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Non Markovian Priority Queuing Model with Compulsory Vacation and Restricted Admissibility

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Abstract: In this paper, we survey a non markovian Queuing model with single server serving two unique lines of priority and non priority customers. Clients landing take after a poisson procedure. The server gives a general support of the priority and non priority units under the supposition of non pre-emptive run the show. We additionally expect that the server takes a necessary vacation of irregular length soon after serving the last client in the priority unit follows a general distribution. In addition, not every one of the clients is permitted to go into the system and join the line of priority and non priority unit. A vital idea of confined acceptability is executed in client's entry. The model is very much characterized and supplementary variable strategy is connected to take care of the issue. We discover the time subordinate probability producing function regarding Laplace changes.
Key words: Non markovian queue, single arrival, restricted admissibility, compulsory vacation, transient solution, queue performance measures. Mathematics subject classification: 60K25,68M30

I. INTRODUCTION

The examination on Queuing models has transformed into an indispensable district as a result of its wide propriety, in reality, conditions; each one of the models considered have had the property that units keep on benefitting on a first begin things out served premise. This is unmistakably not quite recently the method for organization, and there are various decisions, for instance, last begin things out served, assurance in subjective demand and decision by need. In ask for to offer diverse attributes of organization for different sorts of customers, we routinely control a coating system by require component. This ponder is fundamental practically speaking. For case, in media transmission trade tradition, for guaranteeing unmistakable layers of organization for different customers, require classes control may appear in the header of an IP package or in an ATM cell. Restricted admissibility is furthermore comprehensively used as a piece of age hone, transportation administration, and so forth.A M/G/1 line with second discretionary association with general organization time scattering. Borthakur A. furthermore, Chaudhury G.[1], examined a group entry lining model with summed up excursion. Choi. B.D.et.al.[3] made a study on a M/G/1 line with various sorts of information, and gated get-always. Cox. D.R[5] made an investigation on Non-Markovian Stochastic Processes by the joining of Supplementary Variables. Cox. D.R and Miller, H .D. [6] concentrated the speculation of Stochastic Processes. Madan and Anabosi [19] concentrated two sorts of associations with single escape and Bernoulli arrangement outing. Maragathasundari[20] concentrated a mass arriving queueing model with three times of associations took after by advantage interference and put off time. Choudhury[4] examined a globule passage line with an excursion time under single vacation procedure.. Maraghi et.al [23] made an examination on the package entry queueing framework with second discretionary association and sporadic breakdown. Maragathasundari and Srinivasan [21] made an examination on a Non Markovian line with three times of organization and different outings. Kavitha and Maragathasundari [13] investigated limited unremarkable ness and optional sorts of repair in a Non Markovian line. A Non Markoviaan line with discretionary service has been examined by Srinivasan and Maragathasundari [26].A Non Markovian line with multi times of service and reneging have been considered by Maragathasundariet.al[22]. Karthikeyan and Maragathasundari [12] influenced an examination on a mass entry of two times of association with standby server in the midst of general vacation time and general repair time. M [9] investigated a working outing queueing model with various sorts of server breakdowns. Time-subordinate properties of symmetric M/G/1 lines are considered by Kella.O et.al [10] Kumar. R and Sharma. S.K [11] examined reneging and Balking in a non markovian line. Madan. K.C. likewise, Chodhury.G[17] made an examination on M[x]/G/1 line with Bernoulli escape timetable under restricted suitability. Madan. K.C [16], dismembered the line with two stage heterogeneous affiliation and binomial date-book server vacation. Ranjitham .A. in addition, Maragathasundari .S[25] ,considered the two times of association in mass landing lining model. In their survey, if an arriving party of customers find the server included or in trip, by then the entire social occasion joins the float with a particular true objective to examine for the relationship over once more. Ebrahimmalalla and MadanK.C[7] considered a two times of affiliations, the central association being optional with impedance and constrained straightforwardness of sections in the season of interruption in pack territory lining models. Sowmiyah and

Maragathasundari[27] inspected a mass queuing models with opened up experience and stages in repair. Kendall. D.G [14] concentrated the stochastic Processes occurring in the speculation of lines and their examination by the system for introduced Markov chains. Suthersan and Maragathasundari[28]investigated a queueing model of restricted admissibility. Murugeswari. N and Maragathasundari.S[29] studied the queuing model of compulsory vacation with stand by server service. In perspective of all the above research work, another queuing model has been enveloped for the above adaptable correspondence framework. In this paper we consider a Priority queuing framework with a solitary server serving two lines M/G1/1 and M/G2/1 with confined suitability and mandatory server get-away in view of depleted administration of the need units. The administration time takes after a general appropriation. Server takes an obligatory excursion of irregular length when the administration towards the last client in the priority unit gets over. In the event that the server occupied or in the midst of some recreation an arriving non priority client join the line with the probability υ .In addition to that, not all the arriving clients are permitted to join the line. An imperative idea of confined admissibility is considered over the landing of clients. For the above arranged model, we decide the time dependent generating function for both priority unit and non require units to the extent Laplace changes.

II. MATHEMATICAL DESCRIPTION AND ASSUMPTION OF THE MODEL

Clients arrive one by one out of a compound Poisson process. The server should serve all the priority customers before beginning the non priority clients. Likewise here we take after the non pre-emptive control over the clients. Every client under their particular units is served by FCSF teach. The administration time takes after general conveyance with a distribution function $M_i(x)$and probability density function $m_i(x)$. $i = 1, 2$. $\mu_i(x)$be the conditional probability of completion of both types of units service, so that

$$\mu_i(x) = \frac{m_i(x)}{1-M_i(x)} \quad i = 1,2 \quad , m_i(v) = \mu_i(v)e^{\left[-\int_0^v \mu_i(x)dx\right]}, \text{where } i = 1,2 \ . \quad \text{(a)}$$

Moreover, we assume that the server goes for a compulsory vacation with probability ζ after the completion of the service for the last customer present in the priority unit.Vacation time follows general distribution with distribution function K(x) and density function k(s).Let $\chi(x)$ be the conditional probability of a completion of a vacation during the interval (x,x+dx) given that the elapsed vacation time is x.Hence $\chi(x) = \frac{k(x)}{1-K(x)}, \quad k(t) = \chi(t)e^{\left[-\int_0^t \chi(x)dx\right]}$ (b)

In the event that the server is busy or on vacation, an arriving non need customer joins the queue with probability υ. Not all the arriving clients are permitted to join the framework .Let α_i be the probability of arriving need clients and non need clients to join the framework amid non get-away and β be the likelihood of arriving need and non need clients to join the framework amid Vacation time.

A. *Equations governing the system*

$$\frac{\partial}{\partial t}W_{a,b}^{(1)}(x,t) + \frac{\partial}{\partial x}W_{a,b}^{(1)}(x,t) + \left(\lambda_1 + \lambda_2 + \mu_1(x)W_{a,b}^{(1)}(x,t)\right) = \lambda_1\alpha_1 W_{a-1,b}^{(1)}(x,t) + \lambda_2\alpha_2 \upsilon W_{a,b-1}^{(1)}(x,t) \quad (1.1)$$

$$\frac{\partial}{\partial t}W_{a,0}^{(1)}(x,t) + \frac{\partial}{\partial x}W_{a,0}^{(1)}(x,t) + \left(\lambda_1 + \lambda_2 + \mu_1(x)W_{a,0}^{(1)}(x,t)\right) = \lambda_1\alpha_1 W_{a-1,0}^{(1)}(x,t) \quad (1.2)$$

$$\frac{\partial}{\partial t}W_{0,b}^{(1)}(x,t) + \frac{\partial}{\partial x}W_{0,b}^{(1)}(x,t) + \left(\lambda_1 + \lambda_2 + \mu_1(x)W_{0,b}^{(1)}(x,t)\right) = \lambda_2\alpha_2 \upsilon W_{0,b-1}^{(1)}(x,t) \quad (1.3)$$

$$\frac{\partial}{\partial t}W_{0,0}^{(1)}(x,t) + \frac{\partial}{\partial x}W_{0,0}^{(1)}(x,t) + \left(\lambda_1 + \lambda_2 + \mu_1(x)W_{0,0}^{(1)}(x,t)\right) = 0 \quad (1.4)$$

$$\frac{\partial}{\partial t}S_{a,b}(x,t) + \frac{\partial}{\partial x}S_{a,b}(x,t) + (\lambda_1 + \lambda_2 + \chi(x))S_{a,b}(x,t) = \lambda_1\beta S_{a-1,b}(x,t) + \lambda_2\beta\upsilon S_{a,b-1}(x,t) \quad (1.5)$$

$$\frac{\partial}{\partial t}S_{a,0}(x,t)+\frac{\partial}{\partial x}S_{a,0}(x,t)+(\lambda_1+\lambda_2+\chi(x))S_{a,0}(x,t)=\lambda_1\beta S_{a-1,0}(x,t) \quad (1.6)$$

$$\frac{\partial}{\partial t}S_{0,b}(x,t)+\frac{\partial}{\partial x}S_{0,b}(x,t)+(\lambda_1+\lambda_2+\chi(x))S_{0,b}(x,t)=\lambda_2 \upsilon b S_{0,b-1}(x,t) \quad (1.7)$$

$$\frac{\partial}{\partial t}S_{0,0}(x,t)+\frac{\partial}{\partial x}S_{0,0}(x,t)+(\lambda_1+\lambda_2+\chi(x))S_{0,0}(x,t)=0 \quad (1.8)$$

$$\frac{\partial}{\partial t}W_{a,b}^{(2)}(x,t)+\frac{\partial}{\partial x}W_{a,b}^{(2)}(x,t)+\left(\lambda_1+\lambda_2+\mu_1(x)W_{a,b}^{(2)}(x,t)\right)$$
$$=\lambda_1\alpha_1 W_{a-1,b}^{(2)}(x,t)+\lambda_2\alpha_2\upsilon W_{a,b-1}^{(2)}(x,t) \quad (1.9)$$

$$\frac{\partial}{\partial t}W_{a,0}^{(2)}(x,t)+\frac{\partial}{\partial x}W_{a,0}^{(2)}(x,t)+\left(\lambda_1+\lambda_2+\mu_1(x)W_{a,0}^{(2)}(x,t)\right)=\lambda_1\alpha_1 W_{a-1,0}^{(2)}(x,t) \quad (1.10)$$

$$\frac{\partial}{\partial t}W_{0,b}^{(2)}(x,t)+\frac{\partial}{\partial x}W_{0,b}^{(2)}(x,t)+\left(\lambda_1+\lambda_2+\mu_1(x)W_{0,b}^{(2)}(x,t)\right)=\lambda_2\alpha_2\upsilon W_{0,b-1}^{(2)}(x,t) \quad (1.11)$$

$$\frac{\partial}{\partial t}W_{0,0}^{(2)}(x,t)+\frac{\partial}{\partial x}W_{0,0}^{(2)}(x,t)+\left(\lambda_1+\lambda_2+\mu_1(x)W_{0,0}^{(2)}(x,t)\right)=0 \quad (1.12)$$

$$\frac{d}{dt}Q(t)+(\lambda_1+\lambda_2)Q(t)=(\lambda_1(1-\alpha_1)+\lambda_2(1-\alpha_2))Q(t)+\int_0^\infty S_{0,0}(x,t)dx+\int_0^\infty W_{0,0}^{(2)}(x,t)\mu_2(x)dx \quad (1.13)$$

B. Boundary conditions

The boundary conditions are given by

$$W_{a,b}^{(1)}(0,t)=\int_0^\infty W_{m+1,n}^{(1)}(x,t)\mu_1(x)dx+\int_0^\infty W_{m+1,n}^{(1)}(x,t)\mu_2(x)dx$$
$$+\int_0^\infty S_{m+1,n}(x,t)\chi(x)dx \quad (1.14)$$

$$W_{a,0}^{(1)}(0,t)=\lambda_1\alpha_1 Q(t)+\int_0^\infty W_{a+1,0}^{(1)}(x,t)\mu_1(x)dx+\int_0^\infty W_{a+1,0}^{(2)}(x,t)\mu_2(x)dx+\int_0^\infty S_{a+1,0}(x,t)\chi(x)dx \quad (1.15)$$

$$W_{0,b}^{(1)}(0,t)=\int_0^\infty W_{1,b}^{(1)}(x,t)\mu_1(x)dx+\int_0^\infty W_{1,b}^{(2)}(x,t)\mu_2(x)dx$$
$$+\int_0^\infty S_{1,b}(x,t)\chi(x)dx \quad (1.16)$$

$$W_{0,0}^{(1)}(0,t)=\lambda_1\alpha_1 Q(t)+\int_0^\infty W_{1,0}^{(1)}(x,t)\mu_1(x)dx+\int_0^\infty W_{1,0}^{(2)}(x,t)\mu_2(x)dx+\int_0^\infty S_{1,0}(x,t)\chi(x)dx$$

$$S_{0,b}(x,t)=\zeta\int_0^\infty W_{0,b}^{(1)}(x,t)\mu_1(x)dx$$
$$W_{0,0}^{(2)}(0,t)=\lambda_2\alpha_2 Q(t)+\int_0^\infty W_{0,1}^{(2)}(x,t)\mu_1(x)dx+\int_0^\infty S_{0,1}(x,t)\chi(x)dx$$
$$W_{0,b}^{(2)}(0,t)=\lambda_2\alpha_2 Q(t)+\int_0^\infty W_{0,b+1}^{(2)}(x,t)\mu_2(x)dx+\int_0^\infty S_{0,b+1}(x,t)\chi(x)dx \quad (1.17)$$

C. Initial conditions

We assume that there are initially no customers in the system.Hence Q(0) =1,
$S_{a,b}(0)=S_{0,b}(0)=S_{a,0}(0)=S_{0,0}(0)=0$and for i = 1,2 we have

$$W_{0,0}^{(i)}(0) = W_{a,0}^{(i)}(0) = W_{0,b}^{(i)}(0) = W_{a,b}^{(i)}(0) = 0$$

D. Probability generating function

The probability generating functions are defined as follows:

$$\sum_{a=0}^{\infty}\sum_{b=0}^{\infty} z_1^a z_2^b\, W_{a,b}^{(1)}(x,t) = W^{(1)}(x,z_1,z_2), \sum_{a=0}^{\infty}\sum_{b=0}^{\infty} z_1^a z_2^b\, W_{a,b}^{(2)}(x,t) = W^{(2)}(x,z_1,z_2),$$

$$\sum_{a=0}^{\infty}\sum_{b=0}^{\infty} z_1^a z_2^b\, S_{a,b}(x,t) = S(x,z_1,z_2), \sum_{a=0}^{\infty} z_1^a W_a^{(1)}(x,t) = W^{(1)}(x,z_1),$$

$\sum_{b=0}^{\infty} z_2^b W_b^{(1)}(x,t) = W^{(1)}(x,z_2)$, $\sum_{a=0}^{\infty} z_1^a W_a^{(2)}(x,t) = W^{(2)}(x,z_1)$,
$\sum_{b=0}^{\infty} z_2^b W_n^{(2)}(x,t) = W^{(2)}(x,z_2)$, $\sum_{a=0}^{\infty} z_1^a S_a(x,t) = S(x,z_1)$, (1.18)
$\sum_{b=0}^{\infty} z_2^b S_b(x,t) = S(x,z_2)$,

All the above are convergent inside the circle given by $|z_1| \le 1, |z_2| \le 1$

III. USAGE OF LAPLACE TRANSFORM

Next we define the Laplace transform of a function f(t) as f*(s) =$\int_0^{\infty} f(t)e^{-st}\, dt$

Applying the process of Laplace transform for the equations defined in 1.1 – 1.17. we get

$$\frac{\partial}{\partial x} W_{a,b}^{(1)*}(x,s) + \left(s + \lambda_1 + \lambda_2 + \mu_1(x) W_{a,b}^{(1)}(x,s)\right) = \lambda_1\alpha_1 W_{a-1,b}^{(1)*}(x,s) + \lambda_2\alpha_2\upsilon W_{a,b-1}^{(1)*}(x,s) \quad (1.19)$$

$$\frac{\partial}{\partial x} W_{a,0}^{(1)*}(x,s) + \left(s + \lambda_1 + \lambda_2 + \mu_1(x) W_{a,0}^{(1)*}(x,s)\right) = \lambda_1\alpha_1 W_{a-1,0}^{(1)*}(x,s) \quad (1.20)$$

$$\frac{\partial}{\partial x} W_{0,b}^{(1)*}(x,s) + \left(s + \lambda_1 + \lambda_2 + \mu_1(x) W_{0,b}^{(1)}(x,s)\right) = \lambda_2\alpha_2\upsilon W_{0,b-1}^{(1)*}(x,s) \quad (1.21)$$

$$\frac{\partial}{\partial x} W_{0,0}^{(1)*}(x,s) + \left(s + \lambda_1 + \lambda_2 + \mu_1(x) W_{0,0}^{(1)*}(x,s)\right) = 0 \quad (1.22)$$

$$\frac{\partial}{\partial x} S^*{}_{a,b}(x,s) + (s + \lambda_1 + \lambda_2 + \chi(x)) S^*{}_{a,b}(x,s) = \lambda_1\beta S^*{}_{a-1,b}(x,t) + \lambda_2\beta\upsilon S^*{}_{a,b-1}(x,s) \quad (1.23)$$

$$\frac{\partial}{\partial x} S^*{}_{a,0}(x,s) + (s + \lambda_1 + \lambda_2 + \chi(x)) S^*{}_{a,0}(x,s) = \lambda_1\beta S^*{}_{a-1,0}(x,s) \quad (1.24)$$

$$\frac{\partial}{\partial x} S^*{}_{0,b}(x,s) + (s + \lambda_1 + \lambda_2 + \chi(x)) S^*{}_{0,b}(x,s) = \lambda_2\beta\upsilon S_{0,b-1}(x,s) \quad (1.25)$$

$$\frac{\partial}{\partial x} S^*{}_{0,0}(x,s) + (s + \lambda_1 + \lambda_2 + \chi(x)) S^*{}_{0,0}(x,s) = 0 \quad (1.26)$$

$$\frac{\partial}{\partial x} W_{a,b}^{(2)*}(x,s) + (s + \lambda_1 + \lambda_2 + \mu_1(x)) W_{a,b}^{(2)*}(x,s) = \lambda_1\alpha_1 W_{a-1,b}^{(2)*}(x,s) + \lambda_2\alpha_2\upsilon W_{a,b-1}^{(2)*}(x,s) \quad (1.27)$$

$$\frac{\partial}{\partial x} W_{a,0}^{(2)*}(x,s) + (\lambda_1 + \lambda_2 + \mu_1(x)) W_{a,0}^{(2)*}(x,s) = \lambda_1\alpha_1 W_{a-1,0}^{(2)*}(x,t) \quad (1.28)$$

$$\frac{\partial}{\partial x} W_{0,b}^{(2)*}(x,s) + (s + \lambda_1 + \lambda_2 + \mu_1(x)) W_{0,b}^{(2)*}(x,s) = \lambda_2\alpha_2\upsilon W_{0,b-1}^{(2)*}(x,s) \quad (1.29)$$

$$\frac{\partial}{\partial x}W_{0,0}^{(2)*}(x,s) + (\lambda_1 + \lambda_2 + \mu_1(x))W_{0,0}^{(2)*}(x,s) = 0 \qquad (1.30)$$

$$(s + \lambda_1\alpha_1 + \lambda_2\alpha_2)Q^*(s) - 1 = \int_0^\infty S^*{}_{0,0}(x,s)dx + \int_0^\infty W_{0,0}^{(2)*}(x,s)\mu_2(x)dx \qquad (1.31)$$

Applying the same for the boundary conditions, we have

$$W_{a,b}^{(1)*}(0,s) = \int_0^\infty W_{a+1,b}^{(1)*}(x,s)\mu_1(x)dx + \int_0^\infty W_{a+1,b}^{(1)*}(x,s)\mu_2(x)dx + \int_0^\infty S^*{}_{a+1,b}(x,s)\chi(x)dx \qquad (1.32)$$

$$W_{a,0}^{(1)*}(0,s) = \lambda_1\alpha_1 Q(s) + \int_0^\infty W_{a+1,0}^{(1)*}(x,s)\mu_1(x)dx + \int_0^\infty W_{a+1,0}^{(2)*}(x,s)\mu_2(x)dx + \int_0^\infty S^*{}_{a+1,0}(x,s)\chi(x)dx \qquad (1.33)$$

$$W_{0,b}^{(1)*}(0,s) = \int_0^\infty W_{1,b}^{(1)*}(x,s)\mu_1(x)dx + \int_0^\infty W_{1,b}^{(2*)}(x,s)\mu_2(x)dx + \int_0^\infty S^*{}_{1,b}(x,s)\chi(x)dx \qquad (1.34)$$

$$W_{0,0}^{(1)*}(0,s) = \lambda_1\alpha_1 Q(s) + \int_0^\infty W_{1,0}^{(1)*}(x,s)\mu_1(x)dx + \int_0^\infty W_{1,0}^{(2)*}(x,s)\mu_2(x) + \int_0^\infty S^*{}_{1,0}(x,s)\chi(x)dx \qquad (1.35)$$

$$S_{0,b}(x,s) = \zeta\int_0^\infty W_{0,b}^{(1)*}(x,s)\mu_1(x)dx \qquad (1.36)$$

$$W_{0,0}^{(2)*}(0,s) = \lambda_2\alpha_2 Q^*(s) + \int_0^\infty W_{0,1}^{(2)*}(x,s)\mu_1(x)dx + \int_0^\infty S^*{}_{0,1}(x,s)\chi(x)dx \qquad (1.37)$$

$$W_{0,b}^{(2)*}(0,s) = \lambda_2\alpha_2 Q^*(s) + \int_0^\infty W_{0,b+1}^{(2)*}(x,s)\mu_2(x)dx + \int_0^\infty S^*{}_{0,b+1}(x,s)\chi(x)dx \qquad (1.38)$$

IV. SUPPLEMENTARY VARIABLE METHOD

Now we multiply equations 1.19, 1.21, 1.23, 1.25, 1.27, 1.29 by z_2^b , summing over n from 1 to ∞ , adding to equations 1.20, 1.22, 1.24, 1.26, 1.28, 1.30 and using the generating function approach (1.18) we get the following :

$$\frac{\partial}{\partial x}W_a^{(1)*}(x,s,z_2) + (s + \lambda_1 + \lambda_2(1-\alpha_2\upsilon z) + \mu_1(x))W_a^{(1)}(x,s,z_2) = \lambda_1\alpha_1 W_{a-1,0}^{(1)*}(x,s,z_2) \qquad (1.39)$$

$$\frac{\partial}{\partial x}W_a^{(2)*}(x,s,z_2) + (s + \lambda_1 + \lambda_2(1-\alpha_2\upsilon z) + \mu_2(x))W_a^{(2)}(x,s,z_2) = \lambda_1\alpha_1 W_{a-1,0}^{(2)*}(x,s,z_2) \qquad (1.40)$$

$$\frac{\partial}{\partial x}W_0^{(1)*}(x,s,z_2) + (s + \lambda_1 + \lambda_2(1-\alpha_2\upsilon z) + \mu_1(x))W_0^{(1)*}(x,s,z_2) = 0 \qquad (1.41)$$

$$\frac{\partial}{\partial x}W_0^{(2)*}(x,s,z_2) + (s + \lambda_1 + \lambda_2(1-\alpha_2 z) + \mu_2(x))W_0^{(2)*}(x,s,z_2) = 0 \qquad (1.42)$$

$$\frac{\partial}{\partial x}S_a{}^*(x,s,z_2) + (s + \lambda_1 + \lambda_2(1-\beta\upsilon z) + \chi(x))S_a{}^*(x,s,z_2) = \lambda_1\alpha_1 S^*{}_{a-1,0}(x,s,z_2) \qquad (1.43)$$

$$\frac{\partial}{\partial x}S_0{}^*(x,s,z_2) + (s + \lambda_1 + \lambda_2(1-\beta\upsilon z) + \chi(x))S_0{}^*(x,s,z_2) = 0 \qquad (1.44)$$

Next, multiply eq.(1.39) , 1.41 and 1.43 by z_1^asumming over from 1 to ∞ and adding to equations 1.40, 1.42 and 1.44 respectively and using the generating function defined in 1.18, we get

$$\frac{\partial}{\partial x}W^{(1)*}(x,s,z_1,z_2) + (s + \lambda_1(1-\alpha_1 z) + \lambda_2(1-\alpha_2\upsilon z) + \mu_1(x))W^{(1)*}(x,s,z_1,z_2) = 0 \qquad (1.45)$$

$$\frac{\partial}{\partial x}W^{(2)*}(x,s,z_1,z_2) + (s + \lambda_1(1-\alpha_1 z) + \lambda_2(1-\alpha_2 \upsilon z) + \mu_2(x))W^{(2)*}(x,s,z_1,z_2) = 0 \quad (1.46)$$

$$\frac{\partial}{\partial x}S^*(x,s,z_1,z_2) + \left(s + \lambda_1(1-\beta z) + \lambda_2(1-\beta \upsilon z) + \chi(x))S^*(x,s,z_1,z_2)\right) = 0 \quad (1.47)$$

For the boundary conditions we multiply both sides of equations (1.32) and (1.33) by z_1^{a+1}, summing over a from 1 to ∞ , adding to equations z_1 × (1.34) and z_1 × (1.35) and use the equation (1.18),we get

$$z_1 W_b^{(1)*}(0,s,z_1) = \lambda_1\alpha_1 z_1 Q^*(s) + \int_0^\infty W_b^{(1)*}(x,s,z_1)\mu_1(x)dx - \int_0^\infty W_{0,b}^{(1)*}(x,s)\mu_1(x)dx + \int_0^\infty S_b{}^*(x,s,z_1)\chi(x)dx + \int_0^\infty W_b^{(2)*}(x,s,z_1)\mu_2(x)dx - \int_0^\infty W_{0,b}^{(2)*}(x,s)\mu_2(x)dx - \int_0^\infty S_{0,b}{}^*(x,s)\chi(x)dx$$

(1.48)

$$z_1 W_0^{(1)*}(0,s,z_1) = \lambda_1\alpha_1 z_1 Q^*(s) + \int_0^\infty W_0^{(1)*}(x,s,z_1)\mu_1(x)dx - \int_0^\infty W_{0,0}^{(1)*}(x,s)\mu_1(x)dx + \int_0^\infty S_0{}^*(x,s,z_1)\chi(x)dx + \int_0^\infty W_0^{(2)*}(x,s,z_1)\mu_2(x)dx - \int_0^\infty W_{0,0}^{(2)*}(x,s)\mu_2(x)dx - \int_0^\infty S_{0,0}{}^*(x,s)\chi(x)dx \quad (1.49)$$

Now multiply equations (1.48) and (1.38) by z_2^b summing over b from 1 to ∞ , adding to equations (1.49) and z_2 × (1.37) and using the equation (1.18), we get

$$z_1 W^{(1)*}(0,s,z_1,z_2) = \lambda_1\alpha_1 z_1 Q(s) + \int_0^\infty W^{(1)*}(x,s,z_1,z_2)\mu_1(x)dx - \int_0^\infty W_0^{(1)*}(x,s,z_2)\mu_1(x)dx + \int_0^\infty S^*(x,s,z_1,z_2)\chi(x)dx + \int_0^\infty W^{(2)*}(x,s,z_1,z_2)\mu_2(x)dx - \int_0^\infty W_0^{(2)*}(x,s,z_2)\mu_2(x)dx - \int_0^\infty S_0{}^*(x,s,z_2)\chi(x)dx \quad (1.50)$$

$$z_2 W_0^{(2)*}(0,s,z_2) = \lambda_2\alpha_2 z_2 Q(s) + \int_0^\infty S_0{}^*(x,s,z_2)\chi(x)dx + \int_0^\infty W_0^{(2)*}(x,s,z_2)\mu_2(x)dx - \int_0^\infty W_{0,0}^{(2)*}(x,s,z_2)\mu_2(x)dx - \int_0^\infty S_{0,0}{}^*(x,s)\chi(x)dx \quad (1.51)$$

Now multiply eq (1.36) by z_2^b summing over b from 0 to ∞ , and using generating function, we get

$$S_{0,b}{}^*(x,s,z_2) = \zeta \int_0^\infty W_0^{(1)*}(x,s,z_2)\mu_1(x)dx \quad (1.52)$$

Integrate equation (1.45) between 0 to x, we obtain,

$$W^{(1)*}(x,s,z_1,z_2) = W^{(1)*}(0,s,z_1,z_2)\exp\left(-\left(s + \lambda_1(1-\alpha_1 z) + \lambda_2(1-\alpha_2 \upsilon z)x - \int_0^x \mu_1(t)dt\right)\right. \quad (1.53)$$

Again integrate equation (1.52) by parts, with respect to x, and we get

$$W^{(1)*}(s,z_1,z_2) = W^{(1)*}(0,s,z_1,z_2)\left[\frac{1-\overline{M_1}[s + \lambda_1(1-\alpha_1 z) + \lambda_2(1-\alpha_2 \upsilon z)]}{s + \lambda_1(1-\alpha_1 z) + \lambda_2(1-\alpha_2 \upsilon z)}\right] \quad (1.54)$$

Next multiply equation (1.52) by $\mu_1(x)$ and integrate with respect to x, we get

$$\int_0^\infty W^{(1)*}(x,s,z_1,z_2)\mu_1(x)dx = W^{(1)*}(0,s,z_1,z_2)\overline{M_1}[s + \lambda_1(1-\alpha_1 z) + \lambda_2(1-\alpha_2 \upsilon z)] \quad (1.55)$$

Performing similar operations on Equations (1.41), (1.46), (1.42) ,(1.47),(1.44), we get

$$\int_0^\infty W_0^{(1)*}(x,s,z_2)\mu_1(x)dx = W^{(1)*}(0,s,z_2)\overline{M_1}[s+\lambda_1+\lambda_2(1-\alpha_2\upsilon z)] \quad (1.56)$$

$$\int_0^\infty W^{(2)*}(x,s,z_1,z_2)\mu_2(x)dx = W^{(2)*}(0,s,z_1,z_2)\overline{M_2}[s+\lambda_1(1-\alpha_1 z)+\lambda_2(1-\alpha_2\upsilon z)] \quad (1.57)$$

$$\int_0^\infty W_0^{(2)*}(x,s,z_2)\mu_2(x)dx = W^{(2)*}(0,s,z_2)\overline{M_2}[s+\lambda_1+\lambda_2(1-\alpha_2\upsilon z)] \quad (1.58)$$

$$\int_0^\infty S^*(x,s,z_1,z_2)\chi(x)dx = S^*(x,s,z_1,z_2)\bar{K}[s+\lambda_1(1-\beta z)+\lambda_2(1-\beta\upsilon z)] \quad (1.59)$$

$$\int_0^\infty S_0{}^*(x,s,z_2)\chi(x)dx = S_0{}^*(x,s,z_2)\bar{K}[s+\lambda_1+\lambda_2(1-\beta\upsilon z)] \quad (1.60)$$

Now substitute equations (1.55) – (1.60) in equation. (1.50), we get

$$(z_1-\overline{M_1}[s+\lambda_1(1-\alpha_1(z_1)+\lambda_2(1-\alpha_2 bz)])W^{(1)}(0,s,z_1,z_2) = \lambda_1\alpha_1 z_1 Q(s) + W_0^{(2)*}(0,s,z_2)([\overline{M_2}[s+\lambda_1(1-\alpha_1 z_1)+\lambda_2(1-\alpha_2 bz_2)]-\overline{M_2}[s+\lambda_1+\lambda_2(1-\alpha_2 bz_2)]) - W_0^{(1)*}(x,s,z_2)(\overline{M_1}[s+\lambda_1+\lambda_2(1-\alpha_2 bz_2)])(1-\zeta\bar{K}[s+\lambda_1(1-\beta z_1)+\lambda_2(1-\beta bz_2)] + \zeta\bar{K}[s+\lambda_1+\lambda_2(1-\beta bz_2)]) \quad (1.61)$$

Next using equations (1.31), (1.56), (1.58) and (1.60) in equation (1.51), we get

$$W_0^{(2)*}(0,s,z_2) = \frac{\lambda_2\alpha_2 z_2\upsilon Q(s) + 1 - (s+\lambda_1+\lambda_2\upsilon)Q(s) + \bar{K}[s+\lambda_1+\lambda_2(1-\beta\upsilon z)]\zeta\overline{M_1}[s+\lambda_1+\lambda_2(1-\alpha_2\upsilon z)]W_0^{(1)*}(0,s,z_2)}{(z_2-\overline{M_2}[s+\lambda_1+\lambda_2(1-\alpha_2\upsilon z)])} \quad (1.62)$$

Using the concept of Rouche's theorem, $(z_1-\overline{M_1}[s+\lambda_1(1-\alpha_1 z)+\lambda_2(1-\alpha_2\upsilon z)]$ has one and only one zero inside the circle $|z_1| = 1$ and $|z_2| \le 1$

Hence we have

$$W_0^{(1)*}(0,s,z_2) = \frac{\lambda_2\alpha_2 z_2\upsilon Q(s) + 1 - (s+\lambda_1+\lambda_2\upsilon)Q(s) + \bar{K}[s+\lambda_1+\lambda_2(1-\beta\upsilon z)]\zeta\overline{M_1}[s+\lambda_1+\lambda_2(1-\alpha_2\upsilon z)]\lambda_1\alpha_1 g(z_2)Q(s)}{(z_2-\overline{M_2}[s+\lambda_1+\lambda_2(1-\alpha_2\upsilon z)])(\overline{M_1}[s+\lambda_1+\lambda_2(1-\alpha_2\upsilon z_2)]1-\zeta\bar{K}[s+\lambda_1(1-\beta z_1)+\lambda_2(1-\beta bz_2)] + \zeta\bar{K}[s+\lambda_1+\lambda_2(1-\beta\upsilon z_2)]) - \bar{K}[s+\lambda_1+\lambda_2(1-\beta\upsilon z_2)]\zeta\overline{M_1}[s+\lambda_1+\lambda_2(1-\alpha_2\upsilon z_2)]\overline{(M_2}[s+\lambda_1(1-\alpha_1 z_1)+\lambda_2(1-\alpha_2\upsilon z_2)]-\overline{M_2}[s+\lambda_1+\lambda_2(1-\alpha_2\upsilon z_2)])} \quad (1.63)$$

Substituting equation (1.62) and (1.63) in equation (161), we get $W^{(1)*}(0,s,z_1,z_2)$

Thus substitute the values of $W^{(1)*}(0,s,z_1,z_2)$, $W_0^{(1)*}(0,s,z_2)$ and $W_0^{(2)*}(0,s,z_2)$ in equations

$$W^{(1)*}(s,z_1,z_2) = W^{(1)*}(0,s,z_1,z_2)\left[\frac{1-\overline{M_1}[s+\lambda_1(1-\alpha_1 z_1)+\lambda_2(1-\alpha_2\upsilon z_2)]}{s+\lambda_1(1-\alpha_1 z_1)+\lambda_2(1-\alpha_2\upsilon z_2)}\right]$$

(1.64)

$$W^{(2)*}(s,z_1,z_2) = W^{(2)*}(0,s,z_2)\left[\frac{1-\overline{M_2}[s+\lambda_1(1-\alpha_1 z_1)+\lambda_2(1-\alpha_2\upsilon z_2)]}{s+\lambda_1(1-\alpha_1 z_1)+\lambda_2(1-\alpha_2\upsilon z_2)}\right]$$

(1.65)

$V^*(s, z_1, z_2) = \zeta W^{(1)*}(0, s, z_1, z_2) =$

$$\overline{M_1}[s + \lambda_1 + \lambda_2(1 - \alpha_2 \upsilon z_2)]\left[\frac{1 - \bar{K}[s + \lambda_1(1 - \alpha_1 z_1) + \lambda_2(1 - \alpha_2 \upsilon z_2)]}{s + \lambda_1(1 - \alpha_1 z_1) + \lambda_2(1 - \alpha_2 \upsilon z_2)}\right]$$

(1.66)

Thus Time dependent solutions are completely derived from equation (1.64) – (1.66)

V. CONCLUSION

In this paper, we have researched a queuing arrangement of priority and non need units in which limited acceptability is mulled over. Clients arrive one by one. When the administration towards the priority unit gets finished, the server needs to experience a necessary get-away of irregular length. If the server is occupied or on anvacation, a non need unit client are permitted to go into the unit .Time subordinate arrangements are determined for the above characterized demonstrate. As a future work consistent state arrangement and other line execution can be inferred, alternate parameters phases of administration, discretionary sorts of administration, separate, discretionary repair can likewise be included.

REFERENCES

[1] A.Borthakur and G.Chaudhury, (1997), “On a batch arrival poisson queue with generalised vacation, sankhya ser.B,59,369-383
[2] Chae.K.C, Lee H.W and Ahn. C.W (2001), “An arrival time approach to M/G/1- type queues with generalized vacations”, Queueing Systems, Vol.38,pp. 91-100
[3] Choi. B.D., Kim. B. and Choi. S.H., (2003), “A M/G/1 queue with multiple types of feedback, gated vacations and FCFS policy”, Computers and Operations Research, Vol.30, No.9, pp.1289-1309
[4] Choudhury, G., (2002), “A batch arrival queue with a vacation time under single vacation policy”, Computers and Operations Research, Vol.29, No.14, pp.1941-1955
[5] Cox. D.R (1955), “The analysis of Non-Markovian Stochastic Processes by the inclusion of Supplementary Variables”, Proceedings of the Cambridge Philosophical Society, Vol.51, pp.433-441
[6] Cox. D.R and Miller, H .D. (1965), “The theory of Stochastic Processes”, Chapman & Hall
[7] Ebrahimmalalla and Madan.K.C (2017), “On a batch arrival queue with second optional sevice, random breakdowns, delay time for repairs to start and restricted availability of arrival during breakdowns”, J.Math.Comput.Sci, Vol.7,No.1,175-188
[8]]Gross C. And C.M.Harris (1985), “ The fundamentals of queuing theory, Second edition, John Wiley and Sons, New York
[9] Jain. M & Jain. A (2010), “Working vacation queueing model with multiple types of server breakdowns”, Applied Mathematical Modelling, Vol.34, pp.1-1
[10] Kella.O, Zwart. B &Boxma. O (2005), “Some time-dependent properties of symmetric M/G/1 queues”, Journal of Applied Probability, Vol.42, No.1, pp.223-234
[11] R and Sharma . S.K (2012), “A Markovian Feedback Queue with Retention of Reneged Customers and Balking”, AMO-Advanced Modeling and Optimization, Vol.14, No.3, pp.681-688
[12] Karthikeyan.K and Maragathasundari.S (2015), Batch arrival of two stages with standby server during general vacation time & general repair time”, International Journal ofMathematical archive, Vol.6, No.4, pp.43-48
[13] Kavitha. K and Maragathasundari.S (2012),“Analysis of Non Markovian queue with restricted admissibility and optional types of repair”, International journal of scientific research and management studies, Vol.2, No.5, pp.244-252.
[14] Kendall. D.G (1953), “Stochastic Processes occurring in the theory of queues and their analysis by the method of embedded Markov chain”, Annals of Mathematical Statistics, Vol.24
[15] Kashyap.B.R.K. and ChaudhryM.L (1988), “An introduction to Queuing theory,A and A publications,Kingston.Ontario,Canada.
[16] Madan. K.C (2000), “An M/G/1 queue with second optional service”, Queueing Systems Vol.34, pp.37-46
[17] Madan. K.C. and Chodhury. G (2004), “An $M^{[x]}$/G/1 queue with Bernoulli vacation schedule under restricted admissibility policy”,Sankhaya,Vol.66, pp.172-193
[18] MadanK.C(2011),A Non preemptive priority queuing system with a single server serving two queues M/G/1 and M/D/1 with optional server vacation based on exhaustive service of priority units,Applied Mathematics,Vol.2,pp.791-799
[19] Madan. K.C and Anabosi. R.F (2003), “A single server queue with two types of service, Bernoulli schedule server vacations and a single vacation policy”, Pakistan Journal ofStatistics, Vol.19, pp.331-442.
[20] Maragathasundari.S (2015), “A bulk arrival queuing model of three stages of service with different vacation policies, service interruption and delay time”, American InternationalJournal of Research in Science, Technology, Engineering& Mathematics, Vol.11, No.1,pp.52-56
[21] Maragathasundari.S and Srinivasan.S (2012), “Analysis of M/G/1 feedback queue with three stage and multiple server vacation”, Applied mathematical sciences, Vol.6, No.125, pp.6221-6240
[22] Maragathasundari.S and Srinivasan.S (2015), “A Non-Markovian Multistage Batch arrival queue with breakdown and reneging”, Mathematical problems in engineering, Volume 2014/16 pages/ Article ID 519579/ http: // dx. Doi. Org / 10.1155/2014/ 519579.
[23] Maraghi. F. A, Madan. K.C and Darby-Dowman. K (2010), “Batch Arrival vacation queue with second optional service and Random Breakdowns”, International Journal of StatisticalTheory and Practice, Vol.4, No.1, pp.137-153
[24] Maraghi. F.A, Madan. K.C and Darby-Dowman. K (2009), “Batch Arrival queuing system with Random Breakdowns and Bernoulli Schedule server vacations having General vacation Time Distribution”, International Journal of Information and Management Sciences, Vol.20, pp.55-70.

[25] Ranjitham .A. and Maragathasundari .S (2014), "Batch arrival queueing system with two stages of service", International journal of Math.Analysis, Vol.8 (6), pp.247-25

[26] Srinivasan. S and Maragathasundari.S (2017), "Optional services in a Non Markovian Queue", International Journal of Operational Research, Vol.28, No.1, pp.1-17

[27] Sowmiyah. S and Maragathasundari. S (2015), "M/G/1 queueing system with extended vacation, service interruption, Delay time and stages in repair", Journal of Computer and Mathematical Sciences, Vol.6 (7), 363-370, July 2015

[28] SuthersanP. and MaragathasundariS.(2017), "), Co axian – 2 general queuing Model with Restricted Admissibility,International journal of scientific research in science ,Engineering and technology,| Volume 3 | Issue 6 | Print ISSN: 2395-1990 | Online ISSN : 2394-4099

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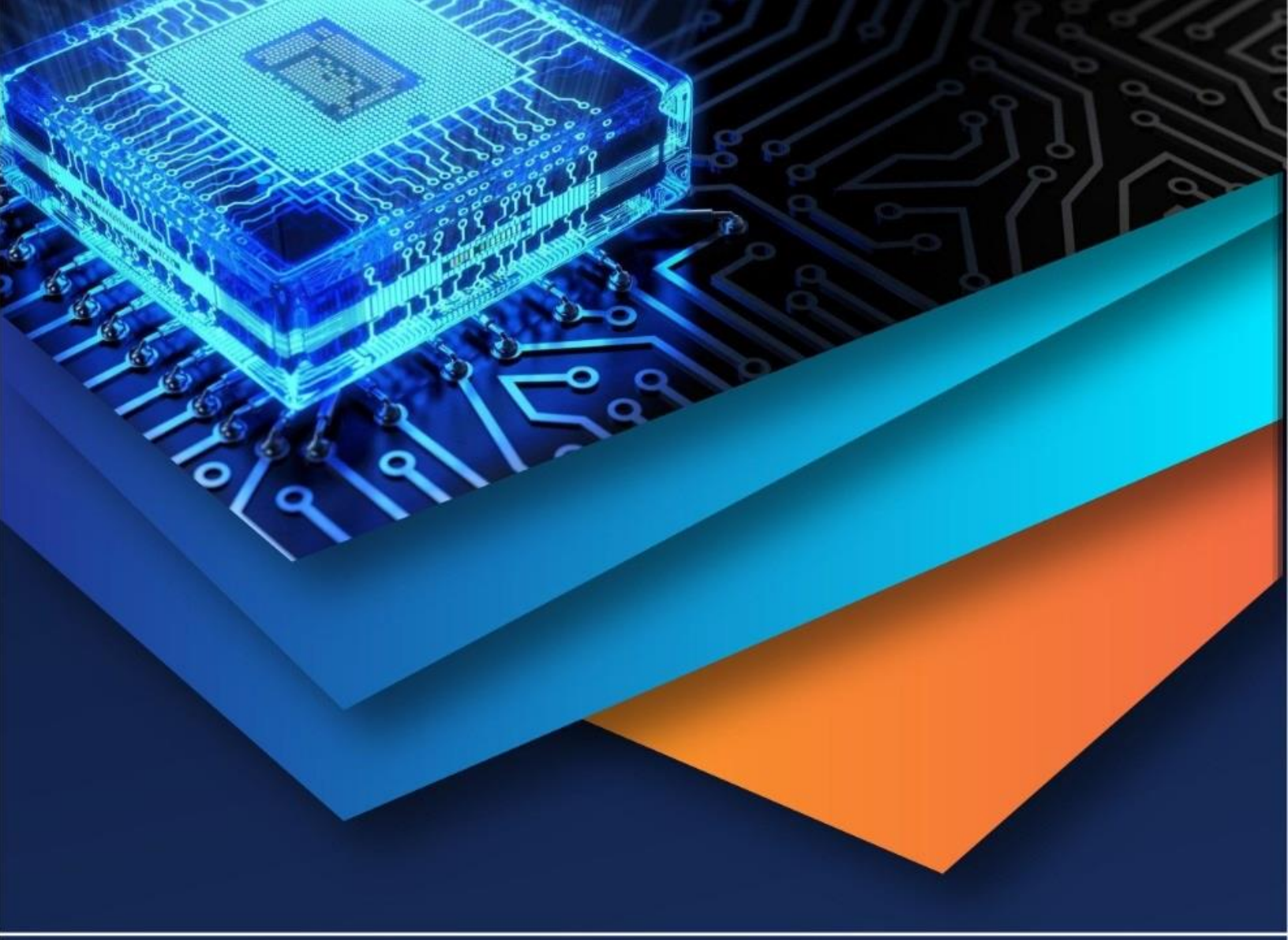

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