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An Application of Fuzzy Tops is Method by Using Epsilon-Delta fuzzy Number

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Abstract: In this paper, we present the real life example of the Fuzzy TOPSIS method. Epsilon- Delta fuzzy numbers are used in the decision matrix. The problem of teaching quality index is discussed. The method is examined in terms of two evaluative criteria.
Keywords: Fuzzy Decision Making; Ranking of Fuzzy Number; Epsilon-delta fuzzy number; Fuzzy TOPSIS Method.

I. INTRODUCTION

In many decision making problem it is crucial to evaluate precisely the pertinent data. Mostly, in real-life decision making problem data are imprecise and fuzzy. Decision maker may encounter difficulty in quantifying and processing the linguistic statements. Therefore it is desirable to develop decision-making methods which take in to account fuzzy data. It is equally important to evaluate the performance of these decision-making methods. Fuzzy numbers were developed by Zadeh [8]. In TOPSIS we have used the epsilon delta fuzzy numbers [1]. In [2-6] a fuzzy version of Saaty's [7] AHP was developed. Fuzzy numbers were used for pair wise comparison to compute the weights of importance of the decision criteria. The fuzzy performance values of the alternatives in terms of each decision criteria were computed by using fuzzy numbers.

II. PRELIMINARIES

In the present paper, we use epsilon delta fuzzy numbers for the pair wise comparison to compute the weights of importance of the decision criteria. We obtain fuzzy performance values of the alternatives in terms of each decision criteria by using fuzzy numbers

A. Definition1

A fuzzy subset A of a set X is a function $A: X \to [0,1]$. For $\alpha \in [0,1]$.

The set $\{x \in X \mid A(x) \geq \alpha\}$ is called α -level cut or α -cut, denoted by A_α .

The strict α -level cut of A is the support of A.

B. Definition 2

If $A(x)=1$ then A is called normal. If each α-cut of A is convex then the fuzzy set *A* is called convex.

We assume $X = ¡$, the set of real numbers. A fuzzy number A is a fuzzy subset of ¡ which is normal, convex and upper semi-continuous with bounded support.

If left and right curves are linear then the fuzzy number is called triangular or a trapezoidal fuzzy number. The triangular fuzzy number is a particular type of a trapezoidal fuzzy number in which core is a singleton set.

C. Definition 3

[3]The membership function of a triangular fuzzy number *A* is of the form

$$A(x)=\begin{cases}\dfrac{(x-l)}{(m-l)}, & \text{if } l<x\le m,\\ \dfrac{u-x}{(u-m)}, & \text{if } m<x\le u,\\ 0, & \text{otherwise.}\end{cases}$$

The above triangular fuzzy number is denoted by *A = (l, m, n).*

D. Definition 4.

[1] If r is a real number then ε-δ fuzzy number $r_{\varepsilon,\delta}$ for some $\varepsilon,\delta\in\mathbb{R}$,$(\varepsilon,\delta>0)$ is a fuzzy set $r_{\varepsilon,\delta}:\mathbb{R}\to[0,1]$ defined by

$$r_{\varepsilon,\delta}(x)=\begin{cases}\dfrac{x-(r-\varepsilon)}{\varepsilon}, & \text{if } r-\varepsilon<x\le r,\\ \dfrac{x-(r+\delta)}{-\delta}, & \text{if } r<x\le r+\delta,\\ 0, & \text{otherwise.}\end{cases}$$

The *support* of ε-δ fuzzy number $r_{\varepsilon,\delta}$ is $\left(r-\varepsilon(1-\alpha),r+\delta(1-\alpha)\right),r\in\mathbb{R},\varepsilon,\delta\in\mathbb{R}$ and $\varepsilon,\delta>0$. The α-*cut* of $r_{\varepsilon,\delta}$ is denoted by $(r_{\varepsilon,\delta})_\alpha=[r-\varepsilon(1-\alpha),r+\delta(1-\alpha)]$. Let $A_L(\alpha)=r-\varepsilon(1-\alpha)$ and $A_U(\alpha)=r+\delta(1-\alpha)$.

E. The TOPSIS method [4]

By TOPSIS (the Technique for Preference by Similarity to Ideal Solution) method we evaluate the following decision matrix, which refers to$m$ alternatives which are evaluated in terms of criteria: x_{ij} is the i^{th} alternative in terms of the j^{th} criterion.

Criterion

Alternative	C_1	C_2	C_3	$\cdots$	C_n
A_1	x_{11}	x_{12}	x_{13}	$\cdots$	x_{1n}
A_2	x_{21}	x_{22}	x_{23}	$\cdots$	x_{2n}
A_3	x_{31}	x_{32}	x_{33}	$\cdots$	x_{3n}
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\cdots$	$\vdots$
A_m	x_{m1}	x_{m2}	x_{m3}	$\cdots$	x_{mn}

Where A_i is the i^{th} alternative C_j is the j^{th} criterion, and x_{ij} is the performance measure of the i^{th} alternative in terms of the j^{th} criterion.

The TOPSIS method consists of the following steps:

1) *Step 1:* Construct the normalized decision matrix. This step converts the various attributes dimensions into non dimensional attributes. Elements r_{ij} of the normalized decision matrix R is calculated as follows:

$$r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{m} x_{ij}^2}}.$$

2) *Step* 2: Construct the weighted normalized decision matrix. A set of weights $W = (w_1, w_2, ...w_n)$ (such that $\sum w_i = 1$), specified by the decision maker. Used in conjunction with the previous matrix to normalize decision matrix to determine the weighted normalized decision matrix V defined as $V = (r_{ij}W_j)$.

3) *Step* 3: Determine the ideal and the negative-ideal solutions:

The ideal (A^+) solution is given by

$$A^+ = \left\{ \left(\max_i v_{ij} : j \in J \right), \left(\min_i v_{ij} : J \in J' \right) \text{ for } i = 1, 2, \cdots, m \right\}$$

$$A^+ = \left\{ v_1^+, v_2^+, \cdots, v_n^+ \right\}$$

The negative-ideal (A^-) solution is given by

$$A^- = \left\{ \left(\min_i v_{ij} : j \in J \right), \left(\max_i v_{ij} : J \in J' \right) \text{ for } i = 1, 2, \cdots, m \right\}$$

$$A^- = \left\{ v_1^-, v_2^-, \cdots, v_n^- \right\}$$

Where

$$J = \left\{ j = 1, 2, \ldots, n : j \text{ is associated with the benifite criteria} \right\}$$

$$J' = \left\{ j = 1, 2, \ldots, n : j \text{ is associated with the cost criteria} \right\}$$

For benefit criteria, the decision maker desire to have a maximum value among the alternatives. for cost criteria, the decision maker desires to have a minimum value among them.

A^+ Indicates the most preferable alternatives or ideal solution. Similarly A^- Indicates the least preferable alternatives or negative ideal solution.

4) *Step* 4: Calculation the separation measure:

In this method using n-dimensional Euclidean distance to measure the separation distance of each alternatives to the ideal solution and negative ideal solution. By using formula

$$S_i^+ = \sqrt{\sum_{j=1}^{n} (v_{ij} - v_j^+)^2}$$ For *i*=1, 2, 3….*m* separation for ideal solution.

$$S_i^- = \sqrt{\sum_{j=1}^{n} (v_{ij} - v_j^-)^2}$$ For *i*=1, 2, 3….*m* separation for negative ideal solution.

5) *Step* 5: Calculate the relative closeness to the ideal solution:

The relative closeness of alternative A_i with respect to the ideal solution A^+ is defined as $C_i^+ = \frac{S_i^-}{S_i^+ + S_i^-}$ $0 \le C_i^+ \le 1$

$i = 1, 2, 3....m$

Evidently, $C_i^+ = 1$ if and only if $A_i = A^+$, and $C_i^- = 0$ if and only if $A_i = A^-$

III. FUZZY TOPSIS METHOD

The fuzzy TOPSIS method is illustrated as follows

A. *Step 1:* Construct the decision matrix: Criterion

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Alternative	C_1	C_2	C_3	$\cdots$	C_n
A_1	$r^{11}_{\varepsilon_{11},\delta_{11}}$	$r^{12}_{\varepsilon_{12},\delta_{12}}$	$r^{13}_{\varepsilon_{13},\delta_{13}}$	$\cdots$	$r^{1n}_{\varepsilon_{1n},\delta_{1n}}$
A_2	$r^{21}_{\varepsilon_{21},\delta_{21}}$	$r^{22}_{\varepsilon_{22},\delta_{22}}$	$r^{23}_{\varepsilon_{23},\delta_{23}}$	$\cdots$	$r^{2n}_{\varepsilon_{2n},\delta_{2n}}$
A_3	$r^{31}_{\varepsilon_{31},\delta_{31}}$	$r^{32}_{\varepsilon_{32},\delta_{32}}$	$r^{33}_{\varepsilon_{33},\delta_{33}}$	$\cdots$	$r^{3n}_{\varepsilon_{3n},\delta_{3n}}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
A_m	$r^{m1}_{\varepsilon_{m1},\delta_{m1}}$	$r^{m2}_{\varepsilon_{m2},\delta_{m2}}$	$r^{m3}_{\varepsilon_{m3},\delta_{m3}}$	$\cdots$	$r^{mn}_{\varepsilon_{mn},\delta_{mn}}$

B. *Step 2.* Construct the weighted decision matrix: A set of weights $W = (w_1, w_2, ...w_n)$ (Such that $\sum w_i = 1$), specified by the decision maker. Used in conjunction with the previous matrix to normalize decision matrix to determine the weighted normalized decision matrix defined as $V^{ij}_{\in_{ij},\delta_{ij}} = (r^{ij}_{\in_{ij},\delta_{ij}} w_j)$. *i= (1, 2, 3….m) j= (1, 2, 3….n)*

Criterion

Alternative	C_1 (w_1)	C_2 (w_2)	C_3 (w_2)	$\cdots$	C_n (w_n)
A_1	$v^{11}_{\varepsilon_{11},\delta_{11}}$	$v^{12}_{\varepsilon_{12},\delta_{12}}$	$v^{13}_{\varepsilon_{13},\delta_{13}}$	$\cdots$	$v^{1n}_{\varepsilon_{1n},\delta_{1n}}$
A_2	$v^{21}_{\varepsilon_{21},\delta_{21}}$	$v^{22}_{\varepsilon_{22},\delta_{22}}$	$v^{23}_{\varepsilon_{23},\delta_{23}}$	$\cdots$	$v^{2n}_{\varepsilon_{2n},\delta_{2n}}$
A_3	$v^{31}_{\varepsilon_{31},\delta_{31}}$	$v^{32}_{\varepsilon_{32},\delta_{32}}$	$v^{33}_{\varepsilon_{33},\delta_{33}}$	$\cdots$	$v^{3n}_{\varepsilon_{3n},\delta_{3n}}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
A_m	$v^{m1}_{\varepsilon_{m1},\delta_{m1}}$	$v^{m2}_{\varepsilon_{m2},\delta_{m2}}$	$v^{m3}_{\varepsilon_{m3},\delta_{m3}}$	$\cdots$	$v^{mn}_{\varepsilon_{mn},\delta_{mn}}$

C. *Step 3:* Determine the ideal and negative- ideal solution:

The ideal (A^+) solution is given by

$$A^+ = \left\{ \left(\max_i V^{ij}_{\in_{ij},\delta_{ij}} \; / \; j \in J \right), \left(\min_i : V^{ij}_{\in_{ij},\delta_{ij}} \; / \; J \in J' \right) \text{ for } i = 1, 2, \cdots, m \right\}$$

$$A^+ = (v^+_{\in_1,\delta_1}, v^+_{\in_2,\delta_2}, v^+_{\in_3,\delta_3}, ..., v^+_{\in_n,\delta_n})$$

The negative-ideal (A^-) solution is given by

$$A^- = \left\{ \left(\min_i v^{ij}_{\in_{ij},\delta_{ij}} \; / \; j \in J \right), \left(\max_i v^{ij}_{\in_{ij},\delta_{ij}} \; / \; J \in J' \right) \text{ for } i = 1, 2, \cdots, m \right\}$$

$$A^- = (v^-_{\in_1,\delta_1}, v^-_{\in_2,\delta_2}, v^-_{\in_3,\delta_3}, ..., v^-_{\in_n,\delta_n})$$

Where

$$J = \{ j = 1, 2, \ldots, n : j \text{ is associated with the benifite criteria} \}$$
$$J' = \{ j = 1, 2, \ldots, n : j \text{ is associated with the cost criteria} \}$$

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A^+ Indicates the most preferable alternatives or ideal solution. Similarly A^- Indicates the least preferable alternatives or negative ideal solution

D. Step 4: Calculate the relative closeness to the ideal solution:

In this method using n-dimensional Euclidean distance to measure the separation distance of each alternatives to the ideal solution and negative ideal solution. By using formula

$$S_i^+ = \sqrt{\sum_{j=1}^{n} (V^{ij}_{\in_{ij},\delta_{ij}} - v^+_{\in_j,\delta_j})^2} \quad \text{For } i=1, 2, 3\ldots m \text{ separation for ideal solution.}$$

$$S_i^- = \sqrt{\sum_{j=1}^{n} (V^{ij}_{\in_{ij},\delta_{ij}} - v^-_{\in_j,\delta_j})^2} \qquad \text{For } i=1, 2, 3\ldots m \text{ separation for negative ideal solution.}$$

E. Step 5: Calculate the relative closeness to the ideal solution:

The relative closeness of alternative A_i with respect to the ideal solution A^+ is defined as $C_i^+ = \dfrac{S_i^-}{S_i^+ + S_i^-}$ $0 \le C_i^+ \le 1$

$i = 1, 2, 3\ldots m$

Evidently, $C_i^+ = 1$ if and only if $A_i = A^+$, and $C_i^- = 0$ if and only if $A_i = A^-$

IV. APPLICATION

Primary Decision Matrix: (feedback for staff given by student)

Alternative	C_1 excellent	C_2 good	C_3 Average	C_4 Below average	C_5 unsatisfactory	Overall Rating Out of 10
A_1	5.6	3.1	0.5	0.49	0.2	8.35
A_2	3	2.3	1.2	1.2	2.24	5.64
A_3	4.8	3.4	0.8	0.34	0.5	7.94
A_4	3.9	3.9	0.97	0.3	0.95	7.35

A. Formulation

We use the following definition of epsilon delta fuzzy number for matrix entries

$$r_{\varepsilon,\delta}(x) = \begin{cases} \dfrac{x-(r-\varepsilon)}{\varepsilon}, & \text{if } r-\varepsilon < x \le r, \\ \dfrac{x-(r+\delta)}{-\delta}, & \text{if } r < x \le r+\delta, \\ 0, & \text{otherwise.} \end{cases}$$

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$$C_i(x) = \begin{cases} \dfrac{x}{10}, \text{ if } 0 < x \le 10, \\ 0, \quad \text{otherwise.} \end{cases} i = 1, 2, \ldots, m$$

$$C_1(x) = \begin{cases} \dfrac{5.6}{10}, \text{ if } 0 < x \le 10, \\ 0, \quad \text{otherwise.} \end{cases} = 0.56$$

And so on

1) *Step 1: Fuzzy Matrix*

Criterion

Alternative	C_1	C_2	C_3	C_4	C_5
A_1	0.56	0.31	0.05	0.049	0.02
A_2	0.3	0.23	0.12	0.12	0.224
A_3	0.48	0.34	0.08	0.034	0.05
A_4	0.39	0.39	0.097	0.03	0.095

2) *Step 2: Weighted decision fuzzy matrix*: A set of weights $W = (0.4, 0.3, 0.2, 0.1, 0)$

Criterion

Alternative	C_1 (0.4)	C_2 (0.3)	C_3 (0.2)	C_4 (0.1)	C_5 (0)
A_1	0.224	0.093	0.010	0.0049	0
A_2	0.12	0.069	0.024	0.012	0
A_3	0.192	0.102	0.016	0.0034	0
A_4	0.156	0.117	0.0194	0.003	0

3) *Step 3: ideal and negative ideal solution*

$$A^+ = \{0.224, 0.117, 0.024, 0.012\}$$

$$A^- = \{0.12, 0.069, 0.01, 0.0034\}$$

4) *Step 4: Calculation of separation measure*

$$S_1^+ = \sqrt{\sum_{j=1}^{5} (V_{\epsilon_{1j},\delta_{1j}}^{1j} - v_{\epsilon_{j},\delta_{j}}^{+})^2}$$ And so on

$$S_1^+ = 0.0286, S_2^+ = 0.108, S_3^+ = 0.037, S_4^+ = 0.068,$$

$$S_1^- = \sqrt{\sum_{j=1}^{5} (V_{\epsilon_{1j},\delta_{1j}}^{11} - v_{\epsilon_{1j},\delta_{1j}}^{-})^2}$$ and so on

$S_1^- = 0.107, S_2^- = 0.016, S_3^- = 0.079, S_4^- = 0.061$

5) *Step 5: Calculation of relative closeness to the ideal solution*

$C_1^+ = \dfrac{S_1^-}{S_1^+ + S_1^-}$ and so on.

$C_1 = 0.789, C_2 = 0.129$

$C_3 = 0.681, C_4 = 0.484$

6) *Step 6: Rankingof preference order:* $C_1 > C_3 > C_4 > C_2$ .Therefore ,the preference order of the four alternatives is $A_1 > A_3 > A_4 > A_2$ .That is the best alternatives is A_1.

V. CONCLUSION

In the present contribution we have considered the problem of fuzzy TOPSIS for selection the best alternative. This method can accommodate more number of alternatives and decision criterion.

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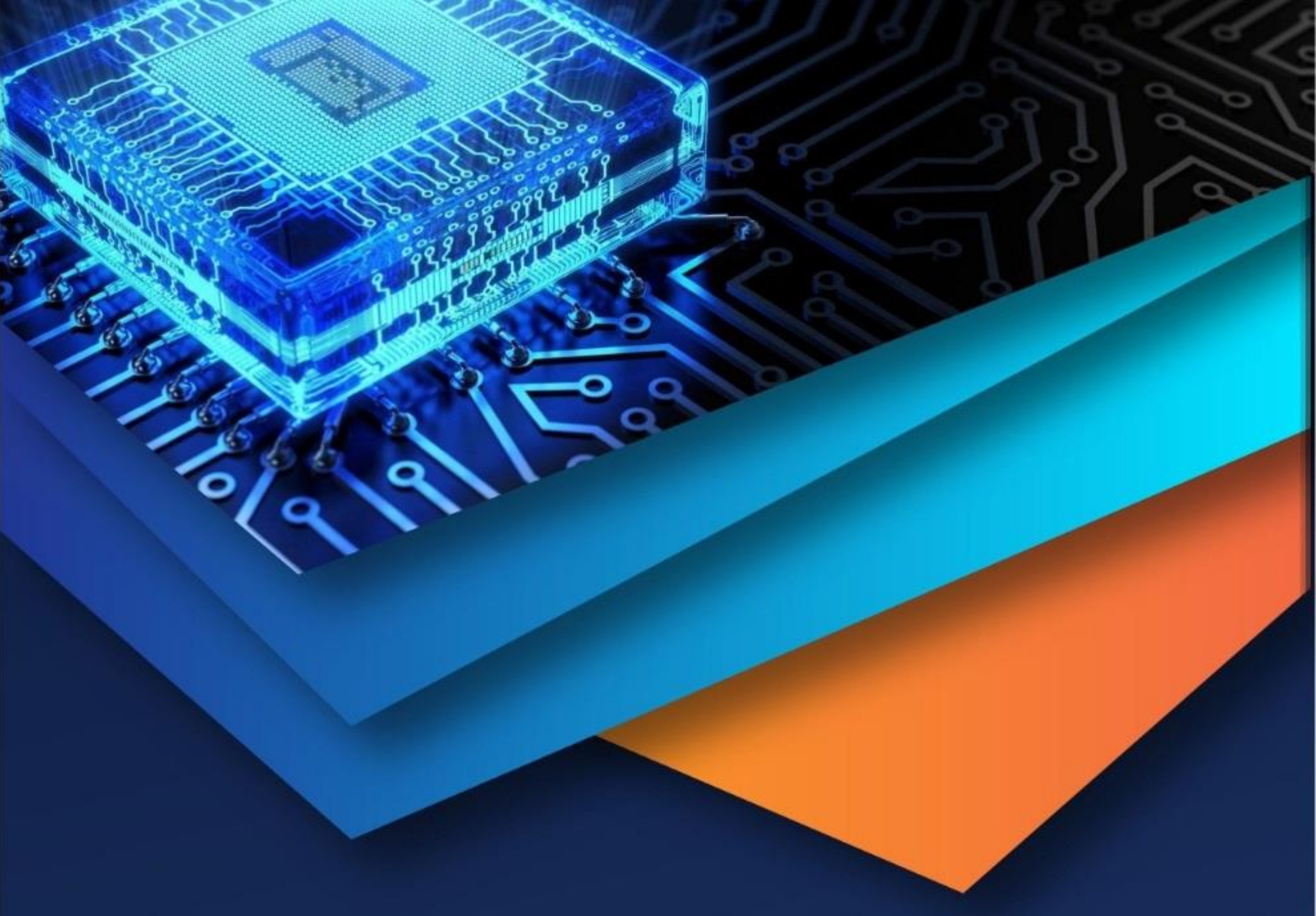

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