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Complementary Tree Domination number and Distance two Complementary Tree Domination number of Cartesian Product of Graphs

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Abstract: A set D of a graph $G = (V, E)$ is a dominating set, if every vertex in $V(G) - D$ is adjacent to some vertex in D . The domination number $\gamma(G)$ of G is the minimum cardinality of a dominating set. A dominating set $D \subseteq V(G)$ is said to be a complementary tree dominating set (ctd-set), if the induced subgraph $\langle V(G) - D \rangle$ is a tree. The minimum cardinality of a ctd-set is called the complementary tree domination number of G and is denoted by $\gamma_{ctd}(G)$. A dominating set D is called a distance two complementary tree dominating set, if for each $u \in V(G) - D$, there exists a vertex $v \in D$ such that $d(u, v) \leq 2$ and also $\langle V(G) - D \rangle$ is a tree. The minimum cardinality of a distance two complementary tree dominating set is said to be distance two complementary tree domination number of G and is denoted by $\gamma_{d2ctd}(G)$. In this paper, complementary tree domination numbers and distance two complementary tree domination number of cartesian product of some standard graphs are found.

Key words: Domination number, complementary tree domination number, distance two complementary tree domination number, Cartesian product.

I. INTRODUCTION

Graphs discussed in this paper are finite, undirected and simple connected graphs. For a graph G , let $V(G)$ and $E(G)$ denote its vertex set and edge set respectively. A graph G with p vertices and q edges is denoted by $G(p, q)$. The concept of domination in graphs was introduced by Ore[5]. A set $D \subseteq V(G)$ is said to be a dominating set of G , if every vertex in $V(G) - D$ is adjacent to some vertex in D . The cardinality of a minimum dominating set in G is called the domination number of G and is denoted by $\gamma(G)$. Muthammai, Bhanumathi and Vidhya[5] introduced the concept of complementary tree dominating set. A dominating set $D \subseteq V(G)$ is said to be a complementary tree dominating set (ctd-set), if the induced subgraph $\langle V(G) - D \rangle$ is a tree. The minimum cardinality of a ctd-set is called the complementary tree domination number of G and is denoted by $\gamma_{ctd}(G)$.

The cartesian product of two graphs G_1 and G_2 is the graph, denoted by $G_1 \times G_2$ with $V(G_1 \times G_2) = V(G_1) \times V(G_2)$ (where \times denotes the cartesian product of sets) and two vertices $u = (u_1, u_2)$ and $v = (v_1, v_2)$ in $V(G_1 \times G_2)$ are adjacent in $G_1 \times G_2$ whenever $[u_1 = v_1 \text{ and } (u_2, v_2) \in E(G_2)]$ or $[u_2 = v_2 \text{ and } (u_1, v_1) \in E(G_1)]$.

The concept of distance two complementary tree dominating set is introduced in [4]. A dominating set D is called a distance two complementary tree dominating set, if for each $u \in V(G) - D$, there exists a vertex $v \in D$ such that $d(u, v) \leq 2$, and also $\langle V(G) - D \rangle$ is a tree. The minimum cardinality of a distance two complementary tree dominating set is said to be distance two complementary tree domination number of G and is denoted by $\gamma_{d2ctd}(G)$.

In this paper, complementary tree domination number and distance two complementary tree domination number of $K_m \times K_n$, $K_m \times P_n$, $K_m \times C_n$, $C_3 \times P_n$, $C_4 \times P_n$, $C_5 \times P_n$, $C_6 \times P_n$, $C_3 \times C_n$, $C_4 \times C_n$, $C_5 \times C_n$ and $C_6 \times C_n$ are found. Any undefined terms in this paper may be found in Harary[2].

II. COMPLEMENTARY TREE DOMINATION NUMBER OF CARTESIAN PRODUCT OF GRAPHS

A. Theorem 2.1

If $G \cong K_m \times K_n$ ($m, n \geq 3$ and $m \leq n$), then $\gamma_{ctd}(G) = \begin{cases} m(n-2) + 1, & \text{if } m = n \\ m(n-2), & \text{if } m < n \end{cases}$

B. Proof.

Let $G \cong K_m \times K_n$.

Let $V(G) = \cup_{i=1}^m \{v_{i1}, v_{i2}, \dots, v_{in}\}$ such that $\langle \{v_{i1}, v_{i2}, \dots, v_{in}\} \rangle \cong K_n^i, i=1,2, \dots, m$ and $\langle \{v_{1j}, v_{2j}, \dots, v_{mj}\} \rangle \cong K_m^j, j=1,2, \dots, n$, where K_n^i is the i^{th} copy of K_n and K_m^j is the j^{th} copy of K_m in $K_m \times K_n$. $|V(G)| = mn$.

C. Case 1: $m = n$.

Let $D' = (\cup_{i=1}^{m-1} \{v_{ii}, v_{i,i+1}\}) \cup \{v_{m,m}\}$ and $D = V(G) - D'$. Then $V(G) - D = D'$ and $|D'| = 2(m-1) + 1 = 2m - 1$. For $i = 1, 2, 3, \dots, m-1$, the vertices $v_{ii}, v_{i,i+1}$ in $V(G) - D$ are adjacent to v_{ii} in D , and the vertex v_{mm} is adjacent to v_{m1} in D . Therefore D is a dominating set of G . $\langle V(G) - D \rangle \cong P_{2(m-1)+1} = P_{2m-1}$. Since D is a ctd-set of G . Hence $\gamma_{\text{ctd}}(G) \leq |D| = |V(G)| - |D'| = mn - (2m - 1) = m(n - 2) + 1$.

It is to be noted that, any tree in G is a path. Let D' be a γ_{ctd} -set of G . The longest path that can be obtained from the subgraph of G induced by the vertices of $V(G) - D'$ is P_{2m-1} . That is, $\langle V(G) - D' \rangle \cong P_{2m-1}$. D' contains atleast $mn - (2m - 1) = m(n - 2) + 1$ vertices. Therefore $\gamma_{\text{ctd}}(G) = |D'| \geq m(n - 2) + 1$.

Hence $\gamma_{\text{ctd}}(G) = m(n - 2) + 1$.

D. Case 2: $m < n$.

Let $D' = \cup_{i=1}^m \{v_{ii}, v_{i,i+1}\}$ and $D = V(G) - D'$. Then $V(G) - D = D'$ and $|D'| = 2m$. The vertices v_{11}, v_{12} are adjacent to v_{1n} and $v_{ii}, v_{i,i+1}$ ($i = 2, 3, \dots, m$) are adjacent to v_{ii} , ($i = 2, 3, \dots, m$) in D . Therefore D is a dominating set of G . Since $\langle V(G) - D \rangle \cong P_{2m}$, D is a ctd-set of G . Therefore $\gamma_{\text{ctd}}(G) \leq |V(G)| - |D'| = mn - 2m = m(n - 2)$.

As in case 1, any tree in G is a path. Let D' be γ_{ctd} -set of G . The longest path that can be obtained from the subgraph of G induced by the vertices of $V(G) - D'$ is P_{2m} .

That is $\langle V(G) - D' \rangle \cong P_{2m}$. Therefore D' contains atleast $mn - 2m = m(n - 2)$ vertices. Therefore $\gamma_{\text{ctd}}(G) = |D'| \geq m(n - 2)$.

Therefore $\gamma_{\text{ctd}}(G) = m(n - 2)$.

Hence $\gamma_{\text{ctd}}(G) = \begin{cases} m(n - 2) + 1, & \text{if } m = n \\ m(n - 2), & \text{if } m < n \end{cases}$

E. Remark 2.1

The set D defined in Case 1 and Case 2 is also a d2ctd-set of G . Since any vertex u in D which is at distance two from a vertex of D , $\langle (V(G) - D) \cup \{u\} \rangle$ either disconnected or contains cycle.

F. Example 2.1

For the graph G given in Figure 1.a and Figure 1.b, the set of vertices marked with \odot is a minimum ctd-set of $K_m \times K_n$ and $\gamma_{\text{ctd}}(K_4 \times K_4) = 9$ and $\gamma_{\text{ctd}}(K_4 \times K_5) = 12$.

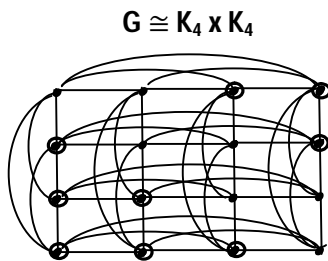


Figure 1.a

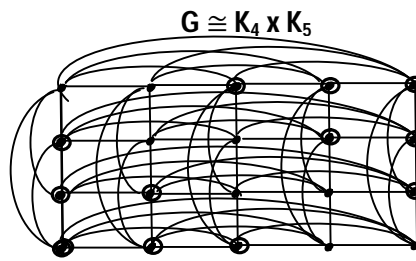


Figure 1.b

G. Theorem 2.2.

If $G \cong K_m \times P_n$ ($4 \leq m \leq n$), then $\gamma_{\text{ctd}}(G) = n(m - 2)$.

1) Proof.

Let $G \cong K_m \times P_n, m, n \geq 4$.

Let $V(G) = \bigcup_{i=1}^m \{v_{i1}, v_{i2}, \dots, v_{in}\}$ such that $\langle \{v_{i1}, v_{i2}, \dots, v_{in}\} \rangle \cong K_n^i, i = 1, 2, \dots, m$ and $\langle \{v_{1j}, v_{2j}, \dots, v_{mj}\} \rangle \cong P_m^j, j = 1, 2, \dots, n$, where K_n^i is the i^{th} copy of K_n and P_m^j is the j^{th} copy of P_m in $K_m \times P_n$.

$$\text{Let } D' = \begin{cases} \{v_{1n}\} \cup [\bigcup_{i=1}^n \{v_{2i}\}] \cup [\bigcup_{i=1}^{\frac{n-1}{2}} \{v_{1,2i-1}, v_{3,2i}\}], & \text{if } n \text{ is odd} \\ [\bigcup_{i=1}^n \{v_{2,i}\}] \cup [\bigcup_{i=1}^{\frac{n}{2}} \{v_{1,2i-1}, v_{3,2i}\}], & \text{if } n \text{ is even} \end{cases}$$

Then $|D'| = 2n$. If $D = V(G) - D'$, then D is a dominating set of G . Also $\langle V(G) - D \rangle = \langle D' \rangle \cong P_n \circ K_1$. Therefore D is a ctd-set of G and $\gamma_{\text{ctd}}(G) \leq |D| = mn - 2n = n(m - 2)$.

Hence $\gamma_{\text{ctd}}(G) \leq n(m - 2)$.

Let D' be a γ_{ctd} -set of G . Since D' is a ctd-set of G , D' contains atleast $(m - 2)$ vertices in each of n copies of K_m . Hence D' contains atleast $n(m - 2)$ vertices. Therefore $\gamma_{\text{ctd}}(G) = |D'| \geq n(m - 2)$.

Hence $\gamma_{\text{ctd}}(G) = n(m - 2)$.

H. Example 2.2.

For the graph G given in Figure 2, the set of vertices marked with \odot is a minimum ctd-set of $K_m \times P_n$ and $\gamma_{\text{ctd}}(K_4 \times K_9) = 18$.

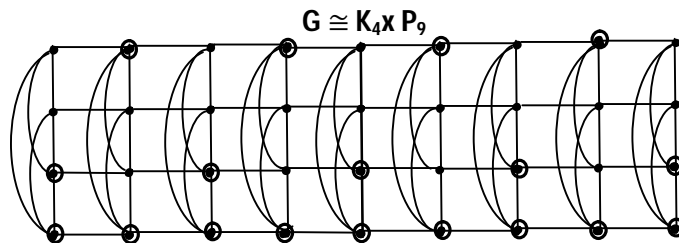


Figure 2

I. Theorem 2.3.

$G \cong C_3 \times P_n$, then $\gamma_{\text{ctd}}(G) = n, n \geq 1$.

1) Proof.

Let $G \cong C_3 \times P_n$.

Let $V(G) = \bigcup_{i=1}^n \{v_{1i}, v_{2i}, v_{3i}\}$ such that $\langle \{v_{1i}, v_{2i}, v_{3i}\} \rangle \cong P_n^i, i = 1, 2, 3$ and $\langle \{v_{1j}, v_{2j}, v_{3j}\} \rangle \cong C_3^j, j = 1, 2, \dots, n$, where P_n^i is the i^{th} copy of P_n and C_3^j is the j^{th} copy of C_3 in $C_3 \times P_n$.

$$\text{Let } D = \begin{cases} \{v_{1n}\} \cup [\bigcup_{i=1}^{\frac{n-1}{2}} \{v_{1,2i-1}, v_{3,2i}\}], & \text{if } n \text{ is odd} \\ \bigcup_{i=1}^{\frac{n}{2}} \{v_{1,2i-1}, v_{3,2i}\}, & \text{if } n \text{ is even} \end{cases}$$

Then D is a dominating set of G . Also $\langle V(G) - D \rangle \cong P_n \circ K_1$. Therefore D is a ctd-set of G and $|D| =$

$$\begin{cases} 2 \left(\frac{n-1}{2} \right) + 1 = n, & \text{if } n \text{ is even} \\ 2 \left(\frac{n}{2} \right) = n, & \text{if } n \text{ is odd} \end{cases} \text{ and } \gamma_{\text{ctd}}(G) \leq |D| = n.$$

Let D' be a γ_{ctd} -set of G . Then D' contains atleast one vertex from each cycle. Since $C_3 \times P_n$ contains n copies of C_3 , D' contains atleast n vertices. $\gamma_{\text{ctd}}(G) = |D'| \geq n$.

Hence $\gamma_{\text{ctd}}(G) = n, n \geq 1$.

J. Theorem 2.4.

If $G \cong C_4 \times P_n$, then $\gamma_{\text{ctd}}(G) = \left\lceil \frac{3n+1}{2} \right\rceil, n \geq 1$.

Let $G \cong C_4 \times P_n$ and $V(G) = \bigcup_{i=1}^n \{v_{1i}, v_{2i}, v_{3i}, v_{4i}\}$ such that $\langle \{v_{1i}, v_{2i}, v_{3i}, v_{4i}\} \rangle \cong P_n^i, i = 1, 2, 3, 4$ and $\langle \{v_{1j}, v_{2j}, v_{3j}, v_{4j}\} \rangle \cong C_4^j, j = 1, 2, \dots, n$, where P_n^i is the i^{th} copy of P_n and C_4^j is the j^{th} copy of C_4 in $C_4 \times P_n$ and $|V(G)| = 4n$.

Let $D' = \{v_{3i}\} \cup [U_{i=1}^{\frac{n-1}{2}}\{v_{1,2i}, v_{4,2i}, v_{3,2i+1}\}] \cup [U_{i=1}^n\{v_{2i}\}]$.

Then $|D'| = 1 + 3\left(\frac{n-1}{2}\right) + n = \frac{5n-1}{2}$ and $D = V(G) - D'$. Then D is a dominating set of G . Also $\langle V(G) - D \rangle = \langle D' \rangle$ is a tree obtained from the path $P_n = \langle \{v_{2i}, i = 1, 2, 3, \dots, n\} \rangle$, ($n \geq 2$) by attaching P_3 at each of the vertices $v_{22}, v_{24}, v_{26}, \dots, v_{2, n-1}$ and attaching a pendant edge at each of the vertices $v_{21}, v_{23}, \dots, v_{2,n}$. Therefore D is a ctd-set of G , and

$$\gamma_{\text{ctd}}(G) \leq |D| = |V(G) - D'| = 4n - \left(\frac{5n-1}{2}\right) = \frac{3n+1}{2}.$$

$$\text{Hence } \gamma_{\text{ctd}}(G) \leq \frac{3n+1}{2} = \left\lfloor \frac{3n+1}{2} \right\rfloor.$$

Let D' be a γ_{ctd} -set of G . The blocks A, B, A^2 and A^2B are constructed as given below.

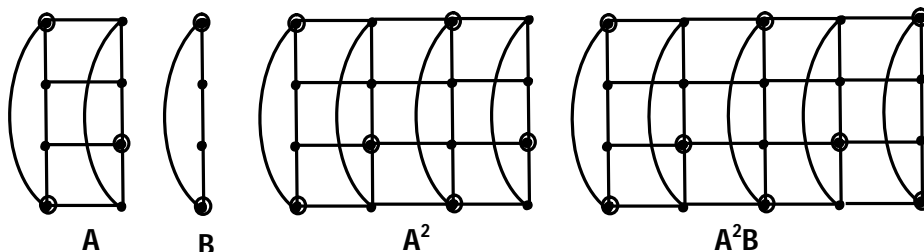


Figure 3

G is obtained by concatenating the blocks $A^{\frac{n-1}{2}}$ and B . That is, $G \cong A^{\frac{n-1}{2}} B$. The vertices with the symbol \bullet in each of the blocks represent the vertices that are to be included in D' .

Therefore D' contains atleast 3 vertices from each block A of $A^{\frac{n-1}{2}}$ and atleast 2 vertices from block B .

$$\text{Therefore } \gamma_{\text{ctd}}(G) = |D'| \geq 2 + 3\left(\frac{n-1}{2}\right) = \frac{3n+1}{2} = \left\lfloor \frac{3n+1}{2} \right\rfloor.$$

$$\text{Hence } \gamma_{\text{ctd}}(G) = \left\lfloor \frac{3n+1}{2} \right\rfloor, n \geq 1.$$

$\cup \{v_{3,2i-1}\} \cup [U_{i=1}^n\{v_{2i}\}]$ and $D = V(G) - D'$. Then $|D'| = 3\left(\frac{n}{2}\right) + n = \frac{5n}{2}$. Then D is a dominating set of G . Also $\langle V(G) - D \rangle = \langle D' \rangle$ is a tree obtained from the path $P_n = \langle \{v_{2i}, i = 1, 2, 3, \dots, n\} \rangle$, ($n \geq 2$) by attaching P_3 at each of the vertices $v_{22}, v_{24}, v_{26}, \dots$, and $v_{2,n}$ and attaching a pendant edge at each of the vertices v_{21}, v_{23}, \dots , and $v_{2, n-1}$. Therefore D is a ctd-set of G and $\gamma_{\text{ctd}}(G) \leq |D| = |V(G) - D'| = 4n - \left(\frac{5n}{2}\right) = \frac{3n}{2}$.

$$\text{Hence } \gamma_{\text{ctd}}(G) \leq \frac{3n}{2} = \left\lfloor \frac{3n+1}{2} \right\rfloor.$$

Let D' be a γ_{ctd} -set of G . The block A is constructed as in Case 1.

Then $G \cong A^{\frac{n}{2}}$. The vertices with the symbol \bullet in each of the blocks represent the vertices that are to be included in D' .

$$\text{Therefore } D' \text{ contains atleast 3 vertices from each block } A \text{ of } A^{\frac{n}{2}}. \text{ Therefore } \gamma_{\text{ctd}}(G) = |D'| \geq 3\left(\frac{n}{2}\right) = \frac{3n}{2} = \left\lfloor \frac{3n+1}{2} \right\rfloor.$$

$$\text{Hence } \gamma_{\text{ctd}}(G) = \left\lfloor \frac{3n+1}{2} \right\rfloor, n \geq 1.$$

K. Theorem 2.5

If $G \cong C_5 \times P_n$, then $\gamma_{\text{ctd}}(G) = 2n, n \geq 3$.

L. Proof

Let $G \cong C_5 \times P_n$ and $V(G) = U_{i=1}^n\{v_{1i}, v_{2i}, v_{3i}, v_{4i}, v_{5i}\}$ such that $\langle \{v_{1i}, v_{2i}, \dots, v_{in}\} \rangle \cong P_n^i, i = 1, 2, 3, 4, 5$ and $\langle \{v_{1j}, v_{2j}, v_{3j}, v_{4j}, v_{5j}\} \rangle \cong C_5^j, j = 1, 2, \dots, n$, where P_n^i is the i^{th} copy of P_n and C_5^j is the j^{th} copy of C_5 in $C_5 \times P_n$. $|V(G)| = 5n$.

$$\text{Let } D = \begin{cases} [U_{i=1}^{\frac{n+1}{2}}\{v_{1,2i-1}, v_{5,2i-1}\}] \cup [U_{i=2}^{\frac{n-1}{2}}\{v_{3,2i}, v_{4,2i}\}], & \text{if } n \text{ is odd} \\ U_{i=2}^{\frac{n}{2}}\{v_{1,2i-1}, v_{3,2i}, v_{4,2i}, v_{5,2i-1}\}, & \text{if } n \text{ is even.} \end{cases}$$

Then D is a dominating set of G and also $\langle V(G) - D \rangle$ is a tree obtained from the path $P_n = \langle \{v_{2i}, i = 1, 2, 3, \dots, n\} \rangle$, ($n \geq 2$) by attaching P_3 at each of the vertices $v_{21}, v_{22}, v_{23}, \dots$, and v_{2n} . Therefore D is a ctd-set of G.

$$\text{and } |D| = \begin{cases} 2\left(\frac{n+1}{2}\right) + 2\left(\frac{n-1}{2}\right) = 2n, & \text{if } n \text{ is even} \\ 4\left(\frac{n}{2}\right) = 2n, & \text{if } n \text{ is odd.} \end{cases}$$

Therefore $\gamma_{\text{ctd}}(G) \leq |D| = 2n$.

Let D' be a γ_{ctd} -set of G. Since $\gamma(C_5) = 2$, D' contains atleast 2 vertices from each of n cycles and hence D' contains atleast $2n$ vertices. Therefore $\gamma_{\text{ctd}}(G) = |D'| \geq 2n$.

Hence $\gamma_{\text{ctd}}(G) = 2n, n \geq 1$.

M. Theorem 2.6

If $G \cong C_6 \times P_n$, then $\gamma_{\text{ctd}}(G) = \lfloor \frac{5n}{2} \rfloor, n \geq 2$.

1) *Proof.*: Let $G \cong C_6 \times P_n$ and $V(G) = U_{i=1}^n\{v_{1i}, v_{2i}, v_{3i}, v_{4i}, v_{5i}, v_{6i}\}$ such that $\langle \{v_{1i}, v_{2i}, \dots, v_{6i}\} \rangle \cong P_n^i, i = 1, 2, 3, 4, 5, 6$ and $\langle \{v_{1j}, v_{2j}, v_{3j}, v_{4j}, v_{5j}, v_{6j}\} \rangle \cong C_6^j, j = 1, 2, \dots, n$, where P_n^i is the i^{th} copy of P_n and C_6^j is the j^{th} copy of C_6 in $C_6 \times P_n$ and $|V(G)| = 6n$.

2) *Case 1: n is odd.*

$$\text{Let } D' = [U_{i=1}^{\frac{n+1}{2}}\{v_{1,2i-1}, v_{5,2i-1}, v_{6,2i-1}\}] \cup [U_{i=1}^n\{v_{2i}\}] [U_{i=1}^{\frac{n-1}{2}}\{v_{3,2i}, v_{4,2i}\}].$$

Then $|D'| = 3\left(\frac{n+1}{2}\right) + n + 2\left(\frac{n-1}{2}\right) = \frac{7n+1}{2}$ and $D = V(G) - D'$. Then D is a dominating set of G. Also $\langle V(G) - D \rangle = \langle D' \rangle$ is a tree obtained from the path $P_n = \langle \{v_{2i}, i = 1, 2, 3, \dots, n\} \rangle$, ($n \geq 2$) by attaching P_4 at each of the vertices $v_{21}, v_{23}, v_{25}, \dots, v_{2n}$ and attaching P_3 at each of the vertices $v_{22}, v_{24}, \dots, v_{2,n-1}$. Therefore D is a ctd-set of G.

$$\gamma_{\text{ctd}}(G) \leq |D| = |V(G) - D'| = 6n - \left(\frac{7n+1}{2}\right) = \frac{5n-1}{2}.$$

$$\text{Hence } \gamma_{\text{ctd}}(G) \leq \frac{5n-1}{2}.$$

Let D' be a γ_{ctd} -set of G. The blocks A, B, A^2 and A^2B are constructed as given below.

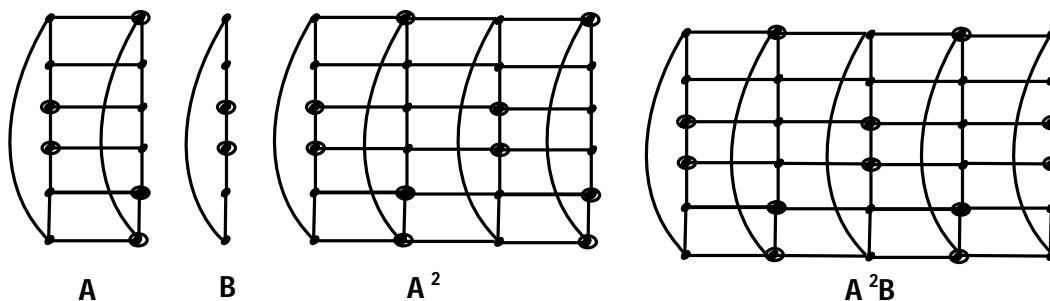


Figure 4

G is obtained by concatenating the blocks $A^{\frac{n-1}{2}}$ and B. That is, $G \cong A^{\frac{n-1}{2}} B$. The vertices with the symbol \bullet in each of the blocks represent the vertices that are to be included in D' .

Therefore D' contains atleast 5 vertices from each block A of $A^{\frac{n-1}{2}}$ and atleast 2 vertices from block B. Therefore $\gamma_{\text{ctd}}(G) = |D'| \geq 5\left(\frac{n-1}{2}\right) + 2 = \frac{5n-1}{2}$ and hence $\gamma_{\text{ctd}}(G) = \frac{5n-1}{2} = \lfloor \frac{5n}{2} \rfloor$.

Case 2: n is even.

Let $D' = [U_{i=1}^n \{v_{1,2i-1}, v_{5,2i-1}, v_{6,2i-1}\}] \cup [U_{i=1}^n \{v_{2i}\}] [U_{i=1}^n \{v_{3,2i}, v_{4,2i}\}]$.

Then $|D'| = 3\binom{n}{2} + n + 2\binom{n}{2} = \frac{7n}{2}$ and $D = V(G) - D'$. Then D is a dominating set of G . Also $\langle V(G) - D \rangle = \langle D' \rangle$ is a tree obtained from the path $P_n = \langle v_{2i}, i = 1, 2, 3, \dots, n \rangle$, ($n \geq 2$) by attaching P_4 at each of the vertices $v_{21}, v_{23}, v_{25}, \dots, v_{2,n-1}$ and attaching P_3 at each of the vertices $v_{22}, v_{24}, \dots, v_{2n}$. Therefore D is a ctd-set of G .

$$\gamma_{\text{ctd}}(G) \leq |D| = |V(G) - D'| = 6n - \left(\frac{7n}{2}\right) = \frac{5n}{2}.$$

$$\text{Hence } \gamma_{\text{ctd}}(G) \leq \frac{5n}{2}.$$

Let D' be a γ_{ctd} -set of G . The block A is constructed as in Case 1.

Then $G \cong A^{\frac{n}{2}}$. The vertices with the symbol $\text{\textcircled{6}}$ in each of the blocks represent the vertices that are to be included in D' .

Therefore D' contains atleast 5 vertices from each block A of $A^{\frac{n}{2}}$.

$$\text{Therefore } \gamma_{\text{ctd}}(G) = |D'| \geq 5 \binom{n}{2} = \frac{5n}{2} \text{ and hence } \gamma_{\text{ctd}}(G) = \frac{5n}{2} = \left\lfloor \frac{5n}{2} \right\rfloor.$$

$$\text{Hence } \gamma_{\text{ctd}}(G) = \left\lfloor \frac{5n}{2} \right\rfloor, n \geq 1.$$

N. Remark 2.2.

In view of Theorem 2.4, Theorem 2.5, Theorem 2.6, and Theorem 2.7,

- 1) $\gamma_{\text{ctd}}(C_3 \times C_n) = n+1, n \geq 2.$
- 2) $\gamma_{\text{ctd}}(C_4 \times C_n) = \left\lfloor \frac{3n+1}{2} \right\rfloor + 1$
- 3) $\gamma_{\text{ctd}}(C_5 \times C_n) = \begin{cases} 2n + 1, & \text{if } n \text{ is even} \\ 2n, & \text{if } n \text{ is odd.} \end{cases}$
- 4) $\gamma_{\text{ctd}}(C_6 \times C_n) = \left\lfloor \frac{5n}{2} \right\rfloor + 1.$

O. Remark 2.3.

- 1) If $G_1 \cong K_m$ and $G_2 \cong K_n$, then $\gamma_{\text{ctd}}(G_1 + G_2) = m + n - 2.$
- 2) If G_1 and G_2 are any two noncomplete connected graphs of order m and n respectively, with minimum degree atleast two, then $\gamma_{\text{ctd}}(G_1 + G_2) \leq m + n - 4.$ Equality holds, if $G_1 \cong K_m - e, G_2 \cong K_n - e.$
- 3) For any two connected graphs G_1 and G_2 of order m and n respectively, $\gamma_{\text{ctd}}(G_1 \circ G_2) \leq m + n - 4.$ Equality holds, if $G_1 \cong P_2$ and $G_2 \cong C_3.$

P. Distance Two Complementary tree domination number of Cartesian product of graphs

In the following, distance two complementary tree domination number of $K_m \times K_n, K_m \times P_n, K_m \times C_n, C_3 \times P_n, C_4 \times P_n, C_5 \times P_n, C_6 \times P_n, C_3 \times C_n, C_4 \times C_n, C_5 \times C_n$ and $C_6 \times C_n$ are given

- 1) If $G \cong K_m \times K_n$ ($m, n \geq 3$ and $m \leq n$), then $\gamma_{\text{d2ctd}}(G) = \begin{cases} m(n-2) + 1, & \text{if } m = n \\ m(n-2), & \text{if } m < n \end{cases}$
- 2) If $G \cong K_m \times P_n$ ($4 \leq m \leq n$), then $\gamma_{\text{d2ctd}}(G) = n(m-2).$
- 3) $\gamma_{\text{d2ctd}}(K_m \times C_n) = n(m-2) + 1.$
- 4) If $G \cong C_3 \times P_n$, then $\gamma_{\text{d2ctd}}(G) = n, n \geq 1.$
- 5) If $G \cong C_4 \times P_n$, then $\gamma_{\text{d2ctd}}(G) = \left\lfloor \frac{3n+1}{2} \right\rfloor, n \geq 1.$
- 6) If $G \cong C_5 \times P_n$, then $\gamma_{\text{d2ctd}}(G) = 2n, n \geq 3.$
- 7) If $G \cong C_6 \times P_n$, then $\gamma_{\text{d2ctd}}(G) = \left\lfloor \frac{5n}{2} \right\rfloor, n \geq 1.$
- 8) $\gamma_{\text{d2ctd}}(C_3 \times C_n) = n+1, n \geq 2.$
- 9) $\gamma_{\text{d2ctd}}(C_4 \times C_n) = \left\lfloor \frac{3n+1}{2} \right\rfloor + 1$
- 10) $\gamma_{\text{d2ctd}}(C_5 \times C_n) = \begin{cases} 2n + 1, & \text{if } n \text{ is even} \\ 2n, & \text{if } n \text{ is odd.} \end{cases}$



$$11) \gamma_{d2ctd}(C_6 \times C_n) = \left\lfloor \frac{5n}{2} \right\rfloor + 1, n \geq 2.$$

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