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# Complementary Tree Domination number and Distance two Complementary Tree Domination number of Cartesian Product of Graphs

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**Abstract:** A set  $D$  of a graph  $G = (V, E)$  is a dominating set, if every vertex in  $V(G) - D$  is adjacent to some vertex in  $D$ . The domination number  $\gamma(G)$  of  $G$  is the minimum cardinality of a dominating set. A dominating set  $D \subseteq V(G)$  is said to be a complementary tree dominating set (ctd-set), if the induced subgraph  $\langle V(G) - D \rangle$  is a tree. The minimum cardinality of a ctd-set is called the complementary tree domination number of  $G$  and is denoted by  $\gamma_{ctd}(G)$ . A dominating set  $D$  is called a distance two complementary tree dominating set, if for each  $u \in V(G) - D$ , there exists a vertex  $v \in D$  such that  $d(u, v) \leq 2$  and also  $\langle V(G) - D \rangle$  is a tree. The minimum cardinality of a distance two complementary tree dominating set is said to be distance two complementary tree domination number of  $G$  and is denoted by  $\gamma_{d2ctd}(G)$ . In this paper, complementary tree domination numbers and distance two complementary tree domination number of cartesian product of some standard graphs are found.

**Key words:** Domination number, complementary tree domination number, distance two complementary tree domination number, Cartesian product.

## I. INTRODUCTION

Graphs discussed in this paper are finite, undirected and simple connected graphs. For a graph  $G$ , let  $V(G)$  and  $E(G)$  denote its vertex set and edge set respectively. A graph  $G$  with  $p$  vertices and  $q$  edges is denoted by  $G(p, q)$ . The concept of domination in graphs was introduced by Ore[5]. A set  $D \subseteq V(G)$  is said to be a dominating set of  $G$ , if every vertex in  $V(G) - D$  is adjacent to some vertex in  $D$ . The cardinality of a minimum dominating set in  $G$  is called the domination number of  $G$  and is denoted by  $\gamma(G)$ . Muthammai, Bhanumathi and Vidhya[5] introduced the concept of complementary tree dominating set. A dominating set  $D \subseteq V(G)$  is said to be a complementary tree dominating set (ctd-set), if the induced subgraph  $\langle V(G) - D \rangle$  is a tree. The minimum cardinality of a ctd-set is called the complementary tree domination number of  $G$  and is denoted by  $\gamma_{ctd}(G)$ .

The cartesian product of two graphs  $G_1$  and  $G_2$  is the graph, denoted by  $G_1 \times G_2$  with  $V(G_1 \times G_2) = V(G_1) \times V(G_2)$  (where  $\times$  denotes the cartesian product of sets) and two vertices  $u = (u_1, u_2)$  and  $v = (v_1, v_2)$  in  $V(G_1 \times G_2)$  are adjacent in  $G_1 \times G_2$  whenever  $[u_1 = v_1 \text{ and } (u_2, v_2) \in E(G_2)]$  or  $[u_2 = v_2 \text{ and } (u_1, v_1) \in E(G_1)]$ .

The concept of distance two complementary tree dominating set is introduced in [4]. A dominating set  $D$  is called a distance two complementary tree dominating set, if for each  $u \in V(G) - D$ , there exists a vertex  $v \in D$  such that  $d(u, v) \leq 2$ , and also  $\langle V(G) - D \rangle$  is a tree. The minimum cardinality of a distance two complementary tree dominating set is said to be distance two complementary tree domination number of  $G$  and is denoted by  $\gamma_{d2ctd}(G)$ .

In this paper, complementary tree domination number and distance two complementary tree domination number of  $K_m \times K_n$ ,  $K_m \times P_n$ ,  $K_m \times C_n$ ,  $C_3 \times P_n$ ,  $C_4 \times P_n$ ,  $C_5 \times P_n$ ,  $C_6 \times P_n$ ,  $C_3 \times C_n$ ,  $C_4 \times C_n$ ,  $C_5 \times C_n$  and  $C_6 \times C_n$  are found. Any undefined terms in this paper may be found in Harary[2].

## II. COMPLEMENTARY TREE DOMINATION NUMBER OF CARTESIAN PRODUCT OF GRAPHS

### A. Theorem 2.1

If  $G \cong K_m \times K_n$  ( $m, n \geq 3$  and  $m \leq n$ ), then  $\gamma_{ctd}(G) = \begin{cases} m(n-2) + 1, & \text{if } m = n \\ m(n-2), & \text{if } m < n \end{cases}$

### B. Proof.

Let  $G \cong K_m \times K_n$ .

Let  $V(G) = \cup_{i=1}^m \{v_{i1}, v_{i2}, \dots, v_{in}\}$  such that  $\langle \{v_{i1}, v_{i2}, \dots, v_{in}\} \rangle \cong K_n^i, i=1,2, \dots, m$  and  $\langle \{v_{1j}, v_{2j}, \dots, v_{mj}\} \rangle \cong K_m^j, j=1,2, \dots, n$ , where  $K_n^i$  is the  $i^{\text{th}}$  copy of  $K_n$  and  $K_m^j$  is the  $j^{\text{th}}$  copy of  $K_m$  in  $K_m \times K_n$ .  $|V(G)| = mn$ .

**C. Case 1:  $m = n$ .**

Let  $D' = (\cup_{i=1}^{m-1} \{v_{ii}, v_{i,i+1}\}) \cup \{v_{m,m}\}$  and  $D = V(G) - D'$ . Then  $V(G) - D = D'$  and  $|D'| = 2(m-1) + 1 = 2m - 1$ . For  $i = 1, 2, 3, \dots, m-1$ , the vertices  $v_{ii}, v_{i,i+1}$  in  $V(G) - D$  are adjacent to  $v_{ii}$  in  $D$ , and the vertex  $v_{mm}$  is adjacent to  $v_{m1}$  in  $D$ . Therefore  $D$  is a dominating set of  $G$ .  $\langle V(G) - D \rangle \cong P_{2(m-1)+1} = P_{2m-1}$ . Since  $D$  is a ctd-set of  $G$ . Hence  $\gamma_{\text{ctd}}(G) \leq |D| = |V(G)| - |D'| = mn - (2m - 1) = m(n - 2) + 1$ .

It is to be noted that, any tree in  $G$  is a path. Let  $D'$  be a  $\gamma_{\text{ctd}}$ -set of  $G$ . The longest path that can be obtained from the subgraph of  $G$  induced by the vertices of  $V(G) - D'$  is  $P_{2m-1}$ . That is,  $\langle V(G) - D' \rangle \cong P_{2m-1}$ .  $D'$  contains atleast  $mn - (2m - 1) = m(n - 2) + 1$  vertices. Therefore  $\gamma_{\text{ctd}}(G) = |D'| \geq m(n - 2) + 1$ .

Hence  $\gamma_{\text{ctd}}(G) = m(n - 2) + 1$ .

**D. Case 2:  $m < n$ .**

Let  $D' = \cup_{i=1}^m \{v_{ii}, v_{i,i+1}\}$  and  $D = V(G) - D'$ . Then  $V(G) - D = D'$  and  $|D'| = 2m$ . The vertices  $v_{11}, v_{12}$  are adjacent to  $v_{1n}$  and  $v_{ii}, v_{i,i+1}$  ( $i = 2, 3, \dots, m$ ) are adjacent to  $v_{ii}$ , ( $i = 2, 3, \dots, m$ ) in  $D$ . Therefore  $D$  is a dominating set of  $G$ . Since  $\langle V(G) - D \rangle \cong P_{2m}$ ,  $D$  is a ctd-set of  $G$ . Therefore  $\gamma_{\text{ctd}}(G) \leq |V(G)| - |D'| = mn - 2m = m(n - 2)$ .

As in case 1, any tree in  $G$  is a path. Let  $D'$  be  $\gamma_{\text{ctd}}$ -set of  $G$ . The longest path that can be obtained from the subgraph of  $G$  induced by the vertices of  $V(G) - D'$  is  $P_{2m}$ .

That is  $\langle V(G) - D' \rangle \cong P_{2m}$ . Therefore  $D'$  contains atleast  $mn - 2m = m(n - 2)$  vertices. Therefore  $\gamma_{\text{ctd}}(G) = |D'| \geq m(n - 2)$ .

Therefore  $\gamma_{\text{ctd}}(G) = m(n - 2)$ .

Hence  $\gamma_{\text{ctd}}(G) = \begin{cases} m(n - 2) + 1, & \text{if } m = n \\ m(n - 2), & \text{if } m < n \end{cases}$

**E. Remark 2.1**

The set  $D$  defined in Case 1 and Case 2 is also a d2ctd-set of  $G$ . Since any vertex  $u$  in  $D$  which is at distance two from a vertex of  $D$ ,  $\langle (V(G) - D) \cup \{u\} \rangle$  either disconnected or contains cycle.

**F. Example 2.1**

For the graph  $G$  given in Figure 1.a and Figure 1.b, the set of vertices marked with  $\odot$  is a minimum ctd-set of  $K_m \times K_n$  and  $\gamma_{\text{ctd}}(K_4 \times K_4) = 9$  and  $\gamma_{\text{ctd}}(K_4 \times K_5) = 12$ .

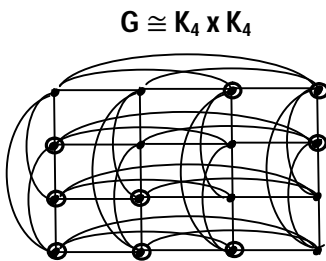


Figure 1.a

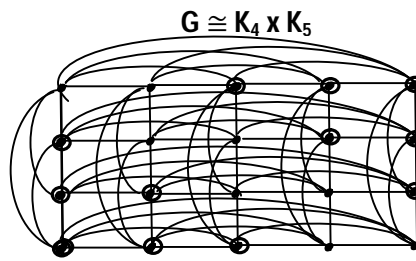


Figure 1.b

**G. Theorem 2.2.**

If  $G \cong K_m \times P_n$  ( $4 \leq m \leq n$ ), then  $\gamma_{\text{ctd}}(G) = n(m - 2)$ .

**1) Proof.**

Let  $G \cong K_m \times P_n, m, n \geq 4$ .

Let  $V(G) = \cup_{i=1}^m \{v_{i1}, v_{i2}, \dots, v_{in}\}$  such that  $\langle \{v_{i1}, v_{i2}, \dots, v_{in}\} \rangle \cong K_n^i, i = 1, 2, \dots, m$  and  $\langle \{v_{1j}, v_{2j}, \dots, v_{mj}\} \rangle \cong P_m^j, j = 1, 2, \dots, n$ , where  $K_n^i$  is the  $i^{\text{th}}$  copy of  $K_n$  and  $P_m^j$  is the  $j^{\text{th}}$  copy of  $P_m$  in  $K_m \times P_n$ .

$$\text{Let } D' = \begin{cases} \{v_{1n}\} \cup [\cup_{i=1}^n \{v_{2i}\}] \cup [\cup_{i=1}^{\frac{n-1}{2}} \{v_{1,2i-1}, v_{3,2i}\}], & \text{if } n \text{ is odd} \\ [\cup_{i=1}^n \{v_{2,i}\}] \cup [\cup_{i=1}^{\frac{n}{2}} \{v_{1,2i-1}, v_{3,2i}\}], & \text{if } n \text{ is even} \end{cases}$$

Then  $|D'| = 2n$ . If  $D = V(G) - D'$ , then  $D$  is a dominating set of  $G$ . Also  $\langle V(G) - D \rangle = \langle D' \rangle \cong P_n \circ K_1$ . Therefore  $D$  is a ctd-set of  $G$  and  $\gamma_{\text{ctd}}(G) \leq |D| = mn - 2n = n(m - 2)$ .

Hence  $\gamma_{\text{ctd}}(G) \leq n(m - 2)$ .

Let  $D'$  be a  $\gamma_{\text{ctd}}$ -set of  $G$ . Since  $D'$  is a ctd-set of  $G$ ,  $D'$  contains atleast  $(m - 2)$  vertices in each of  $n$  copies of  $K_m$ . Hence  $D'$  contains atleast  $n(m - 2)$  vertices. Therefore  $\gamma_{\text{ctd}}(G) = |D'| \geq n(m - 2)$ .

Hence  $\gamma_{\text{ctd}}(G) = n(m - 2)$ .

**H. Example 2.2.**

For the graph  $G$  given in Figure 2, the set of vertices marked with  $\odot$  is a minimum ctd-set of  $K_m \times P_n$  and  $\gamma_{\text{ctd}}(K_4 \times P_9) = 18$ .

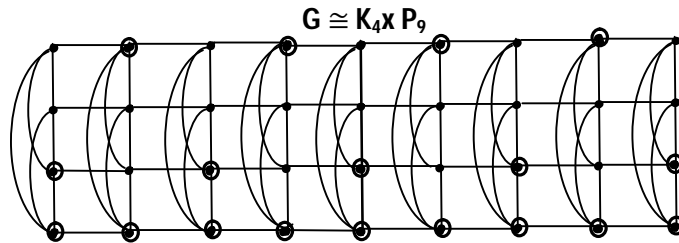


Figure 2

**I. Theorem 2.3.**

$G \cong C_3 \times P_n$ , then  $\gamma_{\text{ctd}}(G) = n, n \geq 1$ .

**1) Proof.**

Let  $G \cong C_3 \times P_n$ .

Let  $V(G) = \cup_{i=1}^n \{v_{i1}, v_{i2}, v_{i3}\}$  such that  $\langle \{v_{i1}, v_{i2}, v_{i3}\} \rangle \cong P_n^i, i = 1, 2, 3$  and  $\langle \{v_{1j}, v_{2j}, v_{3j}\} \rangle \cong C_3^j, j = 1, 2, \dots, n$ , where  $P_n^i$  is the  $i^{\text{th}}$  copy of  $P_n$  and  $C_3^j$  is the  $j^{\text{th}}$  copy of  $C_3$  in  $C_3 \times P_n$ .

$$\text{Let } D = \begin{cases} \{v_{1n}\} \cup [\cup_{i=1}^{\frac{n-1}{2}} \{v_{1,2i-1}, v_{3,2i}\}], & \text{if } n \text{ is odd} \\ \cup_{i=1}^{\frac{n}{2}} \{v_{1,2i-1}, v_{3,2i}\}, & \text{if } n \text{ is even} \end{cases}$$

Then  $D$  is a dominating set of  $G$ . Also  $\langle V(G) - D \rangle \cong P_n \circ K_1$ . Therefore  $D$  is a ctd-set of  $G$  and  $|D| =$

$$\begin{cases} 2 \left( \frac{n-1}{2} \right) + 1 = n, & \text{if } n \text{ is even} \\ 2 \left( \frac{n}{2} \right) = n, & \text{if } n \text{ is odd} \end{cases} \text{ and } \gamma_{\text{ctd}}(G) \leq |D| = n.$$

Let  $D'$  be a  $\gamma_{\text{ctd}}$ -set of  $G$ . Then  $D'$  contains atleast one vertex from each cycle. Since  $C_3 \times P_n$  contains  $n$  copies of  $C_3$ ,  $D'$  contains atleast  $n$  vertices.  $\gamma_{\text{ctd}}(G) = |D'| \geq n$ .

Hence  $\gamma_{\text{ctd}}(G) = n, n \geq 1$ .

**J. Theorem 2.4.**

If  $G \cong C_4 \times P_n$ , then  $\gamma_{\text{ctd}}(G) = \left\lfloor \frac{3n+1}{2} \right\rfloor, n \geq 1$ .

Let  $G \cong C_4 \times P_n$  and  $V(G) = \cup_{i=1}^n \{v_{i1}, v_{i2}, v_{i3}, v_{i4}\}$  such that  $\langle \{v_{i1}, v_{i2}, v_{i3}, v_{i4}\} \rangle \cong P_n^i, i = 1, 2, 3, 4$  and  $\langle \{v_{1j}, v_{2j}, v_{3j}, v_{4j}\} \rangle \cong C_4^j, j = 1, 2, \dots, n$ , where  $P_n^i$  is the  $i^{\text{th}}$  copy of  $P_n$  and  $C_4^j$  is the  $j^{\text{th}}$  copy of  $C_4$  in  $C_4 \times P_n$  and  $|V(G)| = 4n$ .

Let  $D' = \{v_{3i}\} \cup [U_{i=1}^{\frac{n-1}{2}}\{v_{1,2i}, v_{4,2i}, v_{3,2i+1}\}] \cup [U_{i=1}^n\{v_{2i}\}]$ .

Then  $|D'| = 1 + 3\left(\frac{n-1}{2}\right) + n = \frac{5n-1}{2}$  and  $D = V(G) - D'$ . Then  $D$  is a dominating set of  $G$ . Also  $\langle V(G) - D \rangle = \langle D' \rangle$  is a tree obtained from the path  $P_n = \langle \{v_{2i}, i = 1, 2, 3, \dots, n\} \rangle$ , ( $n \geq 2$ ) by attaching  $P_3$  at each of the vertices  $v_{22}, v_{24}, v_{26}, \dots, v_{2, n-1}$  and attaching a pendant edge at each of the vertices  $v_{21}, v_{23}, \dots, v_{2,n}$ . Therefore  $D$  is a ctd-set of  $G$ , and

$$\gamma_{\text{ctd}}(G) \leq |D| = |V(G) - D'| = 4n - \left(\frac{5n-1}{2}\right) = \frac{3n+1}{2}.$$

$$\text{Hence } \gamma_{\text{ctd}}(G) \leq \frac{3n+1}{2} = \left\lfloor \frac{3n+1}{2} \right\rfloor.$$

Let  $D'$  be a  $\gamma_{\text{ctd}}$ -set of  $G$ . The blocks  $A, B, A^2$  and  $A^2B$  are constructed as given below.

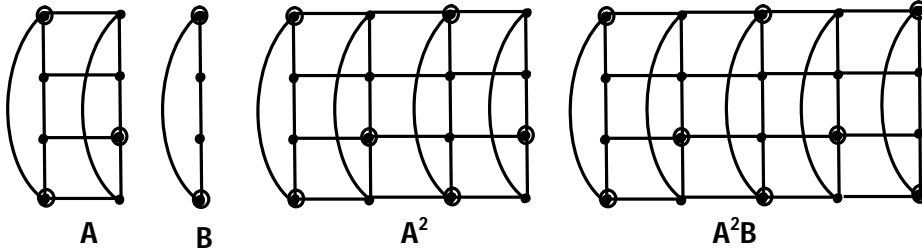


Figure 3

$G$  is obtained by concatenating the blocks  $A^{\frac{n-1}{2}}$  and  $B$ . That is,  $G \cong A^{\frac{n-1}{2}} B$ . The vertices with the symbol  $\bullet$  in each of the blocks represent the vertices that are to be included in  $D'$ .

Therefore  $D'$  contains atleast 3 vertices from each block  $A$  of  $A^{\frac{n-1}{2}}$  and atleast 2 vertices from block  $B$ .

$$\text{Therefore } \gamma_{\text{ctd}}(G) = |D'| \geq 2 + 3\left(\frac{n-1}{2}\right) = \frac{3n+1}{2} = \left\lfloor \frac{3n+1}{2} \right\rfloor.$$

$$\text{Hence } \gamma_{\text{ctd}}(G) = \left\lfloor \frac{3n+1}{2} \right\rfloor, n \geq 1.$$

$\cup \{v_{3,2i-1}\} \cup [U_{i=1}^n\{v_{2i}\}]$  and  $D = V(G) - D'$ . Then  $|D'| = 3\left(\frac{n}{2}\right) + n = \frac{5n}{2}$ . Then  $D$  is a dominating set of  $G$ . Also  $\langle V(G) - D \rangle = \langle D' \rangle$  is a tree obtained from the path  $P_n = \langle \{v_{2i}, i = 1, 2, 3, \dots, n\} \rangle$ , ( $n \geq 2$ ) by attaching  $P_3$  at each of the vertices  $v_{22}, v_{24}, v_{26}, \dots$ , and  $v_{2,n}$  and attaching a pendant edge at each of the vertices  $v_{21}, v_{23}, \dots$ , and  $v_{2, n-1}$ . Therefore  $D$  is a ctd-set of  $G$  and  $\gamma_{\text{ctd}}(G) \leq |D| = |V(G) - D'| = 4n - \left(\frac{5n}{2}\right) = \frac{3n}{2}$ .

$$\text{Hence } \gamma_{\text{ctd}}(G) \leq \frac{3n}{2} = \left\lfloor \frac{3n+1}{2} \right\rfloor.$$

Let  $D'$  be a  $\gamma_{\text{ctd}}$ -set of  $G$ . The block  $A$  is constructed as in Case 1.

Then  $G \cong A^{\frac{n}{2}}$ . The vertices with the symbol  $\bullet$  in each of the blocks represent the vertices that are to be included in  $D'$ .

$$\text{Therefore } D' \text{ contains atleast 3 vertices from each block } A \text{ of } A^{\frac{n}{2}}. \text{ Therefore } \gamma_{\text{ctd}}(G) = |D'| \geq 3\left(\frac{n}{2}\right) = \frac{3n}{2} = \left\lfloor \frac{3n+1}{2} \right\rfloor.$$

$$\text{Hence } \gamma_{\text{ctd}}(G) = \left\lfloor \frac{3n+1}{2} \right\rfloor, n \geq 1.$$

**K. Theorem 2.5**

If  $G \cong C_5 \times P_n$ , then  $\gamma_{\text{ctd}}(G) = 2n, n \geq 3$ .

**L. Proof**

Let  $G \cong C_5 \times P_n$  and  $V(G) = U_{i=1}^n\{v_{1i}, v_{2i}, v_{3i}, v_{4i}, v_{5i}\}$  such that  $\langle \{v_{1i}, v_{2i}, \dots, v_{in}\} \rangle \cong P_n^i, i = 1, 2, 3, 4, 5$  and  $\langle \{v_{1j}, v_{2j}, v_{3j}, v_{4j}, v_{5j}\} \rangle \cong C_5^j, j = 1, 2, \dots, n$ , where  $P_n^i$  is the  $i^{\text{th}}$  copy of  $P_n$  and  $C_5^j$  is the  $j^{\text{th}}$  copy of  $C_5$  in  $C_5 \times P_n$ .  $|V(G)| = 5n$ .

$$\text{Let } D = \begin{cases} [U_{i=1}^{\frac{n+1}{2}}\{v_{1,2i-1}, v_{5,2i-1}\}] \cup [U_{i=2}^{\frac{n-1}{2}}\{v_{3,2i}, v_{4,2i}\}], & \text{if } n \text{ is odd} \\ U_{i=2}^{\frac{n}{2}}\{v_{1,2i-1}, v_{3,2i}, v_{4,2i}, v_{5,2i-1}\}, & \text{if } n \text{ is even.} \end{cases}$$

Then D is a dominating set of G and also  $\langle V(G) - D \rangle$  is a tree obtained from the path  $P_n = \langle \{v_{2,i}, i = 1, 2, 3, \dots, n\} \rangle$ , ( $n \geq 2$ ) by attaching  $P_3$  at each of the vertices  $v_{21}, v_{22}, v_{23}, \dots$ , and  $v_{2,n}$ . Therefore D is a ctd-set of G.

$$\text{and } |D| = \begin{cases} 2\left(\frac{n+1}{2}\right) + 2\left(\frac{n-1}{2}\right) = 2n, & \text{if } n \text{ is even} \\ 4\left(\frac{n}{2}\right) = 2n, & \text{if } n \text{ is odd.} \end{cases}$$

Therefore  $\gamma_{\text{ctd}}(G) \leq |D| = 2n$ .

Let  $D'$  be a  $\gamma_{\text{ctd}}$ -set of G. Since  $\gamma(C_5) = 2$ ,  $D'$  contains atleast 2 vertices from each of n cycles and hence  $D'$  contains atleast  $2n$  vertices. Therefore  $\gamma_{\text{ctd}}(G) = |D'| \geq 2n$ .

Hence  $\gamma_{\text{ctd}}(G) = 2n, n \geq 1$ .

**M. Theorem 2.6**

If  $G \cong C_6 \times P_n$ , then  $\gamma_{\text{ctd}}(G) = \lfloor \frac{5n}{2} \rfloor, n \geq 2$ .

1) **Proof.:** Let  $G \cong C_6 \times P_n$  and  $V(G) = U_{i=1}^n\{v_{1i}, v_{2i}, v_{3i}, v_{4i}, v_{5i}, v_{6i}\}$  such that  $\langle \{v_{1i}, v_{2i}, \dots, v_{6i}\} \rangle \cong P_n^i, i = 1, 2, 3, 4, 5, 6$  and  $\langle \{v_{1j}, v_{2j}, v_{3j}, v_{4j}, v_{5j}, v_{6j}\} \rangle \cong C_6^j, j = 1, 2, \dots, n$ , where  $P_n^i$  is the  $i^{\text{th}}$  copy of  $P_n$  and  $C_6^j$  is the  $j^{\text{th}}$  copy of  $C_6$  in  $C_6 \times P_n$  and  $|V(G)| = 6n$ .

2) **Case 1: n is odd.**

$$\text{Let } D' = [U_{i=1}^{\frac{n+1}{2}}\{v_{1,2i-1}, v_{5,2i-1}, v_{6,2i-1}\}] \cup [U_{i=1}^n\{v_{2i}\}] [U_{i=1}^{\frac{n-1}{2}}\{v_{3,2i}, v_{4,2i}\}].$$

Then  $|D'| = 3\left(\frac{n+1}{2}\right) + n + 2\left(\frac{n-1}{2}\right) = \frac{7n+1}{2}$  and  $D = V(G) - D'$ . Then D is a dominating set of G. Also  $\langle V(G) - D \rangle = \langle D' \rangle$  is a tree obtained from the path  $P_n = \langle \{v_{2,i}, i = 1, 2, 3, \dots, n\} \rangle$ , ( $n \geq 2$ ) by attaching  $P_4$  at each of the vertices  $v_{21}, v_{23}, v_{25}, \dots, v_{2n}$  and attaching  $P_3$  at each of the vertices  $v_{22}, v_{24}, \dots, v_{2,n-1}$ . Therefore D is a ctd-set of G.

$$\gamma_{\text{ctd}}(G) \leq |D| = |V(G) - D'| = 6n - \left(\frac{7n+1}{2}\right) = \frac{5n-1}{2}.$$

$$\text{Hence } \gamma_{\text{ctd}}(G) \leq \frac{5n-1}{2}.$$

Let  $D'$  be a  $\gamma_{\text{ctd}}$ -set of G. The blocks A, B,  $A^2$  and  $A^2B$  are constructed as given below.

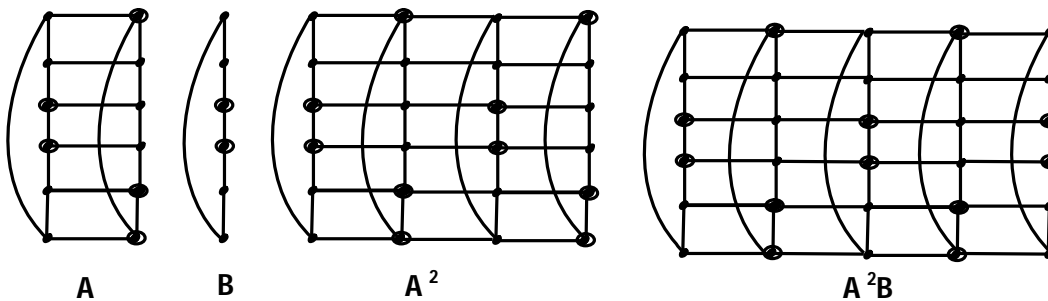


Figure 4

G is obtained by concatenating the blocks  $A^{\frac{n-1}{2}}$  and B. That is,  $G \cong A^{\frac{n-1}{2}} B$ . The vertices with the symbol  $\bullet$  in each of the blocks represent the vertices that are to be included in  $D'$ .

Therefore  $D'$  contains atleast 5 vertices from each block A of  $A^{\frac{n-1}{2}}$  and atleast 2 vertices from block B. Therefore  $\gamma_{\text{ctd}}(G) = |D'| \geq 5\left(\frac{n-1}{2}\right) + 2 = \frac{5n-1}{2}$  and hence  $\gamma_{\text{ctd}}(G) = \frac{5n-1}{2} = \lfloor \frac{5n}{2} \rfloor$ .

Case 2: n is even.

Let  $D' = [U_{i=1}^n \{v_{1,2i-1}, v_{5,2i-1}, v_{6,2i-1}\}] \cup [U_{i=1}^n \{v_{2i}\}] [U_{i=1}^n \{v_{3,2i}, v_{4,2i}\}]$ .

Then  $|D'| = 3\binom{n}{2} + n + 2\binom{n}{2} = \frac{7n}{2}$  and  $D = V(G) - D'$ . Then  $D$  is a dominating set of  $G$ . Also  $\langle V(G) - D \rangle = \langle D' \rangle$  is a tree obtained from the path  $P_n = \langle \{v_{2i}, i = 1, 2, 3, \dots, n\} \rangle$ , ( $n \geq 2$ ) by attaching  $P_4$  at each of the vertices  $v_{21}, v_{23}, v_{25}, \dots, v_{2,n-1}$  and attaching  $P_3$  at each of the vertices  $v_{22}, v_{24}, \dots, v_{2n}$ . Therefore  $D$  is a ctd-set of  $G$ .

$$\gamma_{\text{ctd}}(G) \leq |D| = |V(G) - D'| = 6n - \left(\frac{7n}{2}\right) = \frac{5n}{2}.$$

$$\text{Hence } \gamma_{\text{ctd}}(G) \leq \frac{5n}{2}.$$

Let  $D'$  be a  $\gamma_{\text{ctd}}$ -set of  $G$ . The block  $A$  is constructed as in Case 1.

Then  $G \cong A^{\frac{n}{2}}$ . The vertices with the symbol  $\textcircled{6}$  in each of the blocks represent the vertices that are to be included in  $D'$ .

Therefore  $D'$  contains atleast 5 vertices from each block  $A$  of  $A^{\frac{n}{2}}$ .

$$\text{Therefore } \gamma_{\text{ctd}}(G) = |D'| \geq 5 \left(\frac{n}{2}\right) = \frac{5n}{2} \text{ and hence } \gamma_{\text{ctd}}(G) = \frac{5n}{2} = \left\lfloor \frac{5n}{2} \right\rfloor.$$

$$\text{Hence } \gamma_{\text{ctd}}(G) = \left\lfloor \frac{5n}{2} \right\rfloor, n \geq 1.$$

*N. Remark 2.2.*

In view of Theorem 2.4, Theorem 2.5, Theorem 2.6, and Theorem 2.7,

- 1)  $\gamma_{\text{ctd}}(C_3 \times C_n) = n+1, n \geq 2.$
- 2)  $\gamma_{\text{ctd}}(C_4 \times C_n) = \left\lfloor \frac{3n+1}{2} \right\rfloor + 1$
- 3)  $\gamma_{\text{ctd}}(C_5 \times C_n) = \begin{cases} 2n + 1, & \text{if } n \text{ is even} \\ 2n, & \text{if } n \text{ is odd.} \end{cases}$
- 4)  $\gamma_{\text{ctd}}(C_6 \times C_n) = \left\lfloor \frac{5n}{2} \right\rfloor + 1.$

*O. Remark 2.3.*

- 1) If  $G_1 \cong K_m$  and  $G_2 \cong K_n$ , then  $\gamma_{\text{ctd}}(G_1 + G_2) = m + n - 2.$
- 2) If  $G_1$  and  $G_2$  are any two noncomplete connected graphs of order  $m$  and  $n$  respectively, with minimum degree atleast two, then  $\gamma_{\text{ctd}}(G_1 + G_2) \leq m + n - 4.$  Equality holds, if  $G_1 \cong K_m - e, G_2 \cong K_n - e.$
- 3) For any two connected graphs  $G_1$  and  $G_2$  of order  $m$  and  $n$  respectively,  $\gamma_{\text{ctd}}(G_1 \circ G_2) \leq m + n - 4.$  Equality holds, if  $G_1 \cong P_2$  and  $G_2 \cong C_3.$

*P. Distance Two Complementary tree domination number of Cartesian product of graphs*

In the following, distance two complementary tree domination number of  $K_m \times K_n, K_m \times P_n, K_m \times C_n, C_3 \times P_n, C_4 \times P_n, C_5 \times P_n, C_6 \times P_n, C_3 \times C_n, C_4 \times C_n, C_5 \times C_n$  and  $C_6 \times C_n$  are given

- 1) If  $G \cong K_m \times K_n$  ( $m, n \geq 3$  and  $m \leq n$ ), then  $\gamma_{\text{d2ctd}}(G) = \begin{cases} m(n-2) + 1, & \text{if } m = n \\ m(n-2), & \text{if } m < n \end{cases}$
- 2) If  $G \cong K_m \times P_n$  ( $4 \leq m \leq n$ ), then  $\gamma_{\text{d2ctd}}(G) = n(m-2).$
- 3)  $\gamma_{\text{d2ctd}}(K_m \times C_n) = n(m-2) + 1.$
- 4) If  $G \cong C_3 \times P_n$ , then  $\gamma_{\text{d2ctd}}(G) = n, n \geq 1.$
- 5) If  $G \cong C_4 \times P_n$ , then  $\gamma_{\text{d2ctd}}(G) = \left\lfloor \frac{3n+1}{2} \right\rfloor, n \geq 1.$
- 6) If  $G \cong C_5 \times P_n$ , then  $\gamma_{\text{d2ctd}}(G) = 2n, n \geq 3.$
- 7) If  $G \cong C_6 \times P_n$ , then  $\gamma_{\text{d2ctd}}(G) = \left\lfloor \frac{5n}{2} \right\rfloor, n \geq 1.$
- 8)  $\gamma_{\text{d2ctd}}(C_3 \times C_n) = n+1, n \geq 2.$
- 9)  $\gamma_{\text{d2ctd}}(C_4 \times C_n) = \left\lfloor \frac{3n+1}{2} \right\rfloor + 1$
- 10)  $\gamma_{\text{d2ctd}}(C_5 \times C_n) = \begin{cases} 2n + 1, & \text{if } n \text{ is even} \\ 2n, & \text{if } n \text{ is odd.} \end{cases}$



$$11) \gamma_{d2ctd}(C_6 \times C_n) = \left\lfloor \frac{5n}{2} \right\rfloor + 1, n \geq 2.$$

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