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Construction of special Dio 3-Tuples From $\frac{CC_n}{Gno_n}$ - II

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Abstract: ***We search for special dio 3-tuples from*** $\frac{CC_n}{Gno_n}$***. We also present 6 sets of dio 3-tuples under 3 cases and some numerical examples satisfying the tuples.***
Keywords: Dio 3-Tuples, centered cubic number, Gnomonic number.

I. INTRODUCTION

An n-tuples, sometimes simply called a tuple, when the number n is known implicitly is an ordered set of n-elements. In particular 3-tuples is a set with 3 elements. A set of m distinct positive integers S= {a1,a2,…a_m} satisfies the diaphanous property D(n) of order n if for all i, j = 1,2,…m with $i \neq j, a_i a_j + n = b_{ij}^2$, the b_{ij}'s are integers. The set S is called Diophantine n-tuple. A longstanding conjecture is that no integer Diophantine quintuple exists. Jones derived in 1975, an infinite sequence of polynomials S={x,x+2,c_1(x),c_2(x),…} such that the product of any two consecutive polynomials increased by one is a square of a polynomial.
[1-3] has been studied for basic ideologies.[3-15] has been referred for various concepts and findings of Diophantine triples and quadruples. Recently in [16] special dio 3-tuples is constructed from $\frac{CC_n}{Gno_n}$.

In this paper we search for special dio 3-tuples constructed from $\frac{CC_n}{Gno_n}$ with different method of analysis, where CC_n is the centered cubic number of rank n and Gno_n is the gnomonic number of rank n. Here the product of any two members of the triples with the addition of the same members and the addition with a non-zero integer or a polynomial with integral coefficient satisfies the required property.

II. NOTATIONS

$$CG_n = \frac{CC_n}{Gno_n}$$

Where CC_n is the centered cubic number of rank n and Gno_n is the Gnomonic number of rank n.

III. METHOD OF ANALYSIS

A. *Case(i)*

Let $a = n^2 - 3n + 2$, CG_{n-1} of rank n-1

$b = n^2 - n + 1$, CG_n of rank n

We then have $ab + (a+b) + n - 1 = \alpha^2$ (1)

where $\alpha = n^2 - 2n + 2$

Let c be any non zero integer such that

$ac + (a+c) + n - 1 = \beta^2$ (2)

$bc + (b+c) + n - 1 = \gamma^2$ (3)

Eliminating c from (2) and (3) we get

$(b-a) + (a-b)(n-1) = (a+1)\gamma^2 - (b+1)\beta^2$ (4)

Introducing the linear transformation

$\beta = x + (a+1)y$; $\gamma = x + (b+1)y$; (5)

Hence (4) reduces to

$x^2 = (ab+a+b+1)y^2 + n - 2$

Taking y = 1 we get $x = n^2 - 2n + 2$

Therefore the initial solution is $x_0 = n^2 - 2n + 2$, $y_0 = 1$

Substituting the initial solution in (5) we get $\beta = 2n^2 - 5n + 5$

Using the value of β in (2) we get $c = 4n^2 - 8n + 8 = CG_{2n-3} + 6n - 5$

Therefore the triples $\{n^2 - 3n + 2, n^2 - n + 1, 4n^2 - 8n + 8\}$,ie., $\{CG_{n-1}, CG_n, CG_{2n-3} + 6n - 5\}$ is a special dio 3-tuple with the property $D(n-1)$

Some numerical examples satisfying the above mentioned tuples are listed below

TABLE I

n	a	b	c	a+b	a+c	b+c	D(n)
3	2	7	20	9	22	27	2
4	6	13	40	19	46	53	3
5	12	21	68	33	80	89	4
6	20	31	104	51	124	135	5
7	30	43	148	73	178	191	6

Below we present 5 sets of special dio 3-tuples with their corresponding properties

TABLE III

s.no	A	b	C	D(n)
1	CG_{n-1}	CG_n	$CG_{2n-3} - 6n - 3$	D($2n^2 - 3n + 4$)
2	CG_{n-1}	CG_n	$CG_{2n-3} - 2n + 9$	D($4n^2 - 7n + 11$)
3	CG_{n-1}	CG_n	$CG_{2n-3} + 6n - 13$	D($-8n^2 + 17n - 1$)
4	CG_{n-1}	CG_n	$CG_{2n-3} + 6n - 15$	D($-10n^2 + 21n + 4$)
5	CG_{n-1}	CG_n	$CG_{2n-3} + 6n - 17$	D($-12n^2 + 25n + 11$)

B. Case(ii)

Here we take $a = n^2 - 5n + 7$, CG_{n-2} of rank n-2

$b = n^2 - n + 1$, CG_n of rank n

Proceeding as in case(i) we have $c = 4n^2 - 12n + 15$

Therefore the triples $\{n^2 - 5n + 7, n^2 - n + 1, 4n^2 - 12n + 15\}$,ie., $\{CG_{n-2}, CG_n, CG_{2n-3} + 2n + 2\}$ is a special dio 3-tuple with the property $D(-6)$

Some numerical examples satisfying the above mentioned tuples are listed below

TABLE IIIII

n	a	b	c	a+b	a+c	b+c	D(n)
1	3	1	7	4	10	8	-6
2	1	3	7	4	8	10	-6
3	1	7	15	8	16	22	-6
4	3	13	31	16	34	44	-6
5	7	21	55	28	62	76	-6

Below we present 5 sets of special dio 3-tuples with their corresponding properties

TABLE IVV

s.no	A	b	c	D(n)
1	CG_{n-2}	CG_n	$CG_{2n-3}+2n$	$D(-2n^2+6n-11)$
2	CG_{n-2}	CG_n	$CG_{2n-3}+2n+4$	$D(2n^2-6n+1)$
3	CG_{n-2}	CG_n	$CG_{2n-3}+2n-10$	$D(-12n^2+36n-6)$
4	CG_{n-2}	CG_n	$CG_{2n-3}+2n-12$	$D(-14n^2+42n+1)$
5	CG_{n-2}	CG_n	$CG_{2n-3}+2n-2$	$D(-4n^2+12n-14)$

C. *Case(iii)*

Here we take $a=n^2-5n+7$, CG_{n-2} of rank n-2

$b=n^2-3n+2$, CG_{n-1} of rank n-1

Proceeding as in case(ii) we have $c=4n^2-16n+16=CG_{2n-3}-2n+3$

Therefore the triples $\{n^2-5n+7, n^2-3n+2, 4n^2-16n+16\}$,ie., $\{CG_{n-2}, CG_{n-1}, CG_{2n-3}-2n+3\}$ is a special dio 3-tuple with the property $D(-4n^2+15n-14)$

Some numerical examples satisfying the above mentioned tuples are listed below

TABLE V

n	a	b	c	a+b	a+c	b+c	D(n)
4	3	6	16	9	19	22	-18
5	7	12	36	19	43	48	-39
6	13	20	64	33	77	84	-68
7	21	30	100	51	121	130	-105
8	31	42	144	73	175	186	-150

Below we present 5 sets of special dio 3-tuples with their corresponding properties

TABLE VI

s.no	A	b	c	D(n)
1	CG_{n-2}	CG_{n-1}	$CG_{2n-3}-2n+5$	$D(-2n^2+7n-7)$
2	CG_{n-2}	CG_{n-1}	$CG_{2n-3}-2n+7$	$D(-n+2)$
3	CG_{n-2}	CG_{n-1}	$CG_{2n-3}-2n-13$	$D(-20n^2+79n+2)$
4	CG_{n-2}	CG_{n-1}	$CG_{2n-3}-2n-11$	$D(-18n^2+71n-7)$
5	CG_{n-2}	CG_{n-1}	$CG_{2n-3}-2n-9$	$D(-16n^2+63n-14)$

IV. CONCLUSION

In this paper, we have presented some special dio 3-tuples under 3 cases from $\frac{CC_n}{Gno_n}$ with their corresponding properties. One may also search for similar type of special dio 3-tuples with suitable property.

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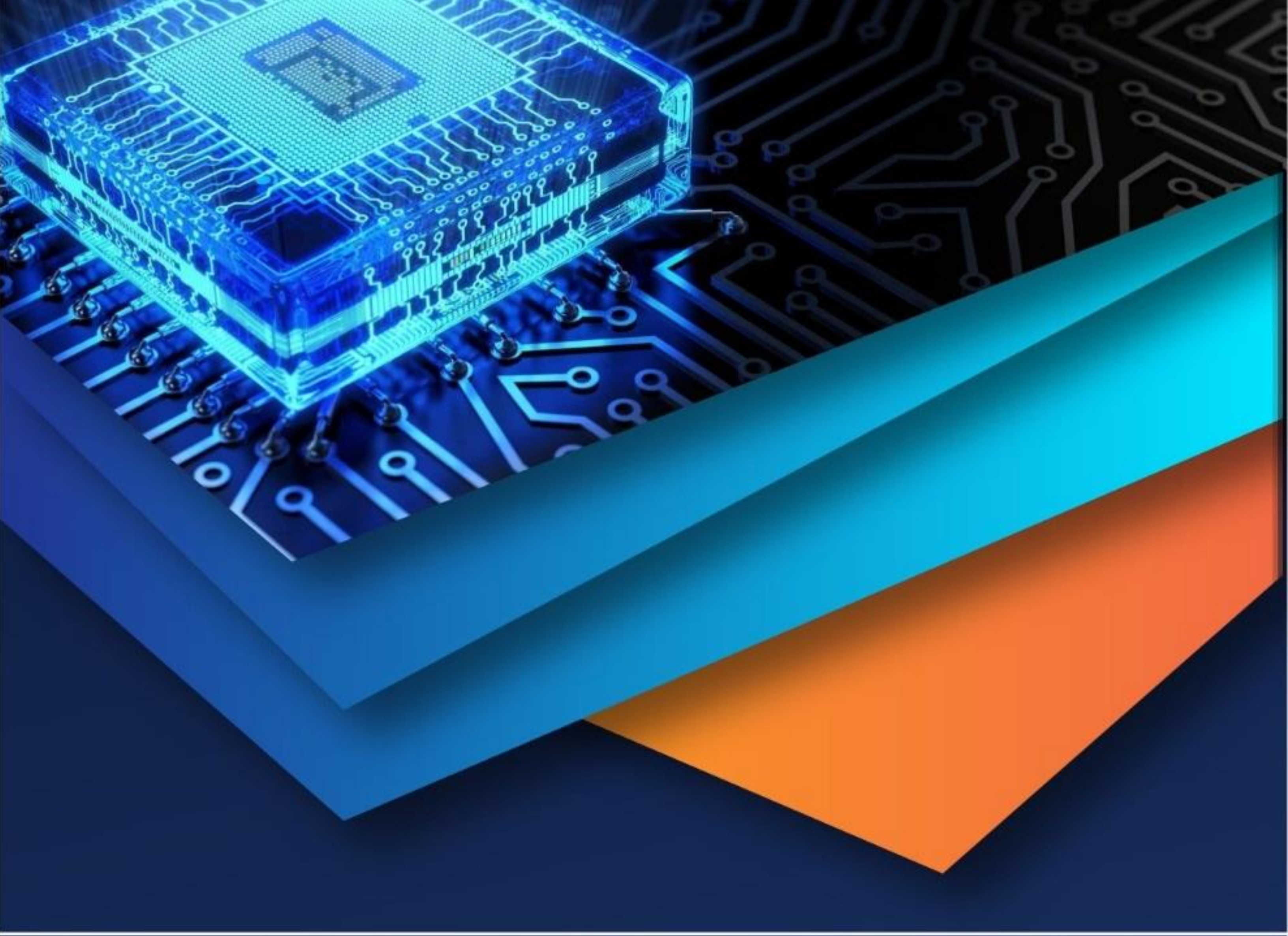

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