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Heat Transfer of a Viscous Fluid with Second Order Slip and Viscous Dissipation over a Shrinking Sheet

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Abstract: We consider the study concerns to flow and heat transfer of viscous fluid over a shrinking sheet with the combined effect of viscous dissipation and second order slip. The governing boundary value problem for flow and heat transfer is transformed in to a set of nonlinear ordinary differential equations using a suitable similarity transformation and is solved analytically using MATHEMATICA software. Effects of various flow and heat transfer charecteristics are analysed with the aid of graphs

Key Words: Second order slip; Boundary value problem; viscous dissipation; Prandtl number; Mathematica.

I. INTRODUCTION

In the present study we have extended the work of Fang [2009] considering heat transfer fluid with second order slip and viscous dissipation over a shrinking sheet. The flow induced by a moving boundary is important in the study of extrusion processes and is a subject of considerable interest in the contemporary literature for both permeable and impermeable stretching sheets.

The flow and heat transfer over a linearly stretching sheet was first considered by Crane [1], who reported the solution in a closed analytical form. Gupta and Gupta [2] extended this problem to a permeable stretching sheet. Grubka and Bobba [3] realized that Crane's solution to the boundary layer equation also happens to be an exact solution to the Navier-Stokes equations. Since then, many authors have considered various aspects of this problem such as Chen and Char [4], Chen [5] and Ishak et al. [6]-[8], among others. Different from the flow over a stretching sheet, the flow over a shrinking sheet only received the attention quite recently. Miklavčič and Wang [9] was the first who studied the properties of a viscous flow due to a shrinking sheet, and found that the solutions are non-unique. The flow is unlikely to exist unless adequate suction on the boundary is imposed, since vorticity of the shrinking sheet is not confined within a boundary layer. This problem was then extended by Fang and Zhang [10] to magneto hydro dynamic flow, and successfully obtained the closed form analytical solution. Moreover, the solution obtained by Fang and Zhang [10] is also an exact solution of the governing Navier-Stokes equations for that problem, and they reported greatly different solution behavior with multiple solution branches compared to the corresponding stretching sheet problem.

In the present study we have extended the work of Fang [10] considering heat transfer fluid with second order slip and viscous dissipation over a shrinking sheet.

II. MATHEMATICAL FORMULATION AND DISCUSSION

Consider a steady, two-dimensional laminar flow over a continuously shrinking sheet in a quiescent fluid. The sheet shrinking velocity is $U_w = -U_0 x$, with U_0 being a constant and the wall mass transfer velocity is $V_w = V_w(x)$, which will be determined later. The x-axis runs along the shrinking surface in the direction opposite to the sheet motion and the y-axis is perpendicular to it. The governing boundary layer equation for the proposed problem can be expressed as

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad (1)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\left(\frac{\partial^2 u}{\partial y^2}\right) \qquad (2)$$

With the boundary conditions $U(x, 0) = -U_0 x+U_{slip}$, $V(x, 0) = U_w(x)$, and $u(x,\infty) = 0$, (3)

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Where u and v are the velocity components in the x and y directions. ν is the kinematic coefficient of viscosity, ρ is the fluid density, and U_{slip} is the velocity slip at the wall. The Wu's slip velocity model used in this paper is valid for arbitrary Knudsen numbers, K_n, and is given as follows [2009]

$$U_{slip} = \frac{2}{3}\left(\frac{3-\alpha l^3}{\alpha} - \frac{3}{2}\frac{1-l^2}{K_n}\right)\lambda\frac{\partial u}{\partial y} - \frac{1}{4}\left[l^4 + \frac{2}{K_n^2}\left(1-l^2\right)\right]\lambda^2\frac{\partial^2 u}{\partial y^2} = A\frac{\partial u}{\partial y} + B\frac{\partial^2 u}{\partial y^2}, \quad (4)$$

Where $l = \min\left[\frac{1}{K_n}, 1\right]$, α, is the momentum accommodation coefficient with $0 \le \alpha \le l$, and λ is the molecular mean free path. Based on the definition of l, it is noticed that any given value of K_n, we have $0 \le l \le l$. The molecular mean free path is always positive. Thus we know that B<0 and positive. The stream function and similarity variable can be assumed in the following form,

$$\psi(x,y) = f(\eta)\, x\sqrt{\nu\, U_0}, \quad \eta = y\sqrt{\frac{U_0}{\nu}} \quad (5)$$

With these transformations, the velocity components are expressed as

$$u = U_0 x f'(\eta) \text{ and } v = -\sqrt{U_0 \nu}\, f(\eta). \quad (6)$$

The wall mass transfer velocity becomes

$$v_w(x) = -\sqrt{U_0 \nu}\, f(0). \quad (7)$$

Using equations (5) and (6) in equations (1) and (2) we obtain the transformed form of boundary layer equations of motion,

$$f''' + ff'' - f'^2 = 0 \quad (8)$$

Similarly, the boundary conditions equations (3) takes the form

$$f(0) = s, \quad f'(0) = -1 + \gamma f''(0) + \delta f'''(0) = 0, \text{ and } f'(\infty) = 0, \quad (9)$$

Where s is the wall mass transfer parameter showing the strength of the mass transfer at the surface, γ is the first order velocity slip parameter with $0 < \gamma = A\sqrt{\frac{U_0}{\nu}}$, and δ is the second order velocity slip parameter with $0 \succ \delta = \frac{BU_0}{\nu}$. we derive a closed form exact solution of Eq.(8) subject to the BCs of Eq. (9). We assume a solution of the form $f(\eta) = a + be^{-\beta\eta}$. The application of boundary condition (9) gives the values for a and b as mentioned below.

$$b = \frac{1}{\beta + \gamma\beta^2 - \delta\beta^3}, \quad (10)$$

$$a = S - \frac{1}{\beta + \gamma\beta^2 - \delta\beta^3}, \quad (11)$$

Substituting the assumed solution into Eq.(9) yields a = β. The use of this relationship in Eq. (11) leads to the following fourth order algebraic equation for β,

$$\delta\beta^4 - (\gamma + \delta s)\beta^3 + (\gamma s - 1)\beta^2 + s\beta - 1 = 0. \quad (12)$$

β should be least positive value.

Then the solution reads as

$$f(\eta) = s - \frac{1}{\beta + \gamma\beta^2 - \delta\beta^3} + \frac{1}{\beta + \gamma\beta^2 - \delta\beta^3} e^{-\beta\eta}, \quad (13)$$

and

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$$f(\eta) = -\frac{1}{1+\gamma\beta-\delta\beta^2} e^{-\beta\eta}, \tag{14}$$

Based on the results in Eq.(14), it is easy to show that

$$f^{//}(0) = \frac{\beta}{1+\gamma\beta-\delta\beta^2} = \beta^2(s-\beta) \tag{15}$$

III. HEAT TRANSFER ANALYSIS

The thermal boundary layer equation, with work done by deformation, and internal heat generation or absorption is given by

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2} + \frac{Q}{\rho c_p}(T - T_\infty) + \frac{\mu}{\rho c_p}\left(\frac{\partial u}{\partial y}\right)^2 \tag{16}$$

Where $\frac{k}{\rho c_p}$ is the thermal diffusivity and c_p is specific heat of a fluid at constant pressure,

A. CASE (I) Prescribed surface temperature (PST)

The boundary conditions in case of PST is given by

$$\left.\begin{aligned} &T = T_w = \mathrm{A}\left(\frac{\mathrm{x}}{\mathrm{l}}\right)^2 + \mathrm{T}_\infty \text{ at } y = 0 \\ &T \to T_\infty \qquad\qquad \text{at } y \to \infty \end{aligned}\right\} \tag{17}$$

Where T_W is the temperature of the sheet and T_∞ is the temperature of the fluid far away from the sheet.

Defining the non-dimensional temperature $\theta(\eta)$ as

$$\theta(\eta) = \frac{T - T_\infty}{T_W - T_\infty} \tag{18}$$

Using (18), Eq. (16) can be written in the form

$$\theta''(\eta) + \Pr(a + be^{-\beta\eta})\theta'(\eta) + \left(2\Pr b\beta e^{-\beta\eta} + \alpha\Pr\right)\theta(\eta) = -\Pr Ec\, b^2\beta^4\left(e^{-\beta\eta}\right)^2 \tag{19}$$

Where $\Pr = \frac{\mu Cp}{k}$ is the Prandtl number, Eckert number $Ec = \frac{u_o^{\,2}\, l^2}{Cp\, A}$

& Heat source /sink $\alpha = \frac{Q}{\rho c_p u_o}$

Consequently the boundary conditions (17) take the form

$$\left.\begin{aligned} &\theta(\eta) = 1 \;at\; \eta = 0 \\ &\theta(\eta) \to 0 \;as\; \eta \to \infty \end{aligned}\right\} \tag{20}$$

Introducing the new independent variable

$$\xi = -\frac{\Pr}{\beta^2}$$

and substituting in Eq. (19) we obtain

$$\xi\theta''(\xi) + (1 - P_{11} + P_{12}\xi)\theta'(\xi) + \frac{P_{13}}{\xi}\theta(\xi) - 2P_{12}\theta(\xi) = -\frac{Ec\, b^2\beta^6}{\Pr}\xi \tag{21}$$

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Where $P_{11} = \dfrac{a\Pr}{\beta}$, $P_{12} = \dfrac{b}{\beta}$,& $P_{13} = \dfrac{\Pr\alpha}{\beta^2}$

The corresponding boundary conditions are

$$\left.\begin{aligned} &\theta(\xi)=1 && at\ \ \xi=\frac{-\Pr e^{-\beta\eta}}{\beta^2} \\ &\theta(\xi)\to 0 && as\ \ \xi\to 0 \end{aligned}\right\} \tag{22}$$

The solution of Eq. (21) subject to the boundary conditions (22) is given by

$$\phi(\eta)=a_0\left(\frac{\Pr e^{-\beta\eta}}{\beta^2}\right)^{\frac{C+D}{2}}+M\left[-P_{12}\left(\frac{C+D}{2}-2\right),1+D;\frac{-\Pr e^{-\beta\eta}}{\beta^2}\right]-\frac{Ecb^2\beta^2}{\Pr(4-2P_{11}+P_{13})}\left(\frac{-\Pr e^{-\beta\eta}}{\beta^2}\right)^2 \tag{23}$$

Where $a_0 = \dfrac{1+\dfrac{Ecb^2\beta^6}{\Pr(4-2P_{11}+P_{13})}\left(\dfrac{-\Pr}{\beta^2}\right)^2}{\left(\dfrac{-\Pr}{\beta^2}\right)^{\frac{C+D}{2}} M\left[-P_{11}\left(\dfrac{C+D}{2}-2\right),1+D;\dfrac{-\Pr}{\beta^2}\right]}$

C = P_{11} & $D=\sqrt{(P_{11})^2-4P_{13}}$

B. CASE (ii) Prescribed power law heat flux (PHF case)

In this heating process we employ the following power law heat flux. Consider boundary conditions given by

$$\left.\begin{aligned} &-k\left(\frac{\partial T}{\partial y}\right)_w = q_w = E_0+\left(\frac{x}{l}\right)^2 && at\ \ y=0 \\ &T=T_\infty && at\ y\to\infty \end{aligned}\right\} \tag{24}$$

Where E_O is constant. T_W is temperature at the wall. T_∞ is temperature away from the sheet.

We define non-dimensional temperature as

$$\phi(\eta)=\frac{T-T_\infty}{T_W-T_\infty} \tag{25}$$

So that the equation (16) reduces to the form

$$\phi''(\eta)=\Pr\left(a+be^{-\beta\eta}\right)\phi'(\eta)+\left[2\Pr\beta be^{-\beta\eta}+\Pr\alpha\right]\phi(\eta)=-Ec\Pr b^2\beta^4e^{-2\beta\eta} \tag{26}$$

the corresponding boundary conditions (24) reduces to

$$\left.\begin{aligned} &\phi(\eta)=-1 && \eta=1 \\ &\phi(\eta)=\infty && \eta=0 \end{aligned}\right\} \tag{27}$$

Using the new independent variable defined as

$$t=\frac{-\Pr e^{-\beta\eta}}{\beta^2}$$

and substituting in equation (26) we obtain

$$t\phi''(t)+[1-P_{11}]\phi'(t)+P_{12}\,t\,\phi'(t)+\left(\frac{P_{13}}{t}-2P_{12}\right)\phi(t)=\frac{Ecb^2\beta^6t}{\Pr} \tag{28}$$

Where $P_{11} = \frac{a\Pr}{\beta}$, $P_{12} = \frac{b}{\beta}$,& $P_{13} = \frac{\Pr\alpha}{\beta^2}$

The corresponding boundary conditions will be

$$\left.\begin{array}{ll} \phi'(t) = -1 & as \quad t = -\Pr e^{-\beta\eta} \\ \phi(t) \to 0 & as \quad t \to \infty \end{array}\right\} \tag{29}$$

We obtain the solution of above equation (26) by using power series method and in terms of Kummer's function is as mentioned below,

$$\phi(t) = a_0\left(\frac{\Pr e^{-\beta\eta}}{\beta^2}\right)^{\frac{C+D}{2}} + M\left[-P_{12}\left(\frac{C+D}{2}-2\right), 1+D; \frac{-\Pr e^{-\beta\eta}}{\beta^2}\right] - \frac{Ecb^2\beta^2}{\Pr(4-2P_{11}+P_{13})}\left(\frac{-\Pr e^{-\beta\eta}}{\beta^2}\right)^2 \tag{30}$$

Using (29), the non-dimensional wall temperature gradient can be written as

$$\phi'(t) = a_0\left(\frac{-\Pr}{\beta^2}\right)^{\frac{C+D}{2}}\left\{-\beta\left(\frac{C+D}{2}\right)\right\} M\left[-P_{12}\left(\frac{C+D-4}{2}\right), 1+D; \frac{\Pr}{\beta^2}\right] +$$

$$a_0\left(\frac{-\Pr}{\beta^2}\right)^{\frac{C+D}{2}}\left\{\frac{-P_{12}(C+D-4)}{2(1+D)}\right\}\frac{\Pr}{\beta} \quad M\left[-P_{12}\left(\frac{C+D-2}{2}\right), 2+D; -\frac{\Pr}{\beta^2}\right]$$

$$+\frac{2Ecb^2\beta^3}{4-2P_{11}+P_{13}}\Pr \tag{31}$$

IV. RESULTS AND DISCUSSION

In this problem, we proposed to investigate the flow and heat transfer viscous fluid with second order slip& viscous dissipation over shrinking sheet. The governing equations for momentum and heat transfer are partial differential equations which are converted into ordinary differential equations by using suitable similarity transformations.

An analytical solution [exponential solution] for flow has been assumed, and this assumed solution is used to solve the heat transfer equations by power series method and expressed in terms of Kummer's hyper geometric functions.

The results are depicted graphically from graph Fig 2 to 12.

Fig 2. Shows the effect of mass suction parameter s on considered flow. It shows that as there is increase in the parameter value of 's' velocity f' is decreases.

Fig 3. Shows the effect of first order slip parameter γ on velocity profile f'. It is noticed that as first order slip parameter γ increases velocity profile f' decreases.

Similarly in Fig 4, we notice that the effect of second order slip parameter δ is to sustain velocity profile f' in the boundary layer.

Fig 5(a) and 5(b), Shows the temperature distribution $\theta(\eta)$ and $\phi(\eta)$ versus η for different values's' PST and PHF cases, respectively. .By analyzing the graphs that the effect of increasing values of's' in both PST and PHF cases, temperature decreases.

Fig 6(a) and 6(b), Shows the temperature distribution $\theta(\eta)$ and $\phi(\eta)$ versus η for different values γ PST and PHF cases, respectively. .By analyzing the graphs that the effect of increasing values of γ in both PST and PHF cases, temperature decreases.

Fig 7(a) and 7(b), Shows the temperature distribution $\theta(\eta)$ and $\phi(\eta)$ versus η for different values δ PST and PHF cases, respectively. .By analyzing the graphs that the effect of decrease values of δ in both PST and PHF cases, temperature decreases

Fig 8(a) and 8(b), Shows the temperature distribution $\theta(\eta)$ and $\phi(\eta)$ versus η for different values α PST and PHF cases, respectively. .By analyzing the graphs that the effect of increase values of α in both PST and PHF cases, temperature increase.

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Fig 9(a) and 9(b), Shows the temperature distribution $\theta(\eta)$ and $\phi(\eta)$ versus η for different values α in PST and PHF cases, respectively. By analyzing the graphs that the effect of increase values of Ec in both PST and PHF cases, temperature increase in PST and decrease in PHF.

Fig 10(a) and 10(b), Shows the temperature distribution $\theta(\eta)$ and $\phi(\eta)$ versus η for different values Pr in PST and PHF cases, respectively. By analyzing the graphs that the effect of increase values of Pr in both PST and PHF cases, temperature decrease.

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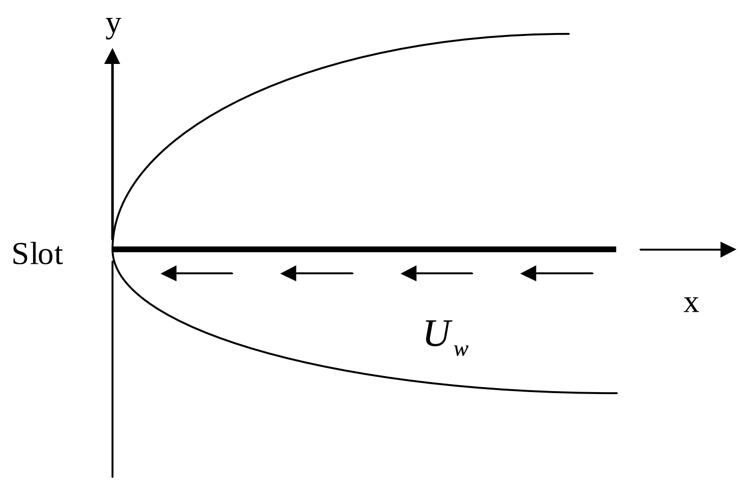
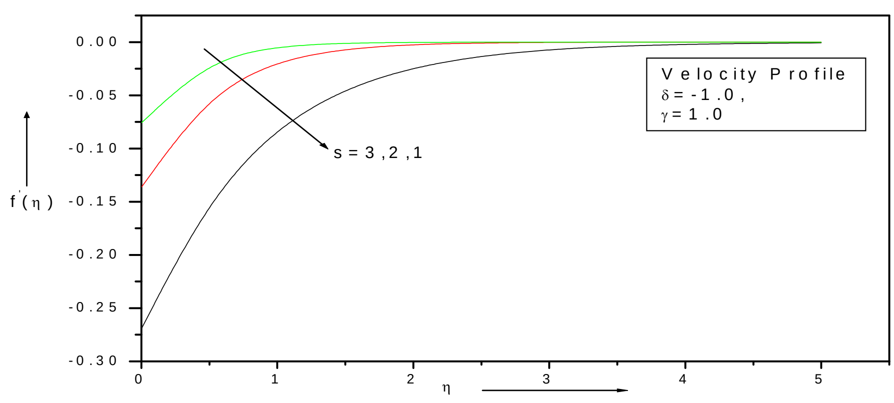


Fig(1) Schematic diagram of boundary layer slip flow past a shrinking sheet



Fig(2) Variation of $f'(\eta)$ versus η for different values of s

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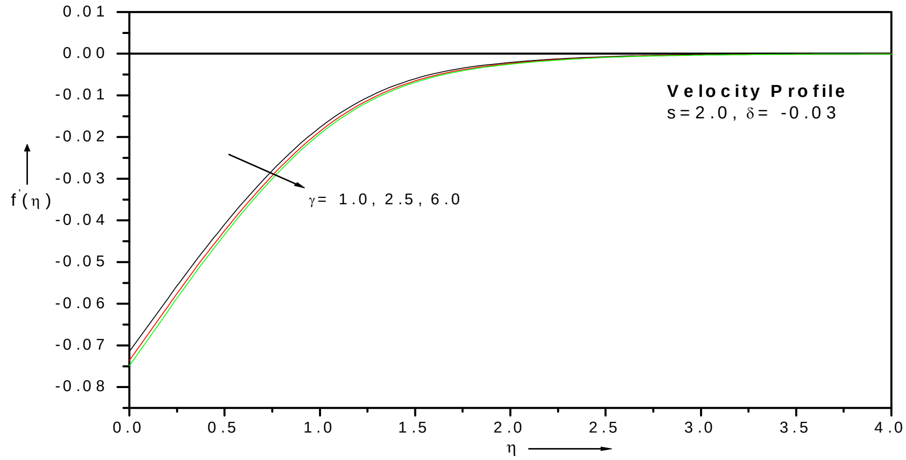

Fig(3) Variation of f'(η) versus η for different values of ν

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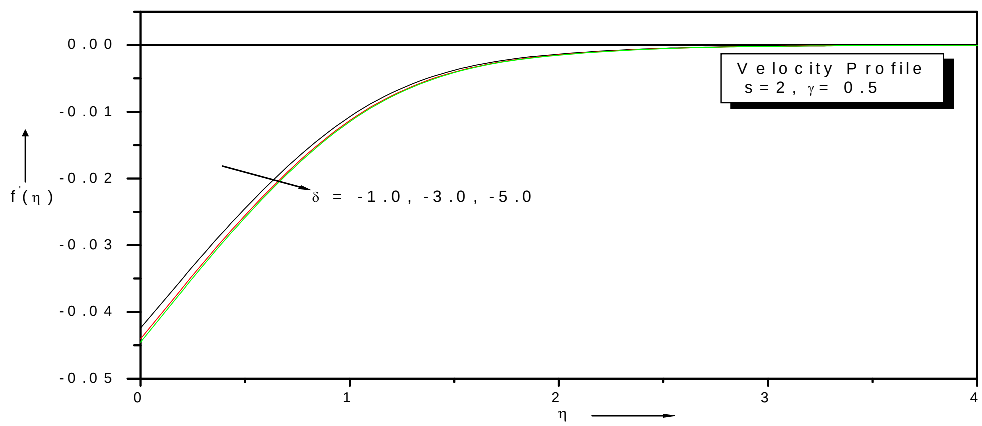

Fig(4) Variation of f'(η) versus η for different values of δ

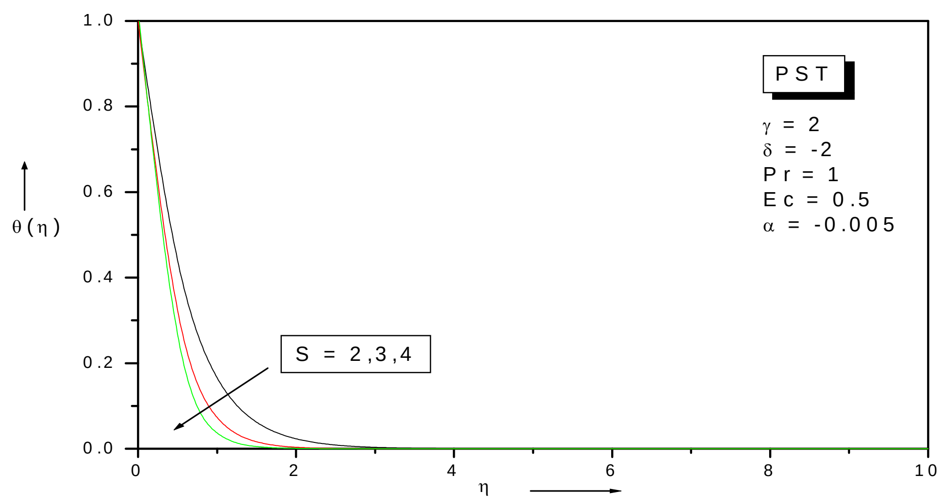

Fig 5(a)Variation of θ(η) versus η for different values of S

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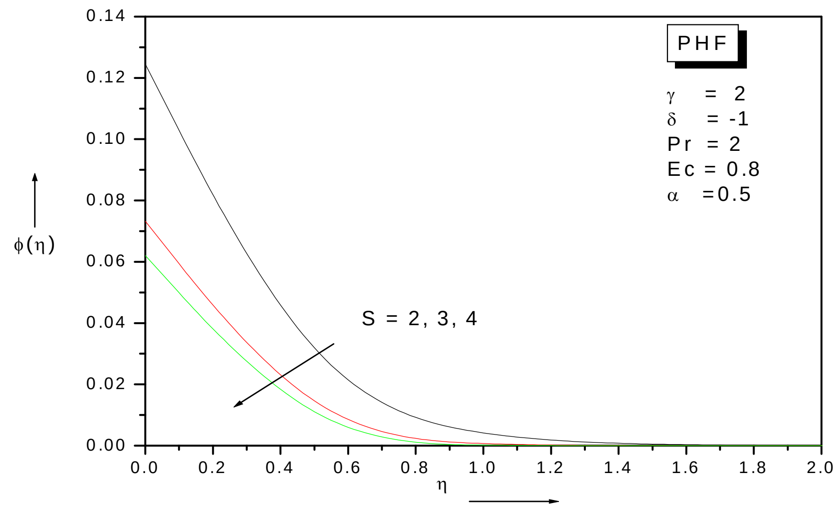


Fig 5(b) Variation of $\phi(\eta)$ versus η for different values of S

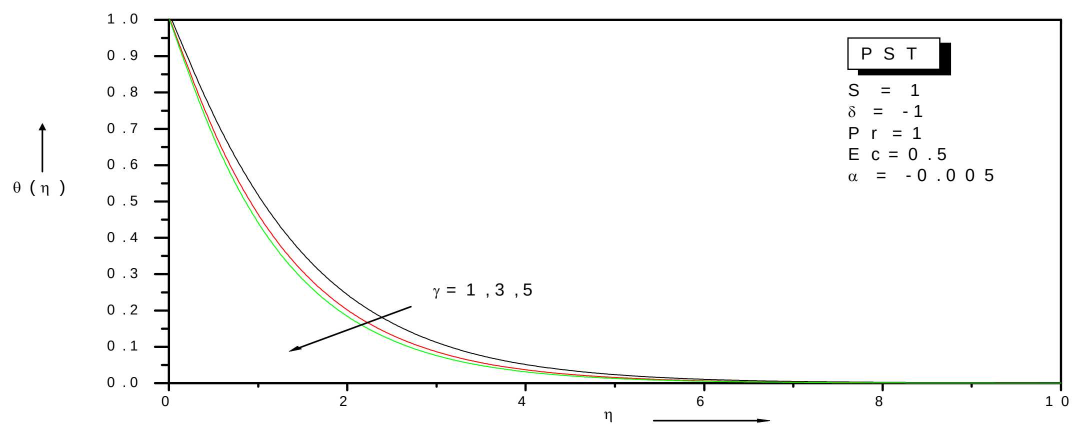


Fig 6(a) Variation of $\theta(\eta)$ versus η for different values of γ

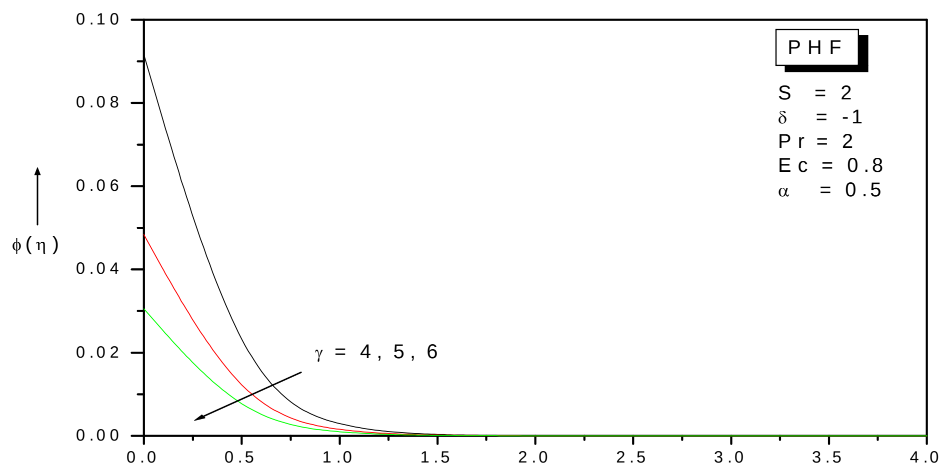


Fig 6(b) Variation of $\phi(\eta)$ versus η for different values of γ

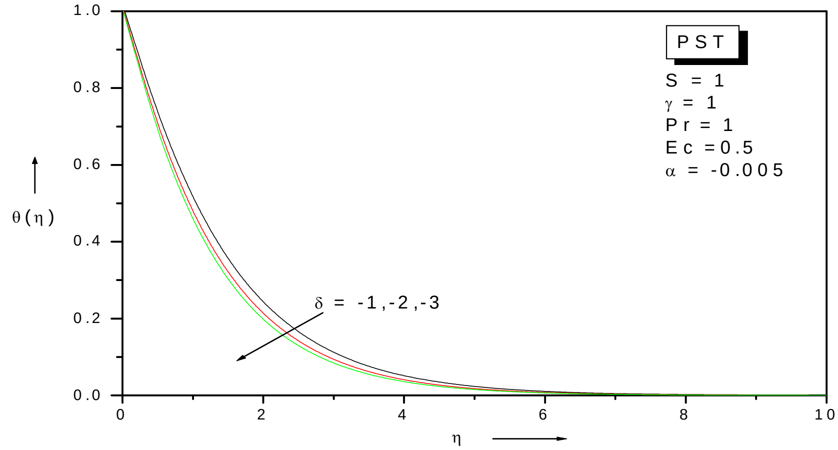


Fig 7(a) Variation of θ(η) versus η for different values of γ

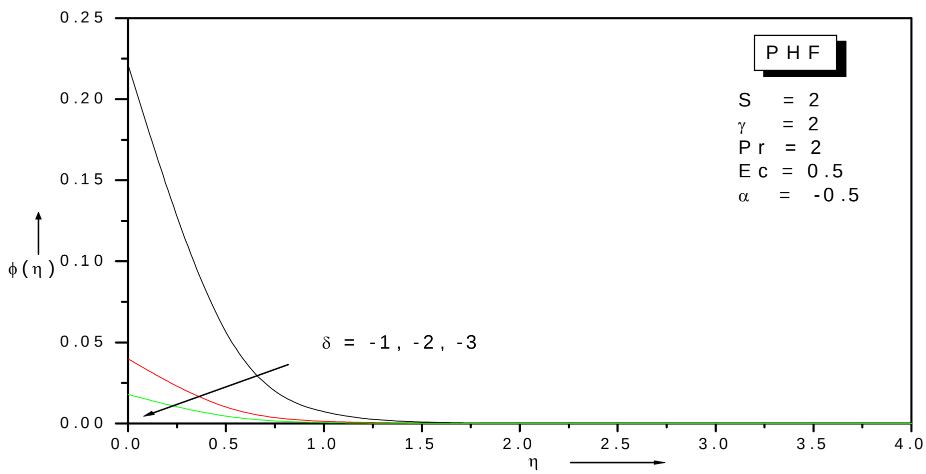


Fig 7(b) Variation of φ(η) versus η for different values of δ

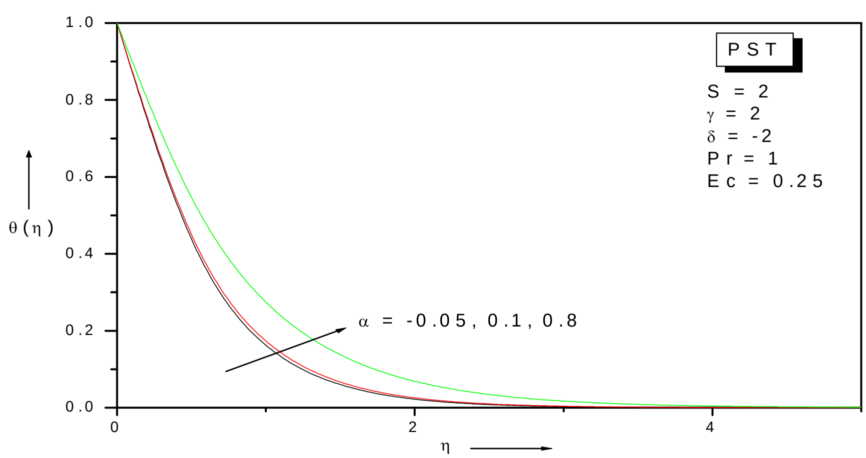


Fig 8(a) Variation of θ(η) versus η for different values of α

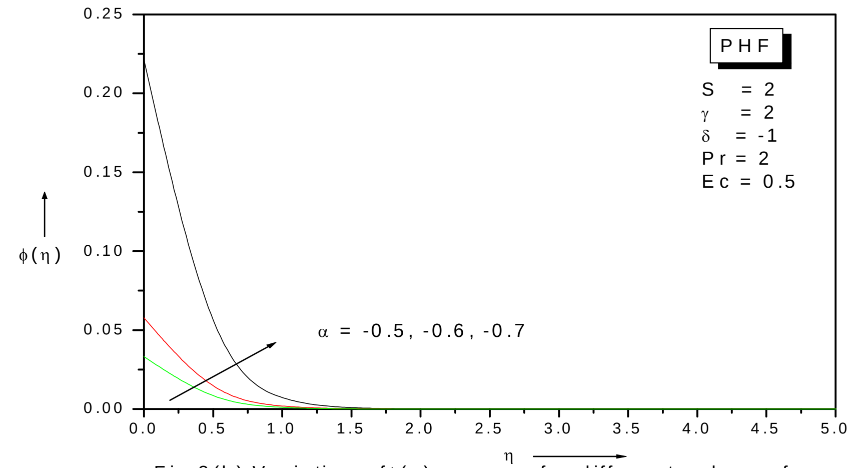


Fig 8(b) Variation of $\phi(\eta)$ versus η for different values of α

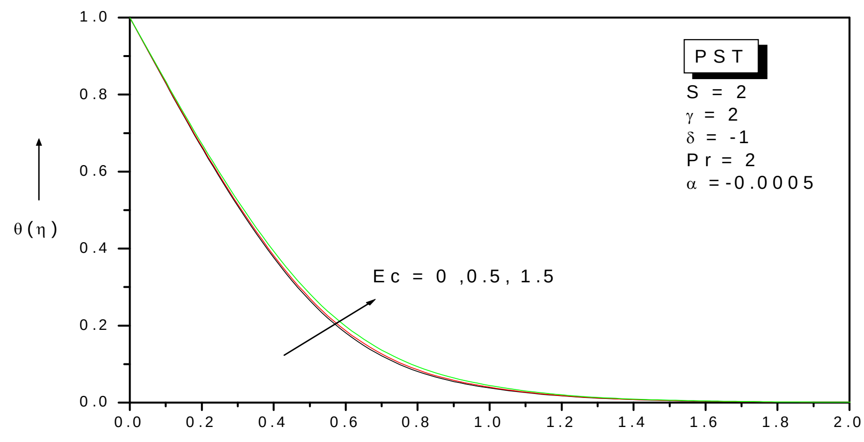


Fig 9(a) Variation of $\theta(\eta)$ versus η for different values of Ec

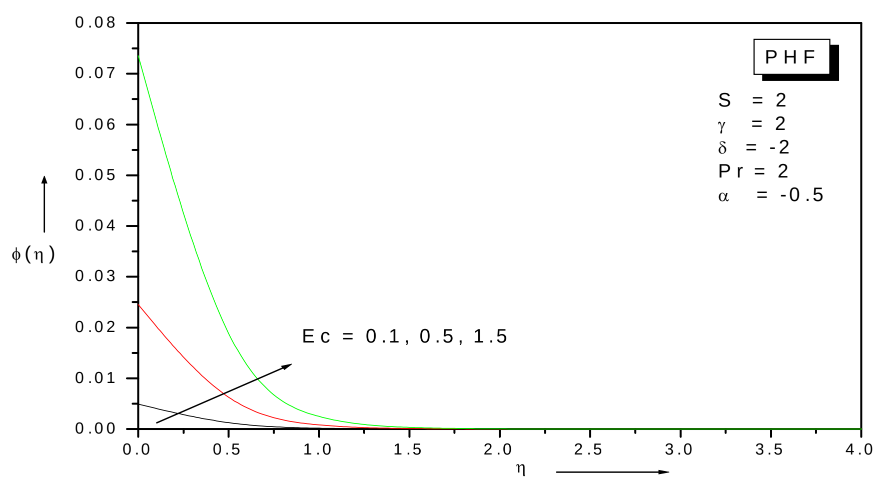


Fig 9(b) Variation of $\phi(\eta)$ versus η for different values of Ec

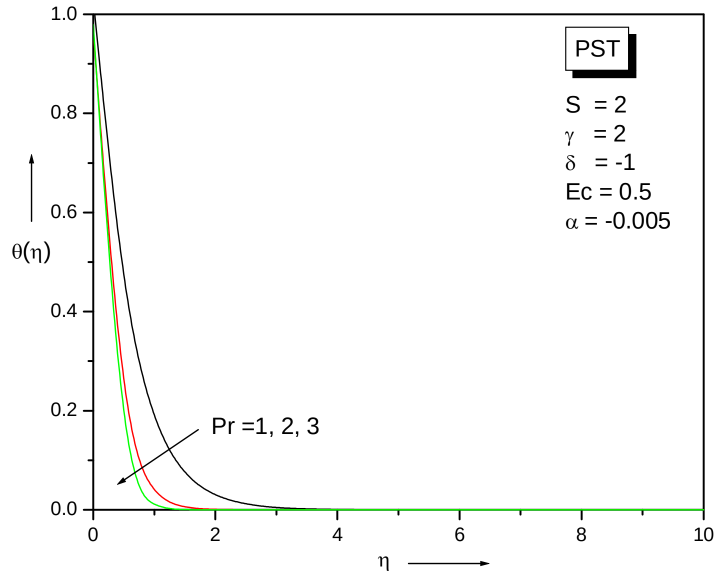


Fig10(a) Variation of $\theta(\eta)$ versus η for different values of Pr

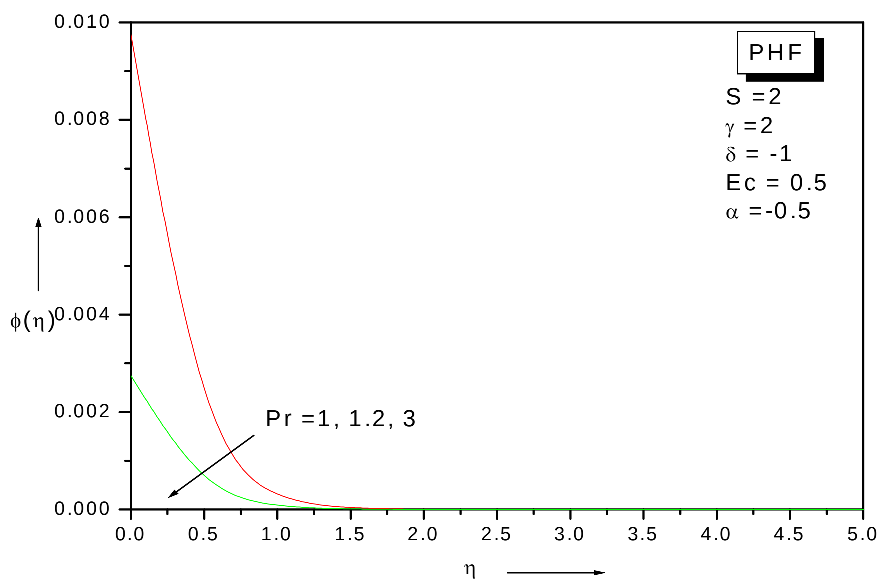
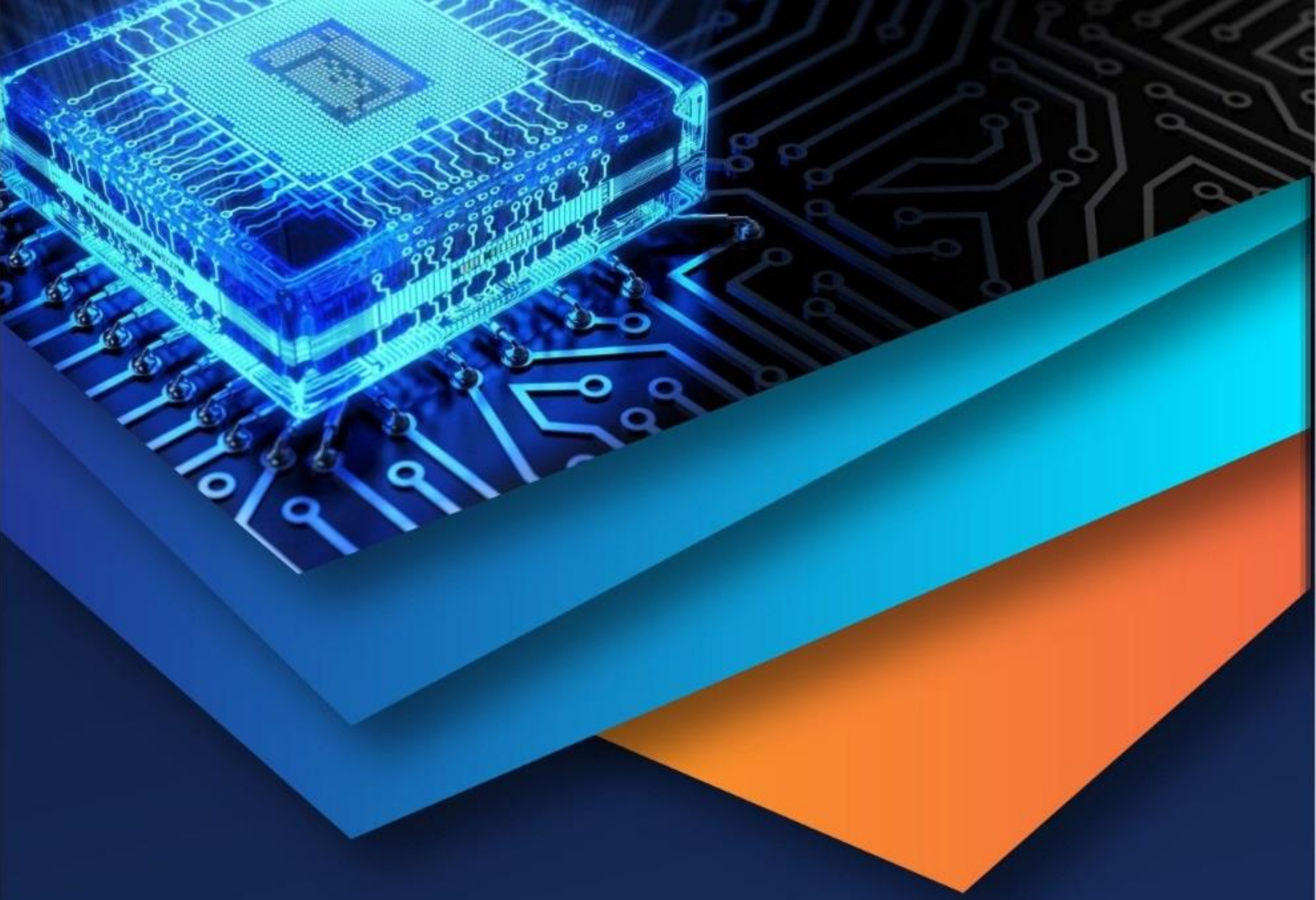


Fig10(b) Variation of $\phi(\eta)$ versus η for different values of Pr



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