iJRASET
International Journal For Research in Applied Science and Engineering Technology

INTERNATIONAL JOURNAL FOR RESEARCH

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY

Volume: 5 **Issue:** XI **Month of publication:** November 2017

DOI: http://doi.org/10.22214/ijraset.2017.11047

International Journal for Research in Applied Science & Engineering Technology (IJRASET)
ISSN: 2321-9653; IC Value: 45.98; SJ Impact Factor:6.887
Volume 5 Issue XI, November 2017- Available at www.ijraset.co

Evaluating Pyramidal Numbers and Pent tope Number Using Initial Value Theorem in Z-Transform

C.Saranya[1], G.Janaki[2],
[1, 2] *Department of Mathematics, Cauvery College for Women, Tiruchirappalli, Tamil Nadu, India.*

***Abstract:** In this communication, We evaluate Pyramidal numbers and Pentatope number by applying the Initial value theorem in Z-Transform.*
***Keywords:** Inverse Z-Transform, Initial value theorem, Pyramidal numbers and Pentatope number.*

I. INTRODUCTION

Mathematics is the language of patterns and relationships, and is used to describe anything that can be quantified. The main goal of Number theory is to discover interesting and unexpected relationships. It is devoted primarily to the study of natural numbers and integers. In [1-7], theory of numbers were discussed. The Z-Transform is a transform for a sequence. The development of communication branch is based on discrete analysis. Z-Transform plays the same role in discrete analysis as in continuous systems. The Z-Transform technique used for time signals and systems. In [8-10], Z-Transform methods were analysed. Recently in [11], the sequence of m-gonal numbers and octahedral numbers was developed.
In this communication, we develop the sequence of pyramidal numbers and pentatope number by using initial value theorem in Z-Transform.

A. Notations

$$P_n^m = \frac{n(n+1)}{6}\left[(m-2)\,n+(5-m)\right]$$ = Pyramidal number of rank ‘n’ with sides ‘m’.

$$PT_n = \frac{1}{24}n(n+1)(n+2)(n+3)$$ = Pentatope number of rank ‘n’.

B. Definition

If the function u_n is defined for discrete values $(n = 0,1,2,3,...)$ and $u_n = 0$ for $n < 0$, then its Z-Transform is defined to be $Z(u_n) = U(z) = \sum u_n z^{-n}$. The inverse Z-transform is written as $Z^{-1}[U(z)] = u_n$.

C. Initial value theorem

If $Z(u_n) = U(z)$, then $u_0 = \underset{z\to\infty}{Lt}\, U(z)$.

D. Method of Analysis

The process of finding the sequence of pyramidal numbers and pentatope number by using initial value theorem in Z-Transform is given in the following theorems.

E. Theorem 1

$$Z^{-1}\left[\frac{z^3-(3-m)\,z^2}{(z-1)^4}\right] = P_n^m$$ (Pyramidal number of rank ‘n’ with sides ‘m’)

F. Proof

Assume that $U(z) = \dfrac{z^3 - (3-m)z^2}{(z-1)^4}$

By initial value theorem, we have

$$u_0 = \underset{z\to\infty}{Lt}\, U(z) = 0$$

$$u_1 = \underset{z\to\infty}{Lt}\left[z\left(U(z)-u_0\right)\right] = \underset{z\to\infty}{Lt}\left[z\left(\frac{z^3-(3-m)z^2}{(z-1)^4}\right)\right] = \underset{z\to\infty}{Lt}\left[\frac{1-(3-m)z^{-1}}{(1-z^{-1})^4}\right] = 1$$

$$u_2 = \underset{z\to\infty}{Lt}\left[z^2\left(U(z)-u_0-u_1 z^{-1}\right)\right] = \underset{z\to\infty}{Lt}\left[\frac{1+m-6z^{-1}+4z^{-2}-z^{-3}}{(1-z^{-1})^4}\right] = m+1 = \frac{2(2+1)}{6}\left[(m-2)2+(5-m)\right]$$

$$u_3 = \underset{z\to\infty}{Lt}\left[z^3\left(U(z)-u_0-u_1 z^{-1}-u_2 z^{-2}\right)\right] = \underset{z\to\infty}{Lt}\left[\frac{(4m-2)-(6m+2)z^{-1}+(4m+3)z^{-2}-(m+1)z^{-3}}{(1-z^{-1})^4}\right]$$

$$= 4m-2 = \frac{3(3+1)}{6}\left[(m-2)3+(5-m)\right]$$

$$u_4 = \underset{z\to\infty}{Lt}\left[z^4\left(U(z)-u_0-u_1 z^{-1}-u_2 z^{-2}-u_3 z^{-3}\right)\right] = 10(m-1) = \frac{4(4+1)}{6}\left[(m-2)4+(5-m)\right]$$

…………

Proceeding in this manner, we obtain

$$u_n = \underset{z\to\infty}{Lt}\left[z^n\left(U(z)-u_0-u_1 z^{-1}-u_2 z^{-2}-u_3 z^{-3}-\cdots-u_{n-1} z^{-(n-1)}\right)\right]$$

$$= \underset{z\to\infty}{Lt}\left[z^n\left(\frac{z^3-(3-m)z^2}{(z-1)^4}-\frac{1}{z}-\frac{m+1}{z^2}-\frac{2(2m-1)}{z^3}-\frac{10(m-1)}{z^4}-\cdots\right)\right]$$

$$= \frac{n(n+1)}{6}\left[(m-2)n+(5-m)\right] = P_n^m$$

International Journal for Research in Applied Science & Engineering Technology (IJRASET)
ISSN: 2321-9653; IC Value: 45.98; SJ Impact Factor:6.887
Volume 5 Issue XI, November 2017- Available at www.ijraset.co

Hence, $Z^{-1}\left[\frac{z^3-(3-m)z^2}{(z-1)^4}\right]=P_n^m$ (Pyramidal number of rank 'n' with sides 'm').

Numerical examples of the above thorem are illustrated in the following table:

m	$u_1=1$	$u_2=m+1$	$u_3=2(2m-1)$	$u_4=10(m-1)$	………	$u_n=\frac{n(n+1)}{6}[(m-2)n+(5-m)]$
3	1	4	10	20	………	$\frac{n(n+1)(n+2)}{6}$
4	1	5	14	30	………	$\frac{n(n+1)(2n+1)}{6}$
5	1	6	18	40	………	$\frac{n^2(n+1)}{2}$
6	1	7	22	50	………	$\frac{n(n+1)(4n-1)}{6}$
7	1	8	26	60	………	$\frac{n(n+1)(5n-2)}{6}$
………	………	………	………	………	………	………
30	1	31	118	290	………	$\frac{n(n+1)(28n-25)}{6}$

We observe that in the above table, the successive rows from the first row represent the sequence of triangular pyramidal number, square pyramidal number, pentagonal pyramidal number, hexagnal pyramidal number, heptagonal pyramidal number & ……… 30-gonal pyramidal number respectively.

G. *Theorem 2*

$$Z^{-1}\left[\frac{z^4+1}{(z-1)^5}\right]=PT_n \text{ (Pentatope number of rank 'n')}$$

H. *Proof*

Assume that $U(z)=\frac{z^4+1}{(z-1)^5}$

ISSN: 2321-9653; IC Value: 45.98; SJ Impact Factor:6.887
Volume 5 Issue XI, November 2017- Available at www.ijraset.co

By initial value theorem, we have

$$u_0 = \underset{z\to\infty}{Lt}\, U(z) = 0$$

$$u_1 = \underset{z\to\infty}{Lt}\left[z\left(U(z)-u_0\right)\right] = \underset{z\to\infty}{Lt}\left[z\left(\frac{z^4+1}{(z-1)^5}-0\right)\right] = 1$$

$$u_2 = \underset{z\to\infty}{Lt}\left[z^2\left(U(z)-u_0-u_1 z^{-1}\right)\right] = \underset{z\to\infty}{Lt}\left[z^2\left(\frac{z^4+1}{(z-1)^5}-\frac{1}{z}\right)\right] = 5 = \frac{1}{24}(2)(3)(4)(5)$$

$$u_3 = \underset{z\to\infty}{Lt}\left[z^3\left(U(z)-u_0-u_1 z^{-1}-u_2 z^{-2}\right)\right] = \underset{z\to\infty}{Lt}\left[z^3\left(\frac{z^4+1}{(z-1)^5}-\frac{1}{z}-\frac{5}{z^2}\right)\right] = 15 = \frac{1}{24}(3)(4)(5)(6)$$

$$u_4 = \underset{z\to\infty}{Lt}\left[z^4\left(U(z)-u_0-u_1 z^{-1}-u_2 z^{-2}-u_3 z^{-3}\right)\right] = \underset{z\to\infty}{Lt}\left[z^4\left(\frac{z^4+1}{(z-1)^5}-\frac{1}{z}-\frac{5}{z^2}-\frac{15}{z^3}\right)\right]$$
$$= 35 = \frac{1}{24}(4)(5)(6)(7)$$

$$\cdots\cdots\cdots\cdots$$

Proceeding in this manner, we obtain

$$u_n = \underset{z\to\infty}{Lt}\left[z^n\left(U(z)-u_0-u_1 z^{-1}-u_2 z^{-2}-u_3 z^{-3}-\cdots\cdots-u_{n-1}z^{-(n-1)}\right)\right]$$
$$= \underset{z\to\infty}{Lt}\left[z^n\left(\frac{z^4+1}{(z-1)^5}-\frac{1}{z}-\frac{5}{z^2}-\frac{15}{z^3}-\frac{35}{z^4}-\cdots\cdots\right)\right]$$
$$= \frac{1}{24}n(n+1)(n+2)(n+3) = PT_n$$

Hence, $Z^{-1}\left[\frac{z^4+1}{(z-1)^5}\right] = PT_n$ (Pentatope number of rank 'n').

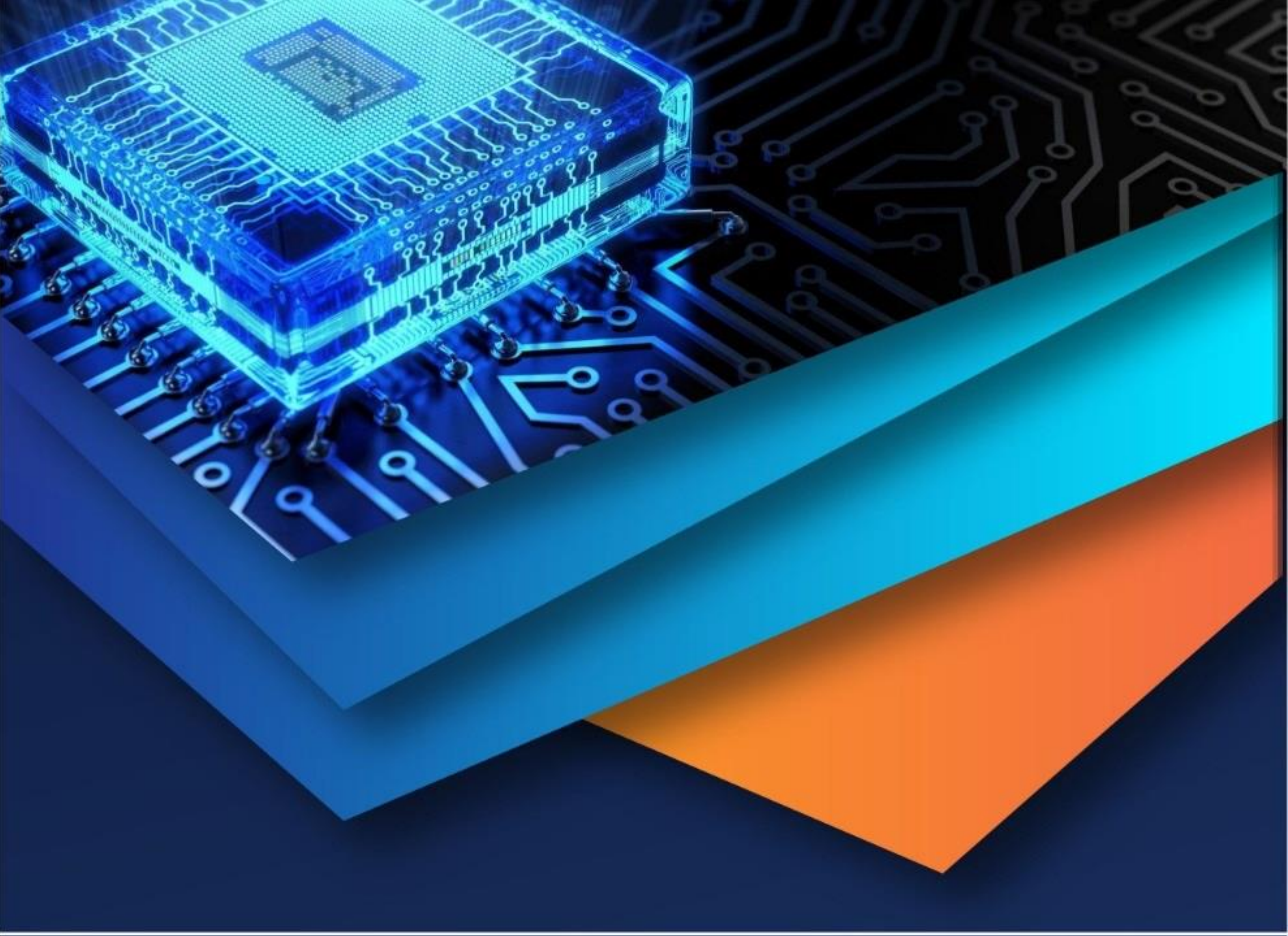
II. CONCLUSION

In this communication, we find pyramidal numbers and pentatope number by applying the initial value theorem in Z-Transform. In this manner, one can find the sequence of other three dimensional and four dimensional numbers by using various properties of Z-Transform.

REFERENCES

[1] R.D.Carmichael, History of Theory of numbers and Diophantine Analysis, Dover Publication, New york, 1959.
[2] L.J.Mordell, Diophantine equations, Academic press, London, 1969.
[3] T.Nagell, Introduction to Number theory, Chelsea publishing company, New york, 1981.
[4] L.K.Hua, Introduction to the Theory of Numbers, Springer-Verlag, Berlin-New york, 1982.

[5] Oistein Ore, Number theory and its History, Dover publications, New york, 1988.
[6] H.John, Conway and Richard K.Guy, The Book of Numbers, Springer-verlag, New york, 1995.
[7] David Wells, The penguin Dictionary of Curious and Interesting numbers, Penguin Book, 1997.
[8] Eliahu Ibrahim Jury, Theory and Application of the Z-Transform Method, John Wiley and Sons, 1964.
[9] Eliahu Ibrahim Jury, Theory and Application of the Z-Transform Method, Krieger Pub Co, 1973.
[10] E.R.Kanasewich, Time sequence Analysis in Geophysics, 3rd Edition, University of Alberta, 1981.
[11] V.Pandichelvi & P.Sivakamasundari, An Innovative Approach of Evaluating Polygonal Numbers and Octahedral Number, International Journal of Development Research, Vol. 7, Issue 5, Pp.12940-12943, May 2017



10.22214/IJRASET

INDEX COPERNICUS 45.98

IMPACT FACTOR: 7.129

IMPACT FACTOR: 7.429

INTERNATIONAL JOURNAL FOR RESEARCH

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY