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Study of Laminar Flow between Parallel Plates via Gupta Integral Transform

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Abstract: Viscosity is the characteristic of a fluid by virtue of which viscous force becomes active when the fluid is in motion and opposes the relative motion of its different layers. This viscous force becomes active when the different layers of the fluid are moving with different velocities, and leads to shearing stress between the layers of the fluid in motion. This paper illustrates the application of Gupta integral transform for studying the unidirectional Laminar Flow between parallel plates directly without finding the general solution of a differential equation relating flow characteristics equation of the viscous liquid. In this paper, Gupta integral transform is applied for solving the differential equation relating flow characteristics of the viscous liquid to obtain the velocity and shear stress distributions of a unidirectional Laminar Flow between the stationary parallel plates as well as between the parallel plates having a relative motion.
Index Terms: Laminar flow; Gupta integral transform; Parallel plates; Shear and Velocity distributions; viscous fluid.

I. INTRODUCTION

The steady flow of a viscous fluid over a horizontal surface in the form of layers of different velocities in which the particles of the fluid move in a regular and well-defined paths is known as laminar flow. A velocity gradient exists between the two layers due to relative velocity and as a result, a shear stress acts on the layers. Seepage through soils, the flow of crude oil and highly viscous fluids through narrow passages are some of the examples of the laminar flow. In such a flow the fluid properties remain unchanged in the directions perpendicular to the direction of flow of the fluid [1-5].

Let g(y) be a continuous function on any interval for y ≥ 0. The Gupta Transform of g(y) is defined as $\dot{R}\{g(y)\} = \frac{1}{q^3}\int_0^\infty e^{-qy}\, g(y)\,dy = G(q)$ ....(1), provided that the integral is convergent, where q may be a real or complex parameter and Ṙ is the Gupta Transform operator [6-12].

The Gupta Transform of some elementary functions [7-11] are

$$\dot{R}\{y^n\} = \frac{n!}{q^{n+4}}\ , where\ n = 0,1,2,3 \ldots\ldots\ldots$$

$$\dot{R}\{e^{by}\} = \frac{1}{q^3(q-b)}\ ,\ r > b$$

$$\dot{R}\{sinby\} = \frac{b}{q^3(q^2+b^2)}\ ,\ r > 0$$

$$\dot{R}\{sinhby\} = \frac{b}{q^3(q^2-b^2)}\ ,\ r > |b|$$

$$\dot{R}\{cosby\} = \frac{1}{q^2(q^2+b^2)}\ ,\ r > 0$$

$$\dot{R}\{coshby\} = \frac{1}{q^2(q^2-b^2)}\ ,\ r > |b|$$

$$\dot{R}\{\delta(y-b)\} = \frac{1}{q^4}\ e^{-bq}$$

A. Gupta Transform of derivatives

Let g(y) is continuous function and is piecewise continuous on any interval, then the Gupta Transform [8-10] of first derivative of g(y) i.e. $\dot{R}\{g'(y)\}$ is given by

$$\dot{R}\{g'(y)\} = \frac{1}{q^3}\int_0^\infty e^{-qy}\, g'(y)\,dy \ldots\ldots..(2)$$

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Integrating by parts and applying limits, we get

$$\dot{R}\{g'(y)\} = \frac{1}{q^3}\{-g(0) - \int_0^\infty -qe^{-qy}\, g(y)\,dy\}$$

$$= \frac{1}{q^3}\{-g(0) + q\int_0^\infty e^{-qy}\, g(y)\,dy\}$$

$$= qG(q) - \frac{1}{q^3}g(0)$$

Hence $\dot{R}\{g'(y)\} = qG(q) - \frac{1}{q^3}g(0)$ ……….(3)

Since $\dot{R}\{g'(y)\} = q\dot{R}\{g(y)\} - \frac{1}{q^3}g(0)$, Therefore, on replacing g(y) by $g'(y)$ and

$g'(y)$ by $g''(y)$, we have

$$\dot{R}\{g''(y)\} = q\dot{R}\{g'(y)\} - \frac{1}{q^3}g'(0)$$

$$= q\left\{q\dot{R}\{g(y)\} - \frac{1}{q^3}g(0)\right\} - \frac{1}{q^3}g'(0)$$

$$= q^2\dot{R}\{g(y)\} - \frac{1}{q^2}g(0) - \frac{1}{q^3}g'(0)$$

$$= q^2G(q) - \frac{1}{q^2}g(0) - \frac{1}{q^3}g'(0)$$

Hence $\dot{R}\{g''(y)\} = q^2G(q) - \frac{1}{q^2}g(0) - \frac{1}{q^3}g'(0)$ and so on.

II. MATERIAL AND METHOD

Consider a steady and uniform laminar flow of the viscous fluid between the two flat parallel plates situated at a perpendicular distance L. Let the distance in which the fluid is flowing be represented by x and the distance which is normal to the flow of fluid and parallel to the plane of paper be represented by y such that the lower plate is situated at y = 0 and the upper plate is situated at y = L [4-5].

The flow characteristics equation of motion of a viscous fluid is given by [5]

$\mu\,\ddot{U}(y) = \frac{dP}{dx}$ ……….. (4)

We will study the laminar flow of viscous fluid on the basis of following assumptions:

i) The flow is steady and incompressible and the properties of the fluid do not vary in the directions normal to the direction of flow of the fluid.
ii) There are no end effects of the surfaces on the viscous fluid.
iii) There is no relative velocity of the fluid with respect to the surfaces of the plates.
iv) There is a uniform effective pressure gradient in the direction of flow of fluid i.e. $\frac{dP}{dx}$ is a constant in the x–direction.

Now taking Gupta Transform of equation (4), we get

$\dot{R}[\mu\,\ddot{U}(y)] = \frac{dP}{dx}\ \dot{R}[1]$

This equation results

$\mu[r^2\overline{U}(r) - \frac{1}{r^2}U(0) - \frac{1}{r^3}\ \dot{U}(0)] = \frac{1}{r^4}\frac{dP}{dx}$ ……… (5)

A. *For Laminar Flow Between Stationary (Fixed) Parallel Plates*

Considering the flow of fluid between two parallel fixed plates, we can write the relevant boundary conditions as given below [5]:

At y = 0 and y = L, $U = 0$.

Applying boundary condition: $U(0) = 0$, equation (5) becomes,

$\mu[r^2\overline{U}(r) - \frac{1}{r^3}\dot{U}(0)] = \frac{1}{r^4}\frac{dP}{dx}$ ….. (6)

In this equation, $\dot{U}(0)$ is some constant so let us substitute $\dot{U}(0) = \varepsilon$. Also, since $\frac{dP}{dx}$ is uniform, therefore, put $\frac{dP}{dx} = -\emptyset$, where $\emptyset$ is a constant and negative sign indicates that the pressure of fluid decreases in the direction of flow of the fluid.

Equation (3) becomes

$\mu[r^2\overline{U}(r) - \frac{1}{r^3}\varepsilon] = -\frac{1}{r^4}\emptyset$

Or $\overline{U}(r) = \frac{1}{r^5}\varepsilon - \frac{1}{r^6}\frac{\emptyset}{\mu}$……….. (7)

Taking inverse Gupta transform [9-10] of equation (7), we get

$U(y) = \varepsilon y - \frac{\emptyset}{2\mu}y^2$……….. (8)

1) Determination of the Constant ε

To find the value of constant ε, applying boundary condition: $U(L) = 0$, equation (8) provides,

$0 = \varepsilon L - \frac{\emptyset}{2\mu}L^2$

Upon rearranging and simplification of the above equation, we get

$\varepsilon = \frac{\emptyset}{2\mu}L$ ……….. (9)

Substitute the value of ε from equation (9) in equation (8), we get

$U(y) = \frac{\emptyset}{2\mu}L y - \frac{\emptyset}{2\mu}y^2$

Or $U(y) = \frac{\emptyset}{2\mu}[L y - y^2]$……… (10)

Differentiating equation (10) w.r.t. y, we get

$\dot{U}(y) = \frac{\emptyset}{2\mu}[L - 2y]$ …….. (11)

For maximum velocity, $\dot{U}(y) = 0$

This results

$y = \frac{L}{2}$……………. (12)

Put the value of y from equation (12) in equation (10), we get

$$U_{max} = \frac{\emptyset}{2\mu}\frac{L^2}{4}$$

Or

$U_{max} = \frac{\emptyset}{8\mu}L^2$ ………. (13)

The shear stress distribution is determined by the application of Newton's law of viscosity as

$\tau(y) = \mu\,\dot{U}(y)$

Using equation (8), we get

$\tau(y) = \frac{\emptyset}{2}[L - 2y]$……….. (14)

At $y = \frac{L}{2}$ i.e. at the mid of the fixed parallel plates, $\tau\left(\frac{L}{2}\right) = \frac{\emptyset}{2}[L - 2\frac{L}{2}] = 0$ i.e. there is no shear stress even when there is constant pressure gradient.

At $y = 0$ i.e. at the surface of the lower fixed plate, $\tau(0) = \frac{\emptyset}{2}L$

At $y = L$ i.e. at the surface of the upper fixed plate, $\tau(L) = -\frac{\emptyset}{2}L$

For a particular case, when $\emptyset = 0, \tau(y) = 0$ i.e. there is no shear stress between the fixed parallel plates if there is no pressure gradient.

B. For Laminar Flow Between Parallel Plates Having Relative Motion

Considering the flow of fluid between the parallel flat plates such that the lower plate is fixed at y = 0 and upper plate is moving uniformly with velocity U_o relative to the lower fixed plate in the direction of flow of the fluid, we can write the relevant boundary conditions as given below [5]:

At y = 0, $U = 0$ and at y = L, $U = U_o$.

Applying boundary condition: $U(0) = 0$, equation (5) becomes,

$\mu[r^2\overline{U}(r) - \frac{1}{r^3}\dot{U}(0)] = -\frac{1}{r^4}\ \emptyset$ ….. (15)

In this equation, $\dot{U}(0)$ is some constant.

Let us substitute $\dot{U}(0) = \delta$,

Equation (15) becomes

$$\mu[r^2\overline{U}(r) - \frac{1}{r^3}\delta\,] = -\frac{1}{r^4}\ \emptyset$$

Or $\overline{U}(r) = \frac{1}{r^5}\ \delta - \frac{\emptyset}{\mu}\frac{1}{r^6}$……….. (16)

Taking inverse Gupta transform [8-9] of equation (16), we get

$U(y) = \delta\ y - \frac{\emptyset}{2\mu}y^2$……….. (17)

1) Determination of the Constant δ

To find the value of constant δ, applying boundary condition: $U(L) = U_o$, equation (17) provides,

$U_o = \delta\ L - \frac{\emptyset}{2\mu}L^2$

Upon rearranging and simplification of the above equation, we get

$\delta = \frac{U_o}{L} + \frac{\emptyset}{2\mu}L$ ……….. (18)

Substitute the value of δ from equation (18) in equation (17), we get

$U(y) = [\frac{U_o}{L} + \frac{\emptyset}{2\mu}L]\ y - \frac{\emptyset}{2\mu}y^2$

Or $U(y) = \frac{U_o}{L}y + \frac{\emptyset}{2\mu}[L\ y - y^2]$ … (19)

This equation (19) confirms that the velocity distribution is parabolic with minimum at the lower fixed plate.

Differentiating equation (19) w.r.t. y, we get

$\dot{U}(y) = \frac{U_o}{L} + \frac{\emptyset}{2\mu}[L - 2y]$…… (20)

For maximum velocity, $\dot{U}(y) = 0$

This results

$y = \frac{L}{2} - \frac{\mu U_o}{L\emptyset}$……………. (21)

Put the value of y from equation (21) in equation (19) and simplifying, we get

$U_{max} = \frac{\mu {U_o}^2}{L^2\ \emptyset}$ ………. (22)

The shear stress distribution is determined by the application of Newton's law of viscosity as

$\tau(y) = \mu\,\dot{U}(y)$

Using equation (20), we get

$\tau(y) = \frac{\mu U_o}{L} + \frac{\emptyset}{2}[L - 2y]$…… (23)

At $y = \frac{L}{2}$ i.e. at the mid of the flow passage, $\tau\left(\frac{L}{2}\right) = \frac{\mu U_o}{L}$

At $y = 0$ i.e. at the surface of the lower plate, $\tau(0) = \frac{\mu U_o}{L} + \frac{\emptyset}{2}L$

At $y = L$ i.e. at the surface of the upper plate, $\tau(L) = \frac{\mu U_o}{L} - \frac{\emptyset}{2}L$

For a particular case, when $\emptyset = 0$, $\tau(y) = \frac{\mu U_o}{L}$ i.e. the shear stress between the plates is not zero and having a constant value even if there is no pressure gradient.

III. CONCLUSION

In this paper, we have studied the unidirectional Laminar Flow between stationary parallel plates and have successfully obtained the velocity and shear stress distributions of a unidirectional Laminar Flow between stationary parallel plates as well as between parallel plates having a relative motion by solving the differential equation describing the flow characteristics of a viscous fluid via Gupta integral transform. Thus, Gupta integral transform has presented a powerful tool for obtaining the solution of the differential equation representing flow characteristics without finding the general solution. It is concluded that, in the case of unidirectional Laminar Flow with constant pressure gradient between stationary parallel plates, the velocity distribution is maximum at the midway between the parallel plates and decreases parabolically with maximum value at the midway between the parallel plates to a minimum value at the lower fixed plate as well as at the upper fixed plate but the shear stress varies linearly with a minimum value at the midway between the parallel plates to a maximum value at the lower fixed plate as well as at the upper fixed plate. In the case of laminar flow with constant pressure gradient between parallel plates having a relative motion, the velocity distribution is parabolic with a minimum at the lower fixed plate but the shear stress varies linearly and at the midway between the parallel plates having a relative motion it is equal to the mean of the values of the shear stresses at the lower fixed plate and at the uniformly moving upper plate, and having a constant value even if there is no pressure gradient between parallel plates having a relative motion.

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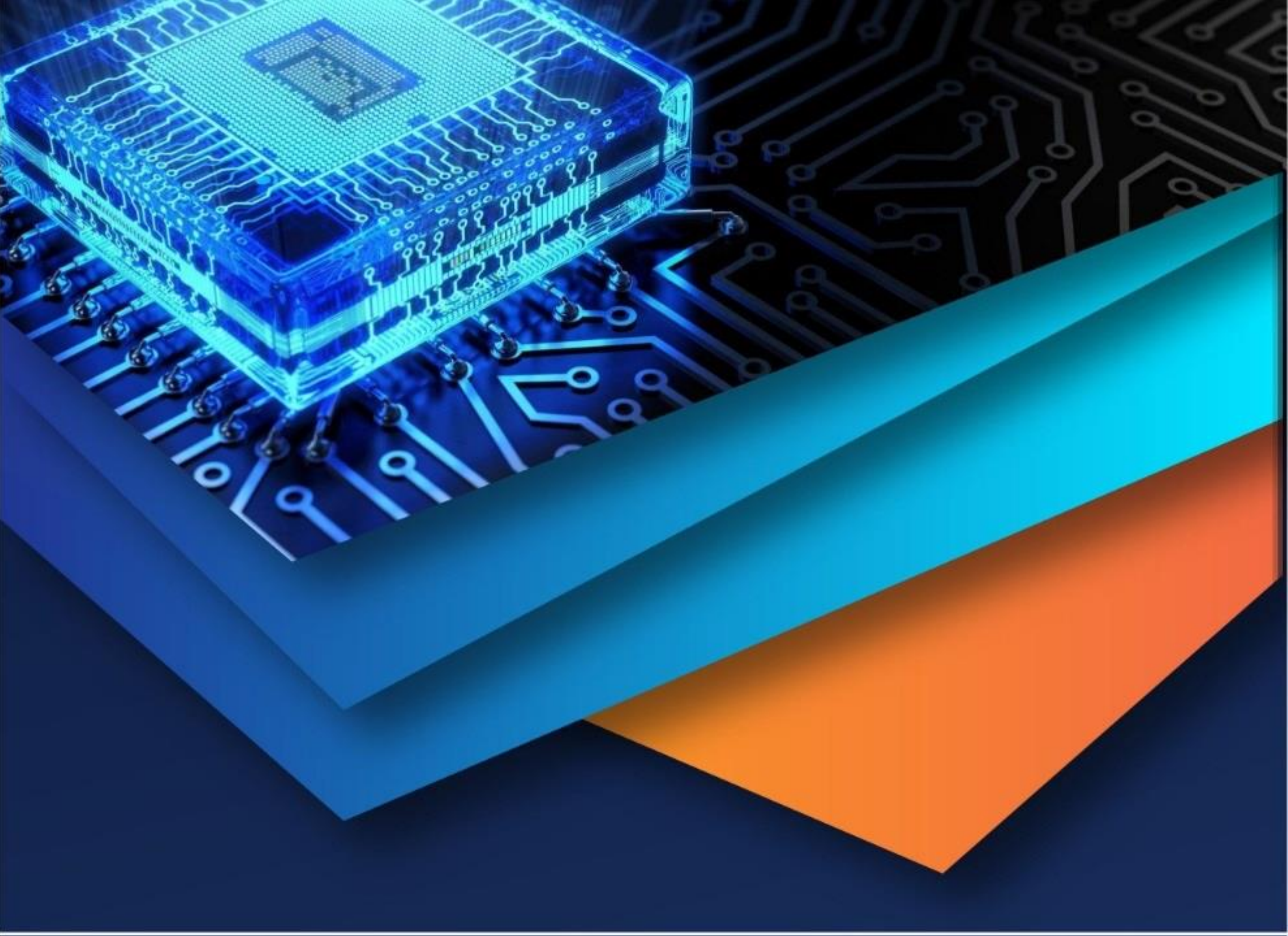

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