iJRASET
International Journal For Research in
Applied Science and Engineering Technology

INTERNATIONAL JOURNAL FOR RESEARCH

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY

Volume: 5 **Issue:** X **Month of publication:** October 2017

DOI: http://doi.org/10.22214/ijraset.2017.10022

International Journal for Research in Applied Science & Engineering Technology (IJRASET)
ISSN: 2321-9653; IC Value: 45.98; SJ Impact Factor:6.887
Volume 5 Issue X, October 2017- Available at www.ijraset.com

A Notes on Bailey's Transform

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***Abstract*: In this paper transformation formulae for poly-basic hypergeometric functions have been established by making use of Bailey's transform.**

Keywords: Hyper geometric functions, Summations, Transformation, Polybasic, Convergence

I. INTRODUCTION

The well-known Bailey's transformation states that, if

$$\beta_n = \sum_{r=0}^{n} \alpha_r\, u_{n-r} v_{n+r} \tag{1.1}$$

$$\text{and} \quad \gamma_n = \sum_{r=n}^{\infty} \delta_r\, u_{n-r} v_{n+r} = \sum_{r=0}^{\infty} \delta_r u_r v_{r+2n} \tag{1.2}$$

where $\alpha_r, \delta_r, u_r, v_r$ are any functions of r only, and that the series for γ_n exists, then, subject to convergence,

$$\sum_{n=0}^{\infty} \alpha_n\, \gamma_n = \sum_{n=0}^{\infty} \beta_n\, \delta_n \tag{1.3}$$

Bailey's transform (1.3) has been exploited, both as a tool to find new transformations of both ordinary and basic, hypergeometric series and also to find new q-identities of Rogers-Ramanujan type. We shall make use of the following summation in our analysis.

$${}_3\Phi_3\left[\begin{matrix} q^a & : & q_1^b & ; & zq^{a+1}q_1^{b+1} : q, q_1 : qq_1 : z \\ zq^{a+1} & : & zq_1^{b+1} & ; & zq^a q_1^b \end{matrix}\right]_N$$

$$= \frac{(1-zq^a)(1-zq_1^b)}{(1-z)(1-zq^aq_1^b)} - \frac{[q^a;q]_{N+1}[q_1^b;q_1]_{N+1} z^{N+1}}{(1-z)(1-zq^aq_1^b)[zq^{a+1};q]_n[zq_1^{b+1};q_1]_N}. \tag{1.4}$$

As $N \to \infty$, yeilds

$${}_3\Phi_3\left[\begin{matrix} q^a & : & q_1^b & ; & zq^{a+1}q_1^{b+1}; q, q_1; qq_1; z \\ zq^{a+1} & : & zq_1^{b+1} & ; & zq^a q_1^b \end{matrix}\right]_N$$

$$= \frac{(1-zq^a)(1-zq_1^b)}{(1-z)(1-zq^aq_1^b)}. \tag{1.5}$$

$$\sum_{k=0}^{n} \frac{(1-ap^kq^k)[a;p]_k[c;q]_k c^{-k}}{(1-a)[q;q]_k[ap/c;p]_k} = \frac{[ap;p]_n[cq;q]_n}{[q;q]_n[ap/c;p]_n} \tag{1.6}$$

$$\sum_{k=0}^{n} \frac{(1-ap^kq^k)(1-bp^kq^{-k})[a,b;p]_k[c,a/bc;q]_k q^k}{(1-a)(1-b)[q,aq/b;q]_k[ap/c,bcp;p]_k}$$

$$= \frac{[ap,bp;p]_n[cq,aq/bc;q]_n}{[q,aq/b;q]_n[ap/c,bcp;p]_n} \tag{1.7}$$

II. NOTATIONS AND DEFINITIONS

A RATIONAL FUNCTION OF Q^N, Q BEING FIXED COMPLEX PARAMETERS CALLED THE BASE OF THE SERIES, USUALLY WITH MODULUS LESS THAN ONE. AN EXPLICIT REPRESENTATION OF SUCH SERIES IS GIVEN BY:

$${}_r\Phi_s\left[\begin{matrix} a_1, & a_{2,\dots,} & a_r; q; z \\ b_1, & b_{2,\dots,} & b_s; q^i \end{matrix}\right]$$

$$= \sum_{n=0}^{\infty} q^{i\binom{n}{2}} \frac{[a_1, a_2, \ldots, a_r; q]_n z^n}{[b_1, b_2, \ldots, b_s; q]_n} \tag{2.1}$$

where $\binom{n}{2} = n(n-1)/2$ And

$[a_1, a_2, \ldots, a_r; q]_n = [a_1, q]_n [a_2, q]_n \ldots [a_r, q]_n$

With the q-shifted factorial defined by

$$[a; q]_n = \begin{cases} 1, & \text{if } n = 0 \\ (1-a)(a-aq)\ldots(1-aq^{n-1}), & \text{if } n = 1,2\ldots \end{cases}$$

(2.2)

For the convergence of the series (2.1) we need $|q| < 1$ and $|z| < \infty$ when $i = 1,2\ldots$ or max. $\{|q|, \ |z|\} < r$. When $i = 0$ provided no zeros appear in the denominator.

${}_r\Phi_s \begin{bmatrix} a_1, & a_2, \ldots, & a_r; q; z \\ b_1, & b_2, \ldots, & b_s; q^i \end{bmatrix}_N$ means that the series runs

From 0 to N only

The poly-basic hypergeometric series is defined as:

$$\Phi \begin{bmatrix} c_{1,1}, \ldots, & c_{1,r_1}; \ldots; c_{m,1}, \ldots, & c_{m,r_m}; q_1, q_2, \ldots, q_m; z \\ d_{1,1}, \ldots, & d_{1,s_1}; \ldots; d_{m,1}, \ldots, & d_{m,s_m} \end{bmatrix}$$

$$= \sum_{n=0}^{\infty} z^n \prod_{j=1}^{m} \frac{\left[c_{j,1}, \ldots, c_{j,r_j}; q_j\right]_n}{\left[d_{j,1}, \ldots, d_{j,s_j}; q_j\right]_n} \tag{2.3}$$

III.MAIN RESULTS

Taking $u_r = v_r = 1$ and

$\delta_r = \frac{[q^a;q]_r [q_1^b;q_1]_r [xq^{a+1}q_q^{b+1};qq_1]_r x^r}{[xq^{a+1};q]_r [xq_1^{b+1};q_1]_r [xq^a q_1^b;qq_1]_r}$ in (1.2) we get

$$\gamma_n = \frac{[q^a;q]_n [q_1^b;q_1]_n [xq^{a+1}q_q^{b+1};qq_1]_n x^n (1-xq^{a+n})(1-xq_1^{b+n})}{[xq^{a+1};q]_n [xq_1^{b+1};q_1]_n [xq^a q_1^b;qq_1]_n (1-x)(1-xq^{a+n}q_1^{b+n})}$$

$$= \frac{(1-xq^a)(1-xq_1^b)(q^a;q)_n (q_1^b;q_1)_n x^n}{(x-1)(1-xq^a q_1^b)(xq^a;q)_n \left[xq_1^b;q_1\right]_n} \tag{3.1}$$

Putting these values in (1.1) (1.2) and (1.3) we get the new form of the Bailey's transform as

If

$$\beta_n = \sum_{r=0}^{n} \alpha_r \tag{3.2}$$

Then

$$\frac{(1-xq^a)(1-xq_1^b)}{(1-x)(1-xq^a q_1^b)} \sum_{n=0}^{\infty} \frac{(q^a;q)_n \left[q_1^b;q_1\right]_n x^n}{(xq^a;q)_n \left[xq_1^b;q_1\right]_n} \alpha_n$$

$$= \sum_{n=0}^{\infty} \frac{(q^a;q)_n \left[q_1^b,q_1\right]_n \left[xq^{a+1}q_1^{b+1};qq_1\right]_n x^n}{(xq^{a+1};q)_n \left[xq^a q_1^b;qq_1\right]_n \left[xq_1^{b+1};q_1\right]_n} \beta_n \tag{3.3}$$

We shall make use of (3.2) and (3.3) in order to establish certain new transformation formulae

(a) Choosing

$\alpha_r = z^r$ in (3.2) we get

$$\beta_n = \frac{1-z^{n+1}}{1-z}$$

Putting these values in (3.3) we get,

$$\frac{(1-xq^a)(1-xq_1^b)}{(1-x)(1-xq^aq_1^b)}$$
$$ {}_3\Phi_2\begin{bmatrix} q^a; & q_1^b; q, q_1; xz \\ xq^a; & xq_1^b \end{bmatrix}$$
$$= \frac{(1-xq^a)(1-xq_1^b)}{(1-z)(1-x)(1-xq^aq_1^b)} - \frac{z}{(1-z)}$$

$$\times {}_3\Phi_3\begin{bmatrix} q^a & : & q_1^b & ; & xq^{a+1}; q_1^{b+1}; q, q_1; qq_1; xz \\ xq^{a+1} & : & xq_1^{b+1}; & & xq^aq_1^b \end{bmatrix},$$
(3.4)

(b) Next, taking

$$\alpha_r = \frac{[q_2^\alpha; q_2]_r [q_3^\beta; q_3]_r [zq_2^{\alpha+1}q_3^{\beta+1}; q_2q_3]_r z^r}{[zq_2^{\alpha+1}; q_2]_r [zq_3^{\beta+1}; q_3]_r [zq_2^\alpha q_3^\beta; q_2q_3]_r}$$

In (3.2) and making use of (1.4) we get:

$$\beta_n = \frac{(1-zq_2^\alpha)(1-zq_3^\beta)}{(1-z)(1-zq_2^\alpha z_3^\beta)} - \frac{[q_2^\alpha; q_2]_{n+1} [q_3^\beta; q_3]_{n+1} z^{n+1}}{(1-z)(1-zq_2^\alpha q_3^\beta)[zq_2^{\alpha+1}; q_2]_n [zq_3^{\beta+1}; q_3]_n} \tag{3.5}$$

Putting these values in (3.3) we have

$$\frac{(1-xq^a)(1-xq_1^b)}{(1-x)(1-xq^aq_1^b)}.$$
$$\begin{bmatrix} q^a; & q_1^b & ; & q_2^\alpha, q_3^\beta; zq_2^{\alpha+1}, zq_3^{\beta+1}; q, q_1, q_2, q_3, q_2q_3; xz \\ xq^a, & xq_1^b; & & zq_2^{\alpha+1}; zq_3^{\beta+1}, zq_2^\alpha q_3^\beta \end{bmatrix}$$

$$= \frac{(1-zq_2^\alpha)(1-zq_3^\beta)(1-xq^a)(1-xq_1^b)}{(1-z)(1-x)(1-zq_2^\alpha q_3^\beta)(1-xq^aq_1^b)} - \frac{z(1-q_2^\alpha)(1-q_3^\beta)}{(1-z)(1-zq_2^\alpha q_3^\beta)}$$
$$\times {}_5\Phi_5\begin{bmatrix} q^a; q_1^b; xq^{a+1}q_1^{b+1}; q_2^{\alpha+1}; q_3^{\beta+1}, q, q_1, qq_1, q_2, q_3; xz \\ xq^{a+1}; xq_1^{b+1}; xq^aq_1^b; zq_2^{\alpha+1}; zq_3^{\beta+1} \end{bmatrix}$$

(3.6)

(c) Again, taking

$\alpha_r = \frac{(1-\alpha P^r Q^r)[\alpha;p]_r [\beta;Q]_r \beta^{-r}}{(1-\alpha)[Q;Q]_r [\alpha P/\beta;P]_r}$ in (3.2)

And making use of (1.6) we get,

$$\beta_n = \frac{[\alpha P; P]_n [\beta Q; Q]_n}{[Q; Q]_n [\alpha P/\beta; P]_n}$$

Putting these values in (3.3) we obtain,

$$\frac{(1-xq^a)(1-xq_1^b)}{(1-x)(1-xq^aq_1^b)} {}_5\Phi_5\begin{bmatrix} q^a; q_1^b; \alpha PQ; \alpha; \beta; q; q_1, PQ, P, P, Q; x/\beta \\ xq^a; xq_1^b; \alpha; \alpha P/\beta; Q; \end{bmatrix}$$

$$\times {}_6\Phi_5\begin{bmatrix} q^a; q_1^b; xq^{a+1}q_1^{b+1}; \alpha P; \beta Q; q, q_1, qq_1, P, Q; x \\ xq^{a+1}; xq_1^{b+1}; xq^aq_1^b; \alpha P/\beta; Q; \end{bmatrix}.$$
(3.7)

(d) Lastly, taking

$$\alpha_r = \frac{(1-\alpha P^r Q^r)(1-\beta P^r Q^{-r})[\alpha,\beta;P]_r[\gamma,\alpha/\beta\gamma;Q]_r Q^r}{(1-\alpha)(1-\beta)[Q,\alpha Q/\beta;Q]_r[\alpha P/\gamma,\beta\gamma P;P]_r}$$

In (3.2) and making use of (1.7) we get,

$$\beta_n = \frac{[\alpha P,\beta P;P]_n[\gamma Q,\alpha Q/\beta\gamma;Q]_n}{[Q,\alpha Q/\beta;Q]_n[\alpha P/\beta,\beta\gamma P;P]_n}$$

Now putting these values in (3.3) we find:

$$\frac{(1-xq^a)(1-xq_1^b)}{(1-x)(1-xq^a q_1^b)} \times {}_9\Phi_8\left[\begin{matrix} q^a;q_1^b;\alpha;\beta;\gamma,a/\beta\gamma;\alpha PQ;\beta P/Q;q,q_1,P,Q,PQ,P/Q;xQ \\ xq^a;xq_1^b;\alpha P/\gamma,\beta\gamma P,Q,\alpha Q/\beta;\alpha;\beta \end{matrix}\right]$$

$$= {}_8\Phi_7\left[\begin{matrix} q^a;q_1^b;xq^{a+1}q_1^{b+1};\alpha P,\beta P;\gamma Q,\alpha Q/\beta\gamma;q,q_1,qq_1,P,Q;x \\ xq^{a+1};xq_1^{b+1};xq^a q_1^b;\alpha P/\gamma,\beta\gamma P,Q,\alpha Q/\beta \end{matrix}\right] \quad (3.8)$$

A number of otherinteresting results can also be obtained by taking suitable values of α_r and β_n.

IV. CONCLUSIONS

Six transformation formulae for poly-basic hypergeometric functions are established by making use of Bailey's Transform.

V. ACKNOWLEDGMENT

My thanks are due to Dr. G.C Chaubey Ex Associate Professor & Head department of Mathematics TDPG College Jaunpur and Professor B. Kunwar Department of Mathematics IET, Lucknow for their encouragement and for providing necessary support. I am extremely grateful for their constructive support.

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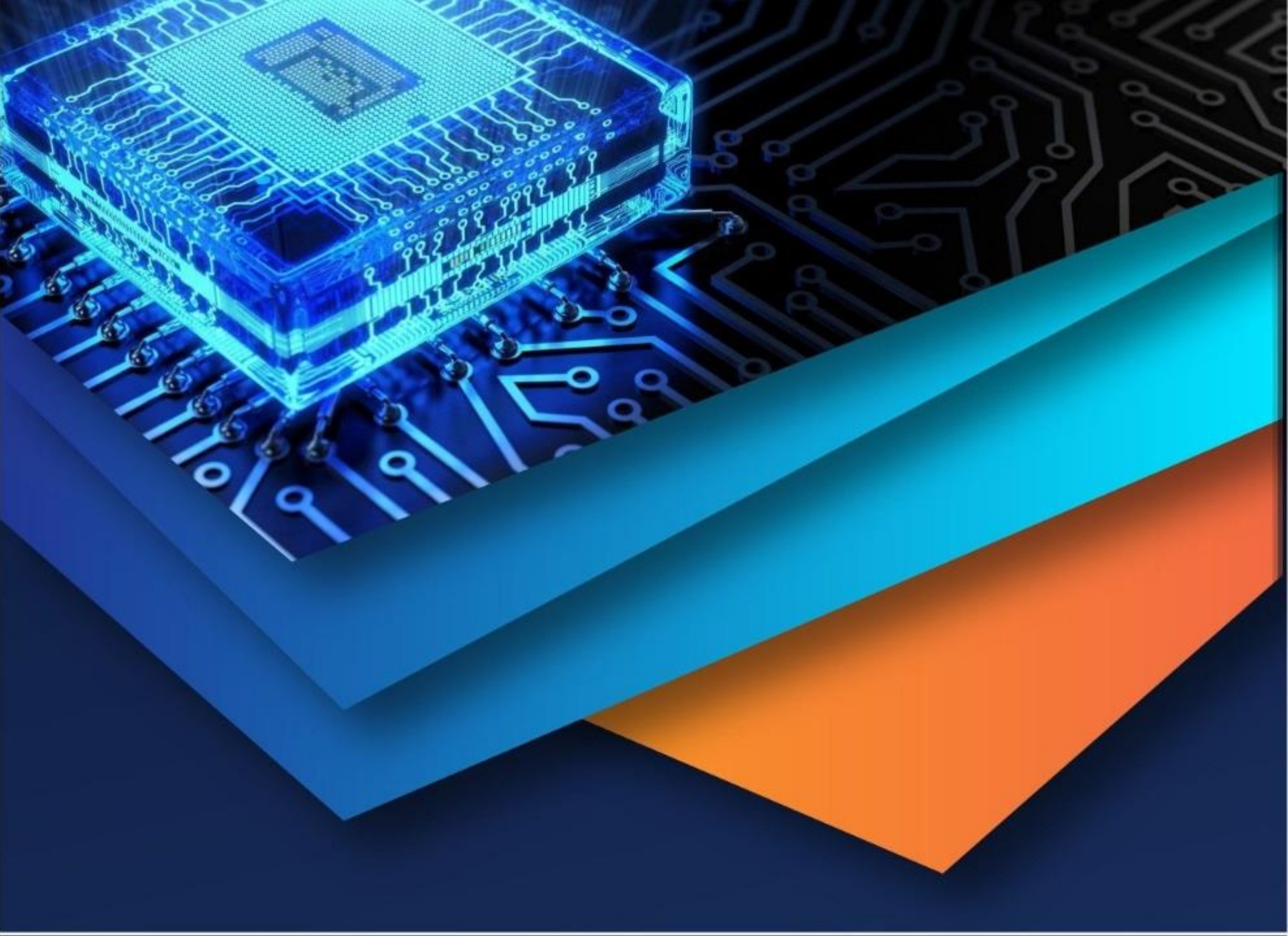

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