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Cubic Harmonious Labeling of Path Related Graphs

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Abstract: ***A (n,m) graph G =(V,E) is said to be Cubic Harmonious Graph(CHG) if there exists an injective function $f:V(G)\to\{1,2,3,\ldots,m^3+1\}$ such that the induced mapping $f^*_{chg}: E(G)\to\{1^3,2^3,3^3,\ldots,m^3\}$ defined by $f^*_{chg}(uv) = (f(u)+f(v)) \bmod (m^3+1)$ is a bijection. The vertex labels are distinct and edge labels are also distinct as well as cubic. In this paper, focus will be given on the result "cubic harmonious labeling of caterpillar graph and (P_n,S_m) graph***

Keywords: Caterpillar graph,Cubic harmonious graph, Cubic harmonious labeling, Harmonious graph, Path graph.

I. INTRODUCTION

Consider a graph $G=(V(G),E(G))$ with m edges. A function f defined by f: $V(G)\to\{1,2,3,\ldots,m-1\}$, is called a harmonious labeling of G if it is an injective and it induces a bijective function f^* defined by $f^*(e) = (f(u)+f(v)) \pmod m$. where $e = uv$ for $u,v \in v(G)$, $e\in E((G)$. By taking the edge labels of a sequentially labeled graph with m edges modulo m, we obviously obtain a harmoniously labeled graph. More than 150 papers have been published on harmonious labeling and comprehensive information can be found in [3].Labeling of special type of Square Harmonious Labeling is examined in [9].Cubic Graceful Labeling is introduced in [5]. Cubic Harmonious Labeling is defined in [6].

In this paper , we study the existence of cubic harmonious labeling for graphs obtained by caterpillar and (P_n,S_m)

A. *Definition 1.1*

The path on n vertices is denoted by P_n.

B. *Definition1.2*

A complete bipartite graph $K_{1,n}$ is called a ***star***and it has (n+1) vertices and n edges

C. *Definition1.3*

The *graph* (P_n,S_m) is obtained from n copies of the star graph S_m and the path P_n: $\{u_1,u_2 \ldots u_n\}$ by joining u_j with the vertex of the j^{th} copy of S_m by means of and edge for $1\le j\le n$.

D. *Definition1.4*

The ***caterpillar*** $S(X_1, X_2\ldots X_m)$ is obtained from the path $P_n$by joining X_i vertices to each of the i^{th} vertex of P_n $(1\le i\le n)$

II.MAIN RESULTS

A. *Theorem 2. 1*

The graph (P_n, S_m) is cubic harmonious graph.

1) *Proof :*

Let $\{u_1,u_2,\ldots u_n\}$ be the vertices of the path P_n and $\{v_{0j},v_{ij},v_{2j},\ldots,v_{mj}\}$ be the vertices of j^{th} copy P_m for $1\le j\le n$.

The graph (P_n,S_m) is obtained from n copies of the star graph S_m and the path P_n: $\{u_1,u_2 \ldots u_n\}$ by joining u_j with the vertex of the j^{th} copy of S_m by means of and edge for $1\le j\le n$.

Then

$$V((P_n,S_m)) = \begin{cases} u_i; & 1\le i\le n \\ v_{0i}; & 1\le i\le n \end{cases}$$

v_{ij}; $1\le i\le m,\ 1\le j\le n$

and

$$E((P_n,S_m)) = \begin{cases} u_i u_{i+1}; & 1\le i\le n-1 \\ u_i v_{0i}; & 1\le i\le n \end{cases}$$

$v_{oj}\,v_{ij}$; $1\le i\le m,\ 1\le j\le n$

Define an injection $f: V((P_n,S_m)) \to \{1,2,\dots\dots,(mn+2n-1)^3+1\}$ by

$f(u_i) = (mn+2n+1-i)^3 + (mn+2n-1)^3 + 1 - f(u_{i-1})$; $2\le i\le n$

$f(V_{oj}) = (mn+j)^3 + (mn+2n-1)^3 + 1 - f(u_j)$; $1\le j\le n$

$f(v_{ij}) = (mn-ni+j)^3 + (mn+2n-1)^3 + 1 - f(V_{oj})$; $1\le i\le m,\ 1\le j\le n$

The induced edge mapping are

$f^*(u_iu_{i+1}) = (mn+2n+1-i)^3$; $1\le i\le n-1$

$f^*(u_iv_{0i}) = (mn+i)^3$; $1\le i\le n$

$f^*(v_{oj}\,v_{ij}) = (mn-ni+j)^3$; $1\le i\le m,\ 1\le j\le n$

The vertex label are in the set $\{1,2,\dots\dots\dots,(mn+2n-1)^3+1\}$ Then the edge labels are distinct and cubic. ie. $\{1^3,2^3,\dots\dots\dots\dots\dots,(mn+2n-1)^3\}$. Hence the graph (P_n, S_m) is cubic harmonious.

B. Theorem 2.2

The caterpillar $S(X_1, X_2\dots X_m)$ is cubic harmonious graph for all n>1.

1) Proof : Let G be the caterpillar S ($x_1, x_2\dots x_m$) is obtained from the path P_n by joining X_i vertices to each of the i^{th} vertex of P_n $(1\le i\le n)$

Let $V(G) = \{v_i;\ 1\le i\le n\}\ U\{v_{ij};\ 1\le i\le n-1,\ 1\le j\le m\}$

and

$E(G) = \{v_iv_{i+1};\ 1\le i\le n-1\}\ U\{v_iv_{ij};\ 1\le i\le n-1,\ 1\le j\le m\}$

Then $|V(G)| = m+n$ and $|E(G)| = m+n-1$

Define an injection $f: V(G)\to\{1,2,\dots\dots,(m+n-1)^3+1\}$ by

$f(v_1) = (m+n-1)^3 + 1$;

$f(v_i) = (m+n+1-i)^3 + (m+n-1)^3 + 1 - f(v_{i-1})$; $2\le i\le n$

$f(v_{ij}) = (m+1-j)^3 + (m+n-1)^3 + 1 - f(v_i)$; $1\le i\le n-1, 1\le j\le m$;

The induced edge mapping are

$f^*(v_i\,v_{i+1}) = (m+n-1)^3$; $1\le i\le n-1$

$f^*(v_iv_{ij}) = (m+1-j)^3$; $1\le j\le m$

The vertex label are in the set $\{1,2,\dots\dots,(m+n-1)^3+1\}$. Then the edge labels are distinct and cubic. The edge sets are $\{1^3,2^3,3^3,\dots\dots,(m+n-1)^3\}$. Hence the caterpillar is cubic harmonious graph.

III.CONCLUSION

The harmonious labeling is one of the most important labeling techniques. As all the graphs are not harmonious, it is very interesting to investigate graphs or graph families which admit harmonious labeling.We have reported the cubic harmonious labeling of different graphs.

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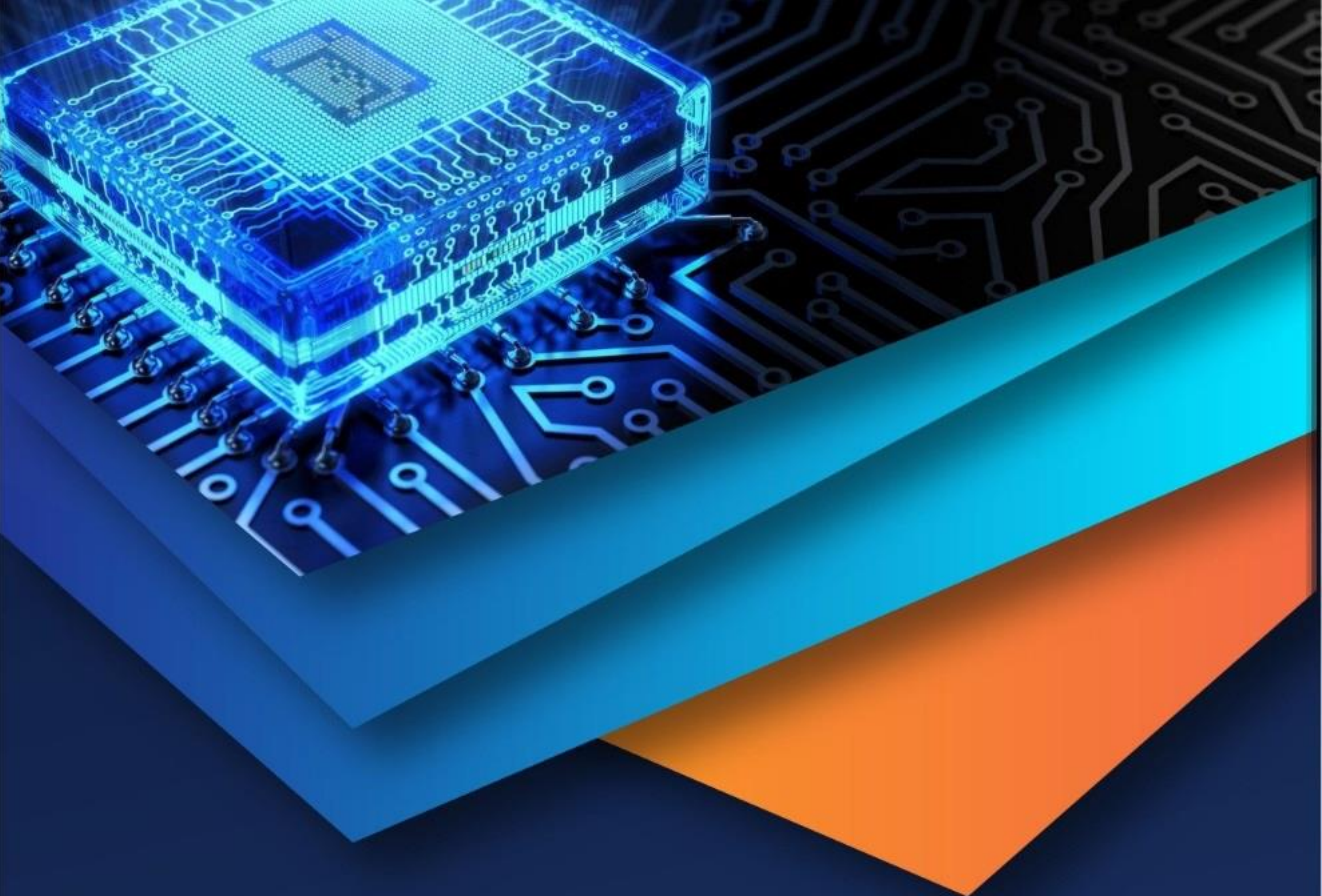

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