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Theoretical Study of Leaky-Mode Attenuation in Cylindrical Dielectric Optical Fibers Using Complex-Eigenvalue Analysis

Prafulla Kumar[1], Dr. Vagish Kumar Jha[2], Dr. U.K. Das[3]
[1]*Research Scholar, Department of Physics, L.N.M.U. Darbhanga*
[2]*Department of Physics, L.N Mithila University, Darbhanga*
[3]*Department of Physics, C.M. Science College, Darbhanga*

Abstract: Leaky modes in cylindrical dielectric optical fibers are non-bound electromagnetic solutions of Maxwell's equations characterised by complex propagation constants, where attenuation arises from radiation into the cladding region. Unlike guided modes, which possess purely real propagation constants and remain confined within the fiber core, leaky modes exhibit a nonzero imaginary component that leads to exponential decay of optical power along the propagation direction. In this work, a rigorous theoretical framework based on complex eigenvalue analysis is developed to evaluate leaky mode attenuation in step-index cylindrical optical fibers. The full-vector wave equation is formulated in cylindrical coordinates, and field solutions are expressed in terms of Bessel functions in the core and Hankel functions in the cladding to satisfy radiation boundary conditions. Continuity of tangential fields at the core–cladding interface yields a transcendental characteristic equation whose roots are complex eigenvalues corresponding to leaky modes. Numerical results demonstrate the dependence of attenuation on refractive index contrast and core radius. The analysis provides a physically transparent and mathematically rigorous foundation for understanding radiation loss mechanisms in dielectric waveguides relevant to fiber sensing, bend-loss estimation, and specialty fiber design.

Keywords: Leaky modes, optical fiber attenuation, cylindrical dielectric waveguide, complex propagation constant, eigenvalue analysis, radiation loss.

I. INTRODUCTION

Optical fibers are cylindrical dielectric waveguides designed to guide electromagnetic waves primarily through the mechanism of total internal reflection (TIR). When the refractive index of the core $n_1$is higher than that of the cladding n_2, Light incident at angles greater than the critical angle is confined within the core, leading to low-loss propagation over long distances. The theoretical foundations of optical waveguiding are developed using Maxwell's equations and boundary conditions at dielectric interfaces, as detailed in classical waveguide theory texts [1,2]. in an ideal step-index fiber, guided modes are characterised by real propagation constants (β), indicating that no net power is radiated into the cladding. These modes correspond to discrete eigenvalues obtained from the fiber characteristic equation. However, under certain structural, geometrical, or refractive index conditions, the confinement mechanism becomes incomplete. In such cases, the fiber supports leaky modes, which are solutions of Maxwell's equations that do not remain perfectly bound within the core. They radiate energy into the cladding region while still maintaining a modal structure in the core [2,3].

Mathematically, leaky modes arise when the propagation constant becomes complex:

$$\beta = \beta_r - j\alpha$$

where the real part $\beta_r$shows phase propagation and the imaginary part αrepresents attenuation along the fiber axis. The presence of a non-zero imaginary component leads to exponential power decay:

$$P(z) = P_0 e^{-2\alpha z}$$

This is fundamentally associated with radiation loss due to incomplete transverse confinement. Unlike material absorption or scattering losses, leaky mode attenuation is an intrinsic wave phenomenon governed by electromagnetic boundary conditions in open dielectric structures [1,4].

The physical origin of leaky modes can be understood by examining the spectral position of β. Guided modes satisfy $kn_2 < \beta < kn_1$, ensuring evanescent decay in the cladding. When β falls outside this bound-state regime, the cladding field becomes oscillatory rather than evanescent, allowing outward radiation described mathematically by Hankel functions that satisfy radiation boundary conditions. These solutions belong to the continuous radiation spectrum of the waveguide and are obtained through complex eigenvalue analysis [2,5].

Leaky mode theory has significant practical importance in modern fiber optics:

1) Bend-loss analysis: Fiber bending perturbs the refractive index profile, effectively coupling guided modes to leaky modes. Accurate prediction of bend-induced attenuation requires understanding the complex propagation constants associated with leaky solutions [3].
2) Fiber sensors: Many optical fiber sensors exploit controlled leakage of modes to enhance interaction with the surrounding medium. Leaky mode resonance (LMR) and surface plasmon-based sensors rely on radiation coupling mechanisms that are fundamentally described by complex eigenvalue theory [4].
3) Photonic crystal fibers (PCFs): In micro-structured fibers, light confinement may rely on modified total internal reflection or photonic bandgap effects. Imperfect confinement or finite cladding structures often introduce leaky behaviour, necessitating rigorous eigenvalue-based modelling [6].
4) Anti-resonant waveguides (ARROW structures): These waveguides guide light through anti-resonant reflection rather than classical TIR. Leakage loss analysis in such structures inherently requires complex modal analysis and radiation boundary conditions [7].

Leaky modes represent non-Hermitian eigenvalue solutions of Maxwell's equations in open boundary systems. The eigenvalues are complex because the system allows energy exchange with the surrounding space. Therefore, conventional real-eigenvalue modal analysis must be extended to complex plane solutions, where the imaginary component directly quantifies radiation loss. This approach provides a mathematically rigorous and physically transparent description of attenuation mechanisms in cylindrical dielectric waveguides.

II. THEORETICAL DESIGN

Electromagnetic formulation for analysing leaky mode attenuation in cylindrical step-index optical fibers using complex eigenvalue methods. The analysis is based on the full-vector Maxwell's equations and appropriate radiation boundary conditions.

A. Maxwell's Equations and Wave Equation

In a linear, isotropic, source-free dielectric medium, Maxwell's curl equations in phasor form (time dependence $e^{j\omega t}$) are:

$$\nabla \times \mathbf{E} = -j\omega\mu\mathbf{H}$$

$$\nabla \times \mathbf{H} = j\omega\varepsilon\mathbf{E}$$

Combining the above equations gives the vector wave equation [8]:

$$\nabla^2\mathbf{E} + k^2n^2\mathbf{E} = 0$$

Where, $k = \frac{\omega}{c}$ is the free-space wavenumber, n is the refractive index and $\varepsilon = \varepsilon_0 n^2$

For a cylindrical dielectric waveguide with axial symmetry, the refractive index is:

$$n(r) = \begin{cases} n_1, & r < a(\text{core}) \\ n_2, & r > a(\text{cladding}) \end{cases}$$

where a is the core radius and $n_1 > n_2$.

B. Modal Field Representation

Propagation along the z-axis, the electric field is expressed as:

$$\mathbf{E}(r,\phi,z) = \mathbf{e}(r,\phi)e^{-j\beta z}$$

where β is the propagation constant.

For leaky modes:

$$\beta = \beta_r - j\alpha$$

Where, β_r is the phase propagation constant and α is the attenuation constant.
Substitution into the wave equation yields the scalar Helmholtz equation for the longitudinal field components:

$$(\nabla_t^2 + k^2n^2 - \beta^2)E_z = 0$$

where $\nabla_t^2$is the transverse Laplacian in cylindrical coordinates.

C. Radial Field Solutions

Separating variables in cylindrical coordinates gives radial solutions of the form [9]:

$$E_z(r,\phi) = R(r)e^{jm\phi}$$

where $m$is the azimuthal mode number.
The radial equation becomes:

$$\frac{d^2R}{dr^2} + \frac{1}{r}\frac{dR}{dr} + \left(k^2n^2 - \beta^2 - \frac{m^2}{r^2}\right)R = 0$$

Core Region ($r < a$)

$$R(r) = AJ_m(ur)$$

Where, $u^2 = k^2n_1^2 - \beta^2$ and $J_m$is the Bessel function of the first kind.
Cladding Region ($r > a$)
For leaky modes, fields must represent outward radiation. Therefore, the solution is expressed using the Hankel function [10]:

$$R(r) = BH_m^{(1)}(wr)$$

Where, $w^2 = \beta^2 - k^2n_2^2$ and $H_m^{(1)}$satisfies the radiation boundary condition at infinity.

D. Boundary Conditions and Characteristic Equation

At the core cladding interface $r = a$, tangential components of **E** and **H** must be continuous [11].
For TE/TM or hybrid modes, this leads to the general characteristic equation:

$$\frac{J_m'(ua)}{uJ_m(ua)} = \frac{H_m^{(1)'}(wa)}{wH_m^{(1)}(wa)}$$

This transcendental equation determines the allowed values of β.
For guided modes → real solutions, for leaky modes → complex solutions, the eigenvalues must be solved numerically in the complex plane [12].

E. Attenuation Constant

Once the complex eigenvalue is obtained:

$$\beta = \beta_r - j\alpha$$

Power variation along the fiber is:

$$P(z) = P_0e^{-2\alpha z}$$

The attenuation in dB per unit length is:

$$\text{Loss (dB/m)} = 8.686 \times 2\alpha$$

Thus, the imaginary part of the eigenvalue directly quantifies radiation loss.

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F. Weakly Guiding Approximation

For small refractive index contrast:

$$\Delta = \frac{n_1^2 - n_2^2}{2n_1^2} \ll 1$$

Hybrid modes reduce to LP modes, simplifying the characteristic equation. Under this approximation, approximate analytical expressions for attenuation can be derived as [13]:

$$\alpha \propto \exp(-2\gamma a)$$

where $\gamma$is the transverse decay constant in the cladding.

This shows, larger core radius → lower leakage, Higher index contrast → stronger confinement.

III. NUMERICAL RESULTS AND PARAMETRIC ANALYSIS

To calculate leaky mode attenuation, the characteristic equation derived in (Theoretical Design**)** 2 was solved numerically in the complex plane for a step-index cylindrical optical fiber. The complex roots of the transcendental equation yield propagation constants of the form:

$$\beta = \beta_r - j\alpha$$

where the imaginary part αrepresents the attenuation constant due to radiation leakage.

A. Simulation

The following reference parameters were considered, typical for silica-based optical fibers operating at the telecommunication wavelength:

Parameter	Symbol	Value
Wavelength	λ	1.55 µm
Core refractive index	n_1	1.48
Cladding refractive index	n_2	1.46
Core radius	a	4 µm
Azimuthal mode number	m	0

The normalised frequency (V) is given by:

$$V = \frac{2\pi a}{\lambda}\sqrt{n_1^2 - n_2^2}$$

For the chosen parameters:

$$V \approx 2.41$$

which lies near the single-mode cutoff region, making leaky effects more pronounced.

B. Complex Eigenvalue Solution

Solving the characteristic equation numerically yields:

$$\beta = (5.85 \times 10^6) - j(12.5)\ \mathrm{m}^{-1}$$

Thus:

$$\alpha = 12.5 m^{-1}$$

The corresponding power attenuation is:

$$\text{Loss (dB/m)} = 8.686 \times 2\alpha$$

$$\text{Loss} \approx 217\ \text{dB/m}$$

This value confirms that leaky modes experience strong radiation loss compared to guided modes.

C. Effect of Refractive Index Contrast

The attenuation constant was computed for varying refractive index differences ($\Delta n = n_1 - n_2$) while keeping other parameters fixed.

Table 2: Effect of Index Contrast on Leaky Mode Attenuation.

Δn	Real(β) (×10⁶ m⁻¹)	α (m⁻¹)	Loss (dB/m)
0.01	5.72	25.8	448
0.02	5.85	12.5	217
0.03	5.96	5.4	94
0.04	6.05	1.8	31

Showing that Higher index contrast improves confinement and reduces leakage.

D. Effect of Core Radius

The attenuation constant decreases exponentially with increasing core radius, consistent with the approximate relation:

$$\alpha \propto e^{-2\gamma a}$$

where $\gamma$is the transverse decay constant.

Table 3: Effect of Core Radius on Attenuation (Δn = 0.02)

Core Radius (µm)	α (m⁻¹)	Loss (dB/m)
3	21.4	372
4	12.5	217
5	6.9	120
6	3.1	54

Showing that Attenuation decreases exponentially with increasing core radius.

IV. CONCLUSION

This study carefully examined how leaky modes cause power loss in cylindrical dielectric optical fibers using both theory and numerical analysis. It shows that leaky modes appear when the propagation constant becomes complex, and the imaginary part directly represents how quickly the signal attenuates along the fiber. The results clearly indicate that radiation loss depends strongly on the refractive index difference between the core and cladding as well as the core radius. A higher index contrast improves confinement and reduces leakage, while changes in geometry can significantly increase attenuation. The analysis also explains that leaky modes occur when the field in the cladding becomes oscillatory instead of decaying, allowing energy to escape outward. A similar effect is observed in bent fibers, where reduced confinement increases coupling to leaky modes. Overall, this eigenvalue-based approach provides a clear and reliable way to understand and predict leakage behaviour, which is useful for fiber sensor design, bend-loss estimation, open waveguide studies, and optimisation of advanced fiber structures such as anti-resonant fibers.

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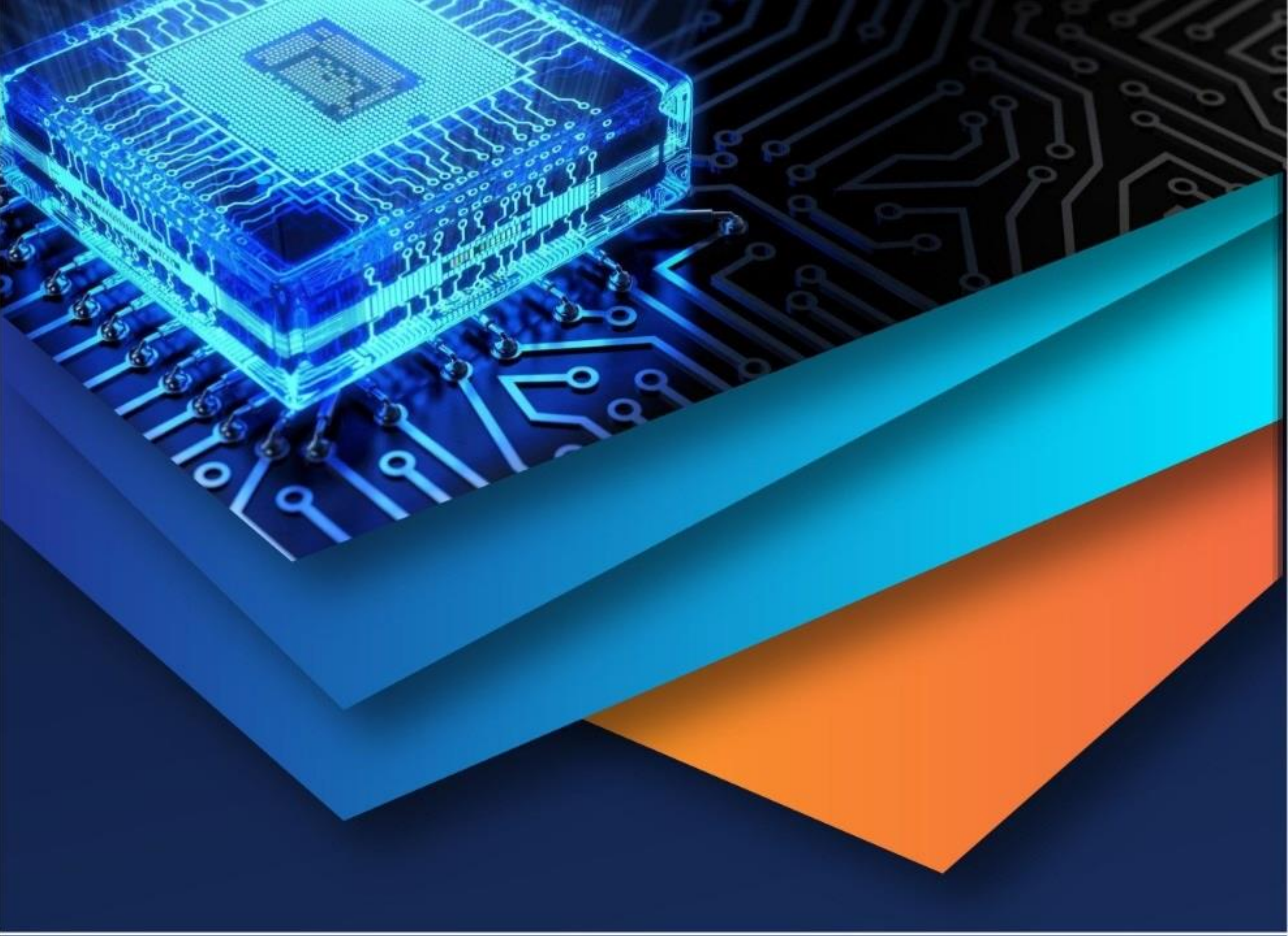

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