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Stochastic Reliability Modelling of H_r/M/1/N Queueing System with Working and Working-Breakdown States

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Abstract: This study models the reliability, availability, and maintainability of a finite capacity H_r/M/1/N phase queueing system in working and working-breakdown states. The arrival times are distributed according to a Hyperexponential distribution with r-parallel phases, whereas the service times are distributed according to an exponential distribution. The differential-difference equations for the transient states are derived from the state-transition diagram for different environmental states, as well as failure and recovery states. For the particular situation of N=4, these governing equations are solved numerically. Sensitivity Analysis was used to investigate RAM performance measures for various parametric variables.
Keywords: Availability, Exponential distribution, Hyperexponential distribution, Maintainability, Reliability, Sensitivity Analysis

I. INTRODUCTION

The study of a series of service stations through which each arriving customer must proceed before leaving the system is one of the most common applications of queueing theory. When the output from one service station becomes the input to the next, i.e., when units departing from one station are not collected and then moved on to the next station in a batch, the analysis of the system's behaviour requires the determination of the inter-departure time distribution at each stage, which is then used as the inter-arrival time distribution to the next station. This tends to be the most challenging component of the analysis in general. Therefore, for such queueing model the arrival or service times can be modelled with Phase type distributions and these models are used to measure the characteristics of queueing system. In such models, finding an ideal solution is very difficult and these models usually consider better approximations. As a result, a new technique to the study of G/M/1 systems with phase type distributions appears to be promising. One of the input distribution G can be assumed to be the blend of two exponential probability distributions, i.e., Hyperexponential distribution H_r. In this context, queuing system with Hyperexponential input distributions is more complex system as compared to Exponential or Erlang distributions and it is challenging to find an analytical solution. A Hyperexponential distribution is a continuous probability distribution which consists of two or more non-identical phases that occur in a exhaustive or parallel manner.

Cosmetatos and Godsave (1986), have developed hyper-exponential Hr/M/1 inter-arrival and exponential service times queueing model with multi-server system. They developed an approximate formula for the equilibrium queue-size and queueing-time distributions. These formulas are based on the combination of heuristic arguments, analytical methods and the model is also dealt with the sensitivity analysis. Tian et al. (1989), examined the G1/M/1 queue with exhaustive service and multiple exponential vacations. The authors represent the embed Markov chain's transition matrix as a block-Jacobi form and provided a matrix-geometric solution. In addition, the limiting behaviour of continuous time queue length processes, probability distributions for the waiting times and the busy period were also explored. Tarasov et al. (2019) studied the H_2/M/1 queueing system and determined the mean delay time for different approaches and compared. Furthermore, the authors were able to optimise the constraints to find the characteristics for the H_2/M/1 queueing system. Reliability theory is a well-developed field and has been researched by several authors. The computation of the Reliability of a system with parts showing dependent failures and repairs for any system is in general complex, and several methods are proposed which are available in the literature. Amiri et al. (2007) presented a methodology for assessing system transient survivability and availability in the presence of identical components and repairmen. To create the technique for the transient reliability of such systems, the authors used the ideas of Markov models, eigen vectors, and eigenvalues. The suggested approach is a more effective method and can be used to analyse a wide range of systems, including series, parallel, and k-out-of-n systems.

The proposed approach may also be used to calculate MTTFs and the time required for the system to attain steady state. There hasn't been much work done on the RAM analysis of the $H_r/M/1$ queueing system with working and working-breakdown states. As a result, this inspires to study the reliability, availability and maintainability analysis for hyper exponential single server finite capacity queuing system with working, working-breakdowns, failure and recovery policies.

In this chapter the RAM analysis of the $H_r/M/1/N$ queueing model with two different environmental states is analysed. The arrival process is modelled by Hyperexponential random variable with r phases of arrival with rate λ_r. The unit is arrived only by one phase of arrival that is chosen according to a discrete distribution p_i, $\sum_{i=1}^{r} p_i = 1$. This Hyperexponential distribution has rate λ_1 with probability p_1 and rate λ_2 with probability $p_2$and so on. Let S(t) denote the environmental states such as the working and working breakdown states at time t. When the system is in the working state failure occurs in the queueing system with the failure rate α. The system is immediately switched to the working-breakdown state where the recovery process takes place with the recovery rate β. During this process, the system did not terminate its function but it continues its process at a slow pace. The differential-difference equations are formed from the state-transition diagram for the $H_r/M/1/N$ queueing model and this model is numerically illustrated for the special case. The Fourth-Order Runge-Kutta numerical method is used to solve the transient probabilities for the $H_r/M/1/N$ queueing model. The Reliability, Availability and Maintainability of the $H_r/M/1/N$ queueing are analysed numerically and presented graphically. The sensitivity analysis was also performed to study the system change for changes in the model parameters.

II. PRESUMPTIONS AND NOTATIONS

The presumptions that are used in this chapter are:

1) The arrival of units to the system are assumed to follow Hyperexponential distribution with r phases.
2) The service process is exponentially distributed with FCFS (First Come First Serve) queue discipline
3) When the system is not empty (i.e., at least one unit must be present), a failure rate arises with exponential rate α. The system immediately changes to Working-Breakdown mode, where it runs at a reduced rate.
4) The recovery process occurs in the system's working-breakdown, which is exponentially distributed. When the system recovers, it returns to its normal working state of operation
5) All arrival and the service times are independent of each other

The following are the notations that are used in this chapter:

RAM	:	Reliability, Availability and Maintainability
N(t)	:	Total number of machines in the system at any time t
H_r	:	Hyper exponential distribution with r identical parallel phases
S(t)	:	The environmental state at any instant of time t
R(t)	:	Reliability of the system at time t
A(t)	:	Availability of the system at time t
M(t)	:	Maintainability of the system at time t
λ_r	:	Arrival rate for the r^{th} phase
μ_1	:	Service rate for working state
μ_2	:	Service rate for working-breakdown state ($\mu_1 > \mu_2$).
α	:	Failure rate
β	:	Recovery rate
$P_{n,i,j}(t)$	:	Probability that there are n machines in the system with i^{th} phases and j^{th} states at time t
$P_n(t)$	:	Probability that there are n units in the system at time t
$P_w(t)$	:	Probability of n units in the system for the working state at time t
$P_{wb}(t)$	:	Probability of n units in the system for the working-breakdown State at time t
p_k	:	Probability that the arrival choosing k^{th} phase

III. RAM MODELLING OF $H_r/M/1$ QUEUEING SYSTEM

In this section the analysis of the Reliability, Availability, and Maintainability for $H_r/M/1/N$ queueing system with two different environmental states are analysed. In $H_r/M/1/N$ queueing systems, the arrival process is modelled by the Hyperexponential random variable with r phases of arrival with rates λ_r. In this case, the arrival phases should be of parallel structure, i.e., a unit arrive only to one phase, each is chosen according to a discrete distribution p_k, $\sum_{k=1}^{r} p_k = 1$.

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The arrival process is according to hyperexponential distribution has rate λ_1 with probability p_1, rate λ_2 with probability $p_2$and so on. The service process is exponentially distributed with two parameters μ_1 and μ_2 depending on the environmental states such as working and working-breakdown. The failure occurs in the working state of the system with failure rate α, the system immediately moved its process to the working-breakdown state where the recovery process takes place with recovery rate β. Once the recovery occurs the state of the system is transposed to the working state. The failure and the recovery rates are exponentially distributed. Figure 1 exhibits the state-transition diagram for H_r/M/1/N queueing model.

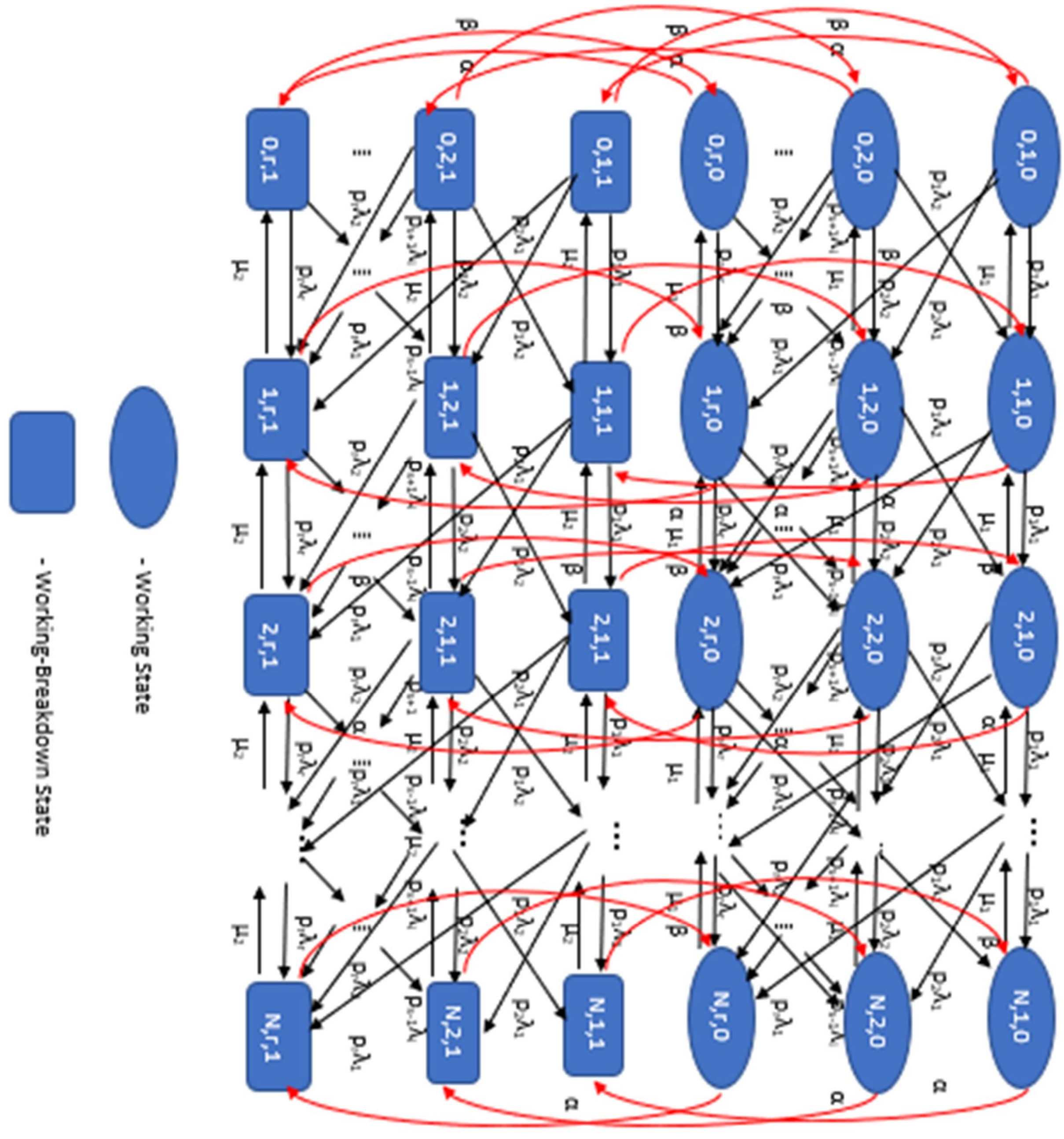


Figure 1: State-transition Diagram of H_r/M/1/N Queueing Model

The differential-difference equations for the time-dependent analysis of working and working-breakdown states for the H_r/M/1/N queueing model are obtained from the state-transition diagram and are given in equations (3.1) - (3.6).

WORKING STATE

$$\frac{dP_{0,i,0}(t)}{dt}=-(\lambda_i+\alpha)P_{0,i,0}(t)+\mu_1 P_{n+1,i,0}(t)+\beta P_{0,i,1}(t) \qquad i=1,2,...,r \qquad (3.1)$$

$$\frac{dP_{n,i,0}(t)}{dt}=-(\lambda_i+\mu_1+\alpha)P_{n,i,0}(t)+\mu_1 P_{n+1,i,0}(t)+p_k\sum_{i=1}^{r}\lambda_i P_{n-1,i,0}(t)+\beta P_{n,i,1}(t) \qquad n=1,2,...,N-1;\ k,i=1,2,...,r \qquad (3.2)$$

$$\frac{dP_{N,i,0}(t)}{dt}=-(\mu_1+\alpha)P_{N,i,0}(t)+p_k\sum_{i=1}^{r}\lambda_i P_{N-1,i,0}(t)+\beta P_{N,i,1}(t) \qquad k,i=2,...,r \qquad (3.3)$$

WORKING-BREAKDOWN STATE

$$\frac{dP_{0,i,1}(t)}{dt}=-(\lambda_i+\beta)P_{0,i,1}(t)+\mu_2 P_{n+1,i,1}(t)+\alpha P_{0,i,0}(t) \qquad i=1,2,...,r \qquad (3.4)$$

$$\frac{dP_{n,i,1}(t)}{dt}=-(\lambda_i+\mu_2+\beta)P_{n,i,1}(t)+\mu_2 P_{n+1,i,1}(t)+p_k\sum_{i=1}^{r}\lambda_i P_{n-1,i,1}(t)+\alpha P_{n,i,0}(t) \qquad n=1,2,...,N-1;\ k,i=1,2,...,r \qquad (3.5)$$

$$\frac{dP_{N,i,1}(t)}{dt}=-(\mu_2+\beta)P_{N,i,1}(t)+p_k\sum_{i=1}^{r}\lambda_i P_{N-1,i,1}(t)+\alpha P_{N,i,0}(t) \qquad k,i=2,...,r \qquad (3.6)$$

The initial conditions for the transient state is
$P_{0,1,0}(t)=1;\ P_{n,i,j}(t)=0,\ \forall\ n=0,1,2\ldots,N;\ i=1,2;\ j=0,1$ (3.7)

The total system probability, which is defined as the probability of having n units in the system at any moment $t \geq 0$, is given by

$$P_n(t)=P_{w,n}(t)+P_{wb,n}(t) \qquad (3.8)$$

The environmental states $P_{w,n}(t)$ and $P_{wb,n}(t)$ are the working normal and working-breakdown states, respectively.
The system reliability at time t is calculated as follows:

$$R(t)=\sum_{n=0}^{N}\sum_{i=1}^{r}\sum_{j=0}^{1}P_{n,i,j}(t) \qquad (3.9)$$

The system Availability at time t is calculated by considering all the working states is as follows:

$$A(t)=\sum_{n=0}^{N}\sum_{i=1}^{r}\sum_{j=0}P_{n,i,j}(t) \qquad (3.10)$$

The system Maintainability at time t is calculated by considering working-breakdown state which is calculated as follows:

$$M(t)=\sum_{n=0}^{N}\sum_{i=1}^{r}\sum_{j=1}P_{n,i,j}(t) \qquad (3.11)$$

IV. SPECIAL CASE –RAM FOR $H_2/M/1/4$ QUEUEING MODEL

The Hyperexponential 2-Phase queueing model with inter-arrival times and exponential service times with one server system and N=4 as the capacity of the system is taken into consideration. The RAM analysis of the $H_2/M/1/4$ queueing model is investigated. The governing equations of the working and working-breakdown states for the $H_2/M/1/4$ queueing model are given below:

WORKING STATE

$$\frac{dP_{0,1,0}(t)}{dt}=-(\lambda_1+\alpha)P_{0,1,0}(t)+\mu_1P_{1,1,0}(t)+\beta P_{0,1,1}(t) \quad (4.1)$$

$$\frac{dP_{0,2,0}(t)}{dt}=-(\lambda_2+\alpha)P_{0,2,0}(t)+\mu_1P_{1,2,0}(t)+\beta P_{0,2,1}(t) \quad (4.2)$$

$$\frac{dP_{1,1,0}(t)}{dt}=-(p_1\lambda_1+p_2\lambda_1+\mu_1+\alpha)P_{1,1,0}(t)+\mu_1P_{2,1,0}(t)+p_1\lambda_2P_{0,2,0}(t)+p_1\lambda_1P_{0,1,0}(t)+\beta P_{1,1,1}(t) \quad (4.3)$$

$$\frac{dP_{1,2,0}(t)}{dt}=-(p_1\lambda_2+p_2\lambda_2+\mu_1+\alpha)P_{1,2,0}(t)+\mu_1P_{2,2,0}(t)+p_2\lambda_1P_{0,1,0}(t)+p_2\lambda_2P_{0,2,0}(t)+\beta P_{1,2,1}(t) \quad (4.4)$$

$$\frac{dP_{2,1,0}(t)}{dt}=-(p_1\lambda_1+p_2\lambda_1+\mu_1+\alpha)P_{2,1,0}(t)+\mu_1P_{3,1,0}(t)+p_1\lambda_2P_{1,2,0}(t)+p_1\lambda_1P_{1,1,0}(t)+\beta P_{2,1,1}(t) \quad (4.5)$$

$$\frac{dP_{2,2,0}(t)}{dt}=-(p_1\lambda_2+p_2\lambda_2+\mu_1+\alpha)P_{2,2,0}(t)+\mu_1P_{3,2,0}(t)+p_2\lambda_1P_{1,1,0}(t)+p_2\lambda_2P_{1,2,0}(t)+\beta P_{2,2,1}(t) \quad (4.6)$$

$$\frac{dP_{3,1,0}(t)}{dt}=-(p_1\lambda_1+p_2\lambda_1+\mu_1+\alpha)P_{3,1,0}(t)+\mu_1P_{4,1,0}(t)+p_1\lambda_2P_{2,2,0}(t)+p_1\lambda_1P_{2,1,0}(t)+\beta P_{3,1,1}(t) \quad (4.7)$$

$$\frac{dP_{3,2,0}(t)}{dt}=-(p_1\lambda_2+p_2\lambda_2+\mu_1+\alpha)P_{3,2,0}(t)+\mu_1P_{4,2,0}(t)+p_2\lambda_1P_{2,1,0}(t)+p_2\lambda_2P_{2,2,0}(t)+\beta P_{3,2,1}(t) \quad (4.8)$$

$$\frac{dP_{4,1,0}(t)}{dt}=-(\mu_1+\alpha)P_{4,1,0}(t)+p_1\lambda_2P_{3,2,0}(t)+p_1\lambda_1P_{3,1,0}(t)+\beta P_{4,1,1}(t) \quad (4.9)$$

$$\frac{dP_{4,2,0}(t)}{dt}=-(\mu_1+\alpha)P_{4,2,0}(t)+p_2\lambda_2P_{3,2,0}(t)+\beta P_{4,2,1}(t) \quad (4.8)$$

WORKING-BREAKDOWN STATE

$$\frac{dP_{0,1,1}(t)}{dt}=-(\lambda_1+\beta)P_{0,1,1}(t)+\mu_2P_{1,1,1}(t)+\alpha P_{0,1,0}(t) \quad (4.9)$$

$$\frac{dP_{0,2,1}(t)}{dt}=-(\lambda_2+\beta)P_{0,2,1}(t)+\mu_2P_{1,2,1}(t)+\alpha P_{0,2,0}(t) \quad (4.10)$$

$$\frac{dP_{1,1,1}(t)}{dt}=-(p_1\lambda_1+p_2\lambda_1+\mu_2+\beta)P_{1,1,0}(t)+\mu_2P_{2,1,1}(t)+p_1\lambda_2P_{0,2,1}(t)+p_1\lambda_1P_{0,1,1}(t)+\alpha P_{1,1,0}(t) \quad (4.11)$$

$$\frac{dP_{1,2,1}(t)}{dt}=-(p_1\lambda_2+p_2\lambda_2+\mu_2+\beta)P_{1,2,1}(t)+\mu_2P_{2,2,1}(t)+p_2\lambda_1P_{0,1,1}(t)+p_2\lambda_2P_{0,2,1}(t)+\alpha P_{1,2,0}(t) \quad (4.12)$$

$$\frac{dP_{2,1,1}(t)}{dt}=-(p_1\lambda_1+p_2\lambda_1+\mu_2+\beta)P_{2,1,0}(t)+\mu_2P_{3,1,1}(t)+p_1\lambda_2P_{1,2,1}(t)+p_1\lambda_1P_{1,1,1}(t)+\alpha P_{2,1,0}(t) \quad (4.13)$$

$$\frac{dP_{2,2,1}(t)}{dt}=-(p_1\lambda_2+p_2\lambda_2+\mu_2+\beta)P_{2,2,1}(t)+\mu_2P_{3,2,1}(t)+p_2\lambda_1P_{1,1,1}(t)+p_2\lambda_2P_{1,2,1}(t)+\alpha P_{2,2,0}(t) \quad (4.14)$$

$$\frac{dP_{3,1,1}(t)}{dt}=-(p_1\lambda_1+p_2\lambda_1+\mu_2+\beta)P_{3,1,0}(t)+\mu_2P_{4,1,1}(t)+p_1\lambda_2P_{2,2,1}(t)+p_1\lambda_1P_{2,1,1}(t)+\alpha P_{3,1,0}(t) \quad (4.15)$$

$$\frac{dP_{3,2,1}(t)}{dt}=-(p_1\lambda_2+p_2\lambda_2+\mu_2+\beta)P_{3,2,1}(t)+\mu_2P_{4,2,1}(t)+p_2\lambda_1P_{2,1,1}(t)+p_2\lambda_2P_{2,2,1}(t)+\alpha P_{3,2,0}(t) \quad (4.16)$$

$$\frac{dP_{4,1,1}(t)}{dt}=-(\mu_2+\beta)P_{4,1,1}(t)+p_1\lambda_2P_{3,2,1}(t)+p_1\lambda_1P_{3,1,1}(t)+\alpha P_{4,1,0}(t) \quad (4.17)$$

$$\frac{dP_{4,2,1}(t)}{dt}=-(\mu_2+\beta)P_{4,2,1}(t)+p_2\lambda_2P_{3,2,1}(t)+\alpha P_{3,2,0}(t) \quad (4.18)$$

A. Numerical Illustration

The basis of this section is to examine the RAM analysis of the transient state of the $H_2/M/1/4$ Queueing model by using the governing equations from (4.1) -(4.18). For ease of computation the capacity of the system is assumed as N=4 for the time range of t=0 to t=200 and for standard parametric values are taken as λ_1=0.05, λ_2=0.03 μ_1=0.09, μ_2=0.07, α=0.009, β=0.007 p_1=0.6 and p_2=0.4. The equations are solved by using fourth order Runge-Kutta numerical method andthe transient state probabilities of the queueing system are obtained. The probability trend curves for $H_2/M/1/4$ queueing model are represented in Figure 2. The RAM analysis for $H_2/M/1/4$ is also obtained and represented in Figures 3-5.

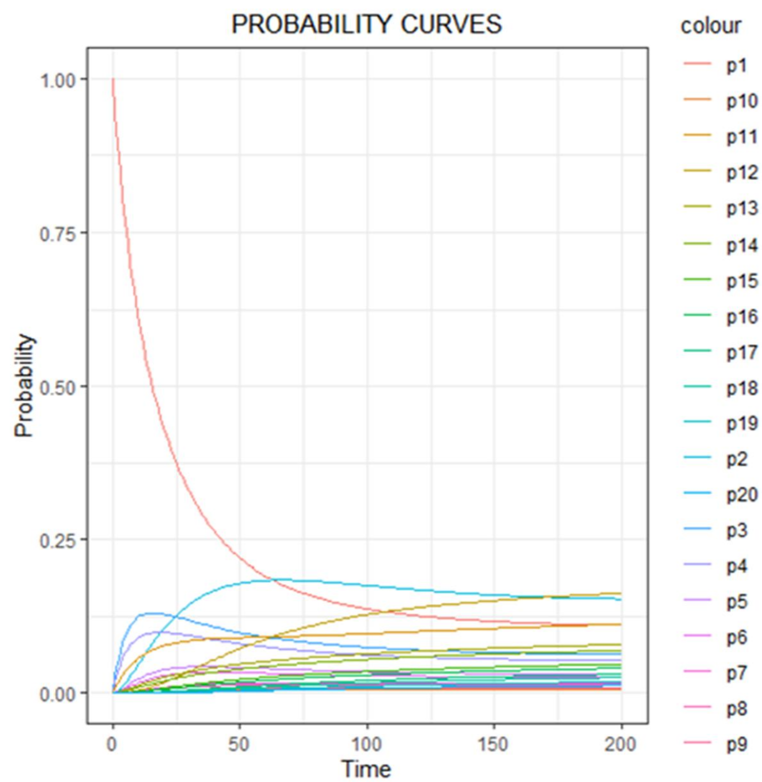
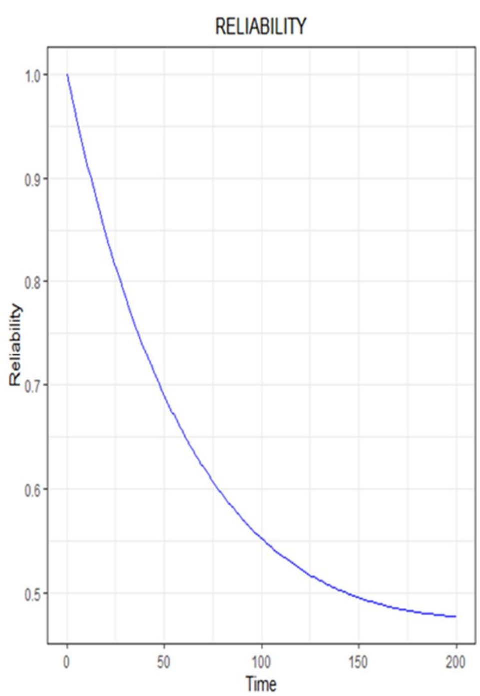


Figure 2: Probability Trend Curves H_2/M/1/4 Queueing Model

Figure 2 shows the probability trend curves that help to understand the distribution trend of the system probabilities for the specified time intervals.



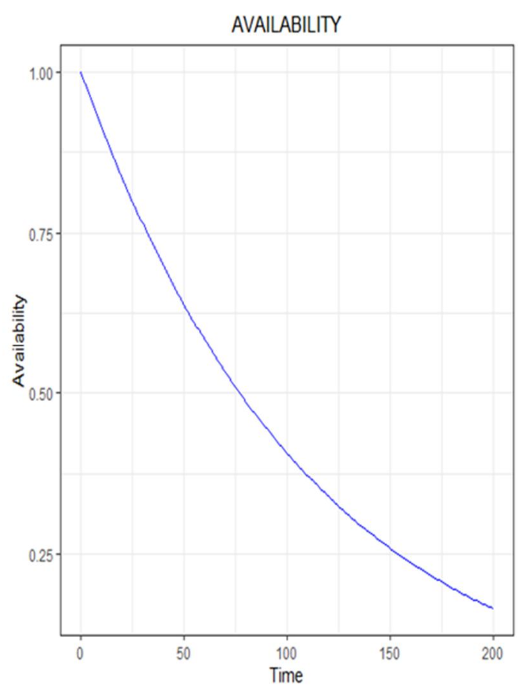


Figure 3: Reliability of H_2/M/1/4 Queueing Model   Figure 4: Availability of H_2/M/1/4 Queueing Model

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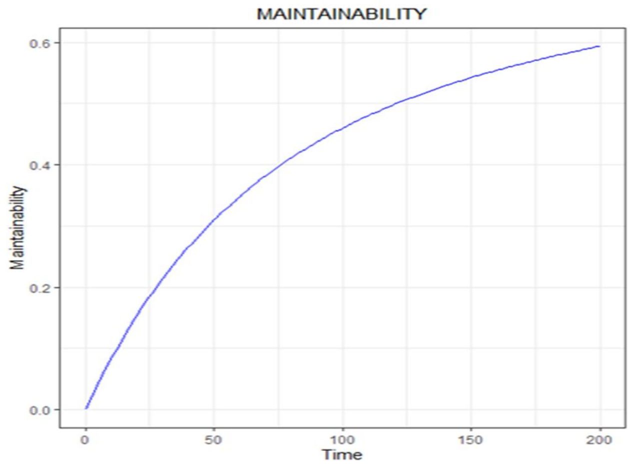

Figure 5: Maintainability of $H_2/M/1/4$ Queueing Model

Figure 3, represents the Reliability of the $H_2/M/1/4$ Queueing model. It was observed from the figure that as the time increases the Reliability of the system decreases. It was also found out that whentime reaches 200 the Reliability of the system will be approximately equal to 57%.

Figure 4, shows the Availability of the $H_2/M/1/4$ Queueing model. It is seen from the figure as the time range increases the Availability of the system decreases. When t=200 the Availability of the system is approximately equal to 17%.

Figure 5, depicts the Maintainability of the $H_2/M/1/4$ Queueing model. From the figure it was seen that as time increases Maintainability of the system increases. It is observed that in t=200, Maintainability is found out to be 42%. The $H_2/M/1/4$ Queueing model's special metrics, such as MTBF and MTTR, were determined. The mean time between failures, or the average time between queueing system breakdowns, is 14 hours, while the average time to recover from a queueing system breakdown is 11 hours.

V. SENSITIVITY ANALYSIS

A. *Sensitivity Analysis For Different Arrival And Service Rates*

In this section, the performance of the system for the Reliability, Availability and Maintainability of the $H_2/M/1/4$ queueing model is measured. This performance measure is studied by using the sensitivityanalysis changing the parametric values of λ_1, λ_2, μ_1, α, β, p_1, p_2 for which the results are attained which are given in Tables 1-4.

Table 1: Sensitivity Analysis for different Arrival and Failure rates

Arrival Rate Vs Failure Rate										
TIME		λ_1=0.05& λ_2=0.02			λ_1=0.06& λ_2=0.03			λ_1=0.07& λ_2=0.04		
		R(t)	M(t)	A(t)	R(t)	M(t)	A(t)	R(t)	M(t)	A(t)
	α=0.008	0.7593	0.2398	0.7261	0.7602	0.2382	0.7289	0.7681	0.2378	0.7727
40	α=0.009	0.7340	0.2650	0.6976	0.7439	0.2645	0.6993	0.7498	0.2620	0.7071
	α=0.01	0.7097	0.2893	0.6703	0.7106	0.2885	0.6728	0.7193	0.2863	0.6877
80	α=0.008	0.6833	0.3157	0.6187	0.6912	0.3016	0.6264	0.7031	0.2960	0.6311
	α=0.009	0.6527	0.3464	0.5827	0.6646	0.3345	0.5984	0.6735	0.3241	0.6050
	α=0.01	0.6237	0.3756	0.5488	0.6316	0.3678	0.5546	0.6426	0.3577	0.5644
120	α=0.008	0.6271	0.3728	0.5273	0.6311	0.3671	0.5321	0.6472	0.3565	0.5427
	α=0.009	0.5937	0.4066	0.4868	0.6038	0.3954	0.4986	0.6199	0.3813	0.5085
	α=0.01	0.5626	0.4382	0.4493	0.5757	0.4203	0.4549	0.5869	0.4140	0.4649
160	α=0.008	0.5856	0.4160	0.4493	0.5979	0.4013	0.4549	0.6056	0.3948	0.4641
	α=0.009	0.5511	0.4513	0.4066	0.5645	0.4454	0.4164	0.5732	0.4369	0.4263
	α=0.01	0.5194	0.4840	0.3679	0.5219	0.4889	0.3768	0.5367	0.4796	0.3867
200	α=0.008	0.5551	0.4487	0.3829	0.5649	0.4356	0.3988	0.5734	0.4539	0.4037
	α=0.009	0.5204	0.4847	0.3396	0.5315	0.4797	0.3495	0.5402	0.5027	0.3534
	α=0.01	0.4890	0.5177	0.3012	0.4974	0.5060	0.3111	0.5093	0.5387	0.3210

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Table 1 depicts the changes in the system's Reliability, Availability, and Maintainability for various sets of arrival and failure rates while holding the other parameters constant. It is discovered that as the failure rate increases while keeping the arrival rate constant, the system's reliability and availability decrease while its maintainability increases. For different time intervals, as the arrival rate increases while the failure rate remains constant, the system's reliability and availability increase while its maintainability decreases.

Table 2: Sensitivity Analysis for different Service and Failure rates

SERVICERATEVs FAILURE RATE										
TIME		μ_1=0.08			μ_1=0.09			μ_1=0.1		
		R(t)	M(t)	A(t)	R(t)	M(t)	A(t)	R(t)	M(t)	A(t)
40	α=0.008	0.7592	0.2396	0.7261	0.7631	0.2389	0.7270	0.7694	0.2302	0.7299
	α=0.009	0.7339	0.2648	0.6977	0.7434	0.2631	0.6986	0.7544	0.2614	0.6995
	α=0.01	0.7094	0.2891	0.6703	0.7197	0.2865	0.6722	0.7298	0.2838	0.6761
80	α=0.008	0.6832	0.3149	0.6188	0.6983	0.3057	0.6217	0.7034	0.2964	0.6386
	α=0.009	0.6526	0.3455	0.5827	0.6657	0.3344	0.5986	0.6758	0.3241	0.6025
	α=0.01	0.6236	0.3745	0.5488	0.6323	0.3674	0.5547	0.6428	0.3572	0.5646
120	α=0.008	0.6269	0.3710	0.5273	0.6327	0.3672	0.5322	0.6423	0.3573	0.5421
	α=0.009	0.5935	0.4045	0.4868	0.6037	0.3958	0.4988	0.6159	0.3969	0.5048
	α=0.01	0.5623	0.4357	0.4493	0.5766	0.4231	0.4542	0.5868	0.4134	0.4641
160	α=0.008	0.5851	0.4128	0.4494	0.5984	0.4043	0.4543	0.6059	0.3956	0.4642
	α=0.009	0.5505	0.4475	0.4066	0.5659	0.4342	0.4105	0.5752	0.4256	0.4204
	α=0.01	0.5187	0.4795	0.3679	0.5211	0.4673	0.3768	0.5314	0.4589	0.3867
200	α=0.008	0.5541	0.4438	0.3829	0.5657	0.4346	0.3788	0.5751	0.4241	0.3827
	α=0.009	0.5193	0.4788	0.3396	0.5322	0.4788	0.3436	0.5424	0.4685	0.3536
	α=0.01	0.4877	0.5108	0.3012	0.4983	0.5019	0.3112	0.5048	0.4957	0.3212

Table 2 shows the changes in the system's Reliability, Availability, and Maintainability for various sets of failure rates and service rates of the Working State while holding the other parameters constant. As the failure rates increase while the service rate remains constant, the system's reliability and availability decrease while its maintainability increases. Furthermore, as the service rates increase while the failure rate remains constant, the system's reliability and availability increase while its maintainability decreases.

Table 3: Sensitivity Analysis for different Arrival and Recovery rates

ARRIVAL RATE Vs RECOVERY RATE							
TIME		λ_1=0.05& λ_2=0.02		λ_1=0.06&λ_2=0.03		λ_1=0.07& λ_2=0.04	
		R(t)	M(t)	R(t)	M(t)	R(t)	M(t)
	β=0.006	0.7292	0.2698	0.7271	0.2709	0.7250	0.2724
40	β=0.007	0.7340	0.2650	0.7329	0.2694	0.7308	0.2702
	β=0.008	0.7387	0.2603	0.7366	0.2668	0.7325	0.2659
80	β=0.006	0.6438	0.3554	0.6347	0.3656	0.6246	0.3753
	β=0.007	0.6527	0.3464	0.6426	0.3545	0.6355	0.3671
	β=0.008	0.6613	0.3376	0.6512	0.3438	0.6461	0.3583
120	β=0.006	0.5807	0.4200	0.5786	0.4329	0.5685	0.4422
	β=0.007	0.5937	0.4066	0.5898	0.4183	0.5799	0.4213
	β=0.008	0.6062	0.3939	0.5963	0.4053	0.5864	0.4181
160	β=0.006	0.5340	0.4689	0.5233	0.4774	0.5131	0.4806
	β=0.007	0.5511	0.4513	0.5455	0.4654	0.5532	0.4769
	β=0.008	0.5673	0.4347	0.5567	0.4433	0.5464	0.4543
200	β=0.006	0.4997	0.5062	0.4857	0.5140	0.4723	0.5261
	β=0.007	0.5204	0.4847	0.5125	0.4917	0.5022	0.5027
	β=0.008	0.5399	0.4646	0.5249	0.4709	0.5147	0.4809

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Table 3 represents the changes in the system's Reliability and Maintainability for various sets of Arrival rate and Recovery rate while holding the other parameters constant. When the recovery rate is increased while keeping the arrival rate constant, the system's reliability increases while its maintainability decreases. Furthermore, for different time intervals, as the arrival rate increases, the system's Reliability decreases, whereas the system's Maintainability increases by maintaining the recovery rate constant.

Table 4: Sensitivity Analysis for differentServiceand Recovery rates

SERVICE RATE Vs RECOVERY RATE							
TIME		μ_1=0.08		μ_1=0.09		μ_1=0.1	
		R(t)	M(t)	R(t)	M(t)	R(t)	M(t)
	β=0.006	0.7287	0.2701	0.7292	0.2693	0.7299	0.2684
40	β=0.007	0.7328	0.2653	0.7339	0.2645	0.7364	0.2636
	β=0.008	0.7365	0.2606	0.7386	0.2598	0.7397	0.2589
80	β=0.006	0.6245	0.3581	0.6337	0.3456	0.6439	0.3327
	β=0.007	0.6353	0.3490	0.6426	0.3365	0.6528	0.3237
	β=0.008	0.6469	0.3402	0.6512	0.3278	0.6614	0.3151
120	β=0.006	0.5680	0.4270	0.5706	0.4129	0.5812	0.4062
	β=0.007	0.5790	0.4132	0.5838	0.4083	0.5944	0.3928
	β=0.008	0.5864	0.4001	0.5963	0.3953	0.6070	0.3801
160	β=0.006	0.5130	0.4819	0.5243	0.4734	0.5356	0.4644
	β=0.007	0.5355	0.4634	0.5415	0.4554	0.5529	0.4467
	β=0.008	0.5466	0.4459	0.5577	0.4383	0.5692	0.4201
200	β=0.006	0.4893	0.5264	0.4957	0.5140	0.5029	0.5011
	β=0.007	0.4959	0.5033	0.5015	0.4917	0.5239	0.4896
	β=0.008	0.5138	0.4818	0.5209	0.4709	0.5435	0.4594

Table 4 depicts the changes in the system's Reliability and Maintainability for various sets of service rates and recovery rates while holding all other parameters constant. The table shows that as the Recovery rate increases while keeping the Service rate constant, the system's reliability increases while its maintainability decreases. It is discovered that for different time intervals while keeping the recovery rate constant, the system's reliability increases as the service rate increases, whereas the system's maintainability decreases.

B. Sensitivity Analysis For Different Failure And Recovery Rates

The graphical representations of RAM analysis for the $H_2/M/1/4$ for different failure and recovery rates are shown in Figures 6-10.

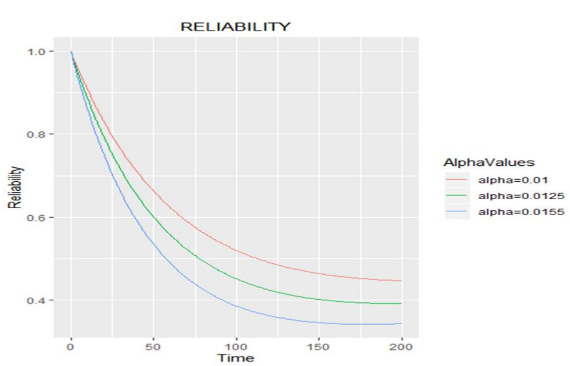

Figure 6: Reliability for Different Failure Rates

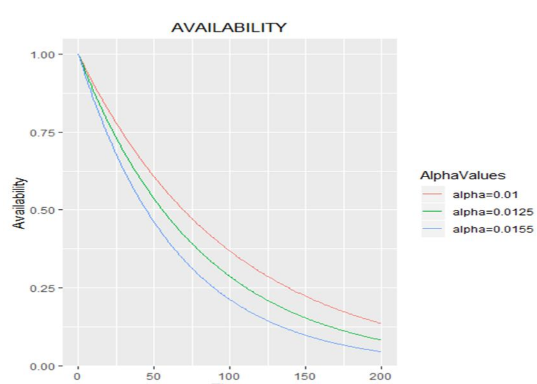

Figure 7: Availability for Different Failure Rates

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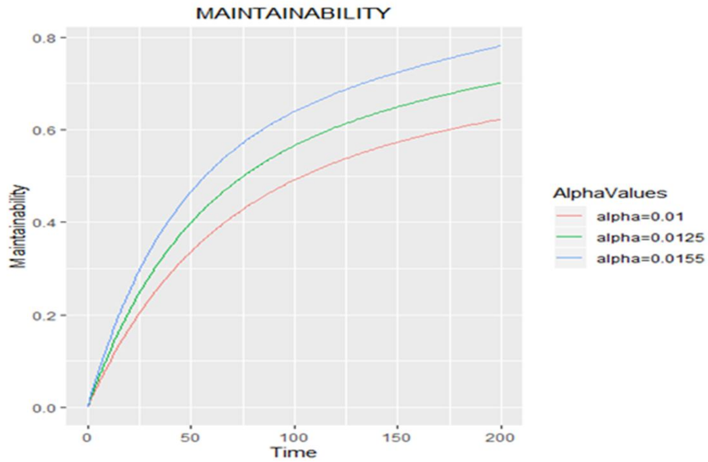
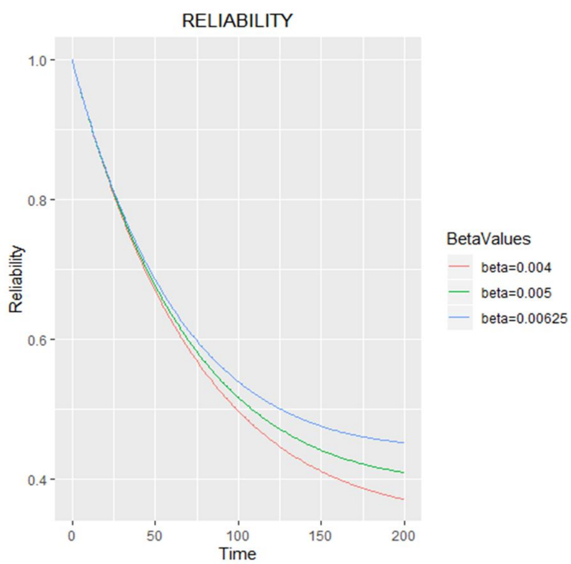


Figure 8: Maintainability for Different Failure Rates

Figures 6, 7, and 8 show the Reliability, Availability, and Maintainability for various failure rates of 0.01, 0.0125, and 0.0155, respectively, while keeping the other parameters constant. The graph shows that as the failure rate is increased, the system's reliability and availability decrease while its maintainability increases. As the failure rate increases by 25%, the average change in system reliability and availability decreases by 12% and 42%, respectively, while maintainability increases by 11%.



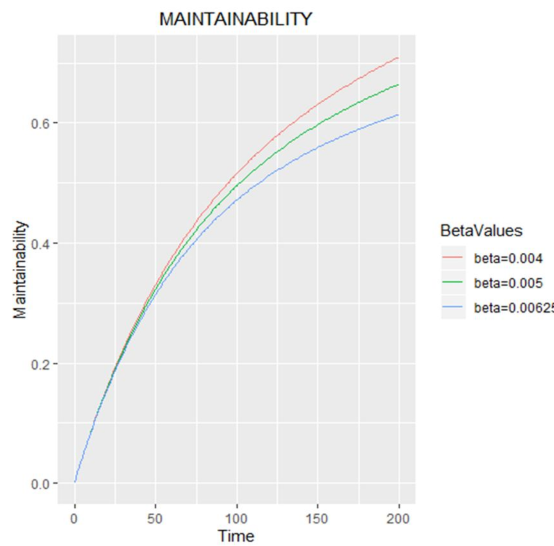


Figure 9:Reliability for Different Recovery Rates   Figure 10:Maintainability for Different Recovery Rates

Figures 9, 10 represents the Reliability and Maintainability for different sets of recovery rates as 0.004, 0.005, 0.00625, while other parameters are kept constant. It has been noticed that when the recovery rate increases, the system's reliability increases yet its maintainability decreases. The average change in the systems Reliability is increased by 10%, while the Maintainability is decreased by 7% when the recovery rates are increased by 25%.

VI. CONCLUSION

The RAM analysis of the $H_r/M/1/N$ Queueing model with two different environmental conditions is examined in this chapter. The state-transition diagram is used to obtain the differential-difference equations for the transient state of the r-phase Hyperexponential queueing model. Hyperexponential 2-phase inter-arrival times and exponential service times are assessed for a single server system with finite capacity as a specific case. The differential-difference equations for the $H_2/M/1/4$ queueing model with working and working-breakdown states are solved using the fourth order Runge-Kutta numerical method.

The following are the outcomes:

1) It was discovered that as time passes, the reliability and availability of the system decreases, while the maintainability of the system increases.
2) When t=200, the system's Reliability and Availability were found to be 57 percent and 17 percent, respectively, whilst the system's Maintainability was determined to be 42 percent.
3) The average time between queueing system failures, or mean time between failures, is 14 hours, whereas the average time to recover from a queueing system failure is 11 hours.
4) For a fixed Failure rate, the Sensitivity Analysis was used to examine various sets of Arrival and Service rates. The H2/M/1/4 Queueing model increases the system's reliability and availability while decreasing its maintainability.
5) For a fixed recovery rate, the system's Reliability decreases as the arrival rates increase, while the system's Maintainability increases, and for a growing service rate, the system's Reliability increases while the Maintainability decreases.
6) When the failure rates increase by 25%, the average change in system reliability and availability decreases by 12% and 42%, respectively, while the system's maintainability increases by 11%.
7) It has been discovered that for every 25% increase in recovery rate, the average change in system reliability increases by 10%, while system maintainability decreases by 7%.

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