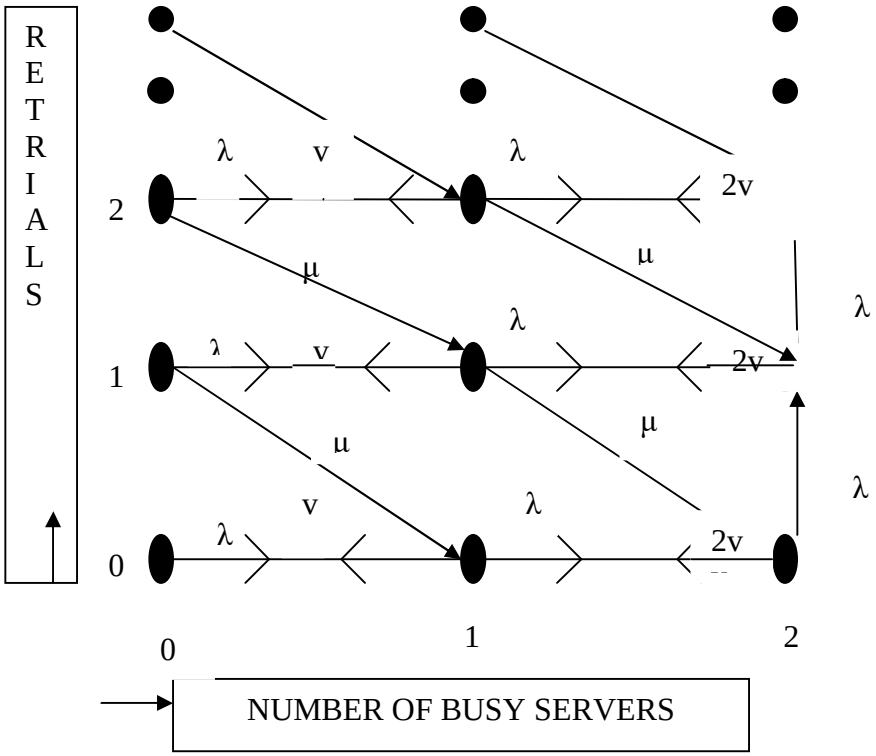



Fig 1

The steady state equations are

$$\left(\lambda+\mu\left(1-\delta_{j0}\right)\right)P_{0j}=v_1 p_{1j};\ j\geq 0\rightarrow(1)$$

$$\left(\lambda+v_1+\mu(1-\delta_{j0})\right)P_{1j}=\lambda P_{0j}+v_2 P_{2j}+\mu P_{0,j+1};\ j\geq 0\rightarrow(2)$$

$$(\lambda+\nu_2)P_{2j}=\lambda P_{1j}+\mu P_{1,j+1}+\lambda P_{2,j-1}(1-\delta_{j0});\ j\geq 0\rightarrow(3)$$

**THEOREM**

A Necessary and sufficient condition for the ergodicity of the M/M/2 constant retrial queue is

$$\frac{\lambda(\lambda+\mu)^2}{\nu_2\mu(\lambda+\nu_1+\mu)}<1$$

From (1),

$$P_{1j}=\frac{(\lambda+\mu(1-\delta_{j0}))}{\nu_1}P_{0j}\rightarrow(I)$$

Using (2) and (1) we can express the probabilities $\{P_{2j}\}$ in terms of the sequence $\{P_{0j}\}$ as follows:

$$\nu_2P_{2j}=(\lambda+\nu_1+\mu(1-\delta_{j0}))\left(\frac{\lambda+\mu(1-\delta_{j0})}{\nu_1}P_{0j}\right)-\lambda P_{0j}-\mu P_{0,j+1}$$

$$\nu_2P_{2j}=(\lambda+\nu_1+\mu(1-\delta_{j0}))\left(\frac{(\lambda+\mu(1-\delta_{j0}))}{\nu_1}\right)P_{0j}-\lambda P_{0j}-\mu P_{0,j+1}$$

$$\nu_2P_{2j}=\frac{[(\lambda+\mu(1-\delta_{j0}))^2+\lambda\nu_1+\nu_1\mu(1-\delta_{j0})]P_{0j}-\lambda\nu_1P_{0j}}{\nu_1}-\mu P_{0,j+1}$$

$$\nu_2P_{2j}=\left(\frac{(\lambda+\mu(1-\delta_{j0}))^2+\lambda\nu_1+\nu_1(1-\delta_{j0})-\lambda\nu_1}{\nu_1}\right)P_{0j}-\mu P_{0,j+1}$$

$$P_{2j} = \frac{(\lambda + \mu(1-\delta_{j0}))^2 + \nu_1\mu(1-\delta_{j0})}{\nu_1\nu_2} P_{0j} - \frac{\mu}{\nu_2} P_{0,j+1} \rightarrow (II)$$

By substituting (I) and (II) in equation (3)

$$(\lambda+\nu_2)\left(\frac{(\lambda + \mu(1-\delta_{j0}))^2 + \nu_1\mu(1-\delta_{j0})}{\nu_1\nu_2} P_{0j} - \frac{\mu}{\nu_2} P_{0,j+1}\right) =$$
$$\lambda\left(\frac{\lambda + \mu(1-\delta_{j0})}{\nu_1} P_{0,j}\right) + \mu\left(\frac{\lambda + \mu(1-\delta_{j+1,0})}{\nu_1} P_{0,j+1}\right) +$$
$$(1-\delta_{j0})\lambda\left(\frac{(\lambda + \mu(1-\delta_{j-1,0}))^2 + \nu_1\mu(1-\delta_{j-1,0})}{\nu_1\nu_2} P_{0,j-1} - \frac{\mu}{\nu_2} P_{0,j}\right)$$

Collecting $P_{0j}$ on L.H.S and the remaining terms on the R.H.S

$$(\lambda+\nu_2)\left(\frac{(\lambda+\mu(1-\delta_{j0}))^2+\nu_1\mu(1-\delta_{j0})}{\nu_1\nu_2} P_{0j}\right) - \lambda\left(\frac{\lambda+\mu(1-\delta_{j0})}{\nu_1} P_{0j}\right) + \left(\frac{\lambda\mu}{\nu_2}(1-\delta_{j0})P_{0j}\right) =$$
$$(\lambda+\nu_2)\frac{\mu}{\nu_2} P_{0,j+1} + \mu\left(\frac{\lambda+\mu(1-\delta_{j+1,0})}{\nu_1}\right) P_{0,j+1} + (1-\delta_{j0})\lambda\left(\frac{(\lambda+\mu(1-\delta_{j-1,0}))^2+\nu_1\mu(1-\delta_{j-1,0})}{\nu_1\nu_2}\right) P_{0,j-1}$$

Taking L.H.S and solving

$$\lambda\left(\frac{(\lambda+\mu(1-\delta_{j0}))^2+\nu_1\mu(1-\delta_{j0})}{\nu_1\nu_2}\right) P_{0j} + \nu_2\left(\frac{\lambda+\mu(1-\delta_{j0}))^2+\nu_1\mu(1-\delta_{j0})}{\nu_1\nu_2}\right) P_{0j} -$$
$$\left(\frac{\left(\lambda^2+\lambda\mu(1-\delta_{j0})\right)}{\nu_1} P_{0j}\right) + \left(\frac{\lambda\mu(1-\delta_{j0})}{\nu_2} P_{0j}\right)$$

$$\alpha_j = \frac{\lambda\left(\left((\lambda+\mu(1-\delta_{j0})\right)^2+\nu_1\mu(1-\delta_{j0})\right)}{\nu_1\nu_2}; j \geq 0$$

$$v_2\left(\frac{(\lambda+\mu(1-\delta_{j0}))^2+v_1\mu(1-\delta_{j0})}{v_1v_2}\right)P_{0j}-\left(\frac{\lambda^2+\lambda\mu(1-\delta_{j0})}{v_1}\right)P_{0j}+\left(\frac{\lambda\mu(1-\delta_{j0})}{v_2}\right)P_{0j}$$

$$v_2\left(\frac{\lambda^2+2\lambda\mu(1-\delta_{j0})+\mu^2(1-\delta_{j0})^2+v_1\mu(1-\delta_{j0})}{v_1v_2}\right)P_{0j}-\left(\frac{\{(v_2(\lambda^2+\lambda\mu(1-\delta_{j0}))P_{j0})+v_1(\lambda\mu(1-\delta_{j0}))}{v_1v_2}\right)P_{0j}$$

$$\left(\frac{\lambda v_2\mu(1-\delta_{j0})+v_2\mu^2(1-\delta_{j0})^2+v_1v_2\mu(1-\delta_{j0})+v_1\lambda\mu(1-\delta_{j0})}{v_1v_2}\right)P_{0j}$$

Taking $\frac{\mu}{v_1}$ and $(1-\delta_{j0})$ as common

$$\Rightarrow \frac{\mu}{v_1}\left(\lambda+\mu(1-\delta_{j0})+v_1+\frac{\lambda}{v_2}\right)(1-\delta_{j0})$$

$$\Rightarrow \frac{\mu}{v_1}\left(\lambda\left(1+\frac{v_1}{v_2}\right)+v_1+\mu(1-\delta_{j0})\right)(1-\delta_{j0})$$

$$\frac{\mu}{v_1}\left(\lambda\left(\frac{v_1+v_2}{v_2}\right)+v_1+\mu(1-\delta_{j0})\right)(1-\delta_{j0});\ j\geq 0$$

$$\beta_j=\frac{\mu}{v_1}\left(\left(\frac{v_1+v_2}{v_2}\right)\lambda+v_1+\mu(1-\delta_{j0})\right)(1-\delta_{j0});\ j\geq 0$$

$\therefore$ L.H.S

$$(\alpha_j+\beta_j)P_{0j}$$

Taking R.H.S and solving

$$(\lambda+\nu_2)\left(\frac{\mu}{\nu_2}\right)P_{0,j+1}+\mu\left(\frac{\lambda+\mu(1-\delta_{j+1,0})}{\nu_1}\right)P_{0,j+1}+$$

$$(1-\delta_{j0})\lambda\left(\frac{(\lambda+\mu(1-\delta_{j0}))^2+\nu_1\mu(1-\delta_{j-1,0})}{\nu_1\nu_2}\right)P_{0,j-1}$$

$$\alpha_{j-1}=\left(\frac{\lambda\left[(\lambda+\mu(1-\delta_{j-1,0})\right]^2+\nu_1\mu(1-\delta_{j-1,0})}{\nu_1\nu_2}\right)$$

$$\left(\frac{\lambda\mu}{\nu_2}+\mu+\frac{\lambda\mu}{\nu_1}+\frac{\mu^2(1-\delta_{j+1,0})}{\nu_1}\right)P_{0,j+1}=\left(\frac{\lambda\mu}{\nu_2}+\frac{\nu_1\mu}{\nu_1}+\frac{\lambda\mu}{\nu_1}+\frac{\mu^2(1-\delta_{j+1,0})}{\nu_1}\right)$$

$$\left(\frac{\lambda\mu}{\nu_2}+\mu+\frac{\lambda\mu}{\nu_1}+\frac{\mu^2(1-\delta_{j+1,0})}{\nu_1}\right)P_{0,j+1}=\left(\frac{\lambda\mu\nu_1}{\nu_1\nu_2}+\frac{\nu_1\nu_2\mu}{\nu_1\nu_2}+\frac{\lambda\mu\nu_2}{\nu_1\nu_2}+\frac{\mu^2\nu_2(1-\delta_{j+1,0})}{\nu_1\nu_2}\right)$$

$$\left(\frac{\lambda\mu}{\nu_2}+\mu+\frac{\lambda\mu}{\nu_1}+\frac{\mu^2(1-\delta_{j+1,0})}{\nu_1}\right)P_{0,j+1}=\frac{\mu}{\nu_1}\left(\frac{\lambda\nu_1}{\nu_2}+\nu_1+\lambda+\mu(1-\delta_{j+1,0}\right)$$

$$\left(\frac{\lambda\mu}{\nu_2}+\mu+\frac{\lambda\mu}{\nu_1}+\frac{\mu^2(1-\delta_{j+1,0})}{\nu_1}\right)P_{0,j+1}=\frac{\mu}{\nu_1}\left(\lambda\left(\frac{\nu_1+\nu_2}{\nu_2}\right)+\nu_1+\mu(1-\delta_{j+1,0}\right)$$

$$\beta_{j+1}=\frac{\mu}{\nu_1}\left(\left(\frac{\nu_1+\nu_2}{\nu_2}\right)\lambda+\nu_1+\mu\left(1-\delta_{j+1,0}\right)\right)$$

$\therefore$ The Equation (3) Becomes

$$(\alpha_j+\beta_j)=P_{0j}=\alpha_{j-1}P_{0,j-1}(1-\delta_{j0})+\beta_{j+1},P_{0,j+1};\ j\geq 0$$

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The sequence $\{P_{0j}\}$ is a Birth and death process with Birth parameter $\alpha_j$ and death parameter $\beta_j$

$$S_1 = \sum_{j=1}^{\infty} \prod_{k=0}^{j-1} \frac{\alpha_k}{\beta_{k+1}}$$

$$S_2 = \sum_{j=1}^{\infty} \left( \alpha_j \prod_{k=0}^{j-1} \frac{\alpha_k}{\beta_{k+1}} \right)^{-1}$$

Defining

$$\alpha = \frac{\lambda((\lambda+\mu)^2 + v_1\mu)}{v_1 v_2}$$

$$\beta = \frac{\mu}{v_1}\left( \left( \frac{v_1 + v_2}{v_2} \right)\lambda + v_1 + \mu \right)$$

$$\gamma = \frac{\lambda^3}{v_1 v_2}$$

Then it follows that,

$$S_1 = \frac{\gamma}{\beta} \sum_{j=0}^{\infty} \left( \frac{\alpha}{\beta} \right)^j$$

$$S_2 = \frac{1}{\beta} \sum_{j=1}^{\infty} \left( \frac{\beta}{\alpha} \right)^j$$

Consequently, $S_1<\infty$ if and only if $\alpha<\beta$, and $S_2<\infty$ if and only if $\beta<\alpha$. Hence the classification of the states in birth and death processes guarantees that $\{P_{0j}\}$ is ergodic in and only $\alpha<\beta$; null recurrent if and only if $\alpha=\beta$ transient if and only if $\alpha>\beta$.

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$\alpha<\beta$ if and only if

$$\frac{\lambda(\lambda+\mu)^2}{\nu_2\mu(\lambda+\nu_1+\mu)}<1$$

$$\frac{\left(\frac{\lambda[(\lambda+\mu)^2+\nu_1\mu}{\nu_1\nu_2}\right)}{\frac{\mu}{\nu_1}\left(\left(\frac{\nu_1+\nu_2}{\nu_2}\right)\lambda+\nu_1+\mu\right)}<1$$

$$\left(\frac{\frac{\lambda(\lambda+\mu)^2}{\nu_1\nu_2}+\frac{\lambda\nu_1\mu}{\nu_1\nu_2}}{\frac{\lambda\mu}{\nu_1}+\frac{\mu}{\nu_1}(\lambda+\nu_1+\mu)}\right)<1$$

$$\frac{\lambda(\lambda+\mu)^2}{\nu_2\mu(\lambda+\nu_1+\mu)}<1$$

This gives the theorem

Since the stationary probabilities for the birth and death sequence $\{P_{0j}\}$ are given by

$$P_{0j}=P_{00}\prod_{k=0}^{j-1}\frac{\alpha_k}{\beta_{k+1}}$$

$$P_{0j} = P_{00} \frac{\alpha_0.\alpha_1.....\alpha_{j-1}}{\beta_1.\beta_2.....\beta_j}$$

$$P_{0j} = P_{00} \frac{\gamma\alpha^{j-1}}{\beta^{j}}$$

$$P_{0j} = \frac{\gamma}{\alpha}\left(\frac{\alpha}{\beta}\right)^{j} P_{00}; j \geq 1$$

$$P_{0j} = \frac{\lambda^3}{v_1 v_2} \frac{v_1 v_2}{\lambda((\lambda+\mu)^2 + v_1\mu)} \left( \frac{\lambda(\lambda+\mu)^2 + v_1\mu}{v_1 v_2} \frac{1}{\frac{\mu}{v_1}\left(\left(\frac{v_1+v_2}{v_2}\right)\lambda + v_1 + \mu\right)} \right)$$

$$P_{0j} = \frac{\lambda^2}{(\lambda+\mu)^2 + v_1\mu} \left( \frac{\lambda[(\lambda+\mu)^2 + v_1\mu}{v_1 v_2} \frac{v_1}{\mu} \frac{1}{\left(1+\frac{v_1}{v_2}\right)\lambda + v_1 + \mu} \right)$$

$$P_{0j} = \frac{\lambda^2}{(\lambda+\mu)^2 + v_1\mu} \left( \frac{\lambda\left[(\lambda+\mu)^2 + v_1\mu\right]}{v_1 v_2} \frac{v_1 v_2}{\mu\left[(v_1+v_2)\lambda + v_2(v_1+\mu)\right]} \right)$$

$$P_{0j} = \frac{\lambda^2}{(\lambda+\mu)^2 + v_1\mu} \left( \frac{\lambda\left[(\lambda+\mu)^2 + v_1\mu\right]}{\mu\left[(v_1+v_2)\lambda + v_2(v_1+\mu)\right]} \right) P_{00}; j \geq 1$$

$$P_{1j} = \frac{\lambda + \mu(1-\delta_{j0})}{v_1} P_{0j}; j \geq 0$$

$$P_{2j} = \left( \frac{(\lambda + \mu(1-\delta_{j0}))^2 + v_1\mu(1-\delta_{j0})}{v_1 v_2} \right) P_{0j} - \frac{\mu P_{0,j+1}}{v_2}; j \geq 0$$

$$P_{2j} = \left( \frac{(\lambda+\mu)^2 + v_1\mu}{v_1 v_2} \right) P_{0j} - \frac{\mu}{v_2} P_{0,j+1}; j \geq 1$$

$$P_{2j} = \left( \frac{(\lambda+\mu)^2 + v_1\mu}{v_1 v_2} \right) \left( \frac{\lambda^2}{(\lambda+\mu)^2 + v_1\mu} \right) \left( \frac{\lambda[(\lambda+\mu)^2 + v_1\mu}{\mu[(v_1+v_2)\lambda + v_2(v_1+\mu)]} \right)^j P_{00} -$$

$$\frac{\mu}{v_2} \left( \frac{\lambda^2}{(\lambda+\mu)^2 + v_1\mu} \left( \frac{\lambda[(\lambda+\mu)^2 + v_1\mu]}{\mu[(v_1+v_2)\lambda + v_2(v_1+\mu)]} \right)^{j+1} P_{00} \right)$$

$$P_{2j} = \left\{ \frac{\lambda^2}{v_1 v_2} - \frac{\lambda^3}{v_2[(v_1+v_2)\lambda + v_2(v_1+\mu)]} \right\} \left( \frac{\lambda((\lambda+\mu)^2 + v_1\mu)}{\mu[(v_1+v_2)\lambda + v_2(v_1+\mu)]} \right)^j P_{00}$$

$$P_{2j} = \left( \frac{\lambda^3 v_1 + \lambda^3 v_2 + \lambda^2 v_2\mu - \lambda^3 v_1}{v_1 v_2[(v_1+v_2)\lambda + v_2(v_1+\mu)]} \right) \left( \frac{\lambda[(\lambda+\nu)^2 + v_1\mu]}{\mu[(v_1+v_2)\lambda + v_2(v_1+\mu)]} \right)^j P_{00}$$

$$P_{2j} = \left( \frac{\lambda^2(\lambda + v_1 + \mu)}{v_1[(v_1+v_2)\lambda + v_2(v_1+\mu)]} \right) \left( \frac{\lambda[(\lambda+\nu)^2 + v_1\mu]}{\mu[(v_1+v_2)\lambda + v_2(v_1+\mu)]} \right)^j P_{00}$$

Using the condition $\sum_{(i,j)} P_{ij} = 1$ , by adding $P_{0j}$,$P_{1j}$ ,$P_{2j}$ and equating to 1.

$P_{00}$ as

$$P_{00} = \left( \frac{v_2\mu(\lambda + v_1 + \mu) - \lambda(\lambda+\mu)^2}{\mu(v_2(\lambda + v_1) + (\lambda + \mu)(\lambda + v_2))} \right)$$

Numerical examples:

The results indicate a clear dependence of system performance on orbit size. When no customers are in orbit ($j = 0$), the probability of both servers being busy ($P_{20}$) is significantly higher than the idle probability ($P_{00}$), suggesting that the system is highly utilized under light retrial conditions.

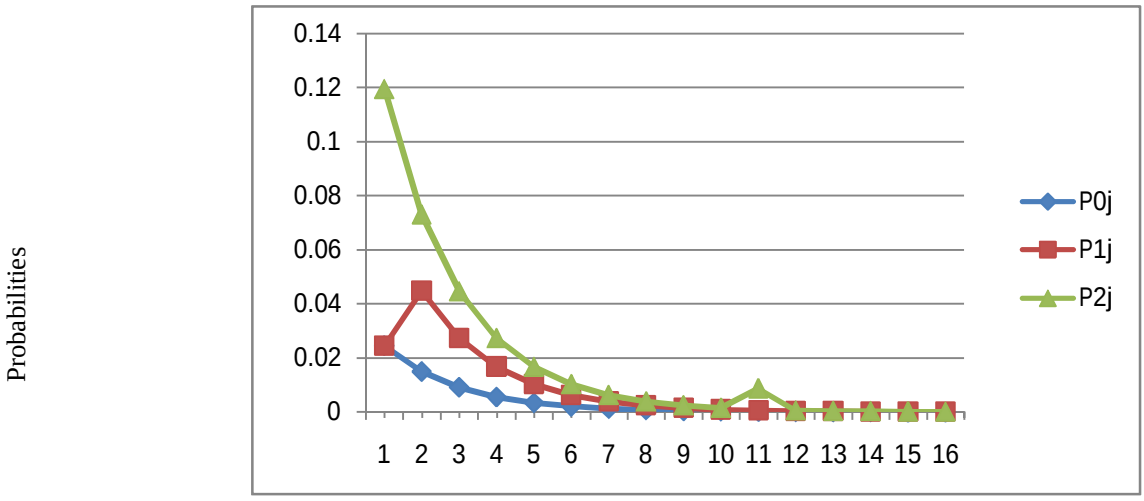
As the number of customers in orbit increases, all three probabilities decrease, reflecting the reduced likelihood of the system being in a low-load state. In particular, $P_{0j}$ (idle probability) decreases rapidly with $j$, while $P_{2j}$ (both servers busy) remains relatively dominant across larger orbit sizes. This highlights that retrial traffic intensifies server utilization, increasing the tendency for both servers to remain busy [1], [2], [5].

Such behavior is consistent with the theoretical expectation that retrials act as a secondary input stream, thereby elevating congestion levels and reducing the probability of idle servers [3], [4]. The incorporation of state-dependent service rates further accentuates this effect, as higher service efficiency under heavier load conditions moderates but does not eliminate congestion growth.

| | |
|---|---|
| **λ** | **1** |
| **μ** | **2** |
| **$V_1$** | **1** |
| **$V_2$** | **2** |

| **j** | **$P_{0j}$** | **$P_{1j}$** | **$P_{2j}$** |
|---|---|---|---|
| 0 | 0.024476 | 0.024476 | 0.119658 |
| 1 | 0.014957 | 0.044872 | 0.073124 |
| 2 | 0.009141 | 0.027422 | 0.044687 |
| 3 | 0.005586 | 0.016758 | 0.027309 |
| 4 | 0.003414 | 0.010241 | 0.016689 |
| 5 | 0.002086 | 0.006258 | 0.010199 |
| 6 | 0.001275 | 0.003824 | 0.006233 |
| 7 | 0.000779 | 0.002337 | 0.003809 |
| 8 | 0.000476 | 0.001428 | 0.002328 |
| 9 | 0.000291 | 0.000873 | 0.001422 |
| 10 | 0.000178 | 0.000533 | 0.00869 |
| 11 | 0.000109 | 0.000326 | 0.000531 |
| 12 | 0.000066 | 0.000199 | 0.000325 |

TABLE VARIATION OF PROBABILITIES



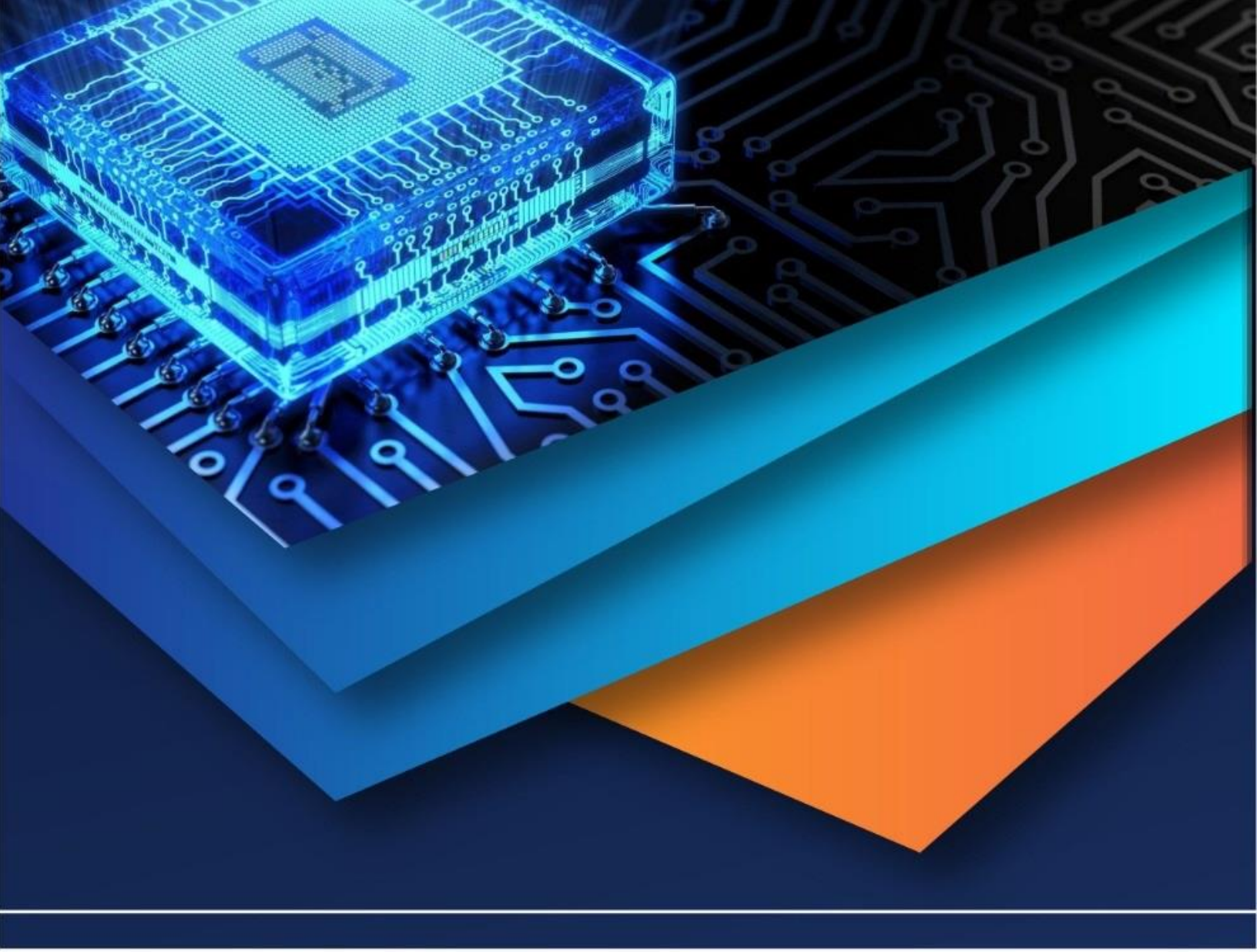
## II. CONCLUSION

Numerical results confirm that as the number of customers in orbit increases, the probability of idle servers decreases, while the likelihood of both servers being busy rises. This demonstrates the direct impact of retrial traffic on system congestion. Moreover, the incorporation of state-dependent service rates offers a more realistic representation of systems where service efficiency is influenced by workload.Overall, the model contributes to the broader understanding of retrial queues by presenting a tractable yet practical framework. Its applicability extends to telecommunication systems, call centers, and computer networks, where managing retrial traffic and varying service capacities is critical for maintaining stability and service quality.

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