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Study of Linear Programming and Its Applications

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Abstract: The linear programming technique is a crucial tool applicable to a wide range of real-world scenarios, from optimising production processes to maximising profits in business and revenue, from planning flight paths to transporting oil from refineries to cities, to discovering affordable diets that fulfil daily nutritional needs. The paper covers the basics of linear programming, its formulation, its applications, its advantages, its limitations and various issues that it can address.
Keywords: Linear programming, objective function, decision variables, constraints, applications

I. INTRODUCTION TO LINEAR PROGRAMMING

A branch of operations research called linear programming uses a variety of techniques to choose system variable values that will optimize solutions to various issues. Linear Programming is also applicable to a variety of management problems, such as investment, Advertising, Production, Refinery Operations, Distribution, and Transportation analysis. The concept of linear programming plays a vital role not only in complex industrial and business systems but also in non-profit sectors such as Education, defence, government, hospitals, and Libraries. [2]

II. LINEAR PROGRAMMING PROBLEM FORMULATION:

A Russian Mathematician, George B. Dantzig, introduced this technique in 1947.[2] The two terms "linear" and "programming" make up "linear programming.". The relationship between the associated variables is referred to as linear, and programming is the systematic planning of a course of action to accomplish some desired ideal results.

A. Basic Concepts used in Linear Programming Problem:

- Optimization
- Decision Variables
- The objective function
- Constraints
- Profit of Cost Coefficient
- Input-Output coefficients
- Capacities

B. Steps Involved in the Formulation of a General Linear Programming Problem: [5] [6]

- Determine decision variables.
- Determine the objective function as a linear equation in terms of the decision variables.
- Write all resource constraints as linear equations or inequalities in terms of the decision variables.
- Consider only those values that are non-negative, as negative values are not allowed.
- Make use of various mathematical techniques to investigate all potential sets of solutions for the variables.
- Select the specific values of variables obtained in the previous step that result in the desired optimal objective function.

The outcome of the initial four steps is referred to as the formulation of a linear programming problem.
The set of feasible solutions obtained in step six is referred to as the final solution. The solution chosen in step six is referred to as the optimal solution of the linear programming problem.
Mathematically, the general LPP can be stated in mathematical terms as follows: [2][3][4]
Suppose

$$oij = \text{Input} - \text{output Coefficients}$$
$$c_j = \text{Cost/Profit Coefficients}$$
$$b_i = \text{Capacities(Right hand side)}$$

x_j = Decision Variables

Identify the values of n decision variables $(x_1, x_2, x_3, x_4, \ldots\ldots\ldots\ldots x_n)$ that optimize (minimize or maximize) a linear objective function $f(x)$ where

$f(x) = c_1x_1 + c_2x_2 + c_3x_3 + c_4x_4, \ldots\ldots\ldots\ldots + c_nx_n$

Subject to the m −linear constraints

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + a_{14}x_4, \ldots\ldots\ldots\ldots + a_{1n}x_n \quad \leq \text{or} \ = \ \text{or} \geq) b_1$$
$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + a_{24}x_4, \ldots\ldots\ldots\ldots + a_{2n}x_n \quad \leq \text{or} \ = \ \text{or} \geq) b_2$$
$$a_{31}x_1 + a_{31}x_2 + a_{33}x_3 + a_{34}x_4, \ldots\ldots\ldots\ldots + a_{3n}x_n \quad \leq \text{or} \ = \ \text{or} \geq) b_3$$

....

....

$$a_{m1}x_1 + a_{m1}x_2 + a_{m3}x_3 + a_{m4}x_4, \ldots\ldots\ldots\ldots + a_{mn}x_n \quad \leq \text{or} \ = \ \text{or} \geq) b_m$$

and non-negative constraints

$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0, x_4 \geq 0, \ldots\ldots\ldots\ldots x_n \geq 0$

III. TYPES OF PROBLEMS WHERE LINEAR PROGRAMMING CAN BE APPLIED:

- Problems of allocation
- Problems of assignment
- Problems of transportation

A. Production Allocation Problem [5][6]

A factory manufactures three stationery products: Pencils, Erasers and Sharpeners. Three different machines are needed to process these stationary items. The table below shows the daily capacity of the three machines as well as the amount of time needed to produce one unit of each of the three stationary products:

Machine	Time per unit (Minutes)			Machine Capacity (Minutes)/daily
	Pencil	Eraser	Sharpener	
A	2	3	2	450
B	4	2	3	490
C	3	5	2	470

For the pencil, eraser, and sharpener products, the profit per unit is Rs 5, Rs 4, and Rs 6, respectively. Formulate the mathematical LP model that will maximize the daily profit.

Solution: We will formulate this Linear Programming Problem using the above steps.

Using the given information, there will be three constraints.

First for machine A Minutes availability

Second for machine B Minutes availability

Third for machine C Minutes availability

Let x_1, x_2 and x_3 be the number of units of Pencils, Erasers and Sharpeners manufactured per month respectively

The objective is to maximize the profit.

$$\max Z = 5x_1 + 4x_2 + 6x_3$$

Subject to the constraints

$2x_1 + 3x_2 + 2x_3 \leq 450$ for machine A

$4x_1 + 2x_2 + 3x_3 \leq 490$ for machine B

$3x_1 + 5x_2 + 2x_3 \leq 470$ for machine C

$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0$ non-negative constraints

B. Product Mix Problem [5] [6]

A factory manufactures two products: Chairs and Tables. To manufacture one Chair, 3 machine hours and 5 labor hours are required and to manufacture one Table, 5 machine hours and 3 labour hours are required. In a month, 400 machine hours and 280 labor hours are available. Profit per unit for Chair is Rs. 500 and for Table is Rs. 750. Formulate the LPP

Solution:

Products	Resource/unit	
	Machine	Labor
Chair	3	5
Table	5	3
Availability	400 hours	280 hours

Using the given information, two constraints are there

First for machine hour's availability and second for labor hour's availability.

Let x_1 and x_2 be the number of units of Chairs and tables respectively

The objective function

$$Max\ Z = 500x_1 + 750x_2$$

Subject to constraints

$3x_1 + 5x_2 \leq 400$ for machine hours

$5x_1 + 3x_2 \leq 280$ for labor hours

$x_1 \geq 0, x_2 \geq 0$ non-negative constraints

C. *Media Selection Problem:[5][6]*

An advertising agency is planning to launch an advertising campaign related to some products on social media. Social media under consideration are Facebook, Instagram, Google and YouTube. Formulate a problem that relates to the advertising social media platform available, the number of people expected to be reached with each medium, the cost (Expenditure) per advertisement, the maximum availability and the expected exposure of each media partner. Expected reach of audience is 500 lakhs and the total advertising budget is Rs. 80 Lakhs.

Advertising Media	Cost per advertisement (in lakh Rs.)	Maximum availability	Number of people expected to reach per advertisement (in lakhs)	Expected Exposure of each media partner
Facebook Ads	12	10	25	75
Instagram Ads	10	08	30	65
Google ads	13	12	50	80
You Tube ads	15	15	60	90

Solution:

As per the given information, we can write

Let x_1 ,x_2, x_3 and x_4 be the number of people to be reached with Facebook ,Instagram, Google and you tube respectively

Objective function

$$Max\ Z = 75\,x_1 + 65x_2 + 80\,x_3 + 90\,x_4$$

Subject to constraints

$25\,x_1 + 30x_2 + 50\,x_3 + 60\,x_4 \leq 500$ (for reach per ad in lakhs)

$12x_1 + 10\,x_2 + 13\,x_3 + 15\,x_4 \leq 80$ (for cost per ad)

$x_1 \leq 10, x_2 \leq 8, x_3 \leq 12, x_4 \leq 15$ (for maximum availabilty)

$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0, x_4 \geq 0$ 0 non-negative constraints

IV. BASIC HYPOTHESIS REQUIRED IN LINEAR PROGRAMMING PROBLEM

It is evident from the examples that a certain hypothesis is followed in linear programming problem:

- Certainty
- Additively
- Proportionality
- Multiplicatively
- Divisibility or Continuity

Since it is quite difficult to encounter a real-life problem that fully meets all the aforementioned assumptions for linear programming, the user needs to be thoroughly informed about the assumptions and approximations involved. The user must ensure that these are justified before proceeding to implement the linear programming technique.

V. APPLICATIONS OF LINEAR PROGRAMMING TECHNIQUES:[1]

Linear programming techniques is among the most widely employed techniques in business organizations, industries, and numerous other fields.

- Assignment Problems: Linear programming techniques can be applied to assignment problems where resources, which may include manpower, machinery, raw materials, finances, etc., need to be allocated to various tasks in a manner that optimizes performance
- Blending Problems: Linear programming techniques can be utilized in blending problems where we determine the quantities of different available components to mix to create a new product, thereby minimizing the total cost of the product and maximizing total profit.
- Transportation Problems: Linear programming techniques can be employed in transportation problems that involve the shipping of available quantities from sources to destinations based on the needs, ensuring that transportation costs are kept to a minimum.
- Military Problems: Linear programming techniques may be utilized by military officers to determine the number of defense units, weapon designs, and optimal bombing strategies based on their availability, ensuring that the outcomes serve the best interests of the nation.
- Production Management Problems: Linear programming techniques can be employed by in determining product mix, smoothing production, and balancing assembly times by considering available inventories, production capacity, labor, various production costs, and constraints, all aimed at maximizing total profit.
- Agricultural Problems: Linear programming techniques can be applied to allocate limited resources like labour, water supply, fertilizers, pesticides, and working capital, with the goal of maximizing net revenue.
- Marketing Management Problems: The linear programming technique serves as an important tool for analyzing the effectiveness of advertising campaigns and timing based on available resources.
- Manpower Management Issues: Linear programming techniques enables the personnel manager within a company to analyze various combinations of personnel policies regarding their suitability for ensuring a consistent flow of individuals into, through and out of the organization.

In addition to the aforementioned, linear programming is applicable in fields such as administration, education, inventory management, contract awarding, capital budgeting, and more. Indian Railways, along with other railway systems, employs linear programming techniques for route selection and train allocation across different routes, as well as for ticket reservations. Similarly, Indian Airlines and other airlines utilize linear programming methods for route selection and aircraft allocation across various routes.

VI. ADVANTAGES OF LINEAR PROGRAMMING TECHNIQUES:[2]

The advatages of linear programming techniques can be summarized as follows:

- The linear programming techniques help in the optimal utilization of productive resources. It also demonstrates how decision-makers can select and distribute these resources to best utilize their productive factors.
- It also demonstrates how decision-makers can select and distribute these resources to best utilize their productive factors.
- Linear programming methods offer practically applicable solutions, as there may be additional constraints outside the problem that need to be considered. Just because a certain number of units must be produced does not imply that all of them can be sold. Therefore, necessary adjustments must be made to account for these factors.

- In production processes, identifying bottlenecks is the primary benefit of this technique. For instance, when bottlenecks arise, certain machines may fail to fulfil the demand while others sit idle for a period of time.
- The required adjustment of the mathematical solutions can also be achieved through linear programming.
- Linear programming assists in the reassessment of a fundamental plan in response to changing conditions.

VII. LIMITATIONS OF LINEAR PROGRAMMING TECHNIQUE:[2]

Despite its extensive range of applications, linear programming techniques come with certain limitations. These limitations are outlined below:

- In various real-life scenarios related to business and industry, the objective function and constraints may not be linearly related to the variables.
- The linear programming technique does not account for the influence of time and uncertainty. Therefore, the model must be structured in a manner that allows for the incorporation of changes arising from both internal and external factors.
- Occasionally, large-scale problems may be unsolvable using linear programming techniques, even with available computational resources. This challenge can be mitigated by breaking down the primary problem into several smaller problems and solving them individually.
- The parameters utilized in the linear programming technique are presumed to be constant and deterministic. However, in real-world scenarios, these parameters are neither constant nor deterministic.
- The linear programming technique focuses solely on a single objective, while real-life situations often present conflicting multi-objective problems. Goal programming and multi-objective programming are designed to address such challenges instead of linear programming.

VIII. CONCLUSION

Linear Programming is an outstanding quantitative technique of decision-making. This technique plays a vital role in the fields of uncertainty, viz., business and commerce. Every company, regardless of its size, has access to resources, money, labour, and equipment however, the supply may be restricted. On the contrary, there will not be any necessity for management tools like linear programming if the supply of these resources remains unrestricted. Due to restricted availability of resources, management must make a decision how to best allocate its resources to maximize profit, reduce loss, or make the utmost use of production capacity to the highest extent. Many problems can be overcome by the linear programming technique.

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