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Vertex Sum Cube Labeling for Split and Mirror Graphs

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Abstract:** In this paper, the new concept vertex sum cube labeling has been introduced and a formula for vertex sum cube labeling has been established. A function θ is called a Vertex sum cube labeling of a graph G with edges, if the vertices of G to the set $\{0,1,2, p-1\}$ such that when each edge uv is assigned the label $\theta(uv)=u^3+v^3+3u^2v+3uv^2$, then the resulting edge labels are distinct cube numbers. In this paper, some families of graphs such as Mirror and Split has been investigated
Keywords: Labeling, Cube labeling, Square labeling, Multiplicative labeling.

I. INTRODUCTION

A study of points and lines is called graph theory. It is a branch of mathematics concerned with graph analysis. The mathematical truth is depicted visually in this illustration. The link between vertices, or nodes, and edges, or lines, is the subject of graph theory.The study of mathematical structures called graphs, which are made up of vertices (or nodes) connected by edges, is known as graph theory.

Every edge in the set of objects called vertices has an unordered pair of vertices connected with it, as does another set whose members are called edges. The symbols and correspond to the vertex set and edge set of a graph G.G's size, denoted by q, is the cardinality of the edge set, and its order, represented by p, is the cardinality of the vertex set. The new idea of vertex sum cube labeling was presented in this chapter. This concept is extended to Weiner index polynomial which is cited as [9,10,11,12,14]. Some basic definitions and notations are referred in [1,2,4,5]**.** Vertex Cube labeling can be applied to different types of graphs which is cited as[13,15,16,17,18,19,20,21,22,23,24,25]. Graph labeling is also extended to domination [3,6,7,8].

II. MAIN RESULT

1) Definition 2.1

A function θ is called a Vertex sum cube labeling of a graph G with edges, if the vertices of G to the set $\{0,1,2,...p-1\}$ such that when each edge uv is assigned the label $\theta(uv)=u^3+v^3+3u^2v+3uv^2$, then the resulting edge labels are distinct cube numbers.

2) Definition 2.2

A Graph G is said to be vertex sum cube graph if it admits vertex sum cube labeling.

3) Theorem: 2.1

The Split $Spl(K_{1,n})$ is a vertex sum cube graph for $n \geq 3$

Proof:

Let G be a graph of Split $Spl(K_{1,n})$

Let $\{u, v, u_1, u_2,..., u_n, v_1, v_2,..., v_n\}$ be the vertices of $Spl(K_{1,n})$ and $\{e_1, e_2,..., e_{n-1}, e_n, e_{n+1}, e_{2n-1}, e_{2n}, e_1', e_2',..., e_n'\}$ be the edges of $Spl(K_{1,n})$ which are denoted as in the figure 2.3

Let $|V(G)| = 2n+2$ and $|E(G)| = 3n$ of Split $Spl(K_{1,n})$

Label the vertices and edges as follows:

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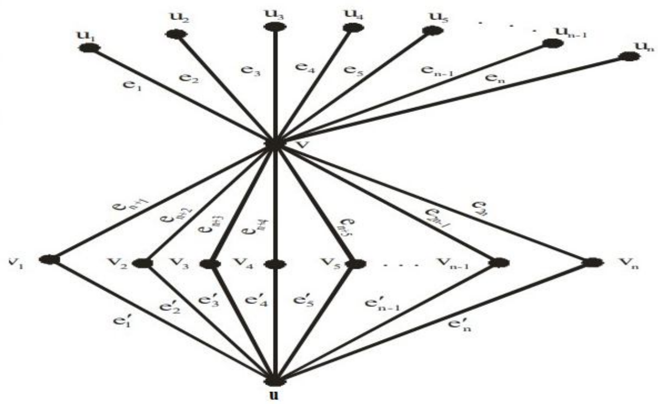

Fig.2.1 Split $Spl(K_{1,n})$graph with ordinary labeling

The function $\theta : V(G) \to \{0,1,2,...,n\}$ is defined by

$\theta(v) = 0$

$\theta(u_i) = i \quad ; \quad 1 \le i \le n$

$\theta(v_i) = n + i \quad ; \quad 1 \le i \le n$

$\theta(u) = 2n + 1$

Then the edge labels are

$\theta(e_i) = i^3 \quad ; \quad 1 \le i \le 2n$

$\theta(e_i^{'}) = 27n^3 + 1^3 + i^3 + 3(3n+1)(1+i)(i+3n) \quad ; \quad 1 \le i \le n$

The edges of the Split $Spl(K_{1,n})$ graph receive distinct cube numbers.

Clearly, the Split $Spl(K_{1,n})$ is a vertex sum cube graphs.

4) Example: 2.1

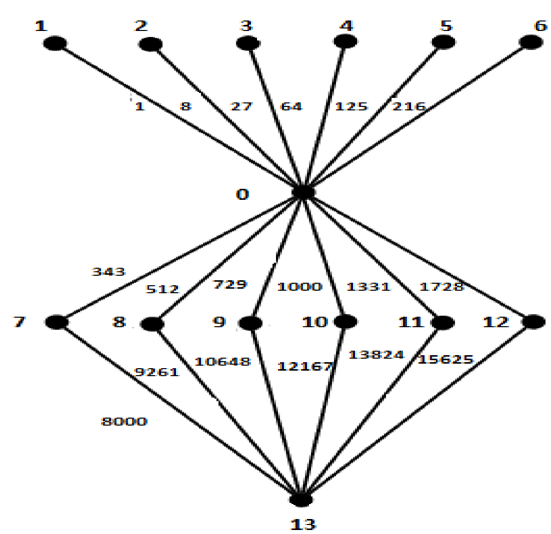

Fig: 2.2 $Spl(K_{1,6})$

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5) Example: 2.2

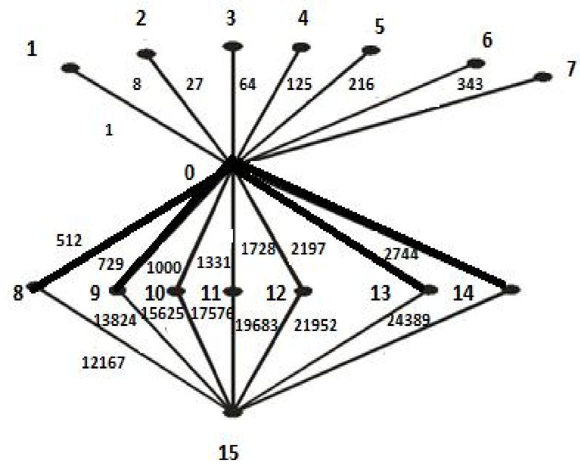

Fig: 2.3 $Spl(K_{1,7})$

6) Theorem: 2.2

The Mirror $M(P_n)$ is a vertex sum cube graphs for $n \geq 2$

Proof:

Let $\{v_1, v_2, v_3, \ldots, v_n, v_1^{'}, v_2^{'}, \ldots, v_n^{'}\}$ be the vertices of $M(P_n)$ and $\{e_1, e_2, \ldots, e_{n-1}, e_1^{'}, e_2^{'}, \ldots, e_{n-1}^{'}, a_1, a_2, \ldots, a_n\}$ be the edges of $M(P_n)$ which are denoted as in the figure 2.4

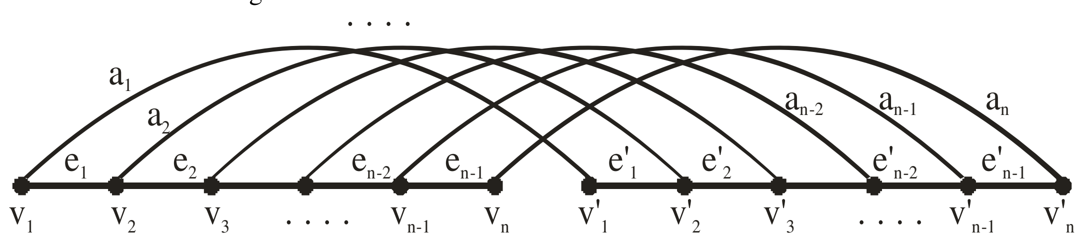

Fig. 2.4 : Mirror $M(P_n)$ with ordinary labeling

Let $|V(G)| = 2n$ and $|E(G)| = 3n-2$ of Mirror $M(P_n)$

Label the vertices and edges as follows:

The function $\theta : V(G) \rightarrow \{0,1,2,\ldots,2n\}$ is defined by

$\theta(v_i) = 2i-2 \quad ; \quad 1 \leq i \leq n$

$\theta(v_i^{'}) = 2i-1 \quad ; \quad 1 \leq i \leq n$

Then the induced edge labels are

$\theta(e_i) = 64i^3 - 96i^2 + 48i - 8 \quad ; \quad 1 \leq i \leq n-1$

$\theta(e_i^{'}) = 4i \quad ; \quad 1 \leq i \leq n-1$

$\theta(a_i) = 64i^3 - 144i^2 + 108i - 27 \quad ; \quad 1 \leq i \leq n$

The edges of the Mirror graph receive distinct cube numbers.

Clearly, the Mirror $M(P_n)$ $(n \geq 2)$ are the vertex sum cube graphs.

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7) Example: 2.3

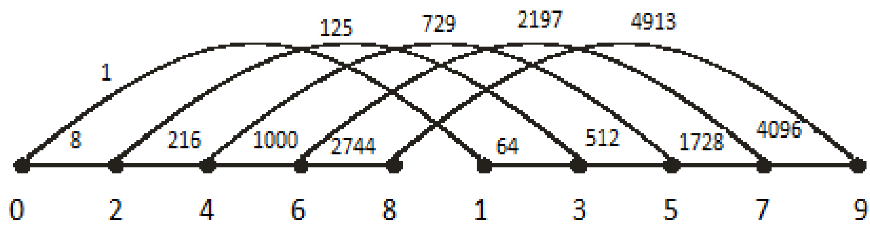

Fig. 2.5 : Mirror $M(P_5)$

8) Example: 2.4

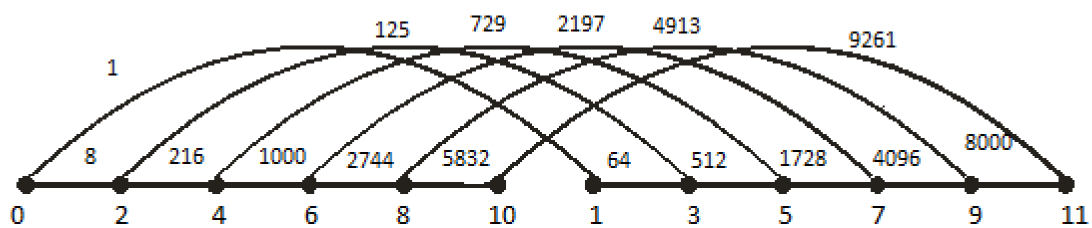

Fig. 2.6 : Mirror $M(P_6)$

9) Corollary: 2.3

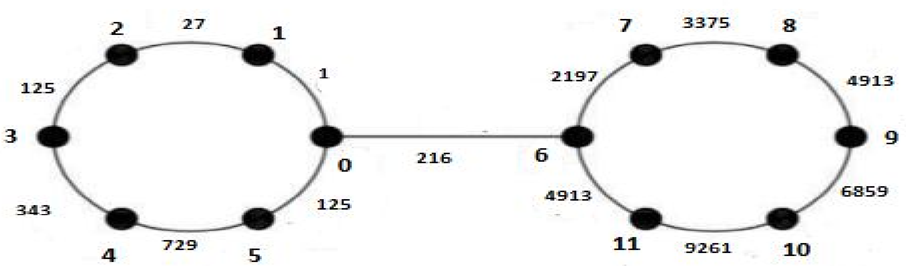

Fig:2.7 Specs $K_2\Theta C_6$

The Specs $K_2\Theta C_n$ consists of $2n$ vertices and $2n+1$ edges. The vertex receives label as $\theta : V(G) \rightarrow \{0,1,2,...,n-1\}$. The edges of the specs graph does not receive distinct numbers. That is, any one of the edge receive same number. So that it does not satisfies the condition of vertex sum cube labeling.

Therefore, the Specs $K_2\Theta C_n$ is not a vertex sum cube graph.

III. CONCLUSION

In this paper, examines the formula for a vertex sum cube labeling and examines several graph families under this labeling scheme. The conclusion is that some graph families, such split $Spl(K_{1,n})$ and mirror $M(P_n)$, are vertex sum cube graphs.

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