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The Integer Solution of the Negative Pell Equation Involving Palindrome Numbers

S. Vidhya[1], V. Devi[2]
Assistant Professor[1], PG Student[2], PG and Research Department of Mathematics, Cauvery College for Women (Autonomous), (Affiliated to Bharathidasan University), Tiruchirapalli-620018

Abstract: The objective of the article is to find an integer solution that is non-zero for the negative Pell's equation involving Palindrome number $x^2 = 33y^2 - 8^t, t \in N$

Concerning the several possibilities for t ***,*** $t = 2k+1, \forall k \in N$ ***, and*** $t = 2k, \forall k \in N$ ***. The recurrence relation on the solutions is retrieved.***

Keywords: Negative Pell's equation,Integer solution, Diophantine equation, Palindrome number, Pell equation

I. INTRODUCTION

In this paper, the negative Pell equation involving Palindrome number $x^2 = 33y^2 - 8^t, t \in N$ is solved for the non-zero distinct integer solutions and the relevant recurrence relation are also retrieved.

Let D be a non-negative integer rather than a non-square number. Then the negative Pell equation $x^2 - Dy^2 = -1$ is solvable only in the event that D can be expressed as $D = a^2 + b^2$ and $\gcd(a,b)=1$.Then the Diophantine equation with a and b positive and with b odd.

A. Definition

A Palindrome number is a number that remains the same when its digits are reversed. (i.e.) it has reflection symmetry across a vertical axis.

Example: 0,1,2,3,4,5,6,7,8,9,11,22,33,44,101,151,202,222,4004… are some of Palindrome number.

II. METHODS OF ANALYSIS

Consider the Pell equation involvingpalindrome numbers

$$x^2 = 33y^2 - 8^t, t \in N \qquad (1)$$

Solvable for integers x and y whose initial solution in case of t=1 is

$$x_0 = 5, y_0 = 1$$

To found other solutions of (1), convert the equation into Pellian equation.

$$x^2 - 33y^2 = 1$$

Whose initial solution is $x_0 = 23, y_0 = 4$ and the general solution x_n, y_n is given by

$$x_n = \frac{1}{2} f_n, y_n = \frac{1}{2\sqrt{33}} g_n$$

Case: 1

$t = 2k+1, \forall k \in N$

The Pellequation is

$$x^2 = 33y^2 - 8^{2k+1} \quad (2)$$

Let the initial solution be

$$x_0 = 4.8^k \ , \ y_0 = 4.8^k$$

By applying the Brahmagupta lemma correlating,the sequence of non-zero solution obtained as

$$x_{n+1} = 2.8^k[f_n + \sqrt{33}g_n]$$

$$y_{n+1} = 2.8^k[f_n + \frac{1}{2\sqrt{33}}g_n]$$

The corresponding recurrence relation for the above solution is

$$x_{n+3} - 46x_{n+2} + x_{n+1} = 0$$

$$y_{n+3} - 46y_{n+2} + y_{n+1} = 0$$

Note

If we choose $t = 2k, \forall k \in N$, then we cannot find non-zero solution because, in this case solution does not exist.

III. CONCLUSION

This paper presents integer solutions to the negative Pell equation involving the palindrome number. Due to the Diophantine equation's wide variety, one can look for other palindrome number to the Diophantine equations .

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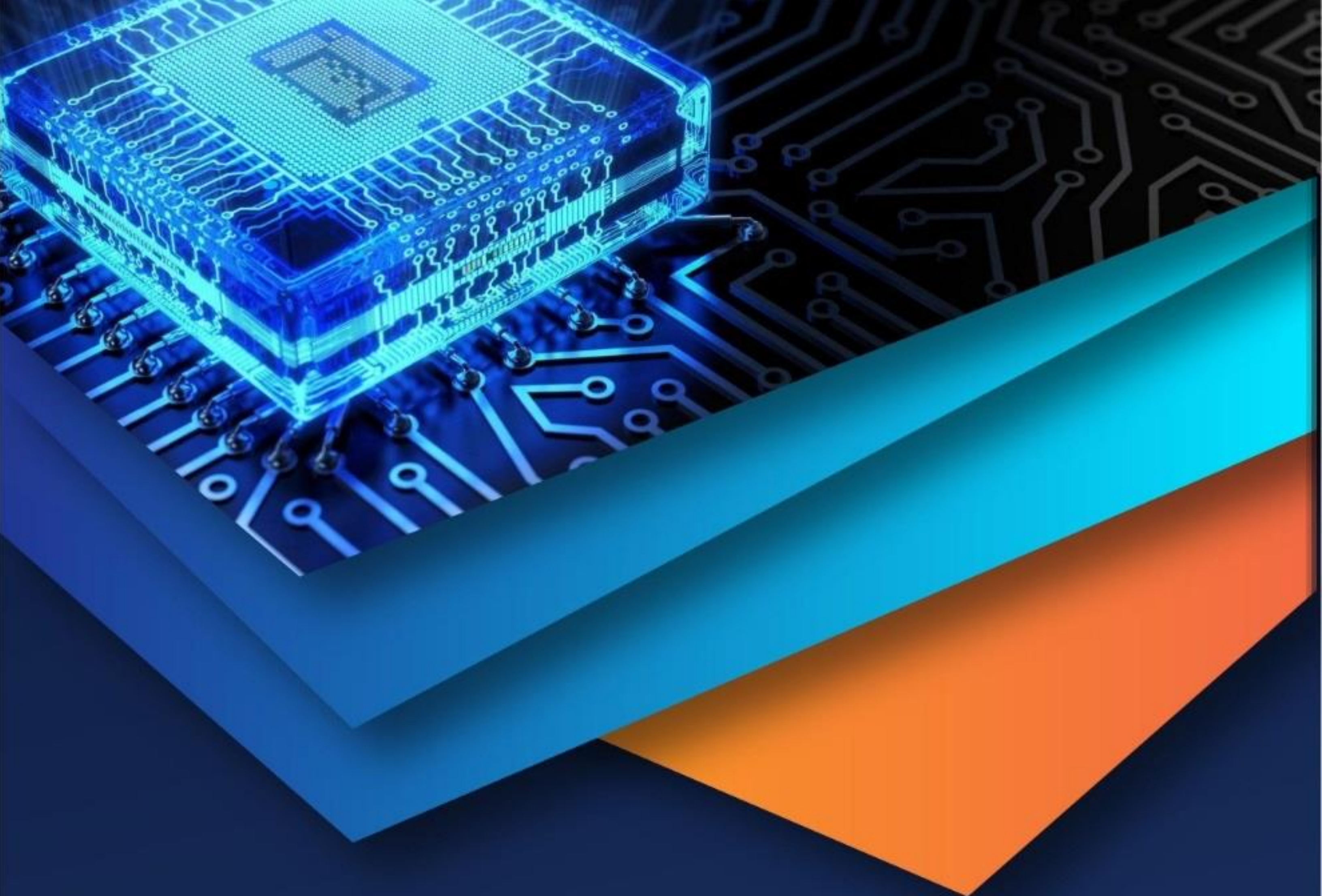

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