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Evaluating Pyramidal Numbers and Pent tope Number Using Initial Value Theorem in Z-Transform

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Abstract: ***In this communication, We evaluate Pyramidal numbers and Pentatope number by applying the Initial value theorem in Z-Transform.***
Keywords: Inverse Z-Transform, Initial value theorem, Pyramidal numbers and Pentatope number.

I. INTRODUCTION

Mathematics is the language of patterns and relationships, and is used to describe anything that can be quantified. The main goal of Number theory is to discover interesting and unexpected relationships. It is devoted primarily to the study of natural numbers and integers. In [1-7], theory of numbers were discussed. The Z-Transform is a transform for a sequence. The development of communication branch is based on discrete analysis. Z-Transform plays the same role in discrete analysis as in continuous systems. The Z-Transform technique used for time signals and systems. In [8-10], Z-Transform methods were analysed. Recently in [11], the sequence of m-gonal numbers and octahedral numbers was developed.
In this communication, we develop the sequence of pyramidal numbers and pentatope number by using initial value theorem in Z-Transform.

A. *Notations*

$P_n^{\,m} = \frac{n(n+1)}{6}\left[(m-2)\,n+(5-m)\right]$ = Pyramidal number of rank 'n' with sides 'm'.

$PT_n = \frac{1}{24}n(n+1)(n+2)(n+3)$ = Pentatope number of rank 'n'.

B. *Definition*

If the function u_n is defined for discrete values $(n = 0,1,2,3,\ldots)$ and $u_n = 0$ for $n < 0$, then its Z-Transform is defined to be $Z(u_n) = U(z) = \sum u_n z^{-n}$. The inverse Z-transform is written as $Z^{-1}[U(z)] = u_n$.

C. *Initial value theorem*

If $Z(u_n) = U(z)$, then $u_0 = \underset{z\to\infty}{Lt}\, U(z)$.

D. *Method of Analysis*

The process of finding the sequence of pyramidal numbers and pentatope number by using initial value theorem in Z-Transform is given in the following theorems.

E. *Theorem 1*

$Z^{-1}\left[\frac{z^3-(3-m)z^2}{(z-1)^4}\right] = P_n^{\,m}$ (Pyramidal number of rank 'n' with sides 'm')

F. *Proof*

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Assume that $U(z)=\dfrac{z^3-(3-m)z^2}{(z-1)^4}$

By initial value theorem, we have

$$u_0=\underset{z\to\infty}{Lt}\,U(z)=0$$

$$u_1=\underset{z\to\infty}{Lt}\left[z\left(U(z)-u_0\right)\right]=\underset{z\to\infty}{Lt}\left[z\left(\frac{z^3-(3-m)z^2}{(z-1)^4}\right)\right]=\underset{z\to\infty}{Lt}\left[\frac{1-(3-m)z^{-1}}{(1-z^{-1})^4}\right]=1$$

$$u_2=\underset{z\to\infty}{Lt}\left[z^2\left(U(z)-u_0-u_1z^{-1}\right)\right]=\underset{z\to\infty}{Lt}\left[\frac{1+m-6z^{-1}+4z^{-2}-z^{-3}}{(1-z^{-1})^4}\right]=m+1=\frac{2(2+1)}{6}\left[(m-2)2+(5-m)\right]$$

$$u_3=\underset{z\to\infty}{Lt}\left[z^3\left(U(z)-u_0-u_1z^{-1}-u_2z^{-2}\right)\right]=\underset{z\to\infty}{Lt}\left[\frac{(4m-2)-(6m+2)z^{-1}+(4m+3)z^{-2}-(m+1)z^{-3}}{(1-z^{-1})^4}\right]$$

$$=4m-2=\frac{3(3+1)}{6}\left[(m-2)3+(5-m)\right]$$

$$u_4=\underset{z\to\infty}{Lt}\left[z^4\left(U(z)-u_0-u_1z^{-1}-u_2z^{-2}-u_3z^{-3}\right)\right]=10(m-1)=\frac{4(4+1)}{6}\left[(m-2)4+(5-m)\right]$$

……..

Proceeding in this manner, we obtain

$$u_n=\underset{z\to\infty}{Lt}\left[z^n\left(U(z)-u_0-u_1z^{-1}-u_2z^{-2}-u_3z^{-3}-\cdots-u_{n-1}z^{-(n-1)}\right)\right]$$

$$=\underset{z\to\infty}{Lt}\left[z^n\left(\frac{z^3-(3-m)z^2}{(z-1)^4}-\frac{1}{z}-\frac{m+1}{z^2}-\frac{2(2m-1)}{z^3}-\frac{10(m-1)}{z^4}-\cdots\right)\right]$$

$$=\frac{n(n+1)}{6}\left[(m-2)n+(5-m)\right]=P_n^m$$

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Hence, $Z^{-1}\left[\frac{z^3-(3-m)z^2}{(z-1)^4}\right]=P_n^m$ (Pyramidal number of rank 'n' with sides 'm').

Numerical examples of the above thorem are illustrated in the following table:

m	$u_1=1$	$u_2=m+1$	$u_3=2(2m-1)$	$u_4=10(m-1)$	.....	$u_n=\frac{n(n+1)}{6}[(m-2)\,n+(5-m)]$
3	1	4	10	20	.....	$\frac{n(n+1)(n+2)}{6}$
4	1	5	14	30	.....	$\frac{n(n+1)(2n+1)}{6}$
5	1	6	18	40	.....	$\frac{n^2(n+1)}{2}$
6	1	7	22	50	.....	$\frac{n(n+1)(4n-1)}{6}$
7	1	8	26	60	.....	$\frac{n(n+1)(5n-2)}{6}$
.....	.....	.....	.....	.....	.....	.....
30	1	31	118	290	.....	$\frac{n(n+1)(28n-25)}{6}$

We observe that in the above table, the successive rows from the first row represent the sequence of triangular pyramidal number, square pyramidal number, pentagonal pyramidal number, hexagnal pyramidal number, heptagonal pyramidal number & ····· 30-gonal pyramidal number respectively.

G. Theorem 2

$$Z^{-1}\left[\frac{z^4+1}{(z-1)^5}\right]=PT_n$$ (Pentatope number of rank 'n')

H. Proof

Assume that $U(z)=\frac{z^4+1}{(z-1)^5}$

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By initial value theorem, we have

$$u_0 = \underset{z\to\infty}{Lt}\, U(z) = 0$$

$$u_1 = \underset{z\to\infty}{Lt}\left[z\left(U(z)-u_0\right)\right] = \underset{z\to\infty}{Lt}\left[z\left(\frac{z^4+1}{(z-1)^5}-0\right)\right] = 1$$

$$u_2 = \underset{z\to\infty}{Lt}\left[z^2\left(U(z)-u_0-u_1 z^{-1}\right)\right] = \underset{z\to\infty}{Lt}\left[z^2\left(\frac{z^4+1}{(z-1)^5}-\frac{1}{z}\right)\right] = 5 = \frac{1}{24}(2)(3)(4)(5)$$

$$u_3 = \underset{z\to\infty}{Lt}\left[z^3\left(U(z)-u_0-u_1 z^{-1}-u_2 z^{-2}\right)\right] = \underset{z\to\infty}{Lt}\left[z^3\left(\frac{z^4+1}{(z-1)^5}-\frac{1}{z}-\frac{5}{z^2}\right)\right] = 15 = \frac{1}{24}(3)(4)(5)(6)$$

$$u_4 = \underset{z\to\infty}{Lt}\left[z^4\left(U(z)-u_0-u_1 z^{-1}-u_2 z^{-2}-u_3 z^{-3}\right)\right] = \underset{z\to\infty}{Lt}\left[z^4\left(\frac{z^4+1}{(z-1)^5}-\frac{1}{z}-\frac{5}{z^2}-\frac{15}{z^3}\right)\right]$$

$$= 35 = \frac{1}{24}(4)(5)(6)(7)$$

……

Proceeding in this manner, we obtain

$$u_n = \underset{z\to\infty}{Lt}\left[z^n\left(U(z)-u_0-u_1 z^{-1}-u_2 z^{-2}-u_3 z^{-3}-\cdots\cdots-u_{n-1}z^{-(n-1)}\right)\right]$$

$$= \underset{z\to\infty}{Lt}\left[z^n\left(\frac{z^4+1}{(z-1)^5}-\frac{1}{z}-\frac{5}{z^2}-\frac{15}{z^3}-\frac{35}{z^4}-\cdots\cdots\right)\right]$$

$$= \frac{1}{24}n(n+1)(n+2)(n+3) = PT_n$$

Hence, $Z^{-1}\left[\frac{z^4+1}{(z-1)^5}\right] = PT_n$ (Pentatope number of rank ‘n’).

II. CONCLUSION

In this communication, we find pyramidal numbers and pentatope number by applying the initial value theorem in Z-Transform. In this manner, one can find the sequence of other three dimensional and four dimensional numbers by using various properties of Z-Transform.

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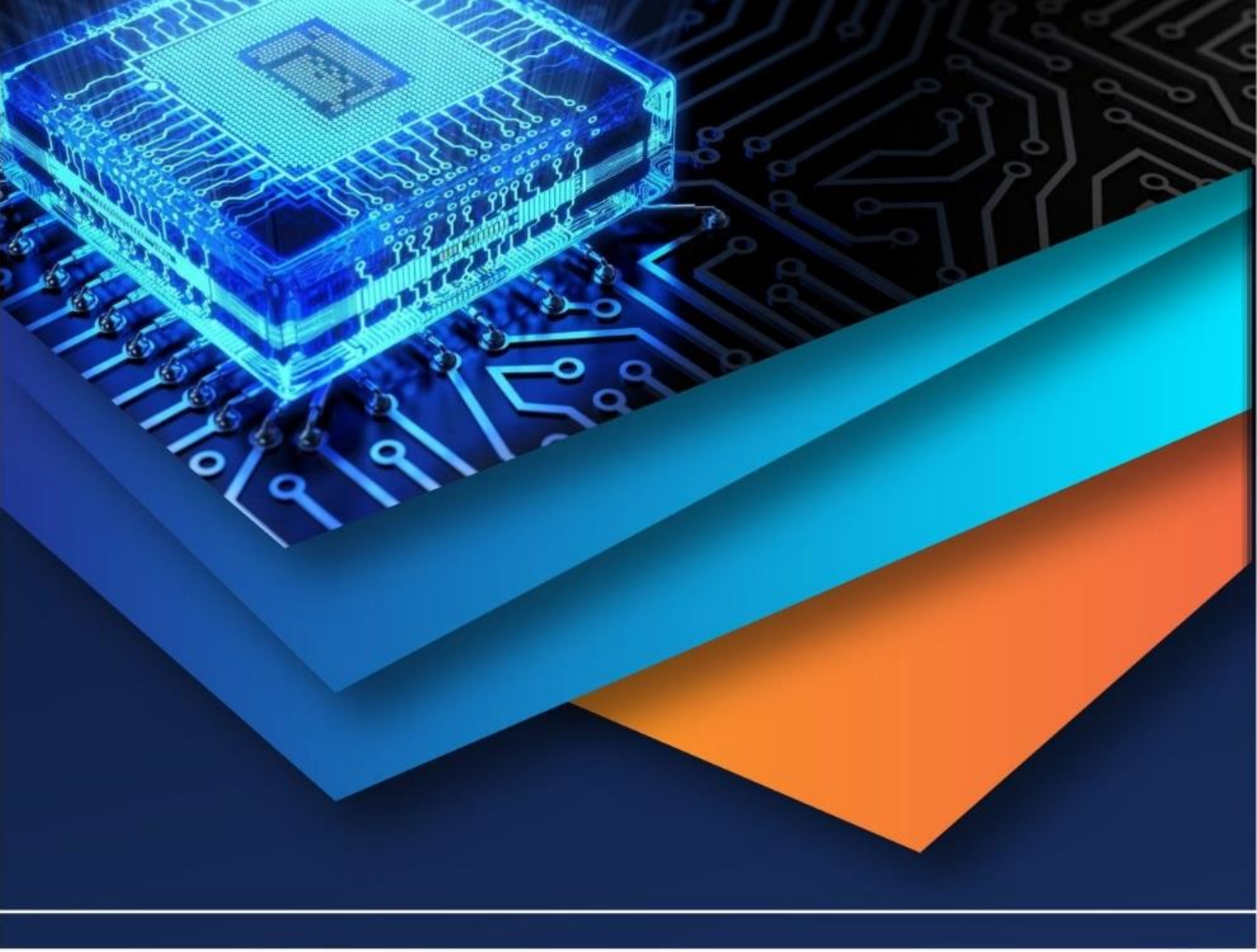

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