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# Distance Two Complementary Tree Domination Number of a Graph

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**Abstract:** A set  $D$  of a graph  $G = (V, E)$  is a dominating set, if every vertex in  $V(G) - D$  is adjacent to some vertex in  $D$ . The domination number  $\gamma(G)$  of  $G$  is the minimum cardinality of a dominating set. A dominating set  $D$  is called a distance two complementary tree dominating set, if for each  $u \in V - D$ ,  $d(u, v) \leq 2$  for some  $v$  in  $D$  and also  $\langle V - D \rangle$  is a tree. The minimum cardinality of a distance two complementary tree dominating set is said to be distance two complementary tree domination number of  $G$  and is denoted by  $\gamma_{d2ctd}(G)$ . In this paper, bounds for  $\gamma_{d2ctd}(G)$  and its exact values for some particular classes of graphs are found. Some results on distance two complementary tree domination number are also established.

**Keywords:** complementary tree domination number, distance two complementary tree domination number.

## I. INTRODUCTION

Graphs discussed in this paper are finite, undirected and simple graphs. For a graph  $G$ , let  $V(G)$  and  $E(G)$  denote its vertex set and edge set respectively. A graph with  $p$  vertices and  $q$  edges is denoted by  $G(p, q)$ . The concept of domination in graphs was introduced by Ore[6]. A set  $D \subseteq V(G)$  is said to be a dominating set of  $G$ , if every vertex in  $V(G) - D$  is adjacent to some vertex in  $D$ . The cardinality of a minimum dominating set in  $G$  is called the domination number of  $G$  and is denoted by  $\gamma(G)$ . The problem of finding a minimal distance  $k$ -dominating set (call  $k$ -basis) was considered by Slater[7] with special reference to communication networks while the distance  $k$ -dominating set was defined by Henning et al. [4]. For an integer  $k \geq 1$ , a set  $D \subseteq V(G)$  is a distance  $k$ -dominating set of  $G$  if every vertex in  $V(G) - D$  is within distance  $k$  from some vertex  $v \in D$ . The minimum cardinality among all distance  $k$ -dominating sets of  $G$  is called the distance  $k$ -domination number of  $G$  and is denoted by  $\gamma_k(G)$ . Muthammai, Bhanumathi and Vidhya[5] introduced the concept of complementary tree dominating set. A dominating set  $D \subseteq V(G)$  is said to be a complementary tree dominating set (ctd-set) if the induced subgraph  $\langle V(G) - D \rangle$  is a tree. The minimum cardinality of a ctd-set is called the complementary tree domination number of  $G$  and is denoted by  $\gamma_{ctd}(G)$ . Any undefined terms in this paper may be found in Harary[1].

In this paper, bounds for  $\gamma_{d2ctd}(G)$  and its exact values for some particular classes of graphs are found. Also, the graphs for which  $\gamma_{d2ctd}(G) = 1, 2, p - 1$  or  $p - 2$  are characterized.

## II. PRIOR RESULTS

Theorem 2.1[5] For any connected graph  $G$ ,  $\gamma(G) \leq \gamma_{ctd}(G)$ .

Theorem 2.2[5] For any connected graph  $G$  with  $p \geq 2$ ,  $\gamma_{ctd}(G) \leq p - 1$ .

Theorem 2.3[5] Let  $G$  be a connected graph with  $p \geq 4$ . Then  $\gamma_{ctd}(G) = p - 1$  if and only if  $G$  is a star on  $p$  vertices.

Theorem 2.4[5] Let  $T$  be a tree with  $p$  vertices which is not a star. Then,  $\gamma_{ctd}(T) = p - 2$  ( $p \geq 5$ ) if and only if  $T$  is a path or  $T$  is obtained from a path by attaching pendant edges at atleast one of the end vertices.

Theorem 2.5[5] Let  $G$  be a connected graph containing a cycle. Then,  $\gamma_{ctd}(G) = p - 2$  ( $p \geq 5$ ) if and only if  $G$  is isomorphic to one of the following graphs  $C_p, K_p$  or  $G$  is the graph obtained from a complete graph by attaching pendant edges at atleast one of the vertices of the complete graph.

## III. MAIN RESULTS

In this section, a new parameter called distance two complementary tree domination number is defined, bounds and exact values of this parameter are found.

**A. Definition 3.1**

A subset  $D \subseteq V(G)$  is said to be a distance two complementary tree dominating set (d2ctd-set), if for each  $u \in V - D$ ,  $d(u,v) \leq 2$  for some  $v$  in  $D$  and also  $\langle V - D \rangle$  is a tree. The minimum cardinality of a distance two complementary tree dominating set is said to be distance two complementary tree domination number of  $G$  and is denoted by  $\gamma_{d2ctd}(G)$ .

**1) Observation 3.1**

a) Since any ctd-set is a d2ctd-set,  $\gamma_{d2ctd}(G) \leq \gamma_{ctd}(G)$ .

Equality holds if,  $G \cong P_n \circ K_1$ .

b) Since d2ctd-set is a distance two dominating set,  $\gamma_2(G) \leq \gamma_{d2ctd}(G)$ .

Equality holds if,  $G \cong P_5, C_5$ .

c) For any connected graph  $G$  with  $p$  vertices,  $1 \leq \gamma_{d2ctd}(G) \leq p - 1$ , since  $\gamma_{ctd}(G) \leq p - 1$ .

2) **Theorem 3.1:** For any connected graph  $G$ ,  $\gamma_{d2ctd}(G) = p - 1$ ,  $p \geq 2$  if and only if  $G \cong K_2$ .

3) **Proof:** By the Observation 3.1,  $\gamma_{d2ctd}(G) \leq \gamma_{ctd}(G)$ .

If  $\gamma_{d2ctd}(G) = p - 1$ , then  $\gamma_{ctd}(G) \geq p - 1$ . But  $\gamma_{ctd}(G) \leq p - 1$  and  $\gamma_{ctd}(G) = p - 1$  if and only if  $G \cong K_{1,p-1}$ . If  $p \geq 3$ , then  $\gamma_{d2ctd}(K_{1,p-1}) = 1 < p - 1$ .

Therefore  $\gamma_{d2ctd}(G) = p - 1$ , if  $p = 2$ , Hence  $G \cong K_2$ .

Conversely if  $G \cong K_2$ , then  $\gamma_{d2ctd}(G) = 1 = p - 1$ .

**B. Observation 3.2**

1) For any path  $P_p$  on  $p$  vertices,  $\gamma_{d2ctd}(G) = p - 4$ ,  $p \geq 6$ .

$\gamma_{d2ctd}(P_3) = 1$ ,  $\gamma_{d2ctd}(P_4) = \gamma_{d2ctd}(P_5) = 2$ .

2) For any cycle  $C_p$  on  $p$  vertices,  $\gamma_{d2ctd}(C_p) = p - 4$ ,  $p \geq 5$ .

$\gamma_{d2ctd}(C_3) = \gamma_{d2ctd}(C_4) = 1$ .

If  $v_1, v_2, v_3, v_4$  be any four consecutive vertices of degree 2 in  $P_p$  (or  $C_p$ ),  $V(P_p) - \{v_1, v_2, v_3, v_4\}$  (or  $V(C_p) - \{v_1, v_2, v_3, v_4\}$ ) is a  $\gamma_{d2ctd}$ -set of  $P_p$  (or  $C_p$ ).

3) For any star  $K_{1,p-1}$ ,  $\gamma_{d2ctd}(K_{1,p-1}) = 1$ .

4) For any complete graph  $K_p$ ,  $\gamma_{d2ctd}(K_p) = p - 2$ ,  $p \geq 3$ .

5) For any complete bipartite graph  $K_{m,n}$ ,  $\gamma_{d2ctd}(K_{m,n}) = m - 1$ ,  $n \geq m \geq 2$ .

6)  $\gamma_{d2ctd}(\overline{mK_2}) = 2m - 3$ ,  $m \geq 2$ .

7) For the graph  $K_p - e$ ,  $\gamma_{d2ctd}(K_p - e) = p - 3$ , where  $e$  is an edge in  $K_p$ .

8) For the graph  $\overline{K_{m,n} - e}$ ,  $\gamma_{d2ctd}(\overline{K_{m,n} - e}) = \min \{m, n\} - 1$ .

9) For the graph  $K_{m,n} - e$ ,  $\gamma_{d2ctd}(K_{m,n} - e) = m + n - 4$ .

10)  $\gamma_{d2ctd}(P_n \circ K_1) = n$ .

11)  $\gamma_{d2ctd}(C_n \circ K_1) = n - 1$ ,  $n \geq 3$ .

**C. Observation 3.3**

1) For the Fan  $F_p$ ,  $\gamma_{d2ctd}(F_p) = 1$ , where  $F_p = P_{p-1} + K_1$  ( $p \geq 3$ ).

2) For the Wheel  $W_p$ ,  $\gamma_{d2ctd}(W_p) = 2$ , where  $W_p = C_{p-1} + K_1$  ( $p \geq 3$ ).

**D. Definition 3.2**

The one point union  $C_n^{(t)}$  of  $t$ -copies of cycle  $C_n$  is the graph obtained by taking a new vertex  $u$  as a common vertex such that any two distinct cycles  $C_i$  and  $C_j$  are edge disjoint and do not have any vertex in common except  $u$ .

**E. Observation 3.4**

For  $t \geq 2$  and  $n \geq 4$ ,  $\gamma_{ctd}(C_n^{(t)}) = \begin{cases} t, & \text{if } n \leq 5 \\ (n - 5)t + 1, & \text{if } n > 5. \end{cases}$

**F. Definition 3.3**

Let  $G_1, G_2, \dots, G_k$  be  $k$  copies of a graph  $G$ , where  $k \geq 2$ .  $G(k)$  is a graph obtained by adding an edge from  $G_i$  to  $G_{i+1}$ ;  $i = 1, 2, 3, \dots, k-1$  and the graph  $G(k)$  is called the path union of  $k$  copies of the graph  $G$ .

**G. Observation 3.5**

Let  $C_n(t)$ ,  $t \geq 2, n \geq 3$  be the path union of  $t$  cycles of length  $n$ . Then  $\gamma_{ctnd}(C_n(t)) = \begin{cases} t, & \text{if } n \leq 5 \\ (n-4)t, & \text{if } n > 5. \end{cases}$

**H. Definition 3.4**

A  $t$ -ply  $P_t(u,v)$  is a graph with  $t$  paths joining vertices  $u$  and  $v$ , each of length atleast two and no two paths have a vertex in common except the end vertices  $u$  and  $v$  in  $P_t(u,v)$ .

**I. Observation 3.6**

$\gamma_{ctnd}(P_t(u, v)) = p - 2t$ , where  $p$  is the number of vertices in  $P_t(u,v)$ .

**J. Observation 3.7**

For the graph  $G$ , there is no relationship between  $\gamma(G)$  and  $\gamma_{d2ctd}(G)$  is not true in general. This is illustrated by the following example.

**K. Example 3.1.**

For the graph given in Figure 3.1,  $\{v_1, v_2, v_4\}$  is a  $\gamma$ -set of  $G$  and hence  $\gamma(G) = 3$  and  $\{v_1, v_2\}$  is a  $\gamma_{d2ctd}$ -set of  $G$  and  $\gamma_{d2ctd}(G) = 2$ . Therefore  $\gamma(G) > \gamma_{d2ctd}(G)$ .

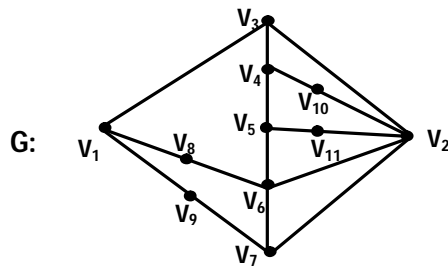


Figure 3.1

For the graph given in Figure 3.2,  $\{v_3, v_{10}\}$  is a  $\gamma$ -set of  $G$  and hence  $\gamma(G) = 2$  and  $\{v_1, v_2\}$  is a  $\gamma_{d2ctd}$ -set of  $G$  and  $\gamma_{d2ctd}(G) = 2$ . Therefore  $\gamma(G) = \gamma_{d2ctd}(G)$ .

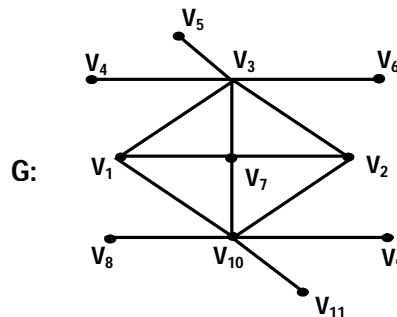


Figure 3.2

For the graph given in Figure 3.3,  $\{v_7\}$  is a  $\gamma$ -set of  $G$  and hence  $\gamma(G) = 1$  and  $\{v_1, v_7\}$  is a  $\gamma_{d2ctd}$ -set of  $G$  and  $\gamma_{d2ctd}(G) = 2$ . Therefore  $\gamma(G) < \gamma_{d2ctd}(G)$ .

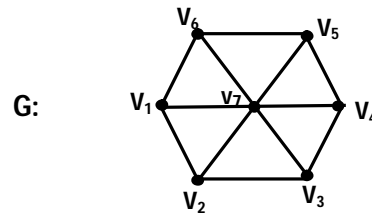


Figure 3.3

*L. Theorem 3.2*

Let  $G$  be a connected graph. Then  $\gamma_{d2ctd}(G) = 1$  if and only if there exists a vertex  $u \in G$  such that  $G - u$  is a tree and  $d(u,v) \leq 2$ , for all  $v \in G - u$ .

*M. Proof.*

Let there exist vertex  $u \in G$  such that  $G - u$  is a tree and  $d(u,v) \leq 2$ , for all  $v \in G - u$ . Let  $D = \{u\}$ . Since  $d(u,v) \leq 2$ , for all  $v \in G - u$ ,  $D$  is a 2-distance dominating set of  $G$ . Also, since  $G - u$  is a tree,  $D$  is a complementary tree dominating set. Therefore  $D$  is a  $d2ctd$ -set of  $G$  and hence  $\gamma_{d2ctd}(G) = 1$ .

Conversely, assume  $\gamma_{d2ctd}(G) = 1$ . Let  $D$  be a  $d2ctd$ -set of  $G$  such that  $|D| = 1$ . Then  $\langle V(G) - D \rangle$  is a tree and all the vertices of  $V(G) - D$  are at a distance  $\leq 2$  from the vertex of  $D$ .

*N. Example 3.2*

For the graphs given in Figure 3.4,  $\gamma_{d2ctd}(G) = 1$ .

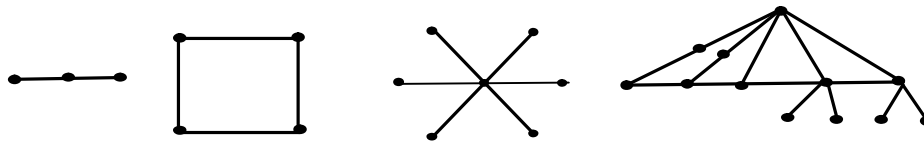


Figure 3.4

*O. Remark 3.1*

Let  $v$  be a support of  $G$  with minimum number  $t$  of pendant vertices  $v_1, v_2, \dots, v_t$  such that  $T = G - \{v_1, v_2, \dots, v_t\}$  is a tree and each vertex of  $T$  is of distance atleast 2 from  $v$ . Then  $\gamma_{d2ctd}(G) \leq t + 1$ .

*P. Theorem 3.3*

Let  $G$  be connected graph with atleast three vertices. Then  $\gamma_{d2ctd}(G) = 2$  if and only if there exist two vertices  $u$  and  $v$  such that  $G - \{u,v\}$  is a tree and each vertex in  $G - \{u,v\}$  is at distance at most 2 from atleast one  $u$  and  $v$  and if atleast one of the following holds

- (i)  $G - \{u\}$  or  $G - \{v\}$  is not a tree.
- (ii)  $d(u,v) \geq 3$ .

1) *Proof.* Assume  $\gamma_{d2ctd}(G) = 2$ .

Then there exists a  $d2ctd$ -set  $D$  of  $G$  such that  $|D| = 2$ .

Let  $D = \{u,v\}$ , where  $u, v \in V(G)$ .

Then  $G - \{u,v\}$  is a tree and each vertex in  $G - \{u,v\}$  is at distance atmost 2 from atleast one of  $u$  and  $v$ . If  $G - \{u\}$  or  $G - \{v\}$  is tree or  $d(u,v) \leq 2$ , then  $\{u\}$  or  $\{v\}$  is a  $d2ctd$ -set of  $G$ .

Therefore  $G - \{u\}$  or  $G - \{v\}$  is not a tree and  $d(u,v) \geq 3$ .

Conversely, if the conditions given in the theorem holds, then  $D = \{u,v\}$  is a  $d2ctd$ -set of  $G$  and hence  $\gamma_{d2ctd}(G) \leq 2$ . Also  $\gamma_{d2ctd}(G) \neq 1$ . Hence  $\gamma_{d2ctd}(G) = 2$ .

*Q. Example 3.3*

For the graphs given in Figure 3.5,  $\gamma_{d2ctd}(G) = 2$ .

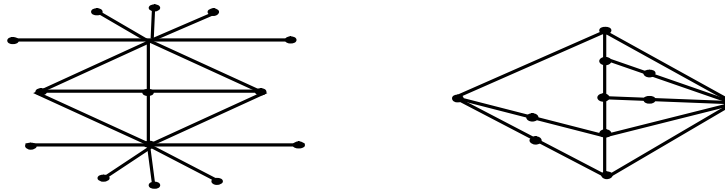


Figure 3.5

R. Theorem 3.4

For any connected graph G with atleast three vertices  $\gamma_{d2ctd}(G) = p - 2$  if and only if  $G \cong P_3, P_4, K_p, p \geq 3$ .

1) Proof. Assume  $\gamma_{d2ctd}(G) = p - 2$ .

Since by the Observation 3.1,  $\gamma_{d2ctd}(G) \leq \gamma_{ctd}(G)$ . That is,  $p - 2 \leq \gamma_{ctd}(G)$ . Therefore,  $\gamma_{ctd}(G) = p - 1$  or  $p - 2$ , Since  $\gamma_{ctd}(G) \leq p - 1$ .

2) Case 1.  $\gamma_{ctd}(G) = p - 1$ .

Then  $G \cong K_{1,p-1}, p \geq 3$

But  $\gamma_{d2ctd}(G) = 1 = p - 2$  implies  $p = 3$ .

Therefore  $G \cong K_{1,2} \cong P_3$ .

3) Case 2.  $\gamma_{d2ctd}(G) = p - 2$ .

a) Subcase 2.1. G is a tree.

By Theorem 2.4, G is a path or G is a tree obtained from a path by attaching pendant edges at atleast one of the end vertices.

$$\text{If } G \text{ is a path } P_p, \text{ then } \gamma_{d2ctd}(G) = \begin{cases} p - 4 & \text{if } p \geq 6 \\ 2 & \text{if } p = 4, 5 \\ 1 & \text{if } p = 3. \end{cases}$$

$\gamma_{d2ctd}(G) = p - 2$  implies  $G \cong P_4$  or  $P_3$ .

Let G be a tree obtained from a path  $P_n, n < p$  by attaching pendant edges at atleast one of the end vertices.

If  $n \leq 4$ , then  $\gamma_{d2ctd}(G) = 2 \neq p - 2$ .

If  $n \geq 5$ , then  $\gamma_{d2ctd}(G) = p - 5$ .

b) Subcase 2.2.

By Theorem 2.5, G is isomorphic to (i)  $C_p$  (ii)  $K_p$  or (iii) G is the graph obtained from a complete graph by attaching pendant edges at atleast one of the vertices of the complete graph.

$$\text{If } G \cong C_p, \gamma_{d2ctd}(G) = \begin{cases} p - 4, & \text{if } p \geq 5 \\ 1 & \text{if } p = 3, 4. \end{cases}$$

Therefore  $G \cong C_3$ .

If  $G \cong K_p$ , then  $\gamma_{d2ctd}(G) = p - 2$ .

Let G be the graph obtained from a complete graph by attaching pendant edges at atleast one of the vertices of the complete graph  $K_n, n < p$ . Then G contains an induced  $P_3$  and  $V(G) - V(P_3)$  is a  $d2ctd$ -set of G and hence  $\gamma_{d2ctd}(G) \leq p - 3$ .

From Case 1 and Case 2,  $G \cong P_3, P_4, K_p, p \geq 3$ .

c) Remark 3.2

Let G be a graph such that both G and its complement  $\bar{G}$  are connected. Then,

$$a) 4 \leq \gamma_{d2ctd}(G) + \gamma_{d2ctd}(\bar{G}) \leq 2(p - 2)$$

$$b) 4 \leq \gamma_{d2ctd}(G) \cdot \gamma_{d2ctd}(\bar{G}) \leq (p - 2)^2$$

Both lower and upper bounds are attained, if G is a path on 4 vertices.

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