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# Construction of special Dio 3-Tuples From $\frac{CC_n}{Gno_n}$ - II

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**Abstract:** We search for special dio 3-tuples from  $\frac{CC_n}{Gno_n}$ . We also present 6 sets of dio 3-tuples under 3 cases and some numerical examples satisfying the tuples.

**Keywords:** Dio 3-Tuples, centered cubic number, Gnomonic number.

## I. INTRODUCTION

An n-tuples, sometimes simply called a tuple, when the number n is known implicitly is an ordered set of n-elements. In particular 3-tuples is a set with 3 elements. A set of m distinct positive integers  $S = \{a_1, a_2, \dots, a_m\}$  satisfies the diaphanous property  $D(n)$  of order n if for all  $i, j = 1, 2, \dots, m$  with  $i \neq j$ ,  $a_i a_j + n = b_{ij}^2$ , the  $b_{ij}$ 's are integers. The set S is called Diophantine n-tuple. A longstanding conjecture is that no integer Diophantine quintuple exists. Jones derived in 1975, an infinite sequence of polynomials  $S = \{x, x+2, c_1(x), c_2(x), \dots\}$  such that the product of any two consecutive polynomials increased by one is a square of a polynomial. [1-3] has been studied for basic ideologies. [3-15] has been referred for various concepts and findings of Diophantine triples and quadruples. Recently in [16] special dio 3-tuples is constructed from  $\frac{CC_n}{Gno_n}$ .

In this paper we search for special dio 3-tuples constructed from  $\frac{CC_n}{Gno_n}$  with different method of analysis, where  $CC_n$  is the centered cubic number of rank n and  $Gno_n$  is the gnomonic number of rank n. Here the product of any two members of the triples with the addition of the same members and the addition with a non-zero integer or a polynomial with integral coefficient satisfies the required property.

## II. NOTATIONS

$$CG_n = \frac{CC_n}{Gno_n}$$

Where  $CC_n$  is the centered cubic number of rank n and  $Gno_n$  is the Gnomonic number of rank n.

## III. METHOD OF ANALYSIS

A. Case(i)

Let  $a = n^2 - 3n + 2$ ,  $CG_{n-1}$  of rank n-1

$$b = n^2 - n + 1, CG_n \text{ of rank } n$$

$$\text{We then have } ab + (a+b) + n - 1 = \alpha^2 \quad (1)$$

$$\text{where } \alpha = n^2 - 2n + 2$$

Let c be any non zero integer such that

$$ac + (a+c) + n - 1 = \beta^2 \quad (2)$$

$$bc + (b+c) + n - 1 = \gamma^2 \quad (3)$$

Eliminating c from (2) and (3) we get

$$(b-a) + (a-b)(n-1) = (a+1)\gamma^2 - (b+1)\beta^2 \quad (4)$$

Introducing the linear transformation

$$\beta = x + (a+1)y ; \quad \gamma = x + (b+1)y ; \quad (5)$$

Hence (4) reduces to

$$x^2 = (ab + a + b + 1)y^2 + n - 2$$

Taking  $y = 1$  we get  $x = n^2 - 2n + 2$

Therefore the initial solution is  $x_0 = n^2 - 2n + 2$ ,  $y_0 = 1$

Substituting the initial solution in (5) we get  $\beta = 2n^2 - 5n + 5$

Using the value of  $\beta$  in (2) we get  $c = 4n^2 - 8n + 8 = CG_{2n-3} + 6n - 5$

Therefore the triples  $\{n^2 - 3n + 2, n^2 - n + 1, 4n^2 - 8n + 8\}$ , i.e.,  $\{CG_{n-1}, CG_n, CG_{2n-3} + 6n - 5\}$  is a special dio 3-tuple with the property  $D(n-1)$

Some numerical examples satisfying the above mentioned tuples are listed below

TABLE I

n	a	b	c	a+b	a+c	b+c	D(n)
3	2	7	20	9	22	27	2
4	6	13	40	19	46	53	3
5	12	21	68	33	80	89	4
6	20	31	104	51	124	135	5
7	30	43	148	73	178	191	6

Below we present 5 sets of special dio 3-tuples with their corresponding properties

TABLE III

s.no	A	b	C	D(n)
1	$CG_{n-1}$	$CG_n$	$CG_{2n-3} - 6n - 3$	$D(2n^2 - 3n + 4)$
2	$CG_{n-1}$	$CG_n$	$CG_{2n-3} - 2n + 9$	$D(4n^2 - 7n + 11)$
3	$CG_{n-1}$	$CG_n$	$CG_{2n-3} + 6n - 13$	$D(-8n^2 + 17n - 1)$
4	$CG_{n-1}$	$CG_n$	$CG_{2n-3} + 6n - 15$	$D(-10n^2 + 21n + 4)$
5	$CG_{n-1}$	$CG_n$	$CG_{2n-3} + 6n - 17$	$D(-12n^2 + 25n + 11)$

### B. Case(ii)

Here we take  $a = n^2 - 5n + 7$ ,  $CG_{n-2}$  of rank  $n-2$

$b = n^2 - n + 1$ ,  $CG_n$  of rank  $n$

Proceeding as in case(i) we have  $c = 4n^2 - 12n + 15$

Therefore the triples  $\{n^2 - 5n + 7, n^2 - n + 1, 4n^2 - 12n + 15\}$ , i.e.,  $\{CG_{n-2}, CG_n, CG_{2n-3} + 2n + 2\}$  is a special dio 3-tuple with the property  $D(-6)$

Some numerical examples satisfying the above mentioned tuples are listed below

TABLE IIIII

n	a	b	c	a+b	a+c	b+c	D(n)
1	3	1	7	4	10	8	-6
2	1	3	7	4	8	10	-6
3	1	7	15	8	16	22	-6
4	3	13	31	16	34	44	-6
5	7	21	55	28	62	76	-6

Below we present 5 sets of special dio 3-tuples with their corresponding properties

TABLE IVV

s.no	A	b	c	D(n)
1	$CG_{n-2}$	$CG_n$	$CG_{2n-3} + 2n$	$D(-2n^2 + 6n - 11)$
2	$CG_{n-2}$	$CG_n$	$CG_{2n-3} + 2n + 4$	$D(2n^2 - 6n + 1)$
3	$CG_{n-2}$	$CG_n$	$CG_{2n-3} + 2n - 10$	$D(-12n^2 + 36n - 6)$
4	$CG_{n-2}$	$CG_n$	$CG_{2n-3} + 2n - 12$	$D(-14n^2 + 42n + 1)$
5	$CG_{n-2}$	$CG_n$	$CG_{2n-3} + 2n - 2$	$D(-4n^2 + 12n - 14)$

### C. Case(iii)

Here we take  $a = n^2 - 5n + 7$ ,  $CG_{n-2}$  of rank n-2

$b = n^2 - 3n + 2$ ,  $CG_{n-1}$  of rank n-1

Proceeding as in case(ii) we have  $c = 4n^2 - 16n + 16 = CG_{2n-3} - 2n + 3$

Therefore the triples  $\{n^2 - 5n + 7, n^2 - 3n + 2, 4n^2 - 16n + 16\}$ , i.e.,  $\{CG_{n-2}, CG_{n-1}, CG_{2n-3} - 2n + 3\}$  is a special dio 3-tuple with the property  $D(-4n^2 + 15n - 14)$

Some numerical examples satisfying the above mentioned tuples are listed below

TABLE V

n	a	b	c	a+b	a+c	b+c	D(n)
4	3	6	16	9	19	22	-18
5	7	12	36	19	43	48	-39
6	13	20	64	33	77	84	-68
7	21	30	100	51	121	130	-105
8	31	42	144	73	175	186	-150

Below we present 5 sets of special dio 3-tuples with their corresponding properties

TABLE VI

s.no	A	b	c	D(n)
1	$CG_{n-2}$	$CG_{n-1}$	$CG_{2n-3} - 2n + 5$	$D(-2n^2 + 7n - 7)$
2	$CG_{n-2}$	$CG_{n-1}$	$CG_{2n-3} - 2n + 7$	$D(-n + 2)$
3	$CG_{n-2}$	$CG_{n-1}$	$CG_{2n-3} - 2n - 13$	$D(-20n^2 + 79n + 2)$
4	$CG_{n-2}$	$CG_{n-1}$	$CG_{2n-3} - 2n - 11$	$D(-18n^2 + 71n - 7)$
5	$CG_{n-2}$	$CG_{n-1}$	$CG_{2n-3} - 2n - 9$	$D(-16n^2 + 63n - 14)$

## IV. CONCLUSION

In this paper, we have presented some special dio 3-tuples under 3 cases from  $\frac{CC_n}{Gno_n}$  with their corresponding properties. One may also search for similar type of special dio 3-tuples with suitable property.

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