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Study on Anti Q-Pythagorean Fuzzy Ideals in Semiring

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Abstract: In this paper, we introduce the notion of anti Q-Pythagorean fuzzy left ideal, anti Q-Pythagorean fuzzy right ideal, anti Q-Pythagorean fuzzy ideal and bi-ideal in semiring some interesting properties, results are discussed in this paper.
Keywords: Pythagorean, Fuzzy set, Semiring

I. INTRODUCTION

Nobusawa[5] studied the concept of gamma semiring as a generalization of ring after that Sen introduced the gamma semigroups as a generalization of gamma groups. Murali Krishna Rao[6] in 1995 introduced the notion of gamma semiring as a generalization of gamma ring, ring, ternary semiring and semiring.The important reason for development of gamma semiring is a generalization of results of rings, gamma rings, semirings, semigroup and ternary semirings.

Zadeh[12] studied the notion of fuzzy set theory. Atanassov [2] introduced intuitionistic fuzzy sets as a generalization of fuzzy sets. In intuitionistic, the sum of membership degree and non-membership degree should not exceed one. Yager [10] initially introduced the concept of Pythagorean fuzzy sets. In a Pythagorean fuzzy sets, the sum of the squared membership and non-membership degrees satisfies the condition. More recently, Yager [10, 11] proposed Pythagorean fuzzy sets as a powerful tool for effectively managing uncertainty or imprecise information in real-world scenarios. These sets enforce a constraint where the sum of squares of membership and non-membership degrees is less than or equal to 1.

Pythagorean fuzzy sets have showcased remarkable efficacy in navigating uncertainties, prompting a surge of scholarly exploration across diverse research avenues, resulting in significant progress. The conceptualization of Pythagorean fuzzy sets facilitates a more comprehensive and accurate portrayal of uncertain information when juxtaposed with intuitionistic fuzzy sets. Across various disciplines, academics have meticulously examined the algebraic attributes of Pythagorean fuzzy sets, shedding light on their practical applications and foundational theoretical constructs. Many authors studied the algebraic structures of Pythagorean fuzzy sets

This paper is structured into three sections. The first and second sections serve as the introduction and cover basic results pertinent to the paper's topic. In the third section, we introduce anti Q-Pythagorean fuzzy ideals in semirings and some interesting properties this ideals are discussed.

II. PRELIMINARIES

In this section we present the basic concepts related to this paper.

1) Definition 2.1 *[6] if* $(R,+)$ *and* $(\Gamma,+)$ *be two commutative semigroups then* R *is called a* Γ *semiring if there exists a mapping* $R\times\Gamma\times R$ *denoted by* $\alpha\gamma\beta$ *for all* $\alpha,\beta\in R$ *and* $\gamma\in\Gamma$ *satisftying the followimg properties,* $\alpha\gamma(\beta+\nu)=\alpha\gamma\beta+\alpha\gamma\nu$, $(\beta+\nu)\gamma\alpha=\beta\gamma\alpha+\nu\gamma\alpha$, $\alpha(\gamma+\gamma_1)\nu=\alpha\gamma\nu+\alpha\gamma_1\nu$, $\alpha\gamma(\beta\gamma_1\nu)=(\alpha\gamma\beta)\gamma_1\nu$ *for all* $\alpha,\beta,\nu\in R$ *and* $\gamma,\gamma_1\in\Gamma$.
2) Definition 2.2 *[6] Define addition in the following way* $A,B\in R,\gamma\in\Gamma$ *, let* $A\gamma B$ *denote the ideal generated by* $\{\alpha\gamma\beta/\alpha,\beta\in R\}$*. Then* R *is a* Γ*-semiring.*
3) Definition 2.3 *[6] A* Γ*-semiring* R *is said to be commutative if* $\alpha\gamma\beta=\beta\gamma\alpha$*, for all* $\alpha,\beta\in R$ *and* $\gamma\in\Gamma$.
4) Definition 2.4 *[6] A* Γ*-semiring* R *is said to have a zero element if* $0\beta\alpha=0=\alpha\beta 0$ *and* $\alpha+0=\alpha=0+\alpha$ *,for all* $\alpha\in R$ *and* $\gamma\in\Gamma$.
5) Definition 2.5 *[6]* R *is said to have a identity element if there exists* $\gamma\in\Gamma$ *such that* $1\gamma\alpha=\alpha=\alpha\gamma 1$ *for all* $\alpha\in R$.
6) Definition 2.6 *[6]* R *is said to have a strong identity element if for all* $\alpha\in R$, $1\gamma\alpha=\alpha=\alpha\gamma 1$ *for all* $\gamma\in\Gamma$.
7) Definition 2.7 *[6] A non empty subset* R *of a* Γ*-semiring* R *is said to be a sub*Γ*semiring of* R *if* $(R,+)$ *is a sub semigroup of* $(R,+)$ *and* $\alpha\gamma\beta\in R$ *for all*$\alpha,\beta\in R$ *and* $\gamma\in\Gamma$.

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8) Definition 2.8 *[6] A non empty subset* R *of a* Γ*-semiring* R *is called an ideal if* $\alpha, \beta \in R$ *implies* $\alpha + \beta \in R$ *and* $a \in R, \alpha \in R$ *and* $\gamma \in \Gamma$ *implies* $\alpha\gamma a \in R$ *and* $a\alpha\gamma \in R$.

9) Definition 2.9 *[10] Let* X *be a non empty set. A Pythagorean Fuzzy Set* $\mathfrak{A}$ *in* X *is given by* $\mathfrak{A} = \{\alpha, \mathfrak{A}_x(\alpha), \mathfrak{A}_y(\alpha) / \alpha \in X\}$ *where* $\mathfrak{A}_x : X \to [0,1]$ *and* $\mathfrak{A}_y : X \to [0,1]$ represent the degree of membership and degree of non membership of $\mathfrak{A}$ respectively. Also, $\mathfrak{A}_x$ and $\mathfrak{A}_y$ satisfies the condition $(\mathfrak{A}_x)^2 + (\mathfrak{A}_y)^2 \leq 1$ for all $\alpha \in X$.

III. ANTI Q - PYTHAGOREAN FUZZY IDEALS IN SEMIRING

In this section S denotes Semiring(S).

1) Definition 3.1 *Let* $P = (\mu_P, \vartheta_P)$ *be an anti Q-Pythagorean fuzzy subset of a semiring* S *and* $\forall x, y \in S, q \in Q$.
   (i) $\mu_P(x + y, q) \leq \max\{\mu_P(x, q), \mu_P(y, q)\}$; $\vartheta_P(x + y, q) \geq \min\{\vartheta_P(x, q), \vartheta_P(y, q)\}$
   (ii) $\mu_P(xy, q) \leq \max\{\mu_P(x, q), \mu_P(y, q)\}$; $\vartheta_P(xy, q) \geq \min\{\vartheta_P(x, q), \vartheta_P(y, q)\}$

Then $P = (\mu_P, \vartheta_P)$ is called an anti Q-Pythagorean fuzzy subsemiring of S.

2) Definition 3.2 *Let* $P = (\mu_P, \vartheta_P)$ *be an anti Q-Pythagorean fuzzy left ideal of* S, *if* P *satisfies the following conditions,* $\forall x, y \in S, q \in Q$.
   (i) $\mu_P(x + y, q) \leq \max\{\mu_P(x, q), \mu_P(y, q)\}$; $\vartheta_P(x + y, q) \geq \min\{\vartheta_P(x, q), \vartheta_P(y, q)\}$
   (ii) $\mu_P(xy, q) \leq \mu_P(y, q)$; $\vartheta_P(xy, q) \geq \vartheta_P(y, q)$

3) Definition 3.3 *Let* $P = (\mu_P, \vartheta_P)$ *be an anti Q-Pythagorean fuzzy right ideal of* S, *if* P *satisfies the following conditions,* $\forall x, y \in S, q \in Q$.
   (i) $\mu_P(x + y, q) \leq \max\{\mu_P(x, q), \mu_P(y, q)\}$; $\vartheta_P(x + y, q) \geq \min\{\vartheta_P(x, q), \vartheta_P(y, q)\}$
   (ii) $\mu_P(xy, q) \leq \mu_P(x, q)$; $\vartheta_P(xy, q) \geq \vartheta_P(x, q)$.

4) Theorem 3.4 *Intersection of non empty collection of anti Q-Pythagorean fuzzy right (resp. left) ideals is also an anti Q-Pythagorean fuzzy right (resp. left) ideal of semiring* S.

Proof. Let $\{P_i = (\mu_i, \vartheta_i) | i \in I\}$ be a non empty family of anti Q-Pythagorean fuzzy right ideals of semiring S and $x, y \in S, q \in Q$.

Then

$$\bigcup_{i \in I} \mu_i(x + y, q) = \sup_{i \in I}\{\mu_i(x + y, q)\}$$
$$\leq \sup_{i \in I}\{\max\{\mu_i(x, q), \mu_i(y, q)\}\}$$
$$= \max\{\sup_{i \in I}\mu_i(x, q), \sup_{i \in I}\mu_i(y, q)\}$$
$$= \max\{\cup_{i \in I} \mu_i(x, q), \cup_{i \in I} \mu_i(y, q)\}.$$

Also

$$\bigcap_{i \in I} \vartheta_i(x + y) = \inf_{i \in I}\{\vartheta_i(x + y, q)\}$$
$$\geq \inf_{i \in I}\{\max\{\vartheta_i(x, q), \vartheta_i(y, q)\}\}$$
$$= \min\{\inf_{i \in I}\vartheta_i(x, q), \inf_{i \in I}\vartheta_i(y, q)\}$$
$$= \min\{\cap_{i \in I} \vartheta_i(x, q), \cap_{i \in I} \vartheta_i(y, q)\}.$$

Moreover

$$\bigcup_{i \in I} \mu_i(xy, q) = \sup_{i \in I}\{\mu_i(xy, q)\}$$
$$\leq \sup_{i \in I}\{\mu_i(x, q)\}$$

$= \bigcup_{i\in I} \mu_i(x,q)$.

Finally

$$\bigcap_{i\in I} \vartheta_i(xy,q) = \inf_{i\in I}\{\vartheta_i(xy,q)\}$$
$$\geq \inf_{i\in I}\{\vartheta_i(x,q)\}$$

$= \bigcap_{i\in I} \vartheta_i(x,q)$.

Hence $\bigcap_{i\in I} P_i$ is an anti Q-Pythagorean fuzzy right ideals of semiring S.
Similarly, we can prove the result for Pythagorean fuzzy left ideal also.

5) Theorem 3.5 *Union of a non empty collection of anti Q-Pythagorean fuzzy right (resp. left) ideals is also an anti Q-Pythagorean fuzzy right (resp. left) ideal of semiring S.*

Proof. Let $\{P_i = (\mu_i, \vartheta_i) | i \in I\}$ be a non empty family of anti Q-Pythagorean fuzzy right ideals of semiring S and $x, y \in S, q \in Q$.

Then

$$\bigcap_{i\in I} \mu_i(x+y,q) = \inf_{i\in I}\{\mu_i(x+y,q)\}$$
$$\geq \inf_{i\in I}\{\min\{\mu_i(x,q), \mu_i(y,q)\}\}$$
$$= \min\{\inf_{i\in I}\mu_i(x,q), \inf_{i\in I}\mu_i(y,q)\}$$

$= \min\{\bigcap_{i\in I} \mu_i(x,q), \bigcap_{i\in I} \mu_i(y,q)\}$.

Also

$$\bigcup_{i\in I} \vartheta_i(x+y,q) = \sup_{i\in I}\{\vartheta_i(x+y,q)\}$$
$$\leq \sup_{i\in I}\{\max\{\vartheta_i(x,q), \vartheta_i(y,q)\}\}$$
$$= \max\{\sup_{i\in I}\vartheta_i(x,q), \sup_{i\in I}\vartheta_i(y,q)\}$$

$= \max\{\bigcup_{i\in I} \vartheta_i(x,q), \bigcup_{i\in I} \vartheta_i(y,q)\}$.

Moreover

$$\bigcap_{i\in I} \mu_i(xy,q) = \inf_{i\in I}\{\mu_i(xy,q)\}$$
$$\geq \inf_{i\in I}\{\mu_i(x,q)\}$$

$= \bigcap_{i\in I} \mu_i(x,q)$.

Finally

$$\bigcup_{i\in I} \vartheta_i(xy,q) = \sup_{i\in I}\{\vartheta_i(xy,q)\}$$
$$\leq \sup_{i\in I}\{\vartheta_i(x,q)\}$$

$= \bigcup_{i\in I} \vartheta_i(x,q)$.

Hence $\bigcup_{i\in I} P_i$ is an anti Q-Pythagorean fuzzy right ideals of S.
Similarly, we can prove the result for an anti Q-Pythagorean fuzzy left ideal also.

6) Definition 3.6 *Let $P_1 = (\mu_1, \vartheta_1)$ and $P_2 = (\mu_2, \vartheta_2)$ be an anti Q-Pythagorean fuzzy subsets of semiring S. The cartesian product of P_1 and P_2 is defined by*

(i) $\mu_1 \times \mu_2((x,y),q) = \max\{\mu_1(x,q), \mu_2(y,q)\}$

(ii) $\vartheta_1 \times \vartheta_2((x,y),q) = \min\{\vartheta_1(x,q), \vartheta_2(y,q)\}$, for all $x, y \in S, q \in Q$.

7) Theorem 3.7 *Let P_1 and P_2 be an anti Q-Pythagorean fuzzy left ideals of semiring S. Then $P_1 \times P_2$ is an anti Q-Pythagorean fuzzy left ideal of semiring $S \times S$.*

Proof. Let $(x_1, x_2), (y_1, y_2) \in S \times S, q \in Q$.

Then

$$(\mu_1 \times \mu_2)((x_1, x_2) + (y_1, y_2), q) = (\mu_1 \times \mu_2)((x_1 + y_1, q), (x_2 + y_2, q))$$

$$= \max\{\mu_1(x_1 + y_1, q), \mu_2(x_2 + y_2, q)\}$$
$$\leq \max\{\max\{\mu_1(x_1, q), \mu_1(y_1, q)\}, \max\{\mu_2(x_2, q), \mu_2(y_2, q)\}\}$$
$$= \max\{\max\{\mu_1(x_1, q), \mu_2(x_2, q)\}, \max\{\mu_1(y_1, q), \mu_2(y_2, q)\}\}$$
$$= \max\{(\mu_1 \times \mu_2)(x_1, x_2, q), (\mu_1 \times \mu_2)(y_1, y_2, q)\}.$$

$$(\mu_1 \times \mu_2)((x_1, x_2)(y_1, y_2), q) = (\mu_1 \times \mu_2)(x_1 y_1, q), (x_2 y_2, q)$$
$$= \max\{\mu_1(x_1 y_1, q), \mu_2(x_2 y_2, q)\}$$
$$\leq \max\{\mu_1(y_1, q), \mu_2(y_2, q)\}$$
$$= (\mu_1 \times \mu_2)((y_1, y_2), q).$$

$$(\vartheta_1 \times \vartheta_2)((x_1, x_2) + (y_1, y_2), q) = (\vartheta_1 \times \vartheta_2)(x_1 + y_1, x_2 + y_2, q)$$
$$= \min\{\vartheta_1(x_1 + y_1, q), \vartheta_2(x_2 + y_2, q)\}$$
$$\geq \min\{\min\{\vartheta_1(x_1, q), \vartheta_1(y_1, q)\}, \min\{\vartheta_2(x_2, q), \vartheta_2(y_2, q)\}\}$$
$$= \min\{\min\{\vartheta_1(x_1, q), \vartheta_2(x_2, q)\}, \min\{\vartheta_1(y_1, q), \vartheta_2(y_2, q)\}\}$$
$$= \min\{(\vartheta_1 \times \vartheta_2)(x_1, x_2, q), (\vartheta_1 \times \vartheta_2)(y_1, y_2, q)\}.$$

$$(\vartheta_1 \times \vartheta_2)((x_1, x_2)(y_1, y_2), q) = (\vartheta_1 \times \vartheta_2)((x_1 y_1, x_2 y_2), q)$$
$$= \min\{\vartheta_1(x_1 y_1, q), \vartheta_2(x_2 y_2, q)\}$$
$$\geq \min\{\vartheta_1(y_1, q), \vartheta_2(y_2, q)\}$$
$$= (\vartheta_1 \times \vartheta_2)((y_1, y_2), q).$$

Therefore $P_1 \times P_2$ is an anti Q-Pythagorean fuzzy left ideal of semiring $S \times S$.

8) Theorem 3.8 *Let P be an anti Q-Pythagorean fuzzy subset of semiring. Then P is an anti Q-Pythagorean fuzzy left ideal of semiring S if and only if $P \times P$ is an anti Q-Pythagorean fuzzy left ideal of semiring $S \times S$.*

Proof. Consider P is an anti Q-Pythagorean fuzzy left ideal of semiring S. Then by Previous theorem $P \times P$.

Conversely $P \times P$ is an anti Q-Pythagorean fuzzy left ideal of $S \times S$, for all $x_1, x_2, y_1, y_2 \in S, q \in Q$.

Then

$$\max\{\mu(x_1 + y_1, q), \mu(x_2 + y_2, q)\} = \mu \times \mu((x_1 + y_1, x_2 + y_2), q)$$
$$= (\mu \times \mu)\{((x_1, x_2) + (y_1, y_2))\}$$
$$\leq \max\{(\mu \times \mu)((x_1, x_2), q), (\mu \times \mu)((y_1, y_2))\}$$
$$= \max\{\max\{\mu(x_1, q), \mu(x_2, q)\}, \max\{\mu(y_1, q), \mu(y_2, q)\}\}$$

Next

we have

$$\max\{\mu(x_1 y_1, q), \mu(x_2 y_2, q)\} = (\mu \times \mu)((x_1 y_1, x_2 y_2), q)$$
$$= (\mu \times \mu)\{((x_1, x_2)(y_1, y_2), q)\}$$
$$= (\mu \times \mu)\{(y_1, y_2), q\}$$
$$= \max\{\mu(y_1, q), \mu(y_2, q)\}$$

Also

$$\min\{\vartheta(x_1 + y_1, q), \vartheta(x_2 + y_2, q)\} = \vartheta \times \vartheta((x_1 + y_1, x_2 + y_2), q)$$
$$= (\vartheta \times \vartheta)\{((x_1, x_2) + (y_1, y_2), q)\}$$
$$\geq \min\{(\vartheta \times \vartheta)((x_1, x_2), q), (\vartheta \times \vartheta)((y_1, y_2), q)\}$$
$$= \min\{\min\{\vartheta(x_1, q), \vartheta(x_2, q)\}, \min\{\vartheta(y_1, q), \vartheta(y_2, q)\}\}$$

And

$$\min\{\vartheta(x_1 y_1, q), \vartheta(x_2 y_2, q)\} = (\vartheta \times \vartheta)((x_1 y_1, x_2 y_2), q)$$

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$$= (\vartheta \times \vartheta)\{((x_1, x_2)(y_1, y_2), q)\}$$
$$= (\vartheta \times \vartheta)\{(y_1, y_2), q\}$$
$$= \min\{\vartheta(y_1, q), \vartheta(y_2, q)\}$$

Hence P is an anti Q-Pythagorean fuzzy left ideal of semiring S.

9) Theorem 3.9 *If P_1, P_2 be any two anti Q-Pythagorean fuzzy ideals of semiring S, then $P_1 + P_2$ is also so.*

Proof. Consider P_1, P_2 are any two anti Q-Pythagorean fuzzy ideals of semiring S and $x, y \in S, q \in Q$.

Then

$$(\mu_1 + \mu_2)(x + y, q) = \inf_{x+y \le c+d} \{\max\{\mu_1(c, q), \mu_2(d, q)\}\}$$
$$\le \inf_{x+y \le (a_1+b_1)+(a_2+b_2)=(a_1+a_2)+(b_1+b_2)} \{\max\{\mu_1(a_1 + a_2, q), \mu_2(b_1 + b_2, q)\}\}$$
$$\le \inf\{\max\{\mu_1(a_1, q), \mu_2(a_2, q)\}, \max\{\mu_2(b_1, q), \mu_2(b_2, q)\}\}$$
$$= \max\{\inf_{x \le a_1+b_1} \{\max\{\mu_1(a_1, q), \mu_2(b_1, q)\}\}, \inf_{y \le a_2+b_2} \{\max\{\mu_1(a_2, q), \mu_2(b_2, q)\}\}\}$$
$$= \max\{(\mu_1 + \mu_2)(x, q), (\mu_1 + \mu_2)(y, q)\}.$$

Also

$$(\vartheta_1 + \vartheta_2)(x + y, q) = \sup_{x+y \le c+d} \{\min\{\vartheta_1(c, q), \vartheta_2(d, q)\}\}$$
$$\ge \sup_{x+y \le (a_1+b_1)+(a_2+b_2)=(a_1+a_2)+(b_1+b_2)} \{\min\{\vartheta_1(a_1 + a_2, q), \vartheta_2(b_1 + b_2, q)\}\}$$
$$\ge \sup\{\min\{\vartheta_1(a_1, q), \vartheta_2(a_2, q)\}, \min\{\vartheta_2(b_1, q), \vartheta_2(b_2, q)\}\}$$
$$= \min\{\sup_{x \le a_1+b_1} \{\min\{\vartheta_1(a_1, q), \vartheta_2(b_1, q)\}\}, \sup_{y \le a_2+b_2} \{\min\{\vartheta_1(a_2, q), \vartheta_2(b_2, q)\}\}\}$$
$$= \min\{(\vartheta_1 + \vartheta_2)(x, q), (\vartheta_1 + \vartheta_2)(y, q)\}.$$

Now let as consider P_1, P_2 are anti Q-Pythagorean fuzzy right ideals and we have

$$(\mu_1 + \mu_2)(xy, q) = \inf_{xy \le cd} \{\max\{\mu_1(c, q), \mu_2(d, q)\}\}$$
$$\le \inf_{xy \le (x_1+x_2)y} \{\max\{\mu_1(x_1 y, q), \mu_2(x_2 y, q)\}\}$$
$$\le \inf_{x \le (x_1+x_2)} \{\max\{\mu_1(x_1, q), \mu_2(x_2, q)\}\}$$
$$= (\mu_1 + \mu_2)(x, q).$$

Then

$$(\vartheta_1 + \vartheta_2)(xy, q) = \sup_{xy \le cd} \{\min\{\vartheta_1(c, q), \vartheta_2(d, q)\}\}$$
$$\ge \sup_{xy \le (x_1+x_2)y} \{\min\{\vartheta_1(x_1 y, q), \vartheta_2(x_2 y, q)\}\}$$
$$\ge \sup_{x \le (x_1+x_2)} \{\min\{\vartheta_1(x_1, q), \vartheta_2(x_2, q)\}\}$$
$$= (\vartheta_1 + \vartheta_2)(x, q).$$

Similarly assuming P_1, P_2 are anti Q-Pythagorean fuzzy left ideal,
we can show that $(P_1 + P_2)(xy) \ge (P_1 + P_2)(y)$.

Also

$$(\mu_1 + \mu_2)(x, q) = \inf_{x \le x_1+x_2} \{\max\{\mu_1(x_1, q), \mu_2(x_2, q)\}\}$$
$$\le \inf_{x \le y \le y_1+y_2} \{\max\{\mu_1(y_1, q), \mu_2(y_2, q)\}\}$$
$$= \inf_{y \le y_1+y_2} \{\max\{\mu_1(y_1, q), \mu_2(y_2, q)\}\}$$
$$= (\mu_1 + \mu_2)(y, q)$$

and

$$(\vartheta_1 + \vartheta_2)(x, q) = \sup_{x \le x_1+x_2} \{\min\{\vartheta_1(x_1, q), \vartheta_2(x_2, q)\}\}$$
$$\ge \sup_{x \le y \le y_1+y_2} \{\min\{\vartheta_1(y_1, q), \vartheta_2(y_2, q)\}\}$$
$$= \sup_{y \le y_1+y_2} \{\min\{\vartheta_1(y_1, q), \vartheta_2(y_2, q)\}\}$$

$= (\vartheta_1 + \vartheta_2)(y, q)$.

Hence $P_1 + P_2$ is an anti Q-Pythagorean fuzzy ideal of semiring S.

10) Theorem 3.10 *If P_1, P_2 be any two anti Q-Pythagorean fuzzy ideals of semiring S, then $P_1 \circ P_2$ is also so.*

Proof. Let P_1, P_2 are any two anti Q-Pythagorean fuzzy ideals of semiring S and $x, y \in S, q \in Q$.

Then

$$(\mu_1 \circ \mu_2)(x + y) = \inf_{x+y\le c+d}\{\max\{\mu_1(c), \mu_2(d)\}\}$$
$$\le \inf_{x+y\le(c_1\gamma d_1)+(c_2\gamma d_2)\le(c_1+c_2)\gamma(d_1+d_2)}\{\max\{\mu_1(c_1 + c_2), \mu_2(d_1 + d_2)\}\}$$
$$\le \inf\{\max\{\mu_1(c_1), \mu_1(c_2)\}, \max\{\mu_2(d_1), \mu_2(d_2)\}\}$$
$$= \max\{\inf_{x\le c_1\gamma d_1}\{\max\{\mu_1(c_1), \mu_2(d_1)\}\}, \inf_{y\le c_2\gamma d_2}\{\max\{\mu_1(c_2), \mu_2(d_2)\}\}\}$$
$$= \max\{(\mu_1 \circ \mu_2)(x), (\mu_1 \circ \mu_2)(y)\}.$$

Also

$$(\vartheta_1 \circ \vartheta_2)(x + y) = \sup_{x+y\le c+d}\{\min\{\vartheta_1(c), \vartheta_2(d)\}\}$$
$$\ge \sup_{x+y\le(c_1\gamma d_1)+(c_2\gamma d_2)\le(c_1+c_2)\gamma(d_1+d_2)}\{\min\{\vartheta_1(c_1 + c_2), \vartheta_2(d_1 + d_2)\}\}$$
$$\ge \sup\{\min\{\vartheta_1(c_1), \vartheta_1(c_2)\}, \min\{\vartheta_2(d_1), \vartheta_2(d_2)\}\}$$
$$= \min\{\sup_{x\le c_1 d_1}\{\min\{\vartheta_1(c_1), \vartheta_2(d_1)\}\}, \sup_{y\le c_2 d_2}\{\min\{\vartheta_1(c_2), \vartheta_2(d_2)\}\}\}$$
$$= \min\{(\vartheta_1 \circ \vartheta_2)(x), (\vartheta_1 circ \vartheta_2)(y)\}.$$

Now let as consider P_1, P_2 are anti Q-Pythagorean fuzzy right ideals and we have

$$(\mu_1 \circ \mu_2)(xy) = \inf_{xy\le c\gamma d}\{\max\{\mu_1(c), \mu_2(d)\}\}$$
$$\le \inf_{xy\le(x_1x_2)\gamma y}\{\max\{\mu_1(x_1\gamma y), \mu_2(x_2\gamma y)\}\}$$
$$\le \inf_{x\le(x_1\gamma x_2)}\{\max\{\mu_1(x_1), \mu_2(x_2)\}\}$$

$= (\mu_1 \circ \mu_2)(x)$.

and

$$(\vartheta_1 \circ \vartheta_2)(xy) = \sup_{xy\le cd}\{\min\{\vartheta_1(c), \vartheta_2(d)\}\}$$
$$\ge \sup_{xy\le(x_1x_2)\gamma y}\{\min\{\vartheta_1(x_1\gamma y), \vartheta_2(x_2\gamma y)\}\}$$
$$\ge \sup_{x\le(x_1\gamma x_2)}\{\min\{\vartheta_1(x_1), \vartheta_2(x_2)\}\}$$

$= (\vartheta_1 \circ \vartheta_2)(x)$.

Similarly assuming P_1, P_2 are anti Q-Pythagorean fuzzy left ideal,
we can show that $(P_1 \circ P_2)(xy) \ge (P_1 \circ P_2)(y)$.

Also

$$(\mu_1 \circ \mu_2)(x) = \inf_{x\le x_1\gamma x_2}\{\max\{\mu_1(x_1), \mu_2(x_2)\}\}$$
$$\le \inf_{x\le y\le y_1\gamma y_2}\{\max\{\mu_1(y_1), \mu_2(y_2)\}\}$$
$$= \inf_{y\le y_1\gamma y_2}\{\max\{\mu_1(y_1), \mu_2(y_2)\}\}$$
$$= (\mu_1 \circ \mu_2)(y)$$

and

$$(\vartheta_1 \circ \vartheta_2)(x) = \sup_{x\le x_1\gamma x_2}\{\min\{\vartheta_1(x_1), \vartheta_2(x_2)\}\}$$
$$\ge \sup_{x\le y\le y_1\gamma y_2}\{\min\{\vartheta_1(y_1), \vartheta_2(y_2)\}\}$$
$$= \sup_{y\le y_1\gamma y_2}\{\min\{\vartheta_1(y_1), \vartheta_2(y_2)\}\}$$
$$= (\vartheta_1 \circ \vartheta_2)(y).$$

Hence $P_1 \circ P_2$ is an anti Q-Pythagorean fuzzy ideal of semiring S.

11) Definition 3.11 *A Pythagorean fuzzy subset* $P = (\mu, \vartheta)$ *is called an anti Q-Pythagorean fuzzy bi-ideal of semiring* S, *for all* $x, y, z \in S, q \in Q$.
   (i) $\mu(x+y) \leq \max\{\mu(x), \mu(y)\}; \vartheta(x+y) \geq \min\{\vartheta(x), \vartheta(y)\}$
   (ii) $\mu(xy) \leq \max\{\mu(x), \mu(y)\}; \vartheta(xy) \geq \min\{\vartheta(x), \vartheta(y)\}$
   (iii) $\mu(xy\beta z) \leq \max\{\mu(x), \mu(z)\}; \vartheta(xy\beta z) \geq \min\{\vartheta(x), \vartheta(z)\}$

12) Theorem 3.12 *Intersection of a non empty collection of anti Q-Pythagorean fuzzy bi-ideals is also anti Q-Pythagorean fuzzy bi-ideal of semiring* S.
   Proof. Let $\{P_i = (\mu_i, \vartheta_i) | i \in I\}$ be a family of anti Q-Pythagorean fuzzy bi-ideals of semiring S and $x, y \in S$, $q \in Q$.
Then

$$\bigcup_{i \in I} \mu_i(xy\beta z) = \sup_{i \in I}\{\mu_i(xy\beta z)\}$$
$$\leq \sup_{i \in I}\{\max\{\mu_i(x), \mu_i(z)\}\}$$
$$= \max\{\sup_{i \in I}\mu_i(x), \sup_{i \in I}\mu_i(z)\}$$
$$= \max\{\cup_{i \in I} \mu_i(x), \cup_{i \in I} \mu_i(z)\}.$$

Finally

$$\bigcap_{i \in I} \vartheta_i(xy\beta z) = \inf_{i \in I}\{\vartheta_i(xy\beta z)\}$$
$$\geq \inf_{i \in I}\{\min\{\vartheta_i(x), \vartheta_i(z)\}\}$$
$$= \min\{\inf_{i \in I}\vartheta_i(x), \inf_{i \in I}\vartheta_i(z)\}$$
$$= \min\{\cap_{i \in I} \vartheta_i(x), \cap_{i \in I} \vartheta_i(z)\}.$$

Hence P_i is an anti Q-Pythagorean fuzzy bi-ideal of semiring S.

13) Theorem 3.13 *Let* P_1 *and* P_2 *be two anti Q-Pythagorean fuzzy bi-ideal of semiring* S. *Then* P *is an anti Q-Pythagorean fuzzy bi-ideal of semiring* S.
   Proof. Let $x, y, z \in S, q \in Q$.

$$(\mu_1 \cdot \mu_2)(x+y) = \max\{\mu_1(x+y), \mu_2(x+y)\}$$
$$\leq \max\{\max\{\mu_1(x), \mu_1(y)\}, \max\{\mu_2(x), \mu_2(y)\}\}$$
$$= \max\{\max\{\mu_1(x), \mu_2(x)\}, \max\{\mu_1(y), \mu_2(y)\}\}$$
$$= \max\{(\mu_1 \cdot \mu_2)(x), (\mu_1 \cdot \mu_2)(y)\}$$

and

$$(\vartheta_1 \cdot \vartheta_2)(x+y) = \min\{\vartheta_1(x+y), \vartheta_2(x+y)\}$$
$$\geq \min\{\min\{\vartheta_1(x), \vartheta_1(y)\}, \min\{\vartheta_2(x), \vartheta_2(y)\}\}$$
$$= \min\{\min\{\vartheta_1(x), \vartheta_2(x)\}, \min\{\vartheta_1(y), \vartheta_2(y)\}\}$$
$$= \min\{(\vartheta_1 \cdot \vartheta_2)(x), (\vartheta_1 \cdot \vartheta_2)(y)\}.$$

Next

$$(\mu_1 \cdot \mu_2)(xy) = \max\{\mu_1(xy), \mu_2(xy)\}$$
$$\leq \max\{\max\{\mu_1(x), \mu_1(y)\}, \max\{\mu_2(x), \mu_2(y)\}\}$$
$$= \max\{\max\{\mu_1(x), \mu_2(x)\}, \max\{\mu_1(y), \mu_2(y)\}\}$$
$$= \max\{(\mu_1 \cdot \mu_2(x)), (\mu_1 \cdot \mu_2(y))\}$$

and

$$(\vartheta_1 \cdot \vartheta_2)(xy) = \min\{\vartheta_1(xy), \vartheta_2(xy)\}$$

$$\geq \min\{\min\{\vartheta_1(x), \vartheta_1(y)\}, \min\{\vartheta_2(x), \vartheta_2(y)\}\}$$
$$= \min\{\min\{\vartheta_1(x), \vartheta_2(x)\}, \min\{\vartheta_1(y), \vartheta_2(y)\}\}$$
$$= \min\{(\vartheta_1 \cdot \vartheta_2(x)), (\vartheta_1 \cdot \vartheta_2(y))\}$$

Also

$$(\mu_1 \cdot \mu_2)(xy\beta z) = \max\{\mu_1(xy\beta z), \mu_2(xy\beta z)\}$$
$$\leq \max\{\max\{\mu_1(x), \mu_1(z)\}, \max\{\mu_2(x), \mu_2(z)\}\}$$
$$= \max\{\max\{\mu_1(x), \mu_2(x)\}, \max\{\mu_1(z), \mu_2(z)\}\}$$
$$= \max\{(\mu_1 \cdot \mu_2(x)), (\mu_1 \cdot \mu_2(x))\}.$$

and

$$(\vartheta_1 \cdot \vartheta_2)(xy\beta z) = \min\{\vartheta_1(xy\beta z), \vartheta_2(xy\beta z)\}$$
$$\geq \min\{\min\{\vartheta_1(x), \vartheta_1(z)\}, \min\{\vartheta_2(x), \vartheta_2(z)\}\}$$
$$= \min\{\min\{\vartheta_1(x), \vartheta_2(x)\}, \min\{\vartheta_1(z), \vartheta_2(z)\}\}$$
$$= \min\{(\vartheta_1 \cdot \vartheta_2(x)), (\vartheta_1 \cdot \vartheta_2(z))\}.$$

Hence P_1 and P_2 is an anti Q-Pythagorean fuzzy bi-ideal of S.

14) Definition 3.14 *The product of* P_1 *and* P_2 *is a Pythagorean fuzzy subset* $P_1 \circ P_2 : S \to [0,1]$ *by*

$$(\mu_1 \circ \mu_2)(a) = \sup_{a=bc}\{\min\{\mu_1(b), \mu_2(c)\}\}$$
$$(\vartheta_1 \circ \vartheta_2)(a) = \inf_{a=bc}\{\max\{\vartheta_1(b), \vartheta_2(c)\}\}$$

15) Theorem 3.15 *If* P_1, P_2 *be any two anti Q-Pythagorean fuzzy bi-ideals of semiring* S, *then* $P_1 \circ P_2$ *is an anti Q-Pythagorean fuzzy bi-ideal of* S.

Proof. Let P_1, P_2 are any two anti Q-Pythagorean fuzzy ideals of semiring S and $x, y \in S$, $\gamma \in \Gamma$.

Then

$$(\mu_1 \circ \mu_2)(x + y) = \inf_{x+y \leq c+d}\{\max\{\mu_1(c), \mu_2(d)\}\}$$
$$\leq \inf_{x+y \leq (c_1\gamma d_1)+(c_2\gamma d_2) \leq (c_1+c_2)\gamma(d_1+d_2)}\{\min\{\mu_1(c_1 + c_2), \mu_2(d_1 + d_2)\}\}$$
$$\leq \inf\{\max\{\mu_1(c_1), \mu_1(c_1)\}, \max\{\mu_2(d_1), \mu_2(d_2)\}\}$$
$$= \max\{\inf_{x \leq c_1\gamma d_1}\{\max\{\mu_1(c_1), \mu_2(d_1)\}\}, \inf_{y \leq c_2\gamma d_2}\{\max\{\mu_1(c_2), \mu_2(d_2)\}\}\}$$
$$= \max\{(\mu_1 \circ \mu_2)(x), (\mu_1 \circ \mu_2)(y)\}$$

Also

$$(\vartheta_1 \circ \vartheta_2)(x + y) = \sup_{x+y \leq c+d}\{\min\{\vartheta_1(c), \vartheta_2(d)\}\}$$
$$\geq \sup_{x+y \leq (c_1\gamma d_1)+(c_2\gamma d_2) \leq (c_1+c_2)\gamma(d_1+d_2)}\{\min\{\vartheta_1(c_1 + c_2), \vartheta_2(d_1 + d_2)\}\}$$
$$\geq \sup\{\min\{\vartheta_1(c_1), \vartheta_1(c_2)\}, \min\{\vartheta_2(d_1), \vartheta_2(d_2)\}\}$$
$$= \min\{\sup_{x \leq c_1\gamma d_1}\{\min\{\vartheta_1(c_1), \vartheta_2(d_1)\}\}, \sup_{y \leq c_2\gamma d_2}\{\min\{\vartheta_1(c_2), \vartheta_2(d_2)\}\}\}$$
$$= \min\{(\vartheta_1 \circ \vartheta_2)(x), (\vartheta_1 circ \vartheta_2)(y)\}.$$

Now let as consider P_1, P_2 are anti Q-Pythagorean fuzzy right ideals and we have

$$(\mu_1 \circ \mu_2)(xy) = \inf_{xy \leq c\gamma d}\{\max\{\mu_1(c), \mu_2(d)\}\}$$
$$\leq \inf_{xy \leq (x_1x_2)\gamma y}\{\max\{\mu_1(x_1\gamma y), \mu_2(x_2\gamma y)\}\}$$
$$\leq \inf_{x \leq (x_1\gamma x_2)}\{\max\{\mu_1(x_1), \mu_2(x_2)\}\}$$
$$= (\mu_1 \circ \mu_2)(x).$$

and

$$(\vartheta_1 \circ \vartheta_2)(xy) = \sup_{xy \leq c\gamma d}\{\min\{\vartheta_1(c), \vartheta_2(d)\}\}$$
$$\geq \sup_{xy \leq (x_1x_2)\gamma y}\{\min\{\vartheta_1(x_1\gamma y), \vartheta_2(x_2\gamma y)\}\}$$

$$\geq \sup_{x \leq (x_1 \gamma x_2)} \{\min\{\vartheta_1(x_1), \vartheta_2(x_2)\}\}$$

$$= (\vartheta_1 \circ \vartheta_2)(x).$$

Similarly assuming P_1, P_2 are anti Q-Pythagorean fuzzy left ideal,
we can show that $(P_1 \circ P_2)(xy) \geq (P_1 \circ P_2)(y)$.
Also

$$(\mu_1 \circ \mu_2)(x) = \inf_{x \leq x_1 \gamma x_2} \{\max\{\mu_1(x_1), \mu_2(x_2)\}\}$$
$$\leq \inf_{x \leq y \leq y_1 \gamma y_2} \{\max\{\mu_1(y_1), \mu_2(y_2)\}\}$$
$$= \inf_{y \leq y_1 \gamma y_2} \{\max\{\mu_1(y_1), \mu_2(y_2)\}\}$$
$$= (\mu_1 \circ \mu_2)(y)$$

and

$$(\vartheta_1 \circ \vartheta_2)(x) = \sup_{x \leq x_1 \gamma x_2} \{\min\{\vartheta_1(x_1), \vartheta_2(x_2)\}\}$$
$$\geq \sup_{x \leq y \leq y_1 \gamma y_2} \{\min\{\vartheta_1(y_1), \vartheta_2(y_2)\}\}$$
$$= \sup_{y \leq y_1 \gamma y_2} \{\min\{\vartheta_1(y_1), \vartheta_2(y_2)\}\}$$
$$= (\vartheta_1 \circ \vartheta_2)(y)$$

Hence $P_1 \circ P_2$ is an anti Q-Pythagorean fuzzy ideal of semiring S.

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