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Velocity-Only Continuous Data Assimilation and Exponential Synchronization for Two-Dimensional Non-Newtonian Micropolar Fluid Flows

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Abstract: A velocity-only continuous data assimilation algorithm is formulated for a two-dimensional incompressible micropolar fluid whose Cauchy stress follows a regularized shear-thinning power law. The reference state consists of a solenoidal velocity and a scalar microrotation, whereas the observer is supplied only with coarse velocity measurements through a linear interpolant. No angular-velocity data are imposed. The error equations retain the nonlinear stress difference and the complete velocity-microrotation coupling. Strong monotonicity on an absorbing strain set, the interpolant approximation inequality, and two-dimensional interpolation estimates yields an explicit differential inequality for a coupled error energy. If the observational spacing is small enough to absorb the interpolation defect and the nudging gain exceeds the reference-gradient production rate, both velocity and microrotation converge exponentially to the reference solution. A twelve-shell Fourier–Galerkin comparison system illustrates the gain–resolution tradeoff. For the baseline shear-thinning index $p = 1.6$, observing the first three shells gives a critical gain of 0.375; gains 0.50, 0.80, and 1.50 produce decay exponents 0.107, 0.300, and 0.407, respectively. The analysis isolates the mechanism by which unobserved microrotation synchronizes through dissipative coupling to the nudged velocity field.

Keywords: continuous data assimilation; velocity-only observations; exponential synchronization; shear-thinning fluid; micropolar continuum; determining modes.

I. INTRODUCTION

Micropolar continua augment the translational velocity with an independent local spin and thereby represent suspensions, polymeric liquids, and fluids containing rigid microstructure more faithfully than a classical Newtonian model. Eringen's balance laws introduced the nonsymmetric stress and couple-stress mechanisms that underlie the modern theory [1]. The mathematical structure, boundary conditions, and dissipative coupling of velocity and microrotation were systematized by Łukaszewicz [2]. For generalized Newtonian constitutive laws, global weak-solution theory relies on monotone stress operators and compactness. Bäuble and Růžička established existence for unsteady micropolar electrorheological flows [3], while Dienes, Růžička, and Wolf treated shear-dependent incompressible fluids under power-law structure assumptions [4]. More recent work has extended the steady micropolar theory to spatially variable shear exponents [5].

Data assimilation asks whether incomplete measurements can determine the evolving state of a dissipative system. The determining-modes program showed that finitely many large scales can determine the long-time dynamics of two-dimensional turbulence [6], [7]. The Azouani–Olson–Titi algorithm turns that principle into a continuous observer: a model trajectory is nudged toward coarse observations by linear feedback [8]. Its attraction is analytical and computational. It requires neither an adjoint optimization nor repeated covariance updates, and its convergence can be expressed directly in terms of the observation scale h and gain μ . Stochastic measurement error produces a controlled asymptotic error floor rather than exact recovery [9].

Partial observation is the decisive issue for multiphysics systems. Velocity data alone can recover temperature in two-dimensional Bénard convection [10], and one velocity component can determine the two-dimensional Navier–Stokes state under suitable resolution conditions [11]. In a velocity–vorticity finite-element formulation, velocity-only nudging also controls the unobserved vorticity [12]. These results suggest that a micropolar observer should not require direct spin measurements when angular momentum remains sufficiently coupled and dissipative. Numerical studies of continuous nudging further show that observed resolutions needed in practice can be substantially milder than worst-case analytic bounds [13], while uniform-in-time finite-element estimates clarify how spatial discretization enters the synchronization error [14].

The non-Newtonian extension is not a formal replacement of viscosity by a scalar coefficient. For $1 < p < 2$, the stress operator is monotone but its quadratic coercivity on a bounded strain set decreases as the admissible shear increases. Consequently, the observation density and nudging gain must be measured against an effective viscosity ν_G rather than the zero-shear viscosity ν_0 . Nonlinear nudging variants can accelerate synchronization [15], and recent analysis has established continuous data assimilation for third-grade non-Newtonian fluids [16]. The present paper addresses a distinct coupled problem: a shear-thinning micropolar model, observations of velocity only, and an explicit exponential estimate for both state variables. The functional setting follows the standard dissipative-systems viewpoint [17], [18].

The contribution is threefold. First, a velocity-only observer is posed without inserting artificial nudging in the microrotation equation. Second, the micropolar cross terms are retained as the nonnegative square $\kappa \| \text{curl} e - 2\eta \|_2^2$, avoiding a loss of coupling in separate estimates. Third, the shear-thinning dependence is exposed through ν_G and tested in a Fourier–Galerkin comparison system. The resulting theorem gives a transparent gain–resolution condition and identifies why coarse velocity measurements can synchronize an unmeasured rotational field.

II. GOVERNING EQUATIONS AND OBSERVATION OPERATOR

A. Reference non-Newtonian Micropolar flow

Let $T^2 = (0, 2\pi)^2$ be the periodic torus. The reference velocity $u(x, t)$ has zero spatial mean and is divergence free; $\omega(x, t)$ is the scalar out-of-plane microrotation. In nondimensional form, the linear momentum equation is

$$\partial_t u + (u \cdot \nabla) u - \nabla \cdot S(Du) - \kappa \Delta u + \nabla p = 2\kappa \nabla^\perp \omega + f, \quad \nabla \cdot u = 0 \quad (1)$$

The angular-momentum balance is

$$j(\partial_t \omega + u \cdot \nabla \omega) - \gamma \Delta \omega + 4\kappa \omega = 2\kappa \text{curl} u + g \quad (2)$$

Here $j > 0$ is the microinertia, $\gamma > 0$ is the spin diffusivity, and $\kappa > 0$ is the vortex viscosity. The forcing pair (f, g) is time independent and mean compatible. The term $-\kappa \Delta u$ and the reciprocal 2κ coupling are written so that the total energy dissipation contains the exact square $\kappa \| \text{curl} u - 2\omega \|_2^2$. This convention is equivalent to the usual incompressible micropolar parameterization after constants are regrouped [1], [2].

The extra stress is regularized at zero shear:

$$S(A) = 2\nu_0 (\delta^2 + |A|^2)^{(p-2)/2} A, \quad 1 < p \leq 2 \quad (3)$$

The exponent $p = 2$ gives the Newtonian limit, whereas $1 < p < 2$ is shear thinning. The positive regularization δ prevents singular viscosity at vanishing strain. This constitutive form satisfies growth, coercivity, and strict monotonicity assumptions used in the weak theory of generalized Newtonian and micropolar electrorheological systems [3]–[5]. For the synchronization proof, the reference and observer are taken as global strong solutions on a common absorbing strain set.

B. Coarse velocity observations

Let I_h be a linear interpolant constructed from low Fourier modes, nodal averages, or local volume averages on scale h . Only two standard properties are needed:

$$\| \varphi - I_h \varphi \|_2 \leq c_0 h \| \nabla \varphi \|_2, \quad \| I_h \varphi \|_2 \leq c_1 \| \varphi \|_2 \quad (4)$$

The constants c_0 and c_1 are independent of h . The first inequality converts observation error into a gradient defect, and the second prevents amplification. These hypotheses are the general-interpolant setting of the continuous nudging method [8]. For Fourier projection onto modes $|k| \leq K_0$, h is proportional to K_0^{-1} ; for cell averages it is the cell diameter.

Table I. Principal variables and dimensionless parameters.

Symbol	Meaning	Assumption
u, v	Reference and assimilated velocities	Divergence free
ω, ξ	Reference and assimilated microrotation	Scalar in 2D
p	Power-law exponent	$1 < p \leq 2$
δ	Zero-shear regularization	$\delta > 0$
κ	Vortex viscosity / spin coupling	$\kappa > 0$
γ	Microrotation diffusivity	$\gamma > 0$
μ, h	Nudging gain and observation scale	$\mu > 0, h > 0$
v_G	Effective monotonicity on strain set	$v_G > 0$

C. Functional setting and solution class

Let H be the closure in L^2 of smooth, mean-zero, divergence-free periodic vector fields and let V be the corresponding H^1 closure. The scalar microrotation belongs to the mean-zero periodic H^1 space. The weak formulation is obtained by testing momentum only with solenoidal functions, so the pressure does not enter the energy estimate. For $1 < p < 2$, the natural stress space is L^p in the symmetric gradient; the regularization $\delta > 0$ and the two-dimensional setting provide the additional regularity used in the strong-solution synchronization argument. The existence theory in [3], [4] justifies Galerkin construction and passage to the nonlinear stress limit, while the observer adds a bounded, dissipative perturbation to the velocity equation.

The theorem is stated after trajectories enter a common absorbing set. This formulation is standard in long-time data assimilation: initial transients are controlled by dissipativity, whereas synchronization uses uniform bounds on the reference solution and, here, on the observer strain. The constant G is not a new constitutive parameter; it summarizes that absorbing-set information. Its appearance is unavoidable for shear thinning because the Hessian of the power-law potential loses uniform quadratic coercivity at large strain. In the Newtonian limit $p = 2$, v_G is independent of G and the familiar AOT scaling is recovered.

III. VELOCITY-ONLY CONTINUOUS DATA ASSIMILATION

The assimilated state (v, ξ) uses the same physical coefficients and forcing as the reference system, but its initial data may be arbitrary. The velocity is corrected continuously by the measured mismatch:

$$\partial_t v + (v \cdot \nabla)v - \nabla \cdot S(Dv) - \kappa \Delta v + \nabla q = 2\kappa \nabla^\perp \xi + f - \mu I_h(v - u), \quad \nabla \cdot v = 0 \quad (5)$$

The microrotation evolves without observation feedback:

$$j(\partial_t \xi + v \cdot \nabla \xi) - \gamma \Delta \xi + 4\kappa \xi = 2\kappa \text{curl} v + g \quad (6)$$

Equations (5)–(6) define a one-channel observer. The only injected data are $I_h u$. The field ξ is reconstructed because velocity and microrotation exchange energy through the reciprocal curl terms, while spin diffusion and the 4κ damping remove rotational error. This architecture is the micropolar analogue of velocity-only recovery in coupled convection and velocity–vorticity formulations [10], [12].

IV. EXPONENTIAL SYNCHRONIZATION

A. Error system and shear-dependent coercivity

Define the velocity, spin, and pressure errors by

$$e = v - u, \quad \eta = \xi - \omega, \quad \pi = q - p \quad (7)$$

Subtracting the reference equations from the observer gives

$$\partial_t e + (v \cdot \nabla)e + (e \cdot \nabla)u - \nabla \cdot [S(Dv) - S(Du)] - \kappa \Delta e + \nabla \pi = 2\kappa \nabla^\perp \eta - \mu I_h e \quad (8)$$

$$j(\partial_t \eta + v \cdot \nabla \eta + e \cdot \nabla \omega) - \gamma \Delta \eta + 4\kappa \eta = 2\kappa \text{curl} e \quad (9)$$

Suppose $|Du| + |Dv| \leq G$ almost everywhere after entry into an absorbing set. The regularized power-law mapping then has the local strong-monotonicity estimate

$$(S(A) - S(B)) : (A - B) \geq \nu_G |A - B|^2, \quad \nu_G = c_p \nu_0 (\delta + G)^{p-2} \quad (10)$$

The constant c_p depends only on p . Because $p - 2 < 0$, ν_G decreases when the admissible strain G increases. This is the exact point at which shear thinning enters the synchronization threshold. It does not alter the feedback architecture; it reduces the amount of gradient error that can absorb the interpolation defect and convective production.

B. Energy inequality

Taking the L^2 inner products of (8) and (9) with e and η , respectively, removes the self-advection terms by incompressibility. The stress difference is bounded below by (10). Integration by parts combines the two micropolar cross terms with the vortex viscosity and spin damping. The nudging contribution obeys

$$\mu(I_h e, e) \geq \frac{\mu}{2} \|e\|_2^2 - \frac{\mu c_0^2 h^2}{2} \|\nabla e\|_2^2 \quad (11)$$

Use the coupled error energy

$$E(t) = \frac{1}{2} (\|e(t)\|_2^2 + j \|\eta(t)\|_2^2) \quad (12)$$

The two-dimensional Ladyzhenskaya inequality controls $((e \cdot \nabla)u, e)$, while the mixed transport term $j((e \cdot \nabla)\omega, \eta)$ is estimated by interpolation followed by Young's inequality. If K_* denotes the resulting uniform production bound determined by $\|\nabla u\|_2$, $\|\nabla \omega\|_2$, ν_G , γ , and the domain constants, then $\mu c_0^2 h^2 \leq \nu_G / 2$ permits absorption of the interpolant defect. One obtains

$$\frac{dE}{dt} + \frac{\nu_G}{2} \|\nabla e\|_2^2 + \gamma \|\nabla \eta\|_2^2 + \kappa \|\text{curl} e - 2\eta\|_2^2 + \left(\frac{\mu}{2} - K_*\right) \|e\|_2^2 \leq 0 \quad (13)$$

The square $\kappa \|\text{curl} e - 2\eta\|_2^2$ is central. Estimating the two curl couplings independently would introduce avoidable constants and could obscure the route by which velocity observations constrain η .

C. Main synchronization theorem

Theorem 1. Let (u, ω) be a global strong solution of (1)–(3) on T^2 and let (v, ξ) solve (5)–(6). Assume both trajectories lie in the strain set used in (10), and let K_* be a uniform upper bound for the convective-production coefficient in (13). If

$$\mu c_0^2 h^2 \leq \frac{\nu_G}{2}, \quad \mu > 2K_*, \quad \lambda = \min\left\{\frac{\mu}{4} - \frac{K_*}{2}, \frac{\gamma \lambda_1}{2j}\right\} > 0 \quad (14)$$

then the complete observer state synchronizes exponentially:

$$E(t) \leq E(0) e^{-2\lambda t}, \quad t \geq 0 \quad (15)$$

Proof. Under the first condition in (14), the gradient defect in (11) is absorbed by monotone stress dissipation. The second condition leaves a strictly positive coefficient on $\|e\|_2^2$. Poincaré's inequality supplies $\gamma \lambda_1 \|\eta\|_2^2$ from spin diffusion, and the nonnegative micropolar square is retained. Hence (13) implies $dE/dt + 2\lambda E \leq 0$. Grönwall's inequality gives (15). The estimate is independent of the observer's initial state and proves that direct observations of ω are unnecessary.

The gain condition is deliberately expressed through K_* rather than a formal Reynolds number. For a given reference trajectory, K_* can be estimated from time-uniform velocity and microrotation gradients; for an attractor-wide statement it can be bounded in terms of the forcing and dissipative parameters. The former is sharper and connects naturally to observed-data criteria, whereas the latter produces a single conservative design value. The rate λ is the smaller of the residual velocity-feedback rate and the spin-diffusion rate. Therefore, arbitrarily large μ does not imply arbitrarily fast complete-state synchronization. Once velocity error has become slaved to the data, γ , κ , and the Poincaré scale govern the remaining angular transient.

Table II. Sufficient conditions and their mathematical roles.

Condition	Role in the estimate	Consequence
$ Du + Dv \leq G$	Fixes a quadratic monotonicity constant	Defines ν_G
$\mu c_0^2 h^2 \leq \nu_G / 2$	Absorbs interpolation-gradient defect	Upper bound on h
$\mu > 2K_*$	Dominates convective error production	Positive velocity damping
$\gamma > 0$	Damps unobserved spin through Poincaré	Complete-state decay

V. FOURIER–GALERKIN COMPARISON EXPERIMENT

A. Reduced system and reproducibility

A comparison experiment was used to examine the gain–resolution mechanism without representing it as a direct numerical simulation of the full nonlinear PDE. Twelve isotropic Fourier shells were retained. The reference-gradient contribution was replaced by its uniform envelope s_0 , the first K_0 velocity shells were observed, and every spin shell remained unobserved. For shell m the error pair satisfies

$$\frac{d}{dt} [e_m \eta_m] = [s_0 - v_G m^2 - \mu 1_{m \leq K_0} 2\kappa m 2\kappa m - \gamma m^2 - 4\kappa] [e_m \eta_m] \quad (16)$$

The spectral decay exponent is

$$\lambda_{FG} = - \max_{1 \leq m \leq M} \max \text{Re} \sigma(M_m) \quad (17)$$

Positive λ_{FG} gives exponential decay of every retained mode. The system was evaluated by exact matrix exponentials on 301 output times over $0 \leq t \leq 12$.

Table III. Baseline parameters for the Fourier–Galerkin comparison.

Quantity	Value	Interpretation
p	1.6	Shear-thinning index
v_0	0.18	Zero-shear consistency
δ	0.25	Stress regularization
G	4.0	Strain-envelope level
κ	0.08	Micropolar coupling
γ	0.12	Spin diffusion
s_0	0.42	Convective envelope
M, K_0	12, 3	Retained and observed shells

Initial modal errors were generated from a fixed random seed and scaled by m^{-1} . This comparison system preserves the feedback location, shear-dependent viscosity, spin damping, and reciprocal micropolar coupling in the theorem. It is intended to test the sufficient mechanism, not to replace a fully dealiased nonlinear simulation.

The comparison is also a diagnostic for the theorem's conservatism. The analytic estimate must control all admissible nonlinear interactions with domain inequalities, whereas the modal calculation uses the actual eigenvalues of each retained two-by-two block. Agreement is therefore expected in the direction of the parameter effects, not in the numerical value of a universal bound. This distinction follows the computational evidence for classical continuous assimilation, where practical determining resolutions are often appreciably smaller than global analytical estimates [13].

B. Error histories and gain threshold

Figure 1 distinguishes subcritical and supercritical nudging. At $\mu = 0.30$, below the computed critical gain, the dominant low shell grows and the total error is not synchronized. Raising the gain to $\mu = 0.50$ changes the leading spectral abscissa to -0.107 . Gains $\mu = 0.80$ and $\mu = 1.50$ accelerate velocity recovery, but the rate eventually approaches a ceiling set by the unnudged spin and the first unobserved dissipative shell. This saturation is expected: increasing velocity feedback cannot remove the intrinsic spin-relaxation timescale.

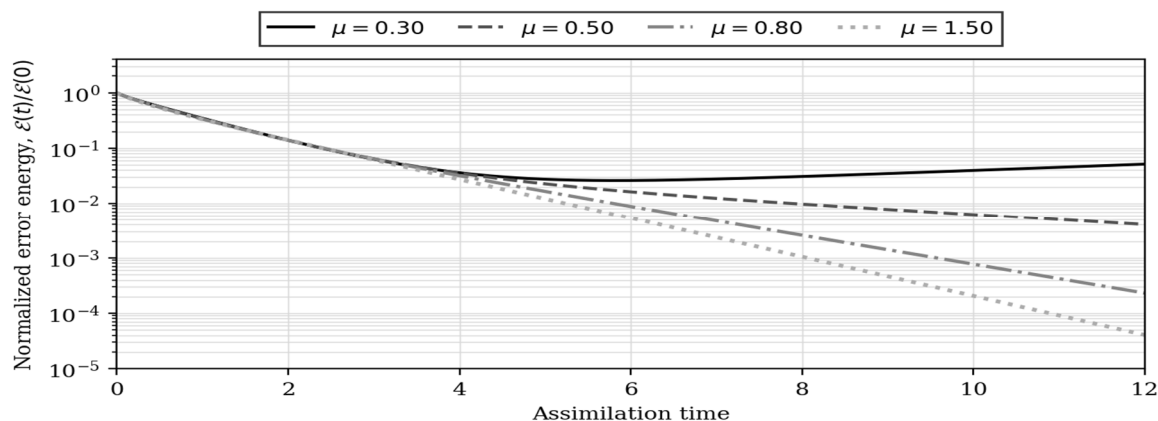


Fig. 1. Normalized comparison-system error for $p = 1.6$, $K_o = 3$, and four velocity nudging gains; microrotation is never nudged.

Table IV. Shear-thinning dependence of the comparison threshold ($K_o = 3$).

p	v_G	Critical μ	λ_{FG} at $\mu = 1.50$
1.4	0.0783	0.400	0.406
1.6	0.1033	0.375	0.407
1.8	0.1364	0.342	0.408
2.0	0.1800	0.298	0.410

Table IV quantifies the constitutive effect. Decreasing p from 2.0 to 1.4 lowers v_G from 0.1800 to 0.0783 and raises the critical gain from 0.298 to 0.400. At the larger gain $\mu = 1.50$, however, all four cases have nearly the same complete-state decay exponent because microrotation, not velocity, controls the slowest mode. Thus shear thinning primarily shifts the entry into the synchronized regime; after strong velocity locking, spin diffusion sets the asymptotic pace.

C. Observation resolution

Figure 2 separates gain from resolution. Observing only the first shell leaves the second shell unstable for every displayed gain, so λ_{FG} remains negative. Observing two shells removes that obstruction and produces a critical gain of 0.375, but the unobserved third shell limits the attainable decay rate to 0.300. Observing three shells yields the same onset gain and a higher saturation level of approximately 0.429. This is the determining-mode mechanism in finite form: feedback must cover every dynamically productive shell, while higher modes can be left to physical dissipation [6]–[8].

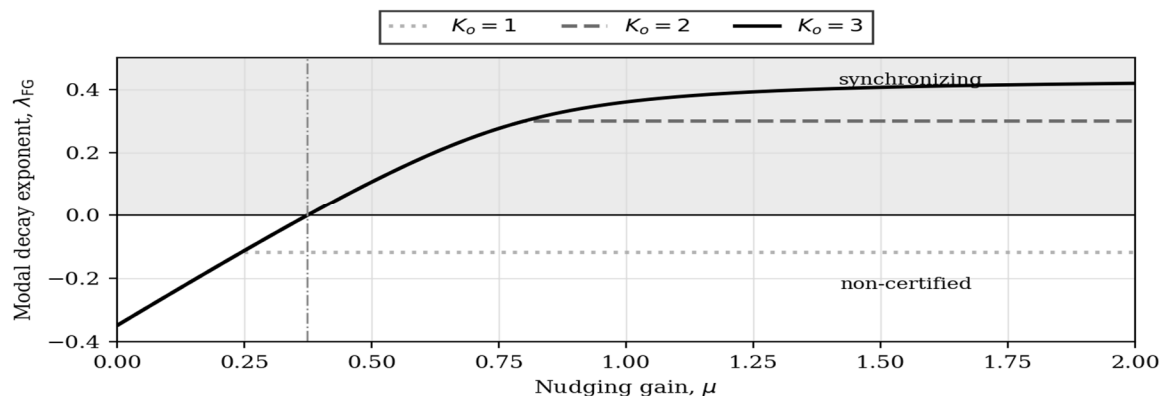


Fig. 2. Modal synchronization exponent versus velocity nudging gain for three observation resolutions at $p = 1.6$; the legend is placed above the axes.

Table V. Gain–resolution outcomes for $p = 1.6$.

Observed shells	Critical μ	λ_{FG} at $\mu = 1.50$	Outcome
$K_o=1$	Not attained	-0.118	Second shell remains unstable
$K_o=2$	0.375	0.300	Synchronization; shell 3 limits rate
$K_o = 3$	0.375	0.407	Synchronization; spin limits rate
$K_o=4$	0.375	0.407	No further gain at baseline

DISCUSSION

A. Why velocity data determine microrotation

The observer does not infer ω algebraically from $\text{curl}u$. Instead, the angular-momentum equation is a stable driven subsystem. Once the velocity error is controlled, the forcing $2\kappa\text{curl}e$ in (9) decays, and spin diffusion plus 4κ damping eliminates η . The energy identity expresses the same fact more sharply: $\text{curl}e$ and 2η are penalized together. This mechanism parallels velocity-only recovery of an unobserved coupled variable in Bénard convection [10] and of vorticity in a velocity–vorticity formulation [12], but here it is embedded in a nonlinear monotone-stress system.

B. Shear thinning and observation design

The condition $\mu c_0^2 h^2 \leq \nu_G/2$ couples rheology to sensor density. At fixed μ , stronger shear thinning reduces ν_G on the absorbing strain set and therefore requires smaller h . At fixed h , it demands a more careful gain choice: μ must be large enough to dominate production but cannot violate the absorption condition unless the measurements are refined. This apparent upper restriction on μ is a feature of the first-order interpolant estimate, not of feedback control itself. Spectral projection, higher-order interpolants, or a sharper stress estimate can relax it, as uniform-in-time discretization analyses for Navier–Stokes assimilation demonstrate [14].

C. Scope of the result

The theorem is deterministic and assumes exact coefficients and noiseless observations. With noisy data, the same energy argument yields a long-time neighborhood whose radius depends on the noise variance, in accord with the stochastic AOT analysis [9]. Coefficient mismatch produces an additional persistent source term and leads to parameter-recovery questions. The strain-bound hypothesis is also substantive: it converts power-law monotonicity into a quadratic constant suited to exponential L^2 decay. A fully degenerate treatment would naturally seek synchronization in a p -energy metric. Recent non-Newtonian CDA results for differential-type fluids [16] indicate that such extensions are analytically accessible, but the micropolar cross-dissipation must be preserved.

VI. CONCLUSIONS

A velocity-only continuous data assimilation scheme has been established for two-dimensional regularized shear-thinning micropolar flow. The observer uses coarse velocity measurements and no microrotation data. Monotonicity of the nonlinear stress, the interpolant approximation property, and the exact micropolar cross-dissipation yield a closed coupled error estimate.

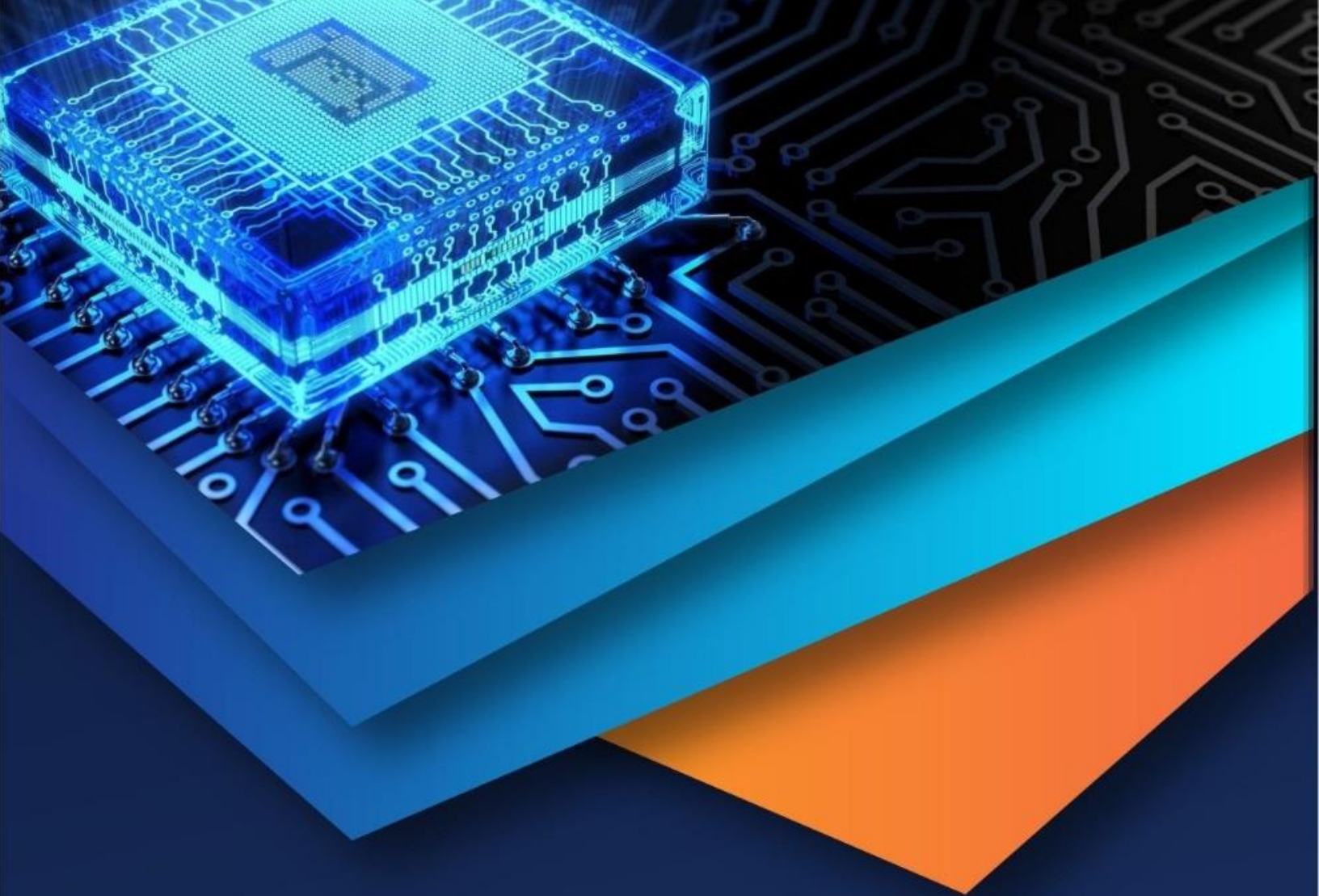
When the observation scale resolves the interpolation defect and the gain exceeds the uniform convective-production coefficient, the velocity and spin errors satisfy $E(t) \leq E(0)e^{-2\lambda t}$. The result explains complete-state recovery as a consequence of velocity feedback, reciprocal curl coupling, spin diffusion, and angular damping.

The Fourier–Galerkin comparison confirms two design principles. Observations must include every productive low shell, and increasing gain beyond velocity locking cannot overcome the intrinsic microrotation timescale. Stronger shear thinning raises the critical gain by reducing ν_G , but it has little influence on the final decay rate once spin relaxation becomes limiting. These conclusions provide a rigorous basis for velocity-based reconstruction in microstructured non-Newtonian flows.

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