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Comparative Analysis and Design of Bridge Pier for Various Geometries Along Height Comparing the Parameters Deflection and Bending Stresses

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***Abstract:** Slender member is subjected to axial load and biaxial bending moment and fails due to buckling. This buckling is caused due to slenderness effect also known as 'PΔ' effect. This buckling gives rise to excessive bending moment occurring at a point of maximum deflection. This additional bending moment is considered in second order analysis. The objective of the research reported in this paper is to formulate bending moment equation by using beam column theory and to study the behaviour of solid circular section and hollow circular section of bridge pier. The optimization in area of cross section is done by providing a combination of solid and hollow circular section in place of a solid circular section of pier within permissible limits. A comparative study on behaviour for all three conditions is been carried out.*
***Keywords:** slender column, buckling, 'PΔ' effect, beam-column, second order analysis, bridge pier.*

I. INTRODUCTION

Piers are not only subjected to axial load but also forces in longitudinal direction as well as in transverse direction. These forces cause moment in longitudinal direction and transverse direction at base of pier. Thus, pier is idealized as a column subjected to axial load and biaxial moment. These forces cause the pier to buckle along its height. The moment due to buckling is not considered in first order analysis. In order to get accurate forces one has to go for second order analysis where in the buckling effect is considered. Beam column theory is one of the methods to calculate the bending moment by second order analysis.

Iterative neutral axis method is used to design the cross section of pier. In a section subjected to axial load combined with two orthogonal moments, by assuming the neutral axis at certain depth and stress at that point is to be calculated. This stress at neutral axis should be zero or else the procedure is revised for another trail.

II. SECOND ORDER ANALYSIS USING BEAM-COLUMN THEORY

Beams subjected to axial compression with lateral loads act as beam-column. The basic equation for analysis of beam-column can be derived by considering a beam as shown in Figure1.

The beam is subjected to an axial compressive force ***P*** and lateral load of intensity '***q***' which varies with the distance '***x***' along the beam. Consider an element of length 'dx' between two cross sections taken normal to the original axis of beam as shown in Figure 2.

The lateral load has a constant intensity 'q' over a distance 'dx' and will be assumed positive when in direction of positive y axis which is downward in this case. The shearing force V and bending moment M acting on either side of the elements are assumed positive in the downward direction. The relation between load, shear force and bending moment are obtained from the equilibrium of the element in Figure 2. On summing forces in the y direction it gives.

$$-V+qdx+(V+dv)=0$$

$$q=-\frac{dV}{dx} \qquad (1)$$

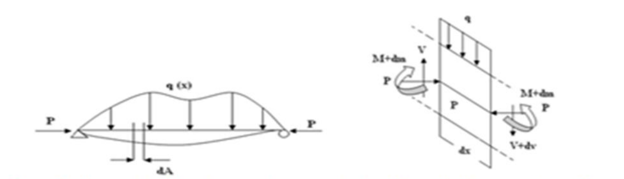

Figure1. General loading beam-column analysis Figure2. Cross section of beam

Fig -1: General Loading for Beam Column analysis

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Taking the moment about point on beam and assuming that angle between the axis of beam and horizontal axis is small, we obtain,

$$M + qdx\frac{dx}{2} + (V + dv) - (M + dM) + P\frac{dy}{dx}dx = 0$$

If terms of second-degree are neglected, this equation becomes

$$V = \frac{dM}{dx} - P\frac{dy}{dx} \qquad (2)$$

If the effects of shearing deformations and shortening of the beam axis are neglected the expression for the curvature of the axis of the beam is,

$$EI\frac{d^2y}{dx^2} = -M \qquad (3)$$

The quantity EI represents the flexural rigidity of beam in a plane of bending, i.e. XY plane, which is assumed to be plane of symmetry. Combining equation (3) with equation (1) and equation (2) we can express the differential equations of the axis of the beam in the following alternate forms:

$$EI\frac{dy}{dx} + P\frac{dy}{dx} = -V \qquad (4)$$

$$EI\frac{dy}{dx} + P\frac{dy}{dx} = q \qquad (5)$$

Equations (1) to (5) are the basic differential equations for bending of beam-column. If the axial force's P equals zero, these equations reduces to the usual equations for bending by lateral loads only. The nature of the axial forces have significant effect on the deflections and ultimately on the secondary moments.

III. ITERATIVE NEUTRAL AXIS METHOD

Iterative neutral axis method is used for design of slender member which are subjected to axial load and biaxial moment. In this method, some percentage of steel is assumed and the moment of inertia of full section is calculated. Then inclination of neutral axis is calculated. Then, moment of inertia and eccentricity of cracked section is computed. Compute stress at neutral axis, if it is zero, and if stresses at extreme fibers are within permissible limit, the assumed percentage of steel is acceptable otherwise the neutral axis has to be shifted and same procedure has to be carried out.

IV. THEOROTICAL FORMULATION

A. *Trapezoidal Load Throughout The Height Of Pier*

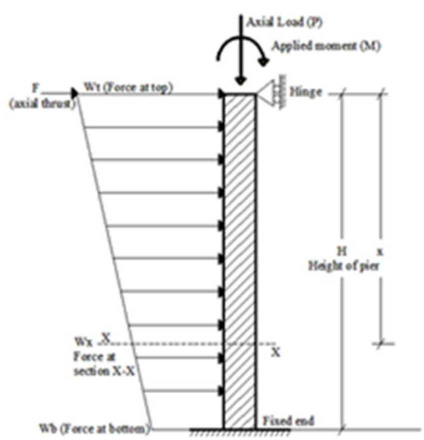

Fig -2: Load acting throughout height of pier

$$W_x = W_T - \left[\frac{\left(W_T - W_B\right)x}{H}\right] \tag{5}$$

1) First Order Analysis Of Pier

Let 'M_x' be the bending moment at a general section '*XX*' at a distance 'x' from top of pier,

$$\therefore M_x = -M - Fx + R_T - \frac{W_T x^2}{2} + \frac{(W_T - W_B)x^3}{6H} \tag{7}$$

$$R_T = \frac{3M}{2H} + F + \frac{11W_T H}{40} + \frac{W_B H}{10}$$

$$M_x = -M + \frac{3M}{2H}x + \frac{11W_T H}{40}x + \frac{W_B H}{10} - \frac{W_T x^2}{2} + \frac{(W_T - W_B)x^3}{6H} \tag{8}$$

2) Second Order Analysis Of Pier

Considering the same values used in first order analysis as given above:

Substituting constant k_w in equation (6)

$$k_w = \frac{\left(W_T - W_B\right)}{H}$$

$$W_x = W_T - k_w x$$

Bending moment at a general section 'x' is given by

$$M_x = Py - M_A + (R_T - F)x - \frac{W_T x^2}{2} + \frac{k_w x^3}{6} \tag{9}$$

$$y = complementary\ \ solution + particular$$

$$y_c = A\sin(\alpha x) + B\cos(\alpha x)$$

$$\backslash y_p = -\frac{k_w}{6P}x^3 + \frac{W_T}{2P}x^2 + \frac{x}{P}\left[F - R_T + \frac{k_w}{\alpha^2}\right] + \frac{1}{P}\left[M_A - \frac{W_T}{\alpha^2}\right]$$

Complete solution,

$$y = A\sin(\alpha x) + B\cos(\alpha x) - \frac{k_w}{6P}x^3 + \frac{W_T}{2P}x^2 + \frac{x}{P}\left[F - R_T + \frac{k_w}{\alpha^2}\right] + \frac{1}{P}\left[M_A - \frac{W_T}{\alpha^2}\right] \tag{10}$$

On substituting the boundary condition, $x=0, y=0$

In equation (10) we get,

$$B = -\frac{1}{P}\left[M_A - \frac{W_T}{\alpha^2}\right]$$

On substituting boundary condition, $x=H, y=0$ and $x=H \; \frac{\partial y}{\partial x} = 0$

In equation (10) we get,

$$A = \frac{1}{\sin(\alpha H)}\left[\frac{k_w}{6P}H^3 - B\cos(\alpha H) - \frac{W_T}{2P}H^2 - \frac{H}{P}\left[F - R_T + \frac{k_w}{\alpha^2}\right] - \frac{1}{P}\left[M_A - \frac{W_T}{\alpha^2}\right]\right] \quad (11)$$

On substituting the values of constants in the deflection equation (10)

$$R_T = \frac{1}{\frac{1}{P}\left[\frac{\tan(\alpha H)}{\alpha} - H\right]} \times \left[\frac{W_T}{P}H\left\{\frac{\tan(\alpha H)}{\alpha} - \frac{H}{2}\right\} - B\left\{\sin(\alpha H)\tan(\alpha H) + \cos(\alpha H)\right\} - \frac{k_w H}{P\alpha}\left\{\frac{H\tan(\alpha H)}{2} + \frac{1}{\alpha}\right\} + \frac{\tan(\alpha H)}{\alpha P}\left\{F + \frac{k_w}{\alpha^2}\right\} + \frac{k_w}{6P}H^3 + \frac{W_T}{P\alpha^2} - \frac{HF}{P} - \frac{M}{P}\right] \quad (12)$$

B. *Validation for the Bending Moment Equation*

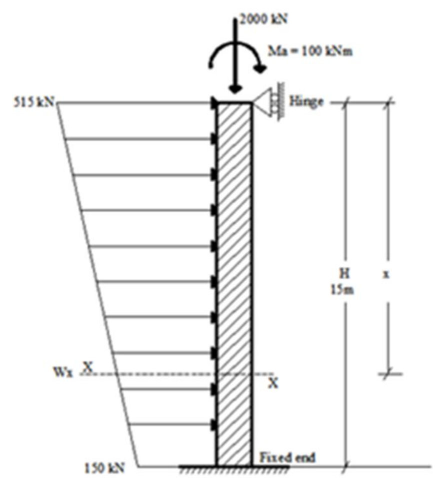

Fig -3: Load acting on partial height of pier

At, x = 0, hinged support

x = H, fixed support

For solid pier ***d*** = 3.0 m

For hollow pier, External diameter = 3.0 m,

Internal diameter = 2.4 m.

Span of bridge = 30m.

The values for base moment obtained by theoretically and by computer application (STAAD) are compared.

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V. PARAMETRIC STUDY

Forces on pier are calculated as specified in IRC and the maximum moment is calculated in ***Table 1*** shown below. Using combined stress equation and keeping the stress constant, behavior of a solid circular and hollow circular section with combination of both is studied. The variation of deflection for combination with solid and hollow pier is plotted for different heights of pier. The variation in bending stresses for different bending moments is also studied.

Table1. Diameter of pier required for critical BM at base

Design Parameters					
Height(m)	Solid (mm)	Hollow (mm)		Combination (mm)	
15	2638	2760	1380	2638	1847
20	3114	3236	1618	3114	2180
25	3551	3675	1837.5	3551	2486
30	3976	4104	2052	3976	2783
35	4381	4514	2257	4381	3067
40	4759	4897	2448.5	4759	3331

A. Comparative Analysis of Deflection in Pier

Table2. Variation of deflection for 15m pier

Deflection (Height 15m)							
Section from top(m)	Solid	Hollow	C-3	C-6	C-9	C-12	C-15
0	0.00	0.00	0.00	0.00	0.00	0.00	0.00
3	25.90	32.50	28.40	29.90	30.50	31.10	32.20
6	35.30	45.60	38.90	42.20	42.60	43.60	44.78
9	31.60	35.10	31.20	31.80	33.40	34.80	35.36
12	12.80	14.80	12.10	12.45	12.90	13.25	14.20
15	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Graph1. Variation of deflection for 15m pier

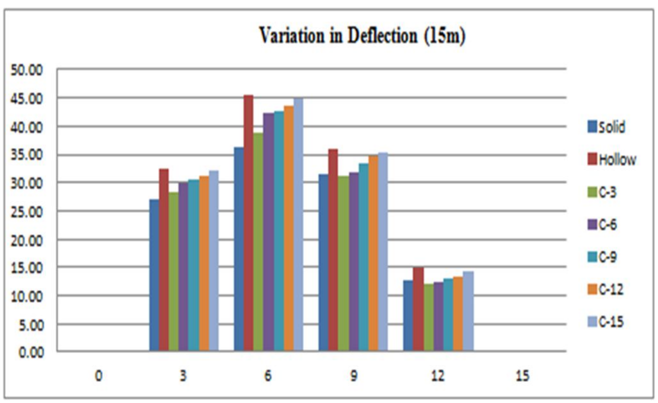


Table3. Variation of deflection for 20m pier

Deflection (Height 20m)							
Section from top(m)	Solid	Hollow	C-3	C-6	C-9	C-12	C-15
0	0.00	0.00	0.00	0.00	0.00	0.00	0.00
4	35.87	43.33	37.87	39.87	40.67	41.47	42.93
8	48.40	60.80	51.87	55.27	55.80	58.13	59.71
12	42.13	48.13	41.60	42.40	44.53	45.40	47.15
16	17.07	19.73	15.13	15.60	17.20	17.67	18.93
20	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Graph2. Variation of deflection for 20m pier

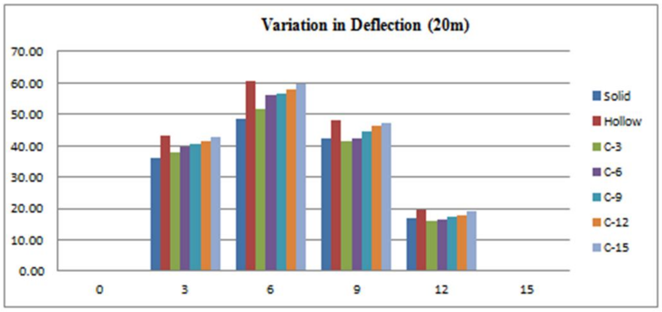


Table4. Variation of deflection for 25m pier

Deflection (Height 25m)							
Section from top(m)	Solid	Hollow	C-3	C-6	C-9	C-12	C-15
0	0.00	0.00	0.00	0.00	0.00	0.00	0.00
5	44.83	54.17	47.33	49.83	50.83	51.83	53.67
10	60.50	75.00	64.83	70.33	71.00	72.67	74.63
15	52.67	60.17	52.00	53.00	55.67	58.00	58.93
20	21.33	24.67	20.17	20.75	21.50	22.08	23.67
25	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Graph3. Variation of deflection for 25m pier

Table5. Variation of deflection for 30m pier

Deflection (Height 30m)							
Section from top(m)	Solid	Hollow	C-3	C-6	C-9	C-12	C-15
0	0.00	0.00	0.00	0.00	0.00	0.00	0.00
6	53.80	65.00	55.80	59.80	61.00	62.20	64.40
12	72.60	91.20	77.80	84.40	85.20	87.20	89.56
18	63.20	72.20	62.40	63.60	65.80	69.60	70.72
24	25.60	29.60	24.20	24.90	25.80	25.50	28.40
30	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Graph4. Variation of deflection for 30m pier

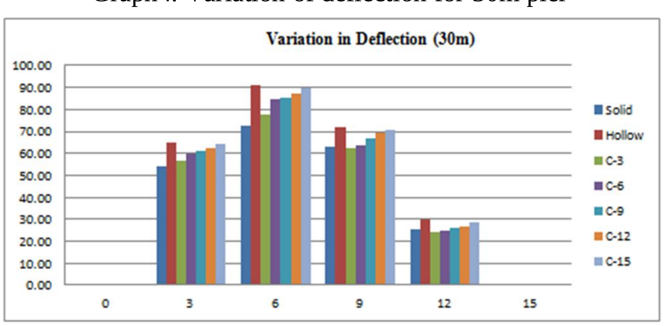


Table6. Variation of deflection for 35m pier

Deflection (Height 35m)							
Section from top(m)	Solid	Hollow	C-3	C-6	C-9	C-12	C-15
0	0.00	0.00	0.00	0.00	0.00	0.00	0.00
7	62.77	75.83	65.27	69.77	71.17	72.57	75.13
14	84.70	105.40	90.77	98.47	99.40	101.73	104.49
21	73.73	84.23	72.80	74.20	77.93	81.20	82.51
28	29.87	34.53	28.23	29.05	30.10	30.92	33.13
35	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Graph5. Variation of deflection for 35m pier

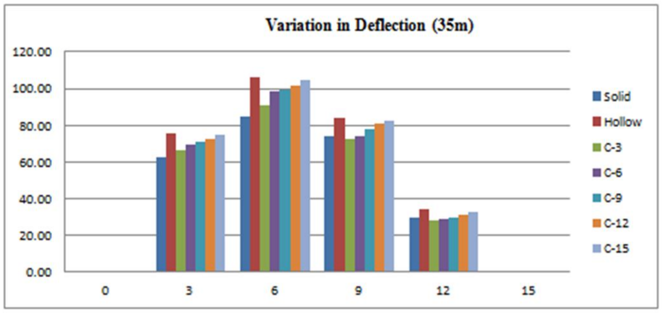
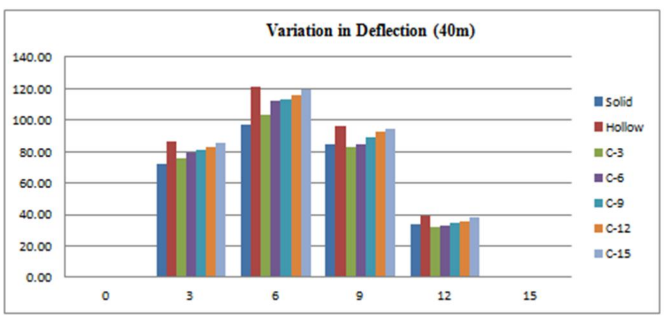


Table7. Variation of deflection for 40m pier

Deflection (Height 40m)							
Section from top(m)	Solid	Hollow	C-3	C-6	C-9	C-12	C-15
0	0.00	0.00	0.00	0.00	0.00	0.00	0.00
8	71.73	85.67	75.73	79.73	81.33	82.93	85.87
16	95.80	121.60	103.73	112.53	113.60	115.27	119.41
24	84.27	95.27	83.20	84.80	89.07	92.80	94.29
32	34.13	39.47	32.27	33.20	34.40	35.33	37.87
40	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Graph6. Variation of deflection for 40m pier



B. Comparative Analysis of Bending Stresses in Pier

Table8. Variation of bending stresses for 15m pier

Bending Stress Distribution (Height 15m)							
Section from top(m)	Solid	Hollow	C-3	C-6	C-9	C-12	C-15
0	0.00	0.00	0.00	0.00	0.00	0.00	0.00
3	-2.57	-2.40	-3.38	-3.36	-3.33	-3.27	-3.21
6	-3.04	-2.84	-3.98	-3.94	-3.89	-3.81	-3.74
9	-1.74	-1.63	-2.26	-2.20	-2.12	-2.06	-2.01
12	0.98	0.90	1.33	1.41	1.52	1.63	1.67
15	4.79	4.44	5.34	5.44	5.57	5.64	5.68

Graph7. Variation of bending stresses for 15m pier

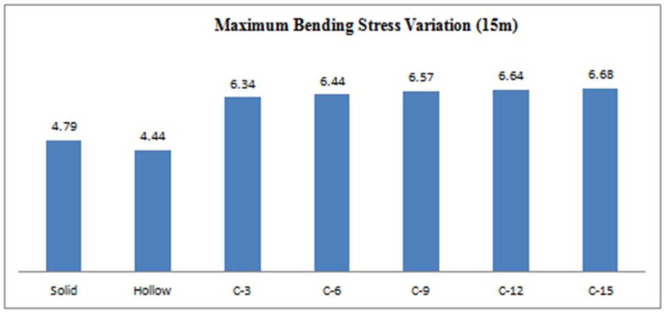


Table9. Variation of bending stresses for 20m pier

Bending Stresses (Height 20m)							
Section from top(m)	Solid	Hollow	C-3	C-6	C-9	C-12	C-15
0	0.00	0.00	0.00	0.00	0.00	0.00	0.00
4	-2.18	-3.34	-2.86	-2.84	-2.82	-2.77	-2.72
8	-2.57	-3.95	-3.37	-3.34	-3.29	-3.23	-3.17
12	-1.47	-2.27	-1.92	-1.86	-1.80	-1.74	-1.70
16	0.83	1.26	1.12	1.20	1.28	1.38	1.41
20	4.05	5.18	5.37	5.46	5.57	5.62	5.66

Graph8. Variation of bending stresses for 20m pier

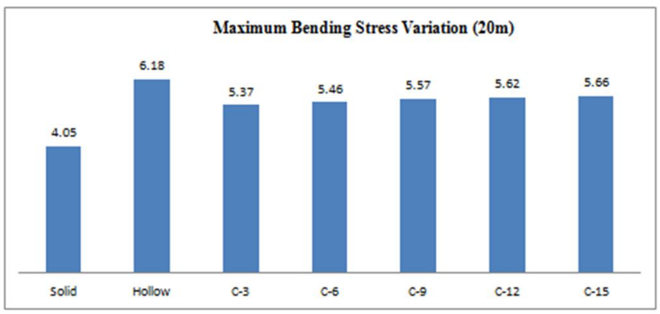


Table10. Variation of bending stresses for 25m pier

Bending Stresses (Height 25m)							
Section from top(m)	Solid	Hollow	C-3	C-6	C-9	C-12	C-15
0	0.00	0.00	0.00	0.00	0.00	0.00	0.00
5	-2.28	-2.20	-3.00	-2.98	-2.96	-2.91	-2.85
10	-2.70	-2.60	-3.54	-3.50	-3.45	-3.38	-3.32
15	-1.54	-1.50	-2.01	-1.95	-1.88	-1.83	-1.78
20	0.87	0.83	1.18	1.26	1.35	1.44	1.48
25	4.25	4.07	5.63	5.72	5.84	5.89	5.93

Graph9. Variation of bending stresses for 25m pier

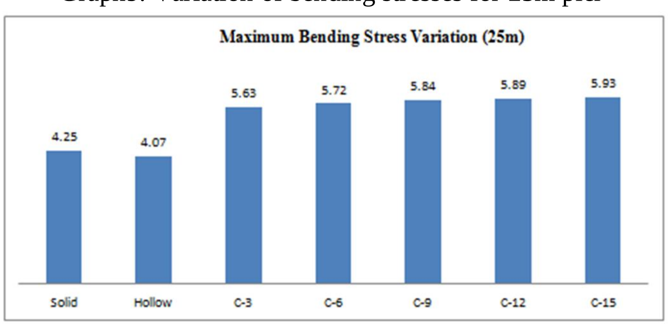


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Table11. Variation of bending stresses for 30m pier

Bending Stresses (Height 30m)							
Section from top(m)	Solid	Hollow	C-3	C-6	C-9	C-12	C-15
0	0.00	0.00	0.00	0.00	0.00	0.00	0.00
6	-2.36	-2.29	-3.09	-3.07	-3.05	-3.00	-2.94
12	-2.78	-2.70	-3.64	-3.61	-3.56	-3.49	-3.42
18	-1.59	-1.55	-2.07	-2.01	-1.94	-1.88	-1.84
24	0.90	0.86	1.22	1.29	1.39	1.49	1.53
30	4.38	4.23	5.80	5.90	5.02	5.07	5.11

Graph10. Variation of bending stresses for 30m pier

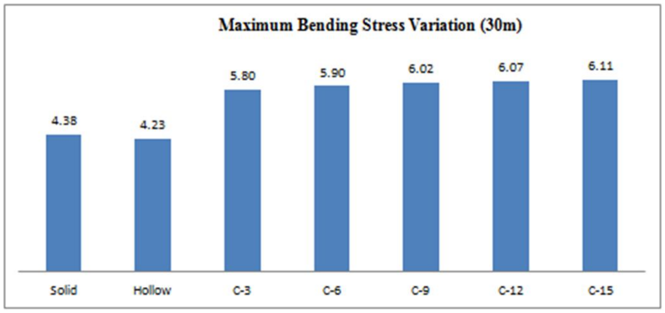


Table12. Variation of bending stresses for 35m pier

Bending Stresses (Height 35m)							
Section from top(m)	Solid	Hollow	C-3	C-6	C-9	C-12	C-15
0	0.00	0.00	0.00	0.00	0.00	0.00	0.00
7	-2.40	-2.35	-3.16	-3.14	-3.11	-3.06	-3.00
14	-2.84	-2.78	-3.72	-3.68	-3.63	-3.56	-3.49
21	-1.62	-1.59	-2.11	-2.05	-1.98	-1.92	-1.88
28	0.92	0.88	1.24	1.32	1.42	1.52	1.56
35	4.47	4.34	5.92	5.02	5.14	5.20	5.24

Graph11. Variation of bending stresses for 35m pier

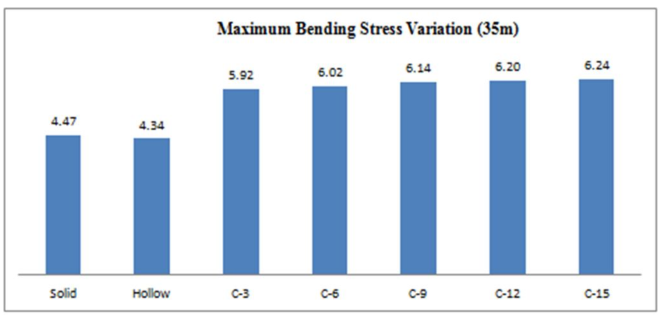
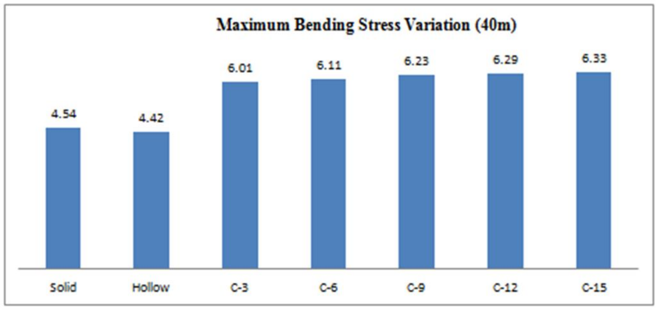


Table13. Variation of bending stresses for 40m pier

Bending Stresses (Height 40m)							
Section from top(m)	Solid	Hollow	C-3	C-6	C-9	C-12	C-15
0	0.00	0.00	0.00	0.00	0.00	0.00	0.00
8	-2.44	-2.39	-3.20	-3.18	-3.16	-3.10	-3.04
16	-2.88	-2.83	-3.78	-3.74	-3.69	-3.61	-3.54
24	-1.65	-1.62	-2.15	-2.08	-2.01	-1.95	-1.91
32	0.93	0.90	1.26	1.34	1.44	1.54	1.58
40	4.54	4.42	5.01	5.11	5.23	5.29	5.33

Graph12. Variation of bending stresses for 40m pier



VI. CONCLUSION

A. As the height of the bridge pier increases the base B.M. value increases and critical B.M develops at the base of the pier.
B. Deflection of section increases as height of pier increase with maximum value for combination of solid and hollow circular section within permissible limits.
C. The maximum deflection developed in C-15 condition increases from 17% to 21% with solid pier and reduces from 1.7% to3.0% with hollow pier for height varying from 15m to 40m respectively.
D. Bending stresses in section increases from 6.11N/mm2 to 6.93 N/mm2 for height varying from 15m to 40m respectively for combination of solid and hollow circular section within permissible limits as per IS 456-2000.
E. Hence it can be concluded that as the height of pier increases the solid circular section and hollow circular section proves to be uneconomical as compared to combination of solid and hollow circular pier in section.

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