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# Vibration Analysis of Straight FGM Beam with Different Parameters

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**Abstract:** *This paper deals with the vibration analysis of Straight functionally Graded Beam (FGM) considering various parameters. It is analysed with different boundary conditions like simply supported, clamped clamped. The FGM beam is analysed for different L/h ratio and different volume fraction index. is analysed with different L/h ratios using higher order shear deformation theory. It was found that for a fixed value of L/h ratio as the volume fraction index is increased the frequency of vibration is reducing. Again, if the value of volume fraction index is kept constant and the L/h ratio is increased the frequency is decreasing. The dependence of support conditions on the frequency response is highlighted. Comparison and convergence study has been performed to validate the present formulation. The result and the analysis of the frequency of vibration can be used to optimize the frequency of a leaf spring treating it as a straight beam to have better ride. Thus, we observe that for both clamped-clamped and simply supported condition, for a fixed value of of L/h ratio as the volume fraction index is increased the frequency of vibration is reducing. Again if the value of volume fraction index is kept constant and the L/h ratio is increased the frequency is decreasing.*

**Keywords:** *Straight Beam, FGM, L/h ratio, Frequency of vibration, Volume fraction*

## I. INTRODUCTION

Spring is very important element of an automobile as it absorbs the vehicle vibration and give the rider a comfortable ride. Leaf spring is one of those springs which are mainly used for heavy vehicles. The advantage of using a leaf spring is that it can be guided along a definite path as it deflects to acts as structural member in addition to energy absorbing device.

Lots of efforts have been made by different people to reduce the vibration of leaf spring so that the ride could be made more comfortable. In this paper the analysis of master leaf was modeled as a curved beam and vibration was studied for different materials and for different L/R ratios with different boundary conditions.

Mahmood M. et. al. [1] did the analysis of composite leaf spring using ANSYS V5.4 software and tried to optimize the weight of the spring. Abdul Rahim Abu Talib et.al [2] tried to analyse the elliptic spring made for both light and heavy trucks .They tried to optimize the spring parameters for different ellipticity ratios. Vinkel Arora et al [3] did the fatigue life assessment of leaf spring of 65Si7 using analytical and graphical methods. They also did the life assessment using SAE design manual approach and it was compared with experimental results.

Mesut Simsek et al [4] studied the vibration of straight beam within the frame work of third order shear deformation theory. They calculated the six dimensionless frequency parameters of different beams having different h/L ratios for different boundary conditions.

Ankit Gupta & Mohammad Talha [5] introduced the geometrically nonlinear vibrations response of FGM plates. They proposed displacement based new hyperbolic higher-order shear and normal deformation theory (HHSNDT). The performance of a curved beam with coupled polynomial distributions was investigated by P. Raveendranath et al [6]. The result shows excellent convergence of natural frequencies even for very thin deep arches and higher vibrational modes.

M. Kawakami [7] presented an approximate method to study the analysis for both the in-plane and out-of-plane free vibration of horizontally curved beams with arbitrary shapes and variable cross-sections.

S.M. Ibrahim et al [8] investigated large amplitude periodic forced vibration of curved beams under periodic excitation using a three-noded beam element.

Mohammad Amir and Mohammad Talha [9] analyzed the thermo-elastic vibration of shear deformable functionally graded material (FGM) curved beams with micro structural defects by the finite element method.. The material properties of FGM beams are allowed to vary continuously in the thickness direction by a simple

power-law distribution in terms of the volume fractions of the constituents. Mohammad Amir and Mohammad Talha [10] studied the imperfection sensitivity in the vibration behavior of functionally graded arches with micro structural defects (porosity).

Vaibhav Ghodge [11] did the modal analysis of a cantilever beam and simply supported beam using ANSYS for Aluminium alloy, Gray cast iron, Structural steel and Copper alloy.

Yiming Fu [12] studied nonlinear free vibration and dynamic stability for the piezoelectric functionally graded beams, subjected to one-dimensional steady heat conduction in the thickness direction. J. Yuan and S. M. Dickinson [13] used Rayleigh-Ritz method for the solution of the free vibration problem of systems comprised of straight and/or curved beam components. Eigen value problems of Timoshenko and shear-deformable curved beams were analyzed by S.Y. Yang & H.C. Sin [14] using the elements with six degrees of freedom. The results of the Eigen-analysis show that the curvature-based beam elements are free of locking and are efficient. Mehdi Hajianmaleki & Mohammad S. Qatu [15] on the basis of exhaustive study on the vibration analysis of composite thin and thick beams explained a simple classic and shear deformation model that can be used for beams with any laminate. P. Chidambaram and A.W. Leissa [16] did a study on the Vibrations of planar curved beams, rings, and arches and special attention was given to the effects of initial static loading, nonlinear vibrations and the application of finite element techniques.

### A. Displacement Field

$$\begin{aligned} u(x, z, t) &= u_0(x, t) + z\phi_x(x, t) - \frac{4z^3}{h^2} \left[ \phi_x(x, t) + \frac{\partial w_0}{\partial x}(x, t) \right] \\ w(x, z, t) &= w_0(x, t) \end{aligned} \quad (3.1)$$

$$\text{let } \frac{\partial w_0}{\partial x} = \theta_x$$

$$u(x, z, t) = u_0(x, t) + z\phi_x(x, t) - \frac{4z^3}{3 \cdot h^2} [\phi_x(x, t) + \theta_x(x, t)]$$

$$u(x, z, t) = u_0(x, t) + \phi_x \left( z - \frac{4z^3}{3h^2} \right) - \frac{4z^3}{3h^2} \theta_x(x, t)$$

$$w(x, z, t) = w_0(x, t)$$

Where,  $u$  and  $w$  represents the displacements of a point along the  $(x, z)$  coordinates.  $u_0, w_0, \phi_x, \theta_x$  are four unknown displacement functions of mid-plane.

$$\begin{bmatrix} u(x, z, t) \\ w(x, z, t) \end{bmatrix} = \begin{bmatrix} 1 & 0 & \left( z - \frac{4z^3}{3h^2} \right) & -\frac{4z^3}{3h^2} \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} u_0 \\ w_0 \\ \phi_x \\ \theta_x \end{bmatrix} \quad (3.2)$$

The basic field variables can be symbolized mathematically as,  $\{\lambda o\} = \{u_0, w_0, \phi_x, \theta_x\}^T$

$$\begin{bmatrix} u \\ w \end{bmatrix} = \begin{bmatrix} 1 & 0 & \left( z - \frac{4z^3}{3h^2} \right) & -\frac{4z^3}{3h^2} \\ 0 & 1 & 0 & 0 \end{bmatrix} [\lambda o]$$

The displacement field in the compressed form can be written as

$$[U] = [\check{N}] [\lambda o]$$

$$\text{Where, } [U] = \begin{bmatrix} u \\ w \end{bmatrix} \text{ and } [\check{N}] = \begin{bmatrix} 1 & 0 & \left( z - \frac{4z^3}{3h^2} \right) & -\frac{4z^3}{3h^2} \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

### B. Strain-displacement Relations

Strain displacement relation is written below for curved beam and for the straight beam the value of  $R$  is taken as infinity.

$$\epsilon_{xx} = 1/\left(1 + \frac{z}{R}\right) \left( \frac{\partial u}{\partial x} + \frac{w}{R} \right)$$

Using linear strain displacement relations, the strain vectors may be defined as

$$\epsilon_{xx} = 1/\left(1 + \frac{z}{R}\right) \left\{ \frac{\partial u_0}{\partial x} + z \frac{\partial \phi_x}{\partial x} - \frac{4z^3}{h^2} \left[ \frac{\partial \phi_x}{\partial x} + \frac{\partial \theta_x}{\partial x} \right] + \frac{w_0}{R} \right\} \quad (3.3)$$

$$\gamma_{xz} = \frac{\partial u}{\partial z} + 1/\left(1 + \frac{z}{R}\right) \left( \frac{\partial w}{\partial x} - \frac{u}{R} \right) \quad (3.4)$$

$$= \left( \frac{\partial u_0}{\partial z} + \phi_x - \frac{4z^2}{h^2} [\phi_x + \theta_x] \right) + 1/\left(1 + \frac{z}{R}\right) \left\{ \frac{\partial w_0}{\partial x} - \frac{u_0}{R} - \frac{z}{R} (\phi_x) - \frac{4z^3}{3h^2} [\phi_x + \theta_x] \right\}$$

As  $u_0$  is a function of  $x$  and  $t$  only .ie  $\frac{\partial u_0}{\partial z} = 0$ .

$$\gamma_{xz} = 1/(1 + \frac{z}{R}) \{ (1 + \frac{z}{R}) (\phi_x - \frac{4z^2}{h^2} [\phi_x + \theta_x]) + \frac{\partial w_0}{\partial x} - \frac{u_0}{R} - \frac{z}{R} (\phi_x) - \frac{4z^3}{3 * R * h^2} [\phi_x + \theta_x] \}$$

$$\gamma_{xz} = 1/(1 + \frac{z}{R}) \{ (\phi_x + \frac{\partial w_0}{\partial x} - \frac{u_0}{R}) - \frac{4z^2}{h^2} [\phi_x + \theta_x] - \frac{8z^3}{3 * R * h^2} [\phi_x + \theta_x] \}$$

$$\begin{bmatrix} \epsilon_{xx} \\ \gamma_{xz} \end{bmatrix} = 1/(1 + \frac{z}{R}) \left( \begin{bmatrix} \frac{\partial u_0}{\partial x} + \frac{w_0}{R} \\ \phi_x + \frac{\partial w_0}{\partial x} - \frac{u_0}{R} \end{bmatrix} + z \begin{bmatrix} \frac{\partial \phi_x}{\partial x} \\ 0 \end{bmatrix} + z^2 \begin{bmatrix} 0 \\ -\frac{4}{h^2} [\phi_x + \theta_x] \end{bmatrix} + z^3 \begin{bmatrix} -\frac{4}{3h^2} (\frac{\partial \phi_x}{\partial x} + \frac{\partial \theta_x}{\partial x}) \\ -\frac{8}{3R * h^2} [\phi_x + \theta_x] \end{bmatrix} \right)$$

$$\begin{bmatrix} \epsilon_{xx} \\ \gamma_{xz} \end{bmatrix} = (1/(1 + \frac{z}{R})) \left( \begin{bmatrix} \epsilon^{\circ}_1 \\ \epsilon^{\circ}_2 \end{bmatrix} + z \begin{bmatrix} k^1_1 \\ 0 \end{bmatrix} + z^2 \begin{bmatrix} 0 \\ k^2_2 \end{bmatrix} + z^3 \begin{bmatrix} k^3_1 \\ k^3_2 \end{bmatrix} \right)$$

$$\begin{bmatrix} \epsilon_{xx} \\ \gamma_{xz} \end{bmatrix} = 1/(1 + \frac{z}{R}) \begin{bmatrix} 1 & 0 & z & 0 & 0 & z^3 & 0 \\ 0 & 1 & 0 & z & z^2 & 0 & z^3 \end{bmatrix} \begin{bmatrix} \epsilon^{\circ}_1 \\ \epsilon^{\circ}_2 \\ k^1_1 \\ k^2_2 \\ k^3_1 \\ k^3_2 \end{bmatrix}$$

$$[T] = 1/(1 + \frac{z}{R}) \begin{bmatrix} 1 & 0 & z & 0 & 0 & z^3 & 0 \\ 0 & 1 & 0 & z & z^2 & 0 & z^3 \end{bmatrix}$$

where,  $\epsilon^{\circ}_1 = \frac{\partial u_0}{\partial x} + w_0/R$

$$\epsilon^{\circ}_2 = \phi_x + \frac{\partial w_0}{\partial x} - \frac{u_0}{R}$$

$$k^1_1 = \frac{\partial \phi_x}{\partial x}$$

$$k^2_2 = -\frac{4}{h^2} [\phi_x + \theta_x]$$

$$k^3_1 = -\frac{4}{3h^2} (\frac{\partial \phi_x}{\partial x} + \frac{\partial \theta_x}{\partial x})$$

$$k^3_2 = -\frac{8}{3R * h^2} [\phi_x + \theta_x]$$

$$\begin{bmatrix} \epsilon^{\circ}_1 \\ \epsilon^{\circ}_2 \\ k^1_1 \\ k^2_2 \\ k^3_1 \\ k^3_2 \end{bmatrix} = \begin{bmatrix} \frac{\partial}{\partial x} & \frac{1}{R} & 0 & 0 \\ -\frac{1}{R} & \frac{\partial}{\partial x} & 1 & 0 \\ 0 & 0 & \frac{\partial}{\partial x} & 0 \\ 0 & 0 & -\frac{4}{h^2} & -\frac{4}{h^2} \\ 0 & 0 & \frac{3h^2}{-4} & \frac{3h^2}{-4} \\ 0 & 0 & \frac{-8}{3 * R * h^2} & \frac{-8}{3 * R * h^2} \end{bmatrix}$$

. The linear constitutive relations of an isotropic beam are given as:

$$\begin{Bmatrix} \sigma_{xx} \\ \tau_{xz} \end{Bmatrix} = \begin{bmatrix} Q_{11} & 0 \\ 0 & Q_{44} \end{bmatrix} \begin{Bmatrix} \epsilon_{xx} \\ \gamma_{xz} \end{Bmatrix}$$

Where  $Q_{11} = E/(1 - \mu^2)$  and  $Q_{44} = E/(2 * (1 + \mu))$ .

$$\begin{Bmatrix} \sigma_{xx} \\ \tau_{xz} \end{Bmatrix} = QT \epsilon^{\circ}$$

For FGM beam E is taken  $E_z$  which varies according to power law along thickness .



## II. FINITE ELEMENT FORMULATION

A two noded element with four degrees of freedom per node is being utilized to discretize the beam geometry. The generalized displacement vector of the model is expressed by

$\{\Lambda\} = [N]\{\Lambda^e\}$  and  $\{\epsilon\} = [B]\{\Lambda^e\}$  where  $\{\Lambda^e\}$  is the nodal displacement vector,  $[N]$  is the shape function matrix and  $[B]$  is the strain displacement matrix.  $N_1$  and  $N_2$  are Lagrangian interpolation function.

The strain energy of the curved beam is given by

$$\begin{aligned} U &= \frac{1}{2} \int \{\epsilon\}^T \{\sigma\} dV = \frac{1}{2} \int \{\epsilon\}^T [Q_i] \{\epsilon\} dV \\ &= \frac{1}{2} \int \{\epsilon^0\}^T [T_{tr}]^T [Q_i] [T_{tr}] \{\epsilon^0\} dV = \frac{1}{2} \int \{\Lambda^e\}^T [B]^T [D] [B] \{\Lambda^e\} dA \\ &= \frac{1}{2} \{\Lambda^e\}^T [K^e] \{\Lambda^e\} \end{aligned}$$

The kinetic energy of the FGM curved beam is given by

$$\begin{aligned} T_k &= \frac{1}{2} \int \rho \{\dot{U}\}^T \{\dot{U}\} dV = \frac{1}{2} \int \rho \{\dot{\Lambda}^e\}^T [\bar{N}]^T [\bar{N}] \{\dot{\Lambda}^e\} dV \\ &= \frac{1}{2} \int_A \{\dot{\Lambda}^e\}^T [m] \{\dot{\Lambda}^e\} dA = \frac{1}{2} \int_A \{\dot{\Lambda}^e\}^T [N]^T [m] [N] \{\dot{\Lambda}^e\} dA \\ &= \frac{1}{2} \{\dot{\Lambda}^e\}^T [M^e] \{\dot{\Lambda}^e\} \end{aligned}$$

Using variational principal the equation for frequency is derived

$$\begin{aligned} \frac{\partial E_T}{\partial x} &= 0 = \int \epsilon^T \sigma dv + \int \rho U^T U dV \\ 0 &= \int \epsilon^0 T^T Q T \epsilon^0 dv + \int \rho \lambda_0^T N^T N \lambda_0 dV \\ &= b \int_0^L \int_{-h/2}^{h/2} \epsilon^0 T^T Q T \epsilon^0 dz dx + b \int_0^L \int_{-h/2}^{h/2} \rho \lambda_0^T N^T N \lambda_0 dz dx \\ &= b \int_0^L \epsilon^0 [D] \epsilon^0 dx + b \int_0^L \lambda_0^T [m] \lambda_0 dx \end{aligned}$$

Where,  $D = \int_{-h/2}^{h/2} T^T Q T dz$  and  $m = \int_{-h/2}^{h/2} N^T N dz$

$$\begin{aligned} &= b \int_0^L \lambda_0^T B^T D B \lambda_0 dx + b \int_0^L \lambda_0^T [m] \lambda_0 dx \\ &= K U + M \ddot{U} \end{aligned}$$

$KU = \lambda MU$ , where  $\lambda = \omega^2$

## III. VIBRATION ANALYSIS OF STRAIGHT FGM BEAM WITH DIFFERENT PARAMETERS

In the present section the variation of frequency of vibration of straight FGM beam is done with different values of L/h ratio and with different volume fraction index for the combination of material mentioned in table 1. The variation is presented both in tabular form and with graphical representation.

L/h ratio	Frequency with diff. volume fraction index, C-C Beam			
	0.5	2	5	10
5	18160	14804	13354	12663
20	5396	4418	4169	4029
50	2187	1792	1697	1643
100	1097	900	852	825

Table 1 Frequency variation with variation in volume fraction index.

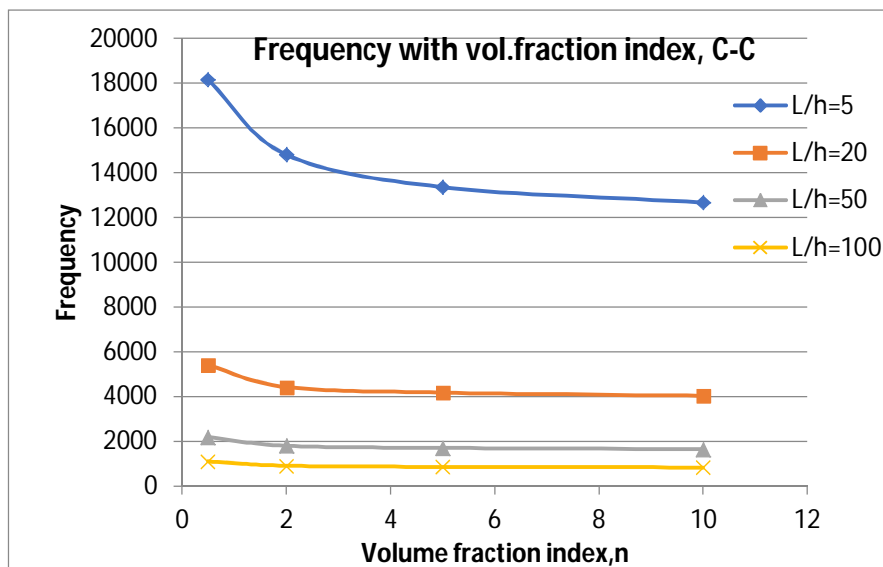


Fig. 1 Frequency variation with variation in volume fraction index for C-C beam.

Volume fraction index	Frequency with diff. L/h ratio, C-C Beam			
	5	20	50	100
0.5	18160	5396	2187	1097
2	14804	4418	1792	900
5	13354	4169	1697	852
10	12663	4029	1643	825

Table 2 Frequency variation with variation in L/h ratio.

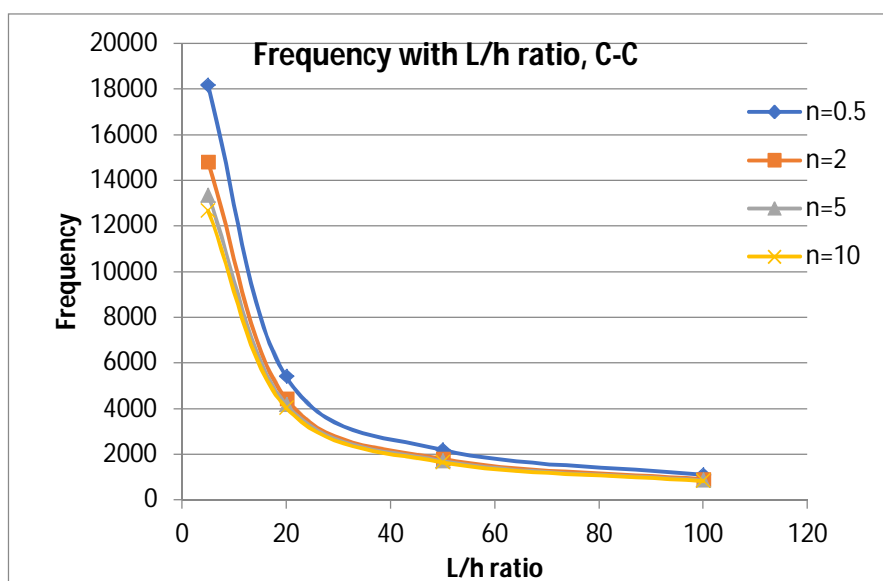


Fig. 2. Frequency variation with variation in L/h ratio for C-C beam.

L/h ratio	Frequency with diff. volume fraction index, S-S Beam			
	0.5	2	5	10
5	9392	8250	7605	7071
20	2476	2192	2050	1909
50	995	881	825	768
100	500	442.6	414	385.5

Table 3 Frequency variation with variation in volume fraction index for S-S beam.

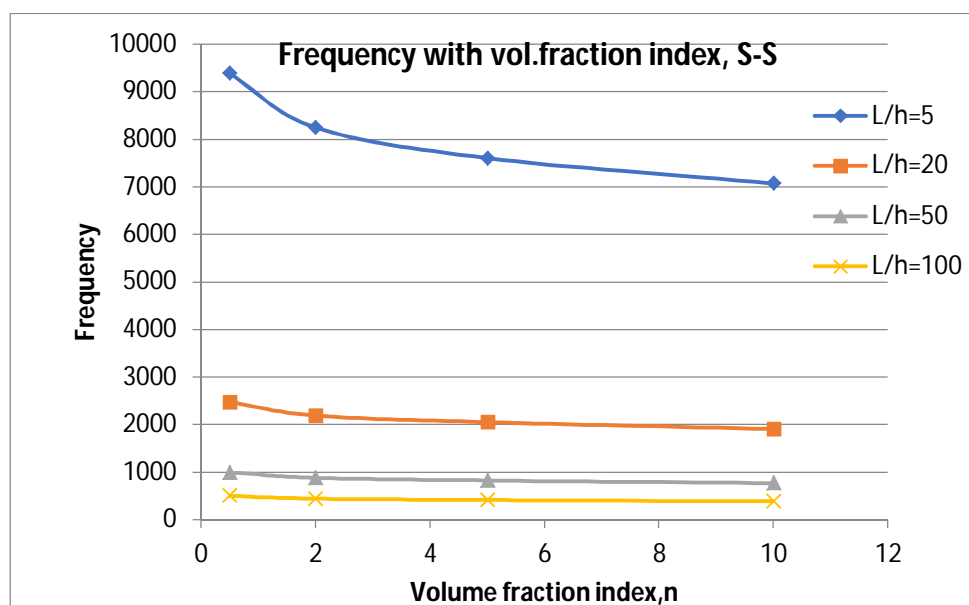


Fig. 3. Frequency variation with variation in volume fraction index for C-C beam.

Volume fraction index	Frequency with diff. L/h ratio, S-S Beam			
	5	20	50	100
0.5	9392	2476	995	500
2	8250	2192	881	442.6
5	7605	2050	825	414
10	7071	1909	768	385.5

Table 4 Frequency variation with variation in L/h ratio for S-S beam

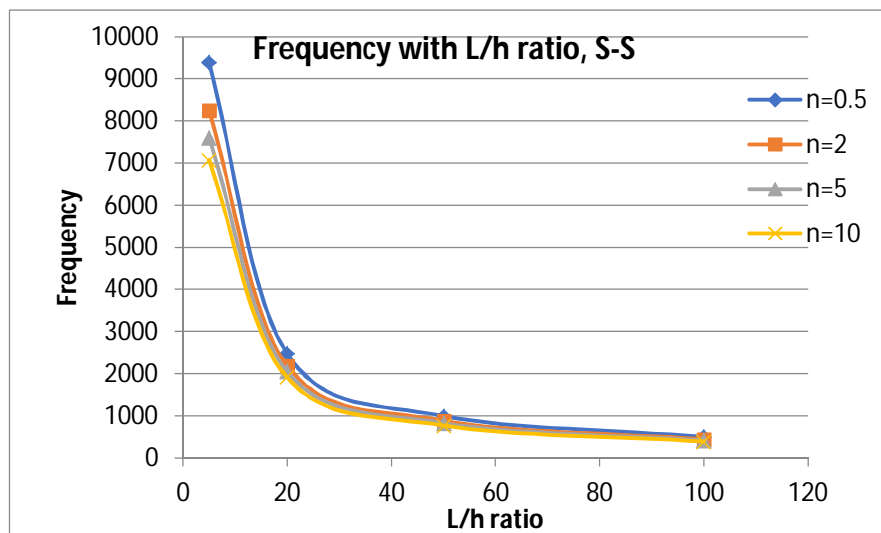


Fig. 4 Frequency variation with variation in L/h ratio for S-S beam

#### IV. CONCLUSION

The vibration analysis straight FGM was done under various conditions. MATLAB was used for the analysis. The effect of end condition on the vibration of straight FGM beam was studied for  $L/h = 5, 20, 50$  and  $100$ . The effect of change in volume fraction index and support condition is also studied. The volume fraction index was taken as  $0.5, 2, 5$  and  $10$ . The result was also validated. During the analysis it was found that as the  $L/h$  ratio is increased the value of frequency of vibration in both the clamped-clamped condition and simply supported condition decreases. but for clamped-clamped condition the frequency decreases much rapidly compared to simply supported condition. We also tried to observe the effect on frequency on volume fraction index and it was found that it decreases with increase in volume fraction index for both clamped and simply supported condition.

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