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# The Study on the Nonlinear Effects of Soliton in Optical Fibre

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**Abstract:** *In optical fibres, the chirps generated by GVD and SPM, which both limit the system's performance when operating independently, balance each other out to produce soliton formation. To comprehend how such a balance is possible, we shall study the nonlinear optical effects and the dispersion-induced pulse broadening. If an optical pulse is not adequately chirped before it propagates inside an optical fibre, the GVD broadens the pulse. More specifically, an early stage of transmission compresses a chirped pulse whenever  $\beta_2$  and the chirp parameter  $C$  have opposite signs and  $\beta_2 C$  is negative. The optical pulse chirps due to SPM, ensuring that  $C > 0$ . If  $\beta_2 < 0$ , it is easy to meet the criteria  $\beta_2 C < 0$ . Moreover, because the SPM-induced chirp depends on power, it is possible that in some situations, SPM and GVD could cooperate to the extent that the SPM-induced chirp is precisely the right amount to cancel out the pulse broadening caused by GVD. Here, an optical pulse propagates distortion-free as a soliton [27].*

**Keywords:** *Soliton, Optical Fiber, Nonlinear Optics, Self Reinforcing, Nonlinear Schrodinger Equation, Dispersion, Self Phase Modulation.*

## I. INTRODUCTION

It is challenging to define soliton in a way that is widely accepted. Three features are offered to soliton by Drazin & Johnson (1989, p. 15). [1]. They can communicate with other solitons, have a permanent shape, and are restricted to a specific area. They do not get affected by collisions other than avoiding phase transitions. There are more formal formulas available, albeit they require a great deal of mathematics. Furthermore, certain scientific occurrences that fail to satisfy these three conditions are classified as soliton phenomena (for example, even though they lose energy during interaction, solitons are sometimes used to characterise "light bullets" in nonlinear optics). [2] Wave patterns can be either permanent or limited, depending on the interplay between dispersion and nonlinearity. Envision a brilliant pulse passing through a piece of glass. This pulse might be made up of several different light frequencies. These different frequencies propagate through glass at different speeds due to its dispersion, which gradually modifies the pulse's structure. There is also the nonlinear Kerr effect, which asserts that the brightness or amplitude of the light determines a material's refractive index at a specific frequency. Because the Kerr effect precisely cancels the dispersion effect, a properly created pulse maintains its shape throughout time. The pulse is a soliton as a result [3]. Solitons can completely solve a number of nonlinear Schrödinger equations, the coupled nonlinear Schrödinger equation, the sine-Gordon equation, the Korteweg-de Vries equation, and other models. The soliton solutions are stable because the field equations are integrable, which is usually achieved using the inverse scattering transform. One vibrant and expansive area of mathematics is the mathematical theory of these equations. [3] A train of solitons moves concurrently with a wavefront in some undular tidal bore events that occur in a few rivers, such as the River Severn. As internal waves travel down the oceanic pycnocline, propelled by the seafloor's topography, more solitons are created. Soliton counts in the atmosphere are higher. Consider the morning beauty cloud in the Gulf of Carpentaria as an illustration. It is produced by massive linear roll clouds caused by pressure solitons travelling across a temperature inversion layer. The signal conduction within neurons is described by pressure solitons in the recently suggested, but not widely accepted, neuroscience soliton model [3]. Any solution of a system of partial differential equations that is persistent against decay to the "trivial solution" is referred to as a topological soliton, also known as a topological defect. The origin of soliton stability is not the integrability of the field equations, but topological constraints. Due to the differential equations' need to uphold the boundary's nontrivial homotopy group and adhere to a set of boundary conditions, constraints are nearly always present. Therefore, solutions to differential equations can be arranged using homotopy classes [3].

More precisely, soliton transmission in optical fibres enhances data transmission quality by mitigating two main forms of pulse degradation. The dispersion that occurs when pulses travel across extended fibre lengths is one kind of degradation. The other is the nonlinear effects resulting from signals interacting with the fibre and with each other in a power-dependent manner. Nonetheless, the effects can cancel each other out, at least to first-order approximation, for certain types and powers of optical pulses.

In general, the two effects exacerbate one another and worsen the illness. These pulse forms are called soliton pulses. One of the key benefits of soliton gearboxes for high-speed, long-distance applications is their inherent endurance. Despite soliton attenuation, soliton can be made fundamentally stable over long fibre lengths. This provides a means of mitigating the significant loss of signal quality at 10 Gbit/s and greater transmission speeds caused by dispersion and nonlinear effects. Because of these characteristics, researchers are creating soliton systems for upcoming 40-Gbit/s and long-haul 10-Gbit/s networks [4].

## II. LITERATURE REVIEW

When John Scott Russell (1808–1882) observed a single wave on Scotland's Union Canal in 1834, he became the first person to record the soliton phenomenon. He called the phenomenon the "Wave of Translation" after he was able to reproduce it in a wave tank. Solutions to the Korteweg–de Vries equation that propagate locally limited and strongly stable can describe waves comparable to the ones Russell observed. Zabusky and Kruskal originally gave these solutions the name "soliton." With the suffix "on" evoking its historical use to identify particles like hadrons, baryons, and electrons and noting their observed particle-like activity, the term was intended to emphasise the solitary nature of the waves[3].

A young engineer named John Scott Russell, engaged for a summer project in 1834 to look at ways to enhance the designs for barges intended to cross canals specifically, the Union Canal in Edinburgh, Scotland—made the first known observation of a lone wave. The barge came to a startling stop one August day when the tow line that held the mules in place broke. However, the body of water ahead of the blunt prow of the barge rolled rapidly, forming a large, solitary elevation and a smooth, rounded mound of water that continued down the canal at the same direction and speed. Russell (1844). Russell rode up to and passed the Wave of Translation as a follow-up to this unintentional discovery. Equation (1) illustrates how the amplitude effect, a nonlinear component, and the dispersive term which permits waves of various wavelengths to travel at different velocities—combine to determine the rate of change of the wave's height over time. Korteweg and de Vries discovered a periodic solution that matched Russell's wave in addition to a solitary-wave solution. A trade-off between dispersion and nonlinearity led to these solutions. Mathematicians, physicists, and engineers researching water waves disregarded Russell's findings and the work of Norman Zabusky and Martin Kruskal, who published their numerical solutions to the KdV equation (and coined the term "soliton"), until 1965 (Zabusky, 1965). Kruskal generated a continuous description of the oscillations of unidirectional waves propagating on the cubic, Fermi–Pasta–Ulam (FPU) nonlinear lattice, according to (1) (Fermi, 1955; Porter, 2009b; Weissert, 1997). Morikazu Toda made history at the same time when he identified a soliton for the first time in the discrete, integrable system that is now known as the Toda lattice and Toda, 1967.[5]

Films of interacting solitary waves in an FPU lattice, the KdV equation, and a modified KdV equation were created by Gary Deem, Zabusky, and Kruskal (1965); refer to the discussion in the review paper (Zabusky, 1984). We illustrate the dynamics of solitons in the space-time diagram shown in Figure 1 using the KdV equation. Robert Miura discovered a precise transition between this modified KdV equation and equation (1) when he understood the significance of this discovery (Miura, 1976). There was a spike in interest in the mathematical study of solitons following the 1967 solution of the initial-value problem of the KdV equation by Clifford Gardner, John Greene, Martin Kruskal, and Robert Miura (Miura, 1968; Gardner, 1967; Gardner, 1974). By demonstrating the integrability of the nonlinear Schrödinger (NLS) problem and the availability of soliton solutions, Vladimir Zakharov and Alexei Borisovich Shabat expanded the use of the inverse scattering approach in 1972. One of the several nonlinear PDEs for which Mark Ablowitz, David Kaup, Alan Newell, and Harvey Segur demonstrated that they had soliton solutions in 1973 was the sine-Gordon equation. Albert Backlund's 19th-century work on surfaces with continuous negative Gaussian curvature established the integrability of this equation. Since then, researchers in one or more spatial dimensions have looked into related soliton solutions and have discovered alternative integrable PDEs.

A more sophisticated definition of a "soliton" over many spatial dimensions is required, as the Kadomtsev-Petviashvili (KP) equation shows.

Analytical approaches for studying solitary waves in nonintegrable equations usually involve the use of asymptotic analysis, variational approximations, and/or perturbative techniques (Kivshar, 1989). (Scott, 2005) The coupled mode equations of the fibre Bragg grating are a well-known illustration of a nonintegrable system having accurate solutions for isolated solitary waves in optics. One of the most active fields in physics and mathematics today is the study of solitons and solitary waves (Scott, 2005). It has affected several academic disciplines, such as experimental science and pure mathematics. Significant advancements have resulted from this in numerous fields, including supersymmetry, nonlinear dynamics, biology, optics, and integrable systems.



### III. MATHEMATICAL EQUATIONS OF SOLITON

Nonlinear Schrodinger equation (NLS)[6],[7]:

$$i \frac{\partial A}{\partial z} - \frac{1}{2} \beta_2 \frac{\partial^2 A}{\partial T^2} + \gamma |A|^2 A = 0$$

$\beta_2$  is the GVD of the optical fiber

$\gamma$  is the nonlinear coefficient of the fiber,

$$\gamma = \frac{n_2 \omega_0}{c A_{\text{eff}}}$$

The effects of dispersion & assuming Gaussian pulse shape, [6],[7]:

$$i \frac{\partial A}{\partial z} - \frac{1}{2} \beta_2 \frac{\partial^2 A}{\partial T^2} = 0 \quad (\text{without the nonlinear term}) \quad (8a)$$

$$\tau_{\text{out}} = \tau_{\text{in}} (1 + (\beta_2 L / \tau^2)^2)^{1/2} \quad (9)$$

$$\tau_{\text{out}} = \tau_{\text{in}} (1 + (L / L_D)^2)^{1/2} \quad (10)$$

Where ,  $L_D = \tau^2 / |\beta_2|$  is the dispersion length.

The effects of nonlinearity, [6],[7]:

$$i \frac{\partial A}{\partial z} + \gamma |A|^2 A = 0 \quad (\text{without the dispersion term}) \quad (8b)$$

The maximum nonlinear phase shift, [6],[7]:

$$\phi_{\text{max}} = \gamma P_0 L = L / L_{\text{NL}}$$

And the nonlinear length, [6],[7]:

$$L_{\text{NL}} = (\gamma P_0)^{-1}$$

For the Self-Phase Modulation,  $I(t)$  gives the intensity of an ultrashort pulse with a Gaussian form and constant phase at time  $t$ , [6][7]:

$$I(t) = I_0 \exp \left( -\frac{t^2}{\tau^2} \right) \quad (11)$$

From the Optical Kerr Effect [6][7]:

$$n(I) = n_0 + n_2 \cdot I \quad (12)$$

This change in refractive index causes a displacement in the pulse's immediate phase [6][7]:

$$\phi(t) = \omega_0 t - kx = \omega_0 t - \frac{2\pi}{\lambda_0} \cdot n(I) L \quad (13)$$

The pulse shifts in frequency as a result of the phase shift. The frequency  $\omega(t)$  at any given instant is provided by [6][7]:

$$\omega(t) = \frac{d\phi(t)}{dt} = \omega_0 - \frac{2\pi L}{\lambda_0} \frac{dn(I)}{dt} \quad (14)$$

$$\omega(t) = \omega_0 + \frac{4\pi L n_2 I_0}{\lambda_0 \tau^2} \cdot t \cdot \exp\left(\frac{-t^2}{\tau^2}\right) \quad (15)$$

Propagation of Solitons in Optical Fibers:

We pick  $s = -1$  in the following equation, assuming that pulses are propagating in the region of anomalous GVD, in order to determine the conditions under which solitons can form[27]:

$$iL_D \frac{\partial U}{\partial z} - \frac{s}{2} \frac{\partial^2 U}{\partial \tau^2} + \frac{L_D}{L_{NL}} p(z) |U|^2 U = 0$$

The equation above assumes the form  $\xi = z/L_D$ , where  $\xi$  is a normalised distance[27].

$$i \frac{\partial U}{\partial \xi} + \frac{1}{2} \frac{\partial^2 U}{\partial \tau^2} + N^2 |U|^2 U = 0$$

wherein the definition of parameter  $N$  is[27]:

$$N^2 = \frac{L_D}{L_{NL}} = \frac{\gamma P_0 T_0^2}{|\beta_2|}$$

It is an arbitrary combination of the fiber's characteristics and the pulse. It is possible to eliminate the single parameter  $N$  entirely by adding  $u = NU$  as a renormalized amplitude. This modification gives the NLS equation its canonical form[27].

$$i \frac{\partial u}{\partial \xi} + \frac{1}{2} \frac{\partial^2 u}{\partial \tau^2} + |u|^2 u = 0$$

This NLS equation is a member of a unique class of nonlinear partial differential equations that may be precisely solved using the inverse scattering method, a mathematical methodology. Using this approach, it was first resolved in 1971. The primary outcome can be summed up as follows. When an initial amplitude pulse in the input[27]:

$$u(0, \tau) = N \operatorname{sech}(\tau)$$

is launched into the fibre; for  $N = 1$ , its shape stays constant during propagation; however, for integer values of  $N > 1$ , it exhibits a periodic pattern that recovers the input shape at  $\xi = m\pi/2$ , where  $m$  is an integer[27]

Without using the inverse scattering method, the following equation can be solved directly to find the solution corresponding to the fundamental soliton[27].

$$i \frac{\partial u}{\partial \xi} + \frac{1}{2} \frac{\partial^2 u}{\partial \tau^2} + |u|^2 u = 0$$

The method is presuming that a formative solution[27]:

$$u(\xi, \tau) = V(\tau) \exp[i\phi(\xi)]$$

The nonlinear differential equation is then found to be satisfied by the function  $V(\tau)$ [27]:

$$\frac{d^2 V}{d\tau^2} = 2V(K - V^2)$$

Thus the Sech Equation gives us[27]:

$$u(\xi, \tau) = \operatorname{sech}(\tau) \exp(i\xi/2)$$

for the fundamental soliton using direct integration of the NLS equation.

The fibre nonlinearity essentially compensates for the effects of fibre dispersion when the input pulse has a "sech" form and its peak power and width are related by[27]:

$$N^2 = \frac{L_D}{L_{NL}} = \frac{\gamma P_0 T_0^2}{|\beta_2|}$$

#### IV. CONCLUSION

In optical fibres, soliton creation is the result of the mutual balancing of the chirps produced by GVD and SPM, which both reduce the system's performance when functioning alone. We will investigate the dispersion-induced pulse broadening and the nonlinear optical effects to understand how such a balance is attainable. The GVD broadens an optical pulse if it is not sufficiently chirped prior to its propagation inside an optical fibre. More precisely, when  $\beta_2$  and the chirp parameter  $C$  have opposite signs and  $c2C$  is negative, an early transmission stage compresses a chirped pulse. SPM causes the optical pulse to chirp, confirming that  $C > 0$ . It is simple to achieve the criterion  $\beta_2 C < 0$  if  $\beta_2 < 0$ . Furthermore, because the SPM-induced chirp is power-dependent, it is feasible that under some circumstances, SPM and GVD will work together to the point where the SPM-induced chirp is just the proper amount to cancel out the GVD-caused pulse broadening. Here, a soliton propagates as an optical pulse without distortion.[27]

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