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# Vertex Sum Cube Labeling for Split and Mirror Graphs

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**Abstract:** In this paper, the new concept vertex sum cube labeling has been introduced and a formula for vertex sum cube labeling has been established. A function  $\theta$  is called a Vertex sum cube labeling of a graph  $G$  with edges, if the vertices of  $G$  to the set  $\{0,1,2,p-1\}$  such that when each edge  $uv$  is assigned the label  $\theta(uv) = u^3 + v^3 + 3u^2v + 3uv^2$ , then the resulting edge labels are distinct cube numbers. In this paper, some families of graphs such as Mirror and Split has been investigated

**Keywords:** Labeling, Cube labeling, Square labeling, Multiplicative labeling.

## I. INTRODUCTION

A study of points and lines is called graph theory. It is a branch of mathematics concerned with graph analysis. The mathematical truth is depicted visually in this illustration. The link between vertices, or nodes, and edges, or lines, is the subject of graph theory. The study of mathematical structures called graphs, which are made up of vertices (or nodes) connected by edges, is known as graph theory.

Every edge in the set of objects called vertices has an unordered pair of vertices connected with it, as does another set whose members are called edges. The symbols and correspond to the vertex set and edge set of a graph  $G$ .  $G$ 's size, denoted by  $q$ , is the cardinality of the edge set, and its order, represented by  $p$ , is the cardinality of the vertex set. The new idea of vertex sum cube labeling was presented in this chapter. This concept is extended to Wiener index polynomial which is cited as [9,10,11,12,14]. Some basic definitions and notations are referred in [1,2,4,5]. Vertex Cube labeling can be applied to different types of graphs which is cited as [13,15,16,17,18,19,20,21,22,23,24,25]. Graph labeling is also extended to domination [3,6,7,8].

## II. MAIN RESULT

### 1) Definition 2.1

A function  $\theta$  is called a Vertex sum cube labeling of a graph  $G$  with edges, if the vertices of  $G$  to the set  $\{0,1,2,\dots,p-1\}$  such that when each edge  $uv$  is assigned the label  $\theta(uv) = u^3 + v^3 + 3u^2v + 3uv^2$ , then the resulting edge labels are distinct cube numbers.

### 2) Definition 2.2

A Graph  $G$  is said to be vertex sum cube graph if it admits vertex sum cube labeling.

### 3) Theorem: 2.1

The Split  $Spl(K_{1,n})$  is a vertex sum cube graph for  $n \geq 3$

Proof:

Let  $G$  be a graph of Split  $Spl(K_{1,n})$

Let  $\{u, v, u_1, u_2, \dots, u_n, v_1, v_2, \dots, v_n\}$  be the vertices of  $Spl(K_{1,n})$  and  $\{e_1, e_2, \dots, e_{n-1}, e_n, e_{n+1}, e_{2n-1}, e_{2n}, e'_1, e'_2, \dots, e'_n\}$  be the edges of  $Spl(K_{1,n})$  which are denoted as in the figure 2.3

Let  $|V(G)| = 2n + 2$  and  $|E(G)| = 3n$  of Split  $Spl(K_{1,n})$

Label the vertices and edges as follows:

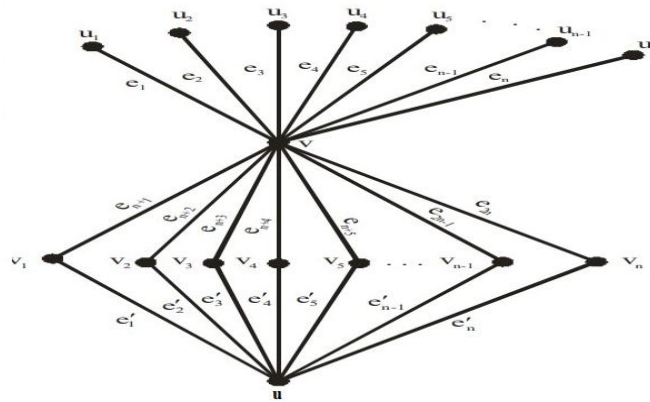


Fig.2.1 Split  $Spl(K_{1,n})$  graph with ordinary labeling

The function  $\theta : V(G) \rightarrow \{0, 1, 2, \dots, n\}$  is defined by

$$\theta(v) = 0$$

$$\theta(u_i) = i \quad ; \quad 1 \leq i \leq n$$

$$\theta(v_i) = n + i \quad ; \quad 1 \leq i \leq n$$

$$\theta(u) = 2n + 1$$

Then the edge labels are

$$\theta(e_i) = i^3 \quad ; \quad 1 \leq i \leq 2n$$

$$\theta(e'_i) = 27n^3 + 1^3 + i^3 + 3(3n+1)(1+i)(i+3n) \quad ; \quad 1 \leq i \leq n$$

The edges of the Split  $Spl(K_{1,n})$  graph receive distinct cube numbers.

Clearly, the Split  $Spl(K_{1,n})$  is a vertex sum cube graphs.

4) Example: 2.1

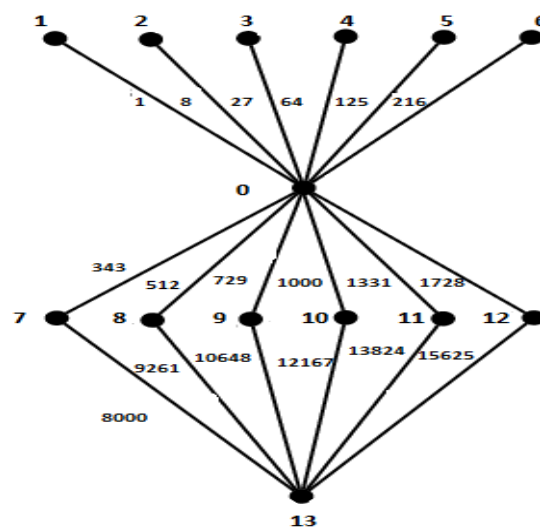
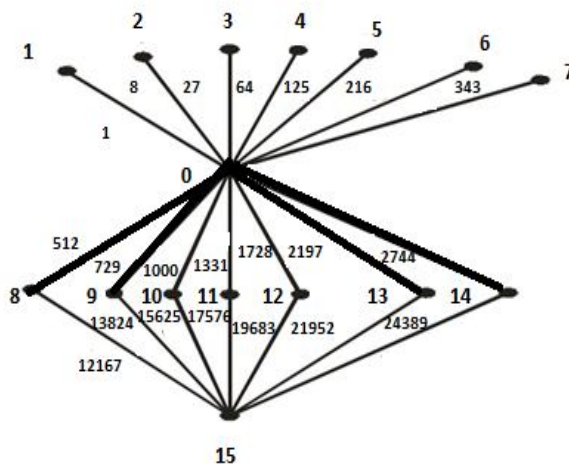


Fig: 2.2  $Spl(K_{1,6})$

5) Example: 2.2



7) Example: 2.3

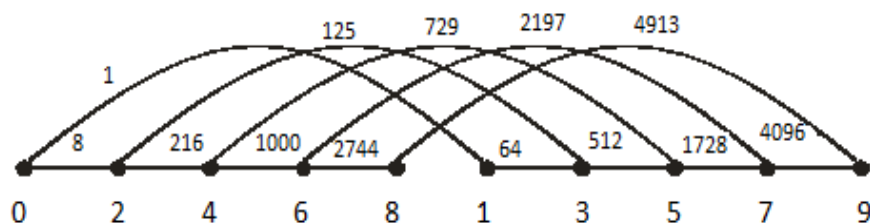


Fig. 2.5 : Mirror  $M(P_5)$

8) Example: 2.4

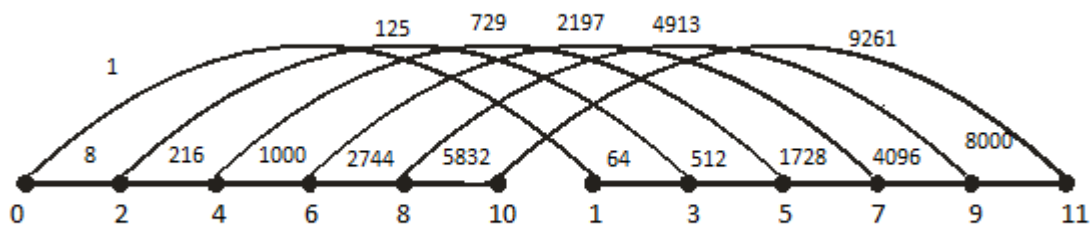


Fig. 2.6 : Mirror  $M(P_6)$

9) Corollary: 2.3

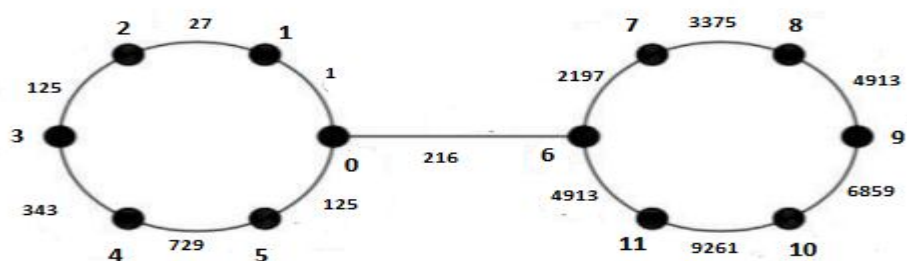


Fig.2.7 Specs  $K_2 \Theta C_6$

The Specs  $K_2 \Theta C_n$  consists of  $2n$  vertices and  $2n + 1$  edges. The vertex receives label as  $\theta : V(G) \rightarrow \{0, 1, 2, \dots, n-1\}$ . The edges of the specs graph does not receive distinct numbers. That is, any one of the edge receive same number. So that it does not satisfies the condition of vertex sum cube labeling.

Therefore, the Specs  $K_2 \Theta C_n$  is not a vertex sum cube graph.

### III. CONCLUSION

In this paper, examines the formula for a vertex sum cube labeling and examines several graph families under this labeling scheme. The conclusion is that some graph families, such split  $Spl(K_{1,n})$  and mirror  $M(P_n)$ , are vertex sum cube graphs.



## REFERENCES

- [1] Bodendick, R. and Walther, G., On number theoretical methods in graph labelings Res.Exp.Maths (2./1995) 3-25.
- [2] Bloom, D.F. Hsu, On graceful directed graphs, SIAMJ, Alg. Discrete Math.,6(1985),519-536.
- [3] Felix, J., Litta, E., Benedict Michael Raj, L., Changing and Unchanging Properties of Single Chromatic Transversal Domination Number of Graphs, International Journal of Mathematics Trends and Technology, Volume 52, Issue 4, December 2017, Pg. No. 262 – 266.
- [4] Gallian, M.A., “A Dynamic survey of graph labelings” Electronic journal, 2000 (Volume-23).
- [5] Harary, F., Graph Theory, New Delhi: Narosa Publishing House, 2001.
- [6] Litta E., Maragatha Dharshini S., Proper Colourings in  $r$  – Regular Modified Zagreb Index Graph, International Journal for Research in Applied Science & Engineering Technology (IJRASET), Volume 11, Issue 3, Mar 2023, P.No.: 1553- 1558.
- [7] Litta, E., Datchini, S, Proper Colourings in  $r$  – Regular Zagreb Index Graph”, Aryabhatta Journal of Mathematics and Informatics, Volume 15, Issue 1, Jan - June 2023. Pg. No. 149-154, Impact Factor: 5.856, ISSN No: 0975-7139(P) 2394-9309.
- [8] Litta, E., Narmadha, S, Proper Colourings in  $r$  – Regular Inverse sum Indeg Index Graph”, Research and Applications Towards Mathematics and Computer Science, Volume 9, February 2024, Pg. No. 66-76, B P International, Print ISBN: 978-81-970187-8-7 eBook ISBN: 978-81-970187-9-4.
- [9] Palanikumar, R, Rameshkumar, A, Wiener Index of Physio-Chemical Labeled Graph, Bulletin of pure and Applied Sciences, Vol. 37E(Math & Stat), No. 2, 2018, PP: 519 – 522.
- [10] Palanikumar, R, Rameshkumar, A, T Wiener Index of Fibonacci Labeled Graph  $P_n F_4$ , Journal of Computer and Mathematical Sciences, Vol. 9(11), 1712 - 1716, November 2018..
- [11] Palanikumar, R, Rameshkumar, A, Labeling on Dragon curve Fractal Graph, Aryabhatta Journal of Mathematics & Informatics, Vol. 10, No. 2, July - Dec 2018.
- [12] Rameshkumar, A, Palanikumar, R, The Wiener Lower Sum and Upper Sum of Friendship Graph  $F_n^r$ , Journal of Indian Acad. Math, Vol. 37, No. 2(2015), pp. 305 – 311.
- [13] Prakash, V., Gopi, R., Shalini, P., Anti Skolem Mean Labeling of Quadrilateral Snake Related Graphs, Tuijin Jishu/Journal of Propulsion Technology, Vol. 44, No. 5(2023), 495 – 500.
- [14] Rameshkumar, A, Palanikumar, R, Wiener Lower Sum of Complete  $K_N^R$  Graph, Aryabhatta Journal of Mathematics & Informatics, Vol. 7, No. 2, July – Dec 2015.
- [15] Shalini, P., Paul Dhayabaran, D., An Absolute Differences of Cubic and Square Difference Labeling, International Journal of Advanced Scientific and Technical Research, May-June 2015, Issue-5, Volume-3, pages 1-8.
- [16] Shalini, P., Paul Dhayabaran, D., A Study on Root Mean Square Labelings in Graphs, International Journal of Engineering Science and Innovative Technology, May 2015, Volume-4, Issue-3, pages 305-309.
- [17] Shalini, P., Gowri, R., Paul Dhayabaran, D., An Absolute Differences of Cubic and Square Difference Labeling For Some Families Of Graphs, The International journal of analytical and experimental modal analysis, Volume XI, Issue 10, October 2019, Page no: 538-544.
- [18] Shalini, P., Paul Dhayabaran, D., Maximization of Multiplicative Labeling, International journal of Research in Advent Technology(IJRAT), Special Issue January 2019, Page no: 209-214.
- [19] Shalini, P., Meena, S. A., Lehmer -4 mean labeling of graphs, International journal for research in Applied Science and Engineering Technology, Volume 10, Issue XII, December 2022, Page no: 1348-1351, ISSN : 2321-9653.
- [20] Shalini, P., Tamizharasi, S., Power-3 Heronian Odd Mean Labeling of Graphs, International Journal for Research in Applied Science and Engineering Technology, Volume 10 Issue XII, December 2022, Page no: 1605-1608.
- [21] Shalini, P., Tamizharasi, S., A Study on Power-3 Heronian Odd Mean Labeling for Some Path Related Graphs, International Journal for Research in Applied Science and Engineering Technology, Volume 11 Issue IV, April 2023, Page no: 1136 – 1139.
- [22] Shalini, P., Madhumitha, D., Root Cube Even Mean Labeling of Graph, Aryabhatta Journal of Mathematics and Informatics, Volume 15, Issue 1, Jan -June 2023. Pg. No. 33-38.
- [23] Shalini, P., Meena, S.A., Lehmer-4 Mean Labeling for Some Path Related Graphs, Aryabhatta Journal of Mathematics and Informatics, Volume 15, Issue 1, Jan -June 2023. Pg. No. 105-110.
- [24] Shalini, P., Skolem Odd Vertex Graceful Signed Graphs for Star Graphs, International Journal of Mathematics Trends and Technology, Volume 69, Issue 8, August 2023, Pg. No. 30 – 35
- [25] Shalini, P., Priyadarshini, K., SP Mean Difference Labeling for Some Families of Graphs”, Research and Applications Towards Mathematics and Computer Science, Volume 9, February 2024, Pg. No. 1-12, B P International, Print ISBN: 978-81-970187-8-7, eBook ISBN: 978-81-970187-9-4.



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