iJRASET
International Journal For Research in
Applied Science and Engineering Technology

INTERNATIONAL JOURNAL FOR RESEARCH
IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY

Volume: 5 Issue: XII Month of publication: December 2017

DOI:

Free Vibration Response of Functionally Graded Material Plate Subjected To Simply Supported and Simply Supported-Clamped Boundary Conditions

Pankaj Bohra[1], Piyush Sharma[2], Dr. Manish Bhandari[3], Shailendra Bohra[4], Ved Prakash[5]
[1]*Lecturer Mechanical Engineering, Government Polytechnic College Bagidora, Distt. Banswara Rajasthan*
[2]*Phd. Scholar, M.B.M Engineering College, JNVU Jodhpur Rajasthan*
[3]*Assistant Professor, Deptt. of Mechanical Engineering, M.B.M Engineering College JNVU Jodhpur Rajasthan*
[4]*Associate Professor Deptt. Of Mechanical Engineering, VIET Jodhpur Rajasthan*
[5]*Lecturer Civil Engineering, Government Polytechnic College Bagidora, Distt. Banswara Rajasthan*

Abstract*: Response of Free Vibration behaviour of Functionally Graded Aluminium / Zirconia plates using finite element method. In this study, the following boundary conditions of FGM plate are considered: (i) all edges are simply supported (SSSS) (ii) two adjacent edges are simply supported and other two adjacent edges are clamped (SSCC). Functionally Graded Material Plate are assumed to be isotropic and effective material properties of FGM plate are graded in thickness direction. Material properties idealization are defined by simple distribution laws which are defined in terms of the volume fractions of the constituent's i.e. Power law function (P-FGM), Sigmoidal-function (S-FGM) and Exponential function (E-FGM). The present finite model is established using ANSYS parametric design language code in the ANSYS platform. An 8-node 3-D solid-shell element (SOLSH190) based on the first-order shear-deformation theory is used for the above analysis. The finite element model of the FGM plate is subdivided into a sufficient number of layers, and its associated material properties are then laminated to establish the through-thickness variation of effective material properties. The layered structure need not necessarily indicate the gradual change in material properties so that a sufficient number of layers can be substantially approximated to the gradation of effective material properties. Comparative studies and convergence tests with different mesh refinements were performed to demonstrate the efficiency / accuracy of the current model. Numerical results are presented in the form of a non-dimensional frequency parameter. Parametric investigation is carried out for different supporting conditions, plate-aspect ratios, effect of material gradient index of FGM plates.
Keywords*: Functionally Graded Plate, First-order shear deformation theory, Non-dimensional frequency parameter, Material-gradation index, aspect ratio (b/a).

I. INTRODUCTION

Materials have played an important role in the development of our society. The scientific use of base materials available in various inorganic and organic compounds has paved the way for the
development of advanced polymers, engineering alloys, structural ceramics, etc. The structure of development of modern materials is illustrated in Fig. 1. Therefore, composites are a class of advanced materials made by combining one or more materials in the solid state with different chemical and physical properties. These composite materials offer superior properties compared to their original materials and are also light in weight. Functionally graded materials (FGMs) are the advanced materials in the family of engineering composites consisting of two constituent phases with continuous and smoothly variable composition. FGMs are an advanced class of composite materials with variable material properties over the change in dimension. De-lamination is a major concern issue in the reliable design of advanced fiber reinforced composite laminates. In laminated composites, the separation of layers caused by high localinter-laminar stresses leads to a destruction of the load transfer mechanism, a reduction in stiffness, and a loss of structural integrity, resulting in a final structural and functional failure [28]. In Functionally Graded Materials, due to the gradual change in the material property from one surface to the other, it can eliminate inter-laminar stresses due to sudden change in material properties. These advanced materials with developed gradients of the composition, structure and properties in preferred direction/ orientation, are superior to a homogeneous material of similar constituents. In functionally graded material mechanical properties such as Young' smodulus of elasticity, Poisson's ratio, modulus of elasticity, and material density varies smoothly and

continuously in the preferred direction of structural member. The FGM materials and their composition are selected based on the function that the material has to perform. Ceramic metal

FGMs are commonly used as thermal barrier coating material, where the ceramic surface will withstand temperature and the metal matrix will provide strength. The idea of FGM originated in Japan in 1984 for space research, in the form of a temperature resistant material that can withstand a temperature of 2000K and a thermal gradient of one thousand Kelvin with thickness less than ten mili-meters. Functionally graded materials can also be used under adverse operating conditions such as high temperature and humid environment. These materials have applications in rocket heat shields, structural walls of thermal and acoustic insulation, wear resistant coatings, thermoelectric generators, heat exchanger tubes, fusion reactor thermal coatings and electrically insulating metal / ceramic joints. FGMs consisting of metal and ceramic components are well known for improving the properties of thermal barrier systems due to cracking or delamination, which is often observed in conventional multilayer systems, and is avoided because of a smooth transition between the components of system properties [28]. The structural unit of a FGM can be represented by the material gradient index. Material gradient index indicates the rate at which material properties are varying. The chemical composition, geometric configuration and physical state of the FGM's depend on the material gradient index. The most common FGMs are metal / ceramic composites wherein the ceramic part has good heat resistance and the metal part has excellent fracture toughness.

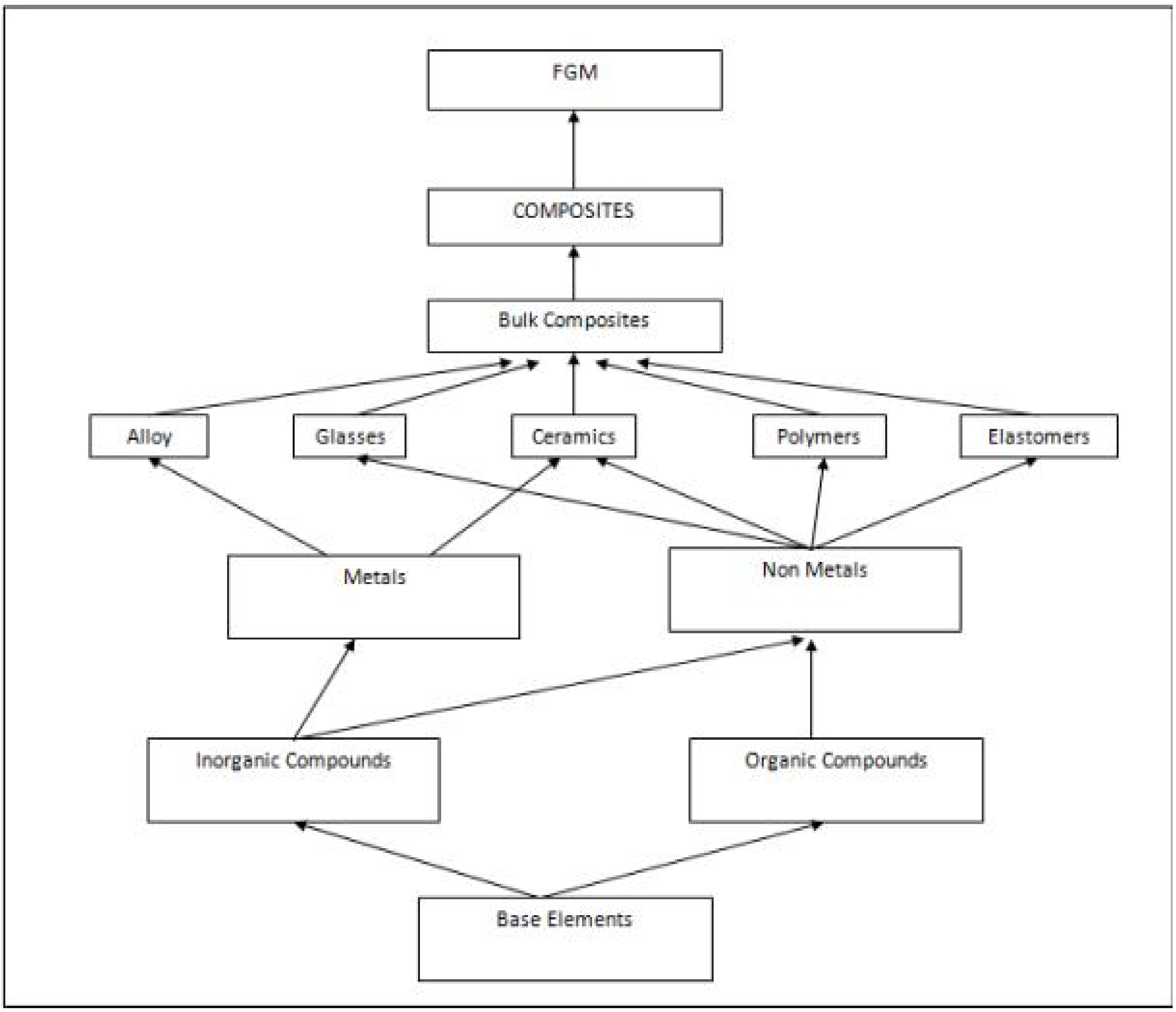


Figure 1: Representation of modern material hierarchy [28]

By varying percentages of volume fractions of two or more materials, spatially FGMs can be formed which have a desired property gradation in spatial directions [28]. The structural unit of FGM is referred to as element or material ingredient. It is a basic unit for the construction of FGM which includes various aspects of its chemical composition, physical state and geometric

ISSN: 2321-9653; IC Value: 45.98; SJ Impact Factor:6.887
Volume 5 Issue XII December 2017- Available at www.ijraset.com

configuration. FGMs were developed by combining the advanced technical material in the form of particulates, fibers, whiskers or platelets. In the continuous drive to improve structural performance, FGM are developed to scale microscopically material architecture to optimize certain functional properties of structures [28].

Table 1
Illustration of a Metal-Ceramic FGM [82]

High temperature side	Ceramic rich	Heat resistant Good anti-oxidant properties Low thermal conductivity
Low temperature side	Metal rich	Mechanical toughness, strength High thermal conductivity High fracture toughness
In between	Ceramic + Metal	Effective thermal stress relaxation throughout

In FGM the gradation of properties may be continuous at the microscopic level, or they may be discontinuous (layered-laminated) which may be formed by gradients of metals, ceramics, polymers or porosity / density variations as shown in Figure 2.

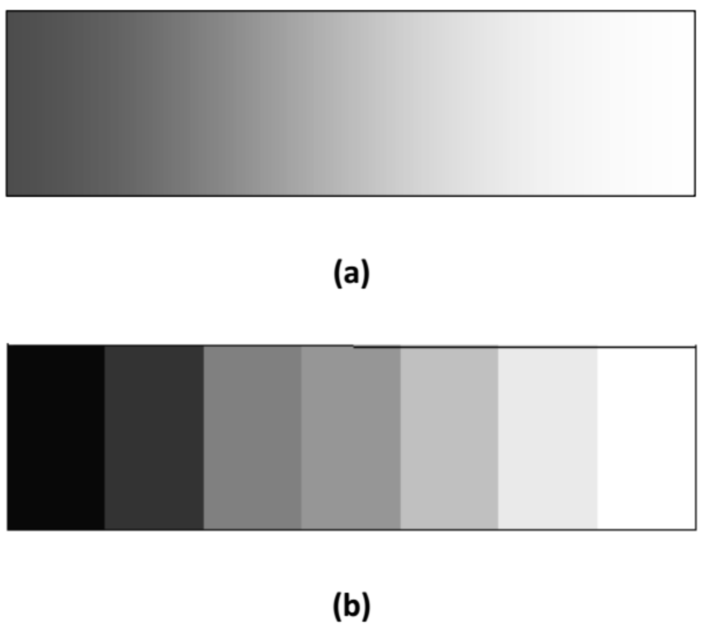


Figure 2: Gradient of FGMs; (a) continuously graded and (b) discretely layered FGMs

The gradation in properties of the material reduces thermal stresses, residual stresses, and stress concentration factors. If two dissimilar materials are bonded together, there is a very high chance that debonding will occur at some extreme loading conditions, be it static, dynamic, or thermal loads.

II. LITERATURE REVIEW

G. N. Praveen and J. N. Reddy [1] examined the static and dynamic thermo-elastic response of functionally graded ceramic metal plates using a plate finite element, which takes into account the effect of transverse shear stresses, rotational inertia and large rotations in von-Kármán type. For the grading of the material properties simple power law distribution of the volume fraction of the metal-ceramic constituents is used. The deflection and loading of the FGM plates is examined under thermal and mechanical stress.

It is concluded that generally the response of the plate corresponding to the properties of the metal and the ceramic is not necessarily between that of the ceramic and the metal. In general, it is stated that gradation in material properties play an essential role in determining the response of the FGM plates. J. N. Reddy [2] presented analytical Navier's solutions for rectangular FGM plates, and third-order shear deformation plate theory was used for developing finite element models. The formulation assumes the parameters of the thermo-mechanical coupling, the time dependency and the geometric non-linearity of Kármán. A simple Power-law distribution with respect to the volume fractions of the constituents is used to vary the material properties of the FGM plate. It is found that fundamental response of FGM plates is not necessarily between ceramic and metal. For all types of boundary conditions, it is concluded that without thermal stress, the response of FGM plate is in between the metal and ceramic plates, but this does not apply when thermo-mechanical stresses are applied. The gradation in the material properties plays an important role in the determination of the response of FGM plates. Z.-Q. Cheng and R. C. Batra [3] used Reddy's third-order shear deformation theory to study steady state free vibration and buckling analysis of simply supported isotropic polygonal FGM plate. The plate rests on the Winkler-Pasternak elastic foundation and is subjected to uniform in-plane hydrostatic loads. To calculate critical buckling load & free vibration frequency of FGM plate is similar to calculate the vibrational frequency of membrane which is clamped at edges and whose shape matches that of the plate. Chakraborty et al. [4] studied static, free vibration and wave propagation problems in bi-material beams fused with functionally graded material layer using new beam finite element method. Formulation of beam element is based on first-order shear deformation theory and it takes into consideration of varying mechanical properties i.e. thermal and elastic. Exact solution of static part of the governing differential equations develops interpolating polynomials for element formulation. For static analysis due to FGM layers variation of stress is smooth. FGM layers in structure causes coupled stiffness and inertial parameters which changes response of base material beams. Limiting frequency of beam having FGM layer lies between parent material beams. Shear speed of mono-material beam is lower than FGM applied beam because FGM applied beam have high coupling inertial terms. L. F. Qian and R. C. Batra [5] investigated transient thermo-elastic deformations of simply supported or clamped thick FGM plates with edges exposed to a uniform temperature. The bottom surface of FGM plate is subjected to uniform temperature or it is thermally insulated while upper surface of plate is subjected to either temperature or heat flux. Simultaneous application of transient thermal and thermo-mechanical loads induce stresses and deformations of FGM plate. Transient thermal and thermo-mechanical problem of FGM plate is solved by higher-order shear and normal deformable plate theory (HOSNDPT) and mesh-less local Petrov-Galerkin (MLPG) method. In MLPG method does not require nodal connectivity & background mesh but only nodal coordinates are needed. Computer code of MLPG method is compared and validated by three dimensional thermo-elastic analytical solution of simply supported plate. Boundary condition applied on edges of plate significantly influence the centroidal deflection and the axial stress which induced at the center of gravity of the top of the plate. Ferreira et al. [6] uses mesh-less collocation method, multi-quad radial basis functions & third order shear deformation theory to analysis static deformations of FGM square plates with varying aspect ratios. The mixture rule and the Mori-Tanaka scheme have been used to calculate material properties. The calculated results are in good agreement with those calculated from the mesh-less local Petrov–Galerkin (MLPG) from Qian, Batra and Chen. Results of collocation and MLPG methods are in asymmetric "stiffness" matrices. Computational time which the collocation method has been significantly less than the MLPG method because there is no numerical integration is necessary in collocation scheme. For nearly equal Poisson's ratios of the two materials, homogenization techniques i.e. rule of mixtures and Mori-Tanaka have nearly the same results, but for widely varying Poisson's ratios the above homogenization techniques give quite different results. A.M. Zenkour et al. [7] introduced the sinusoidal shear formation plate theory to investigate the buckling and free vibration behaviour of the simply supported FGM sandwich plate. The sinusoidal theory comprises the same dependent unknowns as the shear deformation theories of the first and third order type, but transverse shear strain follows cosine-law distribution through thickness direction of FGM plate. By increasing ceramic component i.e. volume fraction exponent decreases in sandwich FGM plate vibration frequencies and critical buckling loads increases. R.C. Batra and J. Jin [8] investigated a free vibration analysis of an anisotropic FGM rectangular plate with the aim of maximizing one of the first five free vibration frequencies by using first-order shear deformation theory (FSDT) using the finite element method. In this study an anisotropic FGM plate was considered in which the orientation of the fibre varied smoothly along the thickness of the plate. Free vibration studies are performed for the following edge conditions (i) all edges simply supported (ii) all edges clamped (iii) two opposite edges simply supported & the other two free (iv)two opposite edges clamped & the other two free. Woo et al. [9] investigated non-linear vibration characteristics of FGM plates. The equations of the motions for the FGM plate are derived by the von Karman theory for large transverse deflections and the solution is derived in terms of mixed Fourier series. Effect of the properties of the material, the boundary conditions and the thermal load on the dynamic behaviour of the plates is studied. It is found that the fundamental frequency of FGM plates is mainly influenced by non-linear coupling effects. Shyang-Ho Chi and Yen-Ling Chung [11] evaluated numerical solutions directly from theoretical formulations or finite element solutions using the MARC program. The mechanical behaviour of the FGM plate is investigated considering the effects of the loading conditions, changing Poisson ratio, aspect ratio of plate and also the variable mechanical properties of FGM defined by the Power-law, Sigmoid Law and exponential law. It is concluded that the changing Poisson's ratio does not affect the mechanical behaviour of FGM plate, so it is assumed to be constant. Variation of FGM properties in the thickness direction or the ratio of Young's modulus for a certain material distribution determines the position of the neutral surface. Serge Abrate [12] studied the free vibrations, buckling and static deflections of the FGM plates in which the properties of the material vary through the thickness. After studying available literature he concluded that the natural frequencies, the critical temperature, in-plane buckling load and deflections of the FGM plates were proportional to those of equivalent homogeneous plates. Extensive studies were performed for thin and thick plates with different aspect ratios, as well as skew and circular plates and many combinations of boundary conditions. These problems were analyzed with classical plate theory, the first and third order shear deformation theories. T. Prakash and M. Ganapathi [13] investigated asymmetric thermal buckling and free vibration characteristics of functionally graded circular plate using finite element method. For gradation of material properties in thickness direction simple power law distribution is used. Based on the principle of field consistency, a flexible three-node shear plate element is used. For numerical calculations it is assumed that the temperature field is uniformly distributed over the plate surface and is only varied in the thickness direction. Effect of material gradient index, temperature field, radius-to-thickness ratio, boundary condition and circumferential wave number is studied on variation of critical buckling load. Non-dimensional frequency and critical buckling temperature decreases by increasing volume fraction index. Roque et al. [14] investigated free vibration response of FGM plates by multi-quadratic radial basis function method with higher order shear deformation theory. The mesh-less, multi-quadratic method allows a fast and easy domain and boundary discretization. Mori-Tanaka scheme is used to homogenize material properties. Hiroyuki Matsunaga et al. [15] studied buckling stress and natural frequency of FGM plates by considering effects of transverse shear, normal deformation and rotary inertia. To vary material properties Simple Power-law distribution in terms of the volume fractions of the constituents is used. Power Series expansion method of displacement components, and set of elementary dynamic equations of two-dimensional (2D) type and higher order shear deformation theory of FGM plates is derived by Hamilton's principle. Several sets of approximate truncated theories are applied to solve the Eigen-value problems of FGM plates with simply supported edges. Nguyen et al. [16] introduced a first-order shear deformation plate theory model for simply supported square FGM plates with improved shear correction coefficient and shear stiffness. Energy equivalence method is used to study transverse shear factors through this model. Equilibrium equations and expressions of membrane stresses are used to calculate transverse shear stresses. Transverse shear stress factor is used to calculate numerical results of simply supported square plate and a cylindrical bending clamped sandwich plate. Numerical simulations showed that the shear correction factor is not the same as that calculated from homogeneous FSDT models, and is a function of the relationship between the elastic modulus of the material constituents and their distribution across thickness of FGM plate. Farzad Ebrahimi and Abbas Rastgo [17] analytically investigated the free vibrational behaviour of thin, circular FGM plates integrated with two evenly distributed piezoelectric (PZT4) material actuator layers using classical plate theory. The properties of the FG substrate plate materials are graded in the thickness direction according to the simple power law distribution in terms of the volume fractions of the constituents while the distribution of the electric potential field in the direction of the thickness of the piezoelectric layers is simulated by a quadratic function in short circuit form. Zhao et al. [18] investigated free vibration characteristic of the metal-ceramic functionally graded material plate with element-free kernel particle Ritz method (kp-Ritz method). First-order shear deformation theory is used which considers effect of rotary-inertia & transverse shear. Two-dimensional displacement field is approximated by Kernel particle estimation. In this study skew FGM plates made of Al/Al2O3, Al/ZrO2, Ti–6Al–4V/Aluminium oxide, and SUS304/ Si3N4 are selected. Eigen-equation is derived by application of energy function of the system using Ritz method. Effects of various parameters i.e. volume fraction index, length-to-thickness ratio & boundary conditions on vibrational frequency are studied. It is concluded that a volume fraction exponent ranging from 0 to 5 has a significant influence on the frequency but the effects of the length-to-thickness ratio on the frequency of a plate are not affected by the volume fraction. Zhang et al. [19] presented an analysis on the non-linear dynamics and the chaos of the simply supported orthotropic FGM rectangular plate in the thermal environment and was applied to parametric and external excitation. Both temperature-dependent ion and heat conduct material properties are taken into account. Equations of motion of rectangular orthotropic FGM plate are based on Reddy's third-order share deformation theory and Hamilton's principle is used to derive this. Galerkin method is applied to the regulating partial differential equations of motion to obtain a non-linear 3-degree of freedom system. Hashemi et al. [20] investigated the analytical vibrational behaviour of piezoelectric coupled thick annular FGM plate based on Reddy's third-order shear deformation theory. They considered various combinations of boundary conditions viz. i.e. soft simply supported, hard simply supported and clamped edges at the inner and outer edges of annular plate.

The properties of host are assumed to vary according to simple force-law distribution of the volume fraction and graded in thickness direction only. Distribution of electric potential is assumed to vary according to sinusoidal function along thickness direction so that it approximately satisfies the Maxwell static electricity equation. Liu et al. [21] investigated free vibration behaviour of FGM plate by considering material inhomogeneity in plane of functionally graded plate. Because of the symmetry of the plate around its median plane, only bending (flexural) vibrations are considered. For simply-supported plate whose edges are parallel to the material gradient direction, a Levy-type solution is derived by using Fourier series expansion and an integration technique that converts the two-point boundary problem into initial value problems. Hashemi and Damavandi Taher [22] analytically solved free vibration problem of reasonably thick quadrilateral FGM plates resting on Winkler or Pasternak elastic foundations and applied to several possible combinations of clamped and simply supported boundary constraints. To determine fundamental frequency of functionally grade plates resting on elastic foundations, equations of motion are based on first-order shear deformation theory (FSDT) and shear correction factor used in Mindlin plate theory. Effect of foundation stiffness parameter, different boundary conditions, plate aspect ratio, material gradient index and thickness-to-length ratio is studied on free vibration of FGM plates. Mohammad Talha and B.N. Singh [23] investigated free vibration & static response of FGM plates with higher-order shear deformation theory considering a specific modification of the transverse displacement in combination with finite element methods. The equations of motion of the FGM plates are derived using a variational approach taken into account traction free boundary conditions on the upper and lower surfaces of the plate. Computational results were attained by means of a continuous iso-parametric Lagrangian finite element with 13 degrees of freedom per node. It is concluded that effect of thickness ratio of plate is independent of the volume fraction index. Gradient in the material properties determines the response of FGM plates. Xiang et al. [24] suggested n-order shear deformation plate theory to study free vibration response of functionally graded composite sandwich plates. Displacement field is represented by n-order polynomial term. Zero transverse shear stress condition are satisfied on the top and bottom of plate. Reddy's third order theory can be regarded as a special case of the current n-order theory. Natural frequencies of the FGM and composite plates with different side-to-thickness ratios, material properties are calculated by norder theory with different material gradation index values. Benachour et al. [25] uses four variable refined plate theory for the free vibration analysis of FGM plates with random material gradation. This theory considers transverse shear effects and the parabolic distribution of transverse shear strains through thickness of the plate, so shear correction factors are not required. In this refined plate theory number of unknown functions are four while in shear deformation theory there are 5 unknown functions. Equations of motion are derived according to Hamilton's principle. Navier method is used to obtain closed form solutions, and fundamental frequencies are then calculated by solving the results of Eigen-value problems. J. Suresh Kumar et al. [26] provided an analytical solution to investigate free vibration of functionally graded plates without imposing zero transverse (cross shear) stress conditions on upper and lower surfaces of plate with the higher order displacement model. Virtual work principle is used to derive motion equations for the higher-order displacement model. Energy principle is used to derive equations of motion of FGM plate and Navier’s solutions. It is found that the natural frequencies of the FGM plate with different material gradient index lie between those of the natural frequencies of metal and ceramics. Y.X. Hao et al. [27] investigated nonlinear transverse vibration response of cantilever type FGM rectangular plates which were subjected to combined transverse and thermal loadings. The nonlinear motion equations for the FGM plate are derived from Reddy's third order shear deformation plate theory and the Hamiltonian principle. The Galerkin method converts the governing partial differential equations into a nonlinear system with two degrees of freedom containing square (quadratic) and cubic nonlinear terms under combined external excitations. Resonance case with 1: 1 internal resonance and sub-harmonic resonance of order ½ are considered for present problem and asymptotic perturbation method develops four nonlinear average equations, which are then solved by the Runge-Kutta method to find out non-linear dynamic response of FGM plate. The chaotic, periodic and quasi-periodic movements of the plate are present under certain conditions, and the forced excitations can alter the shape of the movements of the functionally graded rectangular plate. Prakash et al. [29] investigated the flutter characteristics of a functionally graded plate under high supersonic airflow using a high-precision shear flexible finite element method taking in to account both geometric and aerodynamic nonlinearities. First-order shear deformation theory is used to model functionally graded material plate taking into account the exact position of the neutral surface and the von Kármán theory of large displacement. Aerodynamic pressure is calculated by third-order piston theory. Nonlinear Eigen-value equation is derived using harmonic balanced method which calculates aerodynamic pressure. Time history analysis is done to validate nonlinear Eigen-value equation and to determine the periodic, harmonic, and quasiperiodic characteristic of the flexural vibration under aerodynamic loading load by using plots of transverse displacement, phase portrait, bifurcation diagram & Poincare map. It is apparent that by increasing Mach number & aerodynamic load, the bending (flexural) vibration becomes chaotic and is more resistant to chaotic phenomenon in graded FGM plate having ceramic constituent. Sanjay Anandrao et al. [31] presented the significance of transverse-shear on conditions and length-to-thickness ratios. Two separate finite element formulations, one is Timoshenko beam formulation which considers effect of transverse shear and other is Euler Bernoulli beam formulation which neglects effect of transverse shear. It is noticed that consideration of transverse shear in the formulation of FGM beam increases flexibility of short beams and reduces frequencies. Effect of transverse shear is more dominant for clamped beams as compared with other boundary conditions. Koteswara Rao D et al. [32] developed finite element model of functionally graded composite shell structure using 8 noded layered shell element (SHELL99). Response and characteristic of functionally graded spherical shells were studied and compared with response of homogenous shell structures under static uniform distributed load and thermo-mechanical loading. Structural response of functionally graded shells is found between the responses of the homogeneous shells made of ceramic and metal. Free vibration analysis of thick and thin functionally graded shell structures for only first and second mode shapes. D.K. Jha et al. [33] presented a higher order shear and normal deformation theory (HOSNT) to investigate free vibration behaviour of functionally graded elastic, rectangular plates with simply supported edges. Governing equations of motion are derived by Hamilton's principle using HOSNT. Navier's solution method using double Fourier series is used to compute the result with required accuracy. The results concluded that as the material gradient index increases, the natural frequency decreases significantly. Mohammad Talha and B.N. Singh [34] studied the thermo-mechanical deformation of shear deformable metal ceramic plates (FGM) subjected to various load and boundary conditions. The analysis is performed with higher order shear deformation theory with reasonable improvement in transverse displacement using FEM. The temperature-dependent mechanical properties are assumed and graded according to simple power law distribution of volume constituents in the thickness direction. Fundamental equations for FGM plate are derived using a variational approach taking into account the traction-free boundary condition at the top of plate and a $C0$ continuous iso-parametric Lagrange element with thirteen degree of freedom per node is developed and used to calculate the results. Convergence tests and validation studies were performed to verify the accuracy of the present formulation. It is concluded that in all kinds of boundary conditions when thermal effect is considered, the bending behaviour of the functionally graded plate is not necessarily between that of the metal and the ceramic plate.

K. Swaminathan and D. T. Naveen Kumar [35] presented finite element formulation based on first order shear deformation theory to determine natural frequency of simply supported FGM plates. Results of Exact 3-Dimensional elasticity solution are compared with computational analysis. Results are presented for FGM plates with varying side-to-thickness ratio, plate aspect ratio & material gradation index. Shuohui Yin [36] developed non-uniform rational B-spline (NURBS) based iso-geometric analysis with classical plate theory to investigate free vibration analysis of thin non-homogenous functionally graded material plates and shells. For thin non-homogenous FGM plates, mid-plane displacements are neglected in classical plate theory that's why classical plate theory is not accurate to analysis non-homogenous thin FGM plate with moderate material gradient indexes. Mid-plane displacements effects are not minimized without introducing new unknown variable the physical neutral surface is introduced into CPT which is known as CPT_neu. The NURBS basis function is used to implement geometric description and deflection field approximation. Z.X. Lei et al. [37] investigate free vibration analysis of functionally graded nano-composite plates reinforced by single walled carbon nano-tubes (SWCNTs) using element-free kp-Ritz method. Effective material properties of CNTRCs are approximated by extended rule of mixture or Eshelby–Mori–Tanaka scheme. First-order shear deformation plate theory with considering transverse shear & rotary inertia effect and a kernel particle estimate is used to approximate the two- dimensional displacement field. Various examples are solved in computational simulation to study the effects of carbon nano-tube volume fraction, plate-aspect ratio and width-to-thickness ratio with temperature change on natural vibration frequencies and mode shapes of various FG-CNTRC plates. Alshorbagy et al. [38] studied finite element analysis of thermo-mechanical behaviour of functionally graded material plates using first-order shear deformation theory. A series of simulations are performed to investigate characteristic of FGM plates which are exposed to thermo-mechanical loads and to determine the effect of heat source intensity. The results of the numerical simulation concluded that there is a difference in the plate deflection when one considers the effect of shifting of the neutral plane position. But neutral plane of the FGM plate is shifted toward the surface with higher Young's modulus. But the position of the neutral plane depends mainly on the ratio of the modulus of elasticity of the two plate constituents. The main advantage of the FGM plate versus the conventional laminate plate is that the variation in stresses and strains without any kind of singularities is smooth in the functionally graded plate due to the continuity of the material properties distribution along the thickness of the plates. Natarajan et al. [39] studied mechanical & thermal buckling and free vibration behaviour of FGM plates using cell-based smoothed finite element method with discrete shear gap technique. Plate kinematics are based on first-order shear deformation theory and discrete gap method is used to prevent shear locking. The shear correction factor is evaluated by the use of energy equivalence principle. Mori-Tanaka homogenization method is used to approximate effective for FGM plates with curvilinear stiffeners are greater than those made from a pure metal plate with curvilinear stiffeners but lower than those from a pure ceramic plate with curvilinear stiffeners. It is shown for plates with 1, 2, 3 and 4 curvilinear stiffeners that the gradients in the material properties significantly influence the natural frequencies while the volume fraction exponent is between 0 and 2. Huu-Tai Thai et al. [42] presented a new simple first order shear deformation theory to investigate bending and free vibration response of FGM plates. The present FSDT model contains only four unknown variables and resembles the classical theory of plates in many aspects regarding equation of motions, boundary conditions and stress-resultant expressions. Comparative and validation studies show that the present theory can achieve the same accuracy of the conventional FSDT that has more unknown variables.

ERROR! REFERENCE SOURCE NOT FOUND.. MATHEMATICAL IDEALIZATION OF FUNCTIONALLY GRADED MATERIALS

FGMs are extremely heterogeneous, so it is important to idealize them as continua with their mechanical properties changing gradually with respect to the spatial co-ordinates. In FGM material properties are likely to follow a gradation through the thickness in a continuous profile. Most of the investigations related to functionally graded material, use the power-law function, sigmoid function or exponential function to define the volume fractions functions. So FGM plates with power-law, exponential, or sigmoid function will be considered in this study. Consider a rectangular plate as shown in Fig 3. x & y are the coordinates of the plane of the plate, whereas the z -axis originated at the middle surface of the plate is in the direction of thickness. The material on the top (upper) surface of the plate is ceramic rich whereas the bottom (lower) surface material is metal rich. Effective material properties, such as Young's modulus & the Poisson's ratio, on the top and bottom surfaces are different but are selected according to the performance criteria. However, the Young's modulus & Poisson's ratio of the plates vary continuously only in the thickness direction (z-axis) i.e., $E = E(z)$, but Poisson's ratio has less effect on deformation than that of Young's modulus, so Poisson's ratio is assumed constant $(\nu = \text{constant})$ through thickness of FGM plate. The "z" is varying from "$h/2$" at top surface "0" at the middle of the thickness, to "$-h/2$" at bottom surface and z is the thickness co-ordinate. Young's modulus in the thickness direction of FGM plates vary according to power-law functions (P-FGM), Sigmoid functions (S-FGM) or with exponential functions (EFGM).

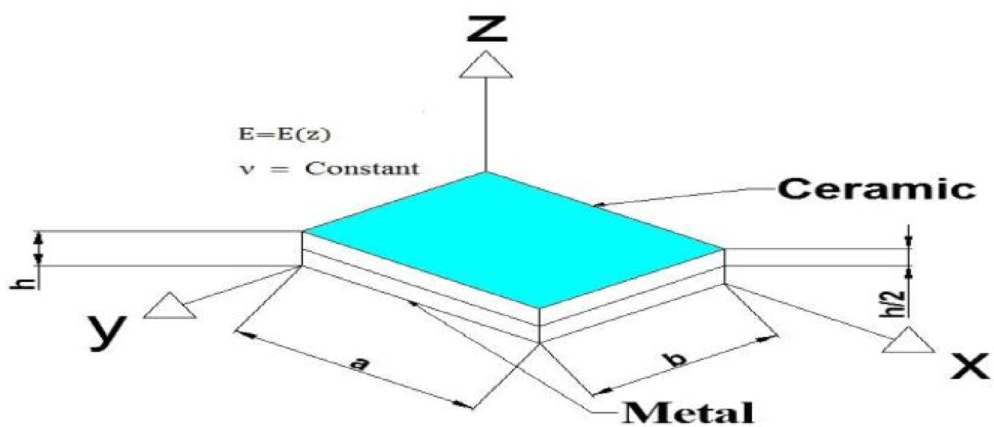


Figure 3: The Geometry of an FGM plate.

A. The material properties of Power Law FGM (P-FGM) plates (Vigot-Model) [30]

The volume fraction of the P-FGM is assumed to vary according to power-law function. The effective material properties at an arbitrary point within the structural domain, like Young's modulus E, Poisson's ratio ν, mass density ρ, coefficient of thermal expansion α of the FGM plate are the effective material properties P. These properties are position dependent & can be expressed according to rule of mixtures.

$$P(z) = P_t V_t(z) + P_b V_b(z) \tag{1}$$

$$V_t(z) + V_b(z) = 1 \tag{2}$$

$$V_t(z) = \left(\frac{z}{h} + \frac{1}{2}\right)^n \text{ Where } (n > 0) \tag{3}$$

Where P denotes a generic material property like elastic modulus, where P_t and P_b denotes the property of the top and bottom faces of the plate respectively & h is the total thickness of the plate. $V_t(z)$ & $V_b(z)$ are the volume fractions of the constituent of the top and bottom faces of the plates. The volume fractions of the constituent of the top (upper) surface of the plate follows a simple power−law as,

$$V_t(z) = \left(\frac{z}{h} + \frac{1}{2}\right)^n \tag{4}$$

Where n is a material gradient index and it is a non-negative volume fraction index which describes the material variation profile through the thickness of the plate and may be adjusted to attain the optimum distribution of the constituent material.

At the bottom surface: $(z/h) = -1/2$; so $V_t(z) = 0$ and $P(z) = P_b$ (Metal)

At the top surface: $(z/h) = 1/2$; so $V_t(z) = 1$ and $P(z) = P_t$ (Ceramic)

At n=0 the plate is a fully ceramic plate while at n=∞ the plate is completely metal. The variation of volume fraction in the thickness direction of the P-FGM plate is depicted in Figure 4, which shows that volume fraction changes rapidly near the lowest surface for n > 1 and increases quickly near the top surface for n < 1.

International Journal for Research in Applied Science & Engineering Technology (IJRASET)
ISSN: 2321-9653; IC Value: 45.98; SJ Impact Factor:6.887
Volume 5 Issue XII December 2017- Available at www.ijraset.com

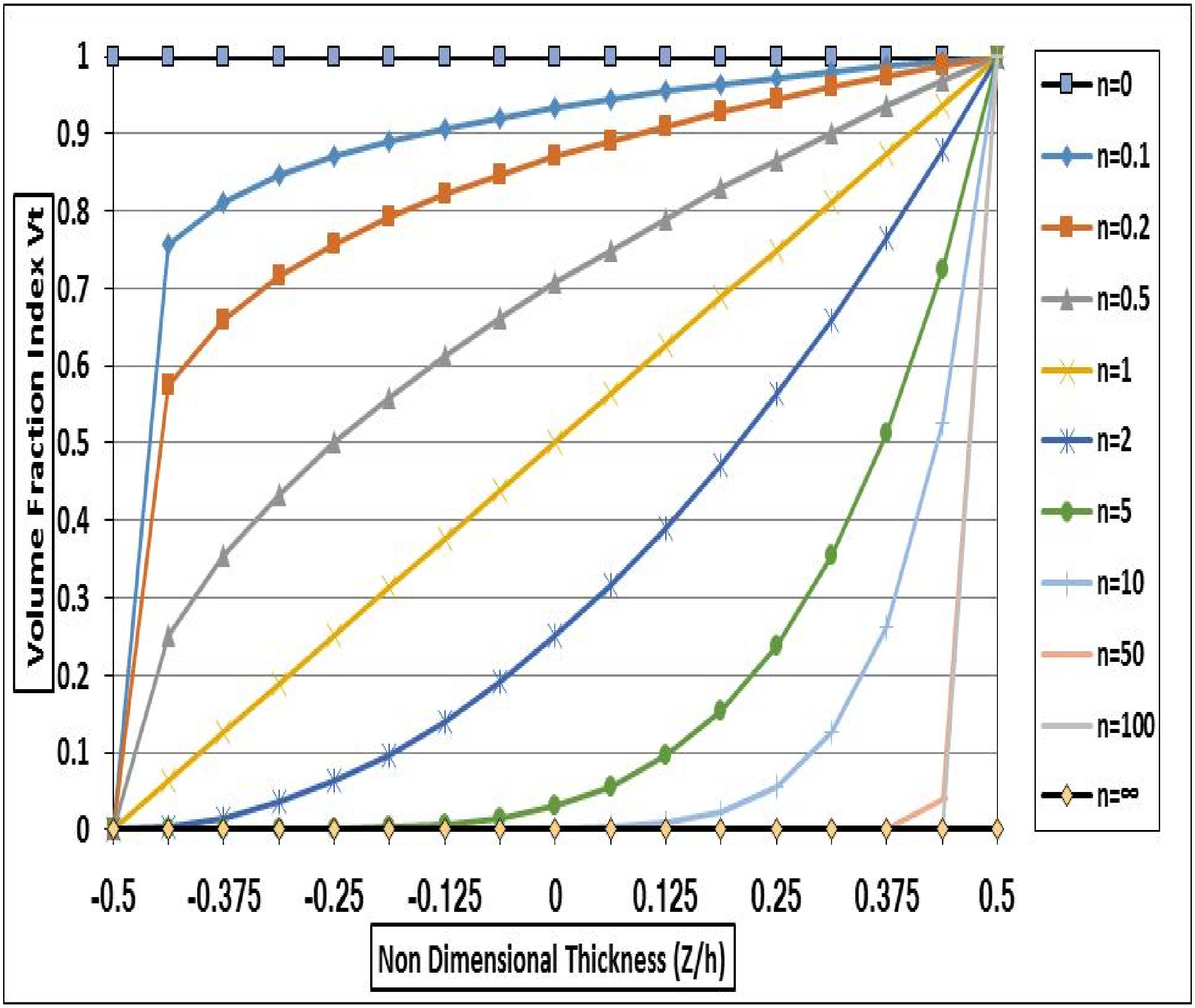


Figure 4: Variation of volume fraction through plate thickness for various values of the power-law index n

It is ascertain that the effective Young's modulus E and coefficient of thermal expansion α dependent on temperature. But, the mass density ρ & the thermal conductivity κ are not temperature dependent. Poisson's ratio ν is assumed to be constant as temperature change has less effect on this. From Eqs. (1) to (3) & Eqs. (5), the effective material properties with two constituents for graded plates can be expressed as

$$E(z) = [E_t - E_b]\left(\frac{2z+h}{2h}\right)^n + E_b$$
$$\alpha(z) = [\alpha_t - \alpha_b]\left(\frac{2z+h}{2h}\right)^n + \alpha_b$$
$$\rho(z) = [\rho_t - \rho_b]\left(\frac{2z+h}{2h}\right)^n + \rho_b \qquad (5)$$
$$\kappa(z) = [\kappa_t - \kappa_b]\left(\frac{2z+h}{2h}\right)^n + \kappa_b$$

B. The Material Properties of Sigmoidal FGM (S-FGM) Plates

In the case of adding an Functionally Graded Material of a single power-law function to the multi-layered composite, stress concentrations induce on one of the interfaces where the material is continuous but changes rapidly [10]. So, Chung & Chi (2001) defined the volume fraction using two power-law functions to make sure smooth distribution of stresses among all the interfaces. Then the two power-law functions are defined by:

$$g_1(z) = 1 - \frac{1}{2}\left(\frac{\frac{h}{2}+z}{\frac{h}{2}}\right)^n \quad \text{for } 0 \le z \le \frac{h}{2} \qquad (6)$$

$$g_2(z) = \frac{1}{2}\left(\frac{\frac{h}{2}+z}{\frac{h}{2}}\right)^n \quad \text{for } -\frac{h}{2} \le z \le 0 \qquad (7)$$

By the use of rule of mixture, the Young’s modulus of elasticity of the S-FGM can be calculated by:

$$E(z) = g_1(z)E_1 + [1-g_1(z)]E_2 \quad \text{for } 0 \le z \le \frac{h}{2} \qquad (8)$$

$$E(z) = g_2(z)E_1 + [1-g_2(z)]E_2 \quad -\frac{h}{2} \le z \le 0 \qquad (9)$$

Fig. 5 shows that the variation of Young’s modulus in Eqs. (8) and (9) represents Sigmoidal distributions, so this FGM plate is thus called a sigmoid FGM plate (S-FGM plates).

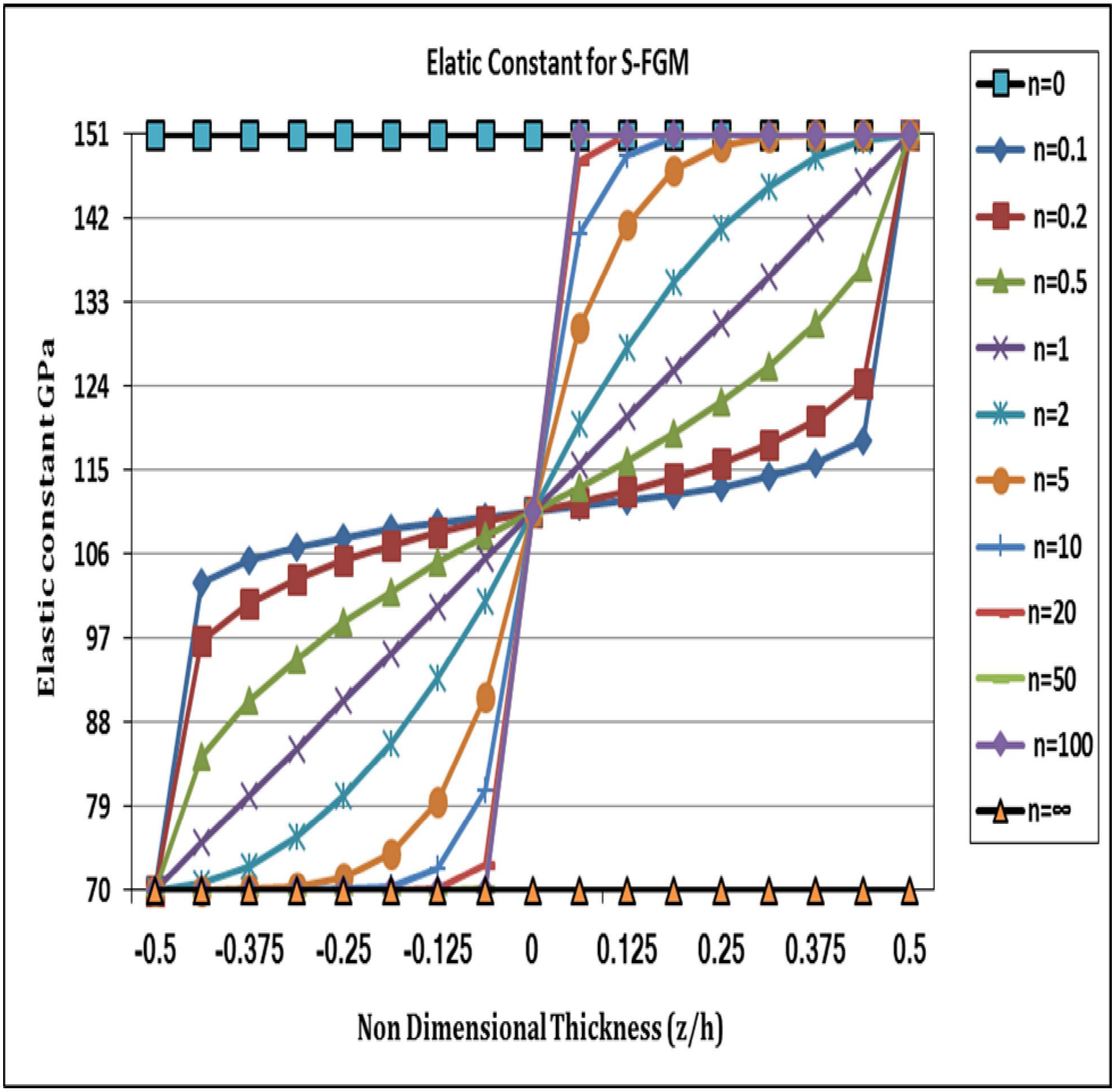


Figure 5: The Variation of Elastic constant for S-FGM

C. The Material Properties of Exponential FGM (E-FGM) Plates

Exponential function used to describe the material properties of FGMs as follows [10]

$$E(z) = Ae^{B\left(z+\frac{h}{2}\right)} \tag{10}$$

With $a = E_2$ and $B = \ln\left(\frac{E_1}{E_2}\right)$

$$E(z) = E_2 e^{\left(\frac{1}{h}\right)\ln\left(\frac{E_1}{E_2}\right)\left(z+\frac{h}{2}\right)} \tag{11}$$

The material distribution in the thickness direction of the E-FGM plates is plotted in Fig. 6.

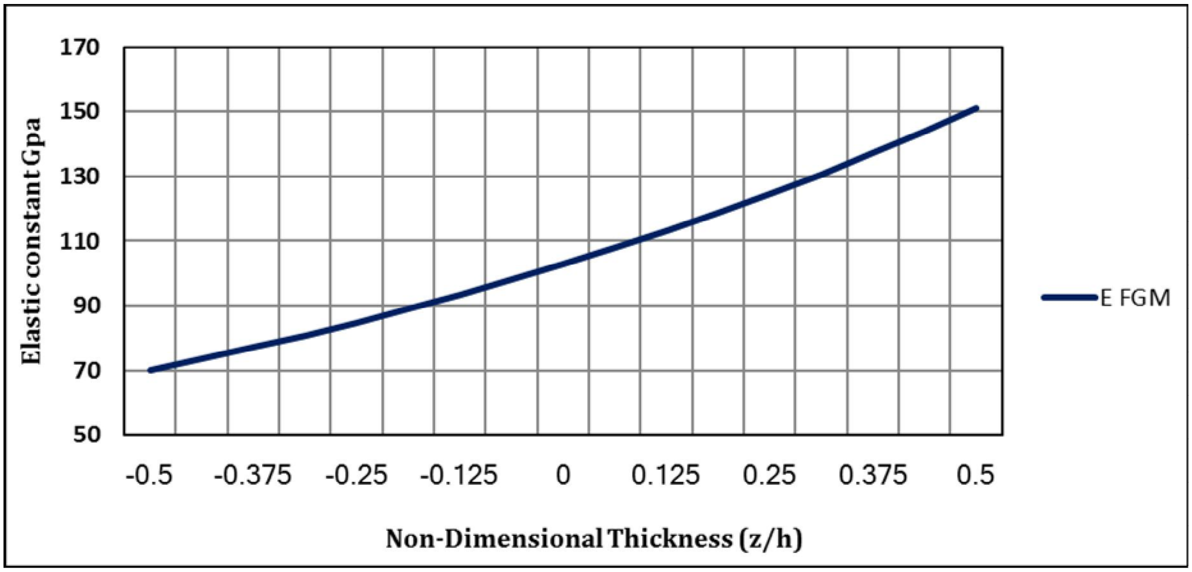


Figure 6: Variation of Elastic constant in E-FGM plate.

IV. MODAL ANALYSIS OF FUNCTIONALLY GRADED MATERIAL PLATE

Modal analysis is used to determine the natural frequencies & mode shapes of a structural element. Modal analysis is a linear analysis in ANSYS. Different mode extraction methods, such as Block Lanczos, Supernode, PCG Lanczos, reduced, un-symmetric, damped, & QR damped are available. Block Lanczos is used in the present analysis. Block Lanczos is used for large symmetric Eigen-value problems and it uses the sparse matrix solver [82]. The solution steps in ANSYS APDL are

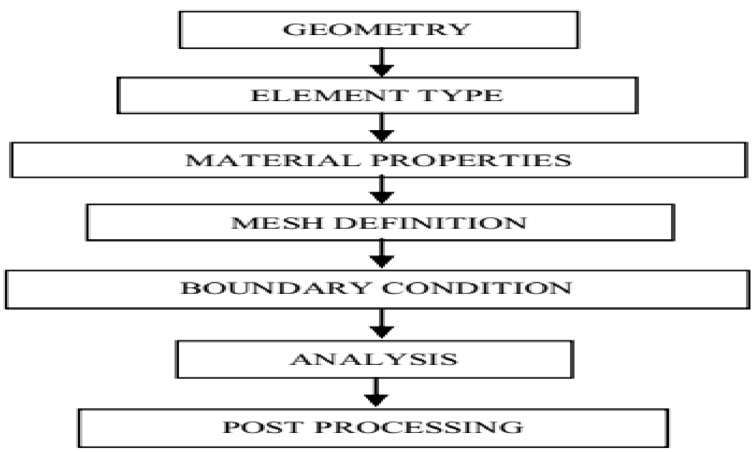


Figure 7: Solution steps in ANSYS mechanical APDL

V. FREE VINRATION ANAYLSIS OF FGM PLATES SUNJECTED TO ONLY MECHANICAL LOADING

Free vibration response of FGM plates subjected to mechanical and thermal loadings are computed using ANSYS 15.0 for Aluminium/Zirconia rectangular plate for various aspect ratios and boundary conditions. Computed results are compared and validated with those available in literature. The analysis is carried out for thickness ratio, and aspect ratio with various volume fraction indices of P-FGM, S-FGM and E-FGM. The following parameters are used in above analysis.

A. Plate dimensions $(a = b = 0.2\,m)$ and thickness $h = 0.01m$.

B. Uniformly distributed loading chosen was equal to $q = -10^6\ N/m^2$

C. The boundary conditions used in analysis are as follows [Figure 8]
   (a) All Edges Simply Supported (SSSS) as shown in Figure 8 (a)

$UY = UZ = 0$ at $x = 0, a$.

$UX = UZ = 0$ at $y = 0, b$.

(b) Simply supported-Simply supported-Clamped-Clamped (SSCC) as shown in Figure 8 (b)

$UY = UZ = 0$ at $x = 0$, and $UX = UZ = 0$ at $y = 0$.

$UX = UY = UZ = 0$ at $x = a$, and $y = b$.

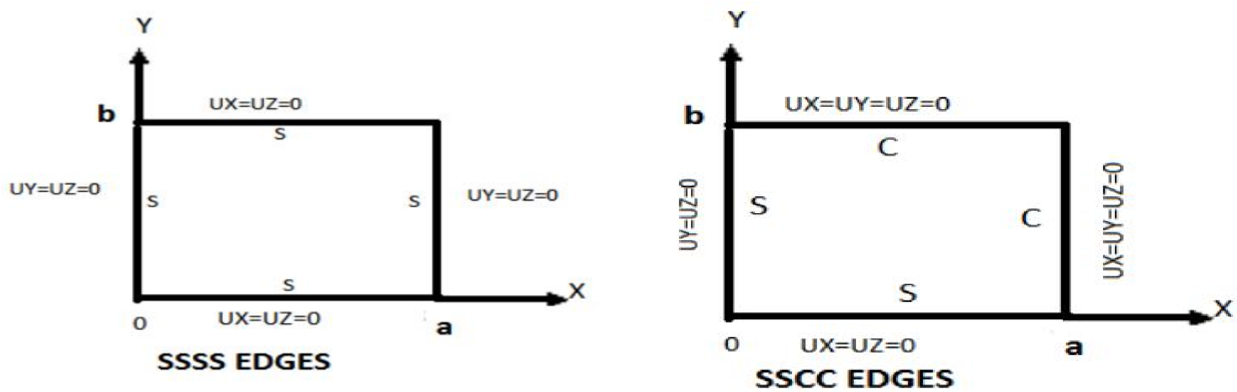


Figure 8: Representation of SSSS and SSCC support condition for Free Vibration analysis

D. The results are presented in terms of Non-dimensional frequency parameter which is obtained by applying 3-dimensional frequency continume method is $\bar{\omega} = \omega\sqrt{\dfrac{12(1-v^2)\rho_c b^2 a^2}{\pi^4 E_c h^2}}$ [23]

VI. CONVERGENCE AND VALIDATION STYDY

The convergence study is performed for the mesh size and number of layers required for finite element analysis. First five non dimensional frequencies of free vibration of the FGM square plate are considered and the results are compared with published results. A 3-D 8-Node Structural Solid Shell element (SOLSH190) is used for analysis in ANSYS APDL. The functionally graded material section is considered as an equivalent laminate section for the finite element modelling. In this analysis 16 number of laminate layers section are considered approximately to represent the FGM. For this study an Al/ZrO_2 FGM Square plate having $b/a = 1.0$, $a/h = 20$ (where a, b, h are width, length and thickness of the plate respectively) was taken. The material properties of metal and ceramic are:-

Table 2 Material Property of Metal and Ceramic FGM

Material Property	Aluminium (metal)	Zirconia (ceramic)
Young's Modulus (E)	70 GPa	151 GPa
Density	2707 Kg/m^3	3000 Kg/m^3
Thermal Conductivity K	204 W/mK	2.09 W/mk
Coefficient of Thermal Expansion α	$23x10^{-6}/{}^0C$	$10x10^{-6}/{}^0C$
Poisson's Ratio υ	0.3	0.3

Boundary Condition- all sides of FGM plate are simply supported (SSSS). Non-dimensional frequency parameter is

$$\bar{\varpi} = \omega \sqrt{\frac{12(1-\upsilon^2)\rho_c a^2 b^2}{\pi^4 E_c h^4}} \quad [23] \tag{12}$$

The convergence studies are conducted for simply supported isotropic FGM plate with $n = 0$ & $n = 1$. First five frequencies are observed with increase in number of mesh size. The Table 5.1 and Table 5.2 show the convergence of non-dimensional frequency parameter with volume fraction index n=0 and n=1 for SSSS Al/ZrO_2 FGM plates. The results show good convergence for mesh size 35X35. The mesh size 35X35 is used in present study.

Table 3: Convergence of Non-Dimensional Frequency Parameter ($\bar{\omega}$) with material gradation index $n = 0$ (ceramic) for SSSS square Al/ZrO_2 FGM Plate $a/h = 20$

Mesh size	$\bar{\omega}_1$	$\bar{\omega}_2$	$\bar{\omega}_3$
10X10	2.2386	5.4941	5.4941
Error	12.250%	9.036%	9.027%
20X20	2.1755	5.324	5.324
Error	9.086%	5.660%	5.652%
30X30	2.013	5.2876	5.2876
Error	0.938%	4.938%	4.929%
35X35	1.9863	4.9078	4.9078
Error	0.401%	2.600%	2.608%
Ref [23]	1.9943	5.0388	5.0392

Table 4: Convergence of Non-Dimensional Frequency Parameter ($\bar{\omega}$) with material gradation index $n=1$ for SSSS square Al/ZrO_2 FGM Plate $a/h = 20$

Mesh size	$\bar{\omega}_1$	$\bar{\omega}_2$	$\bar{\omega}_3$
10X10	1.8553	4.7634	4.7634
Error	6.946%	9.953%	9.953%
20X20	1.8245	4.5309	4.5309
Error	5.171%	4.587%	4.587%
30X30	1.8112	4.4072	4.4072
Error	4.404%	1.731%	1.731%
35X35	1.7386	4.3201	4.3021
Error	0.219%	0.279%	0.695%
Ref [23]	1.7348	4.3322	4.3322

Further, increasing mesh size up to 40X40 increases, computational time. So in this analysis mesh size 35X35 is used for computation of mode frequencies.

VII. RESULTS AND ANALYSIS

Based on the procedures and analyses of foregoing sections, FGM free vibration response of SSSS, SSCC rectangular plates are studied. Natural Frequencies are calculated through Modal analysis using ANSYS and results are presented in non-dimensional frequency parameter.

A. *Variation of Non-Dimensional Frequency Parameter with Aspect Ratio for SSSS FGM Plates*

Table 5: Variation of the Frequency Parameter $\overline{\omega} = \omega\sqrt{12\,(1-\nu^2)\,\rho_c a^2 b^2/\pi^4 E_c h^2}$ with the volume fraction index (n) for (SSSS) Square (b/a=1.0), (Al/ZrO_2) P-FGM plates (a/h=20)

Boundary condition	Mode	n							
		Ceramic(n=0)	0.5	2	5	10	50	100	Metal(n=∞)
SSSS	1	1.9828	1.7143	1.6521	1.5583	1.5106	1.4687	1.4234	1.4212
	2	4.9072	4.2425	4.0886	3.8565	3.7385	3.6346	3.4586	3.5161
	3	4.9072	4.2425	4.0886	3.8565	3.7385	3.6346	3.5227	3.5173
	4	7.7560	6.7055	6.4623	6.0954	5.9088	5.7448	5.5677	5.5581
	5	9.6641	8.3550	8.0527	7.5949	7.3624	7.1580	6.9375	6.9269

Table 6: Variation of the Frequency Parameter $\overline{\omega} = \omega\sqrt{12\,(1-\nu^2)\,\rho_c a^2 b^2/\pi^4 E_c h^2}$ with the volume fraction index (n) for (SSSS) Square (b/a=1.0), (Al/ZrO_2) S-FGM plates (a/h=20)

Boundary condition	Mode	n							
		Ceramic(n=0)	0.5	2	5	10	50	100	Metal(n=∞)
SSSS	1	1.9828	1.7386	1.7371	1.7357	1.7335	1.7313	1.7313	1.4212
	2	4.9072	4.3028	4.2991	4.2954	4.2894	4.2838	4.2838	3.5161
	3	4.9072	4.3028	4.2991	4.2954	4.2894	4.2838	3.7503	3.5173
	4	7.7560	6.8007	6.7948	6.7889	6.7804	6.7716	6.7716	5.5581
	5	9.6641	8.4737	8.4663	8.4579	8.4474	8.4364	8.4363	6.9269

Table 7: Variation of the Frequency Parameter $\overline{\omega} = \omega\sqrt{12\,(1-\nu^2)\,\rho_c a^2 b^2/\pi^4 E_c h^2}$ with the volume fraction index (n) for (SSSS) Square (b/a=1.0), (Al/ZrO_2) E-FGM plates (a/h=20)

Boundary condition	Mode	Exponential FGM
SSSS	1	1.7040
	2	4.2171
	3	4.2171
	4	6.6652
	5	8.3049

For SSSS FGM plates by increasing material gradient index n and aspect ratio value of non-dimensional frequency parameter reduces as shown in figure 9 & figure 10. For S-FGM simply supported plates value of non-dimensional parameter does not change very much by increasing power law index but for P-FGM frequency parameter changes significantly by changing material gradient index between n=0 to ∞. Figure 11 shows comparison of non-dimensional frequency parameter for P-FGM, S-FGM having material gradient index n=2 and E-FGM.

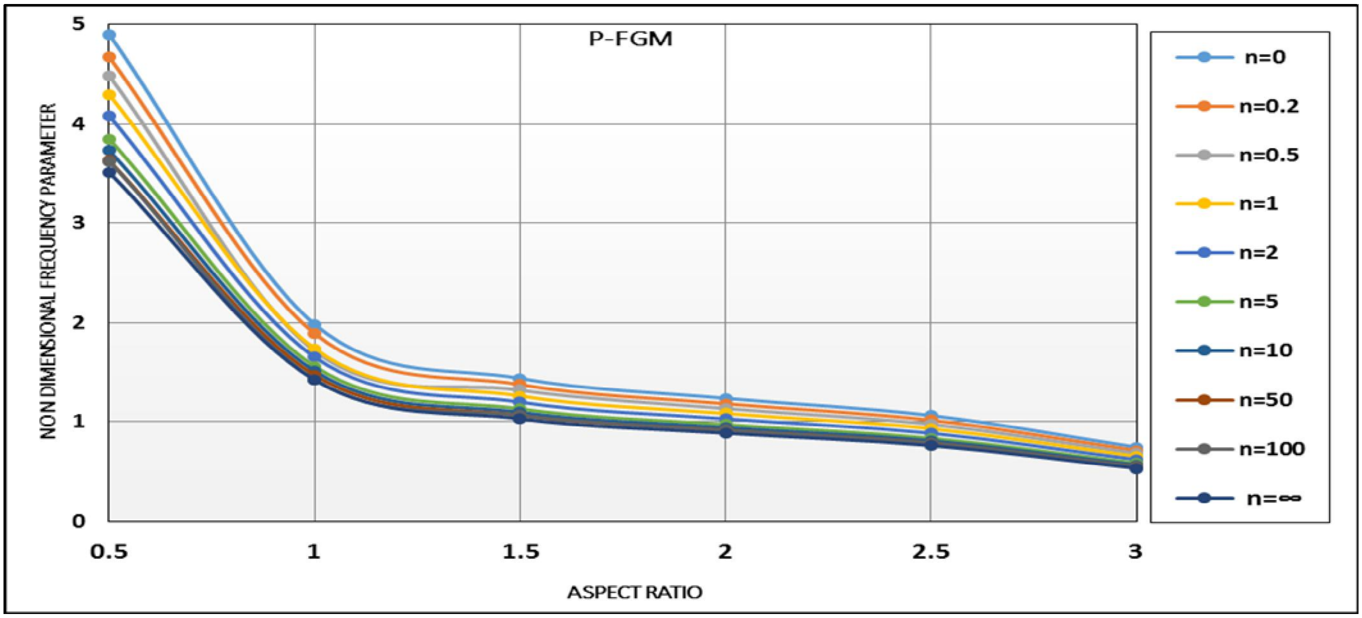


Figure 9: Variation of non-dimensional frequency parameter with material gradient index of P-FGM and aspect ratio b/a=1.0 for SSSS Al/ZrO_2 plates.

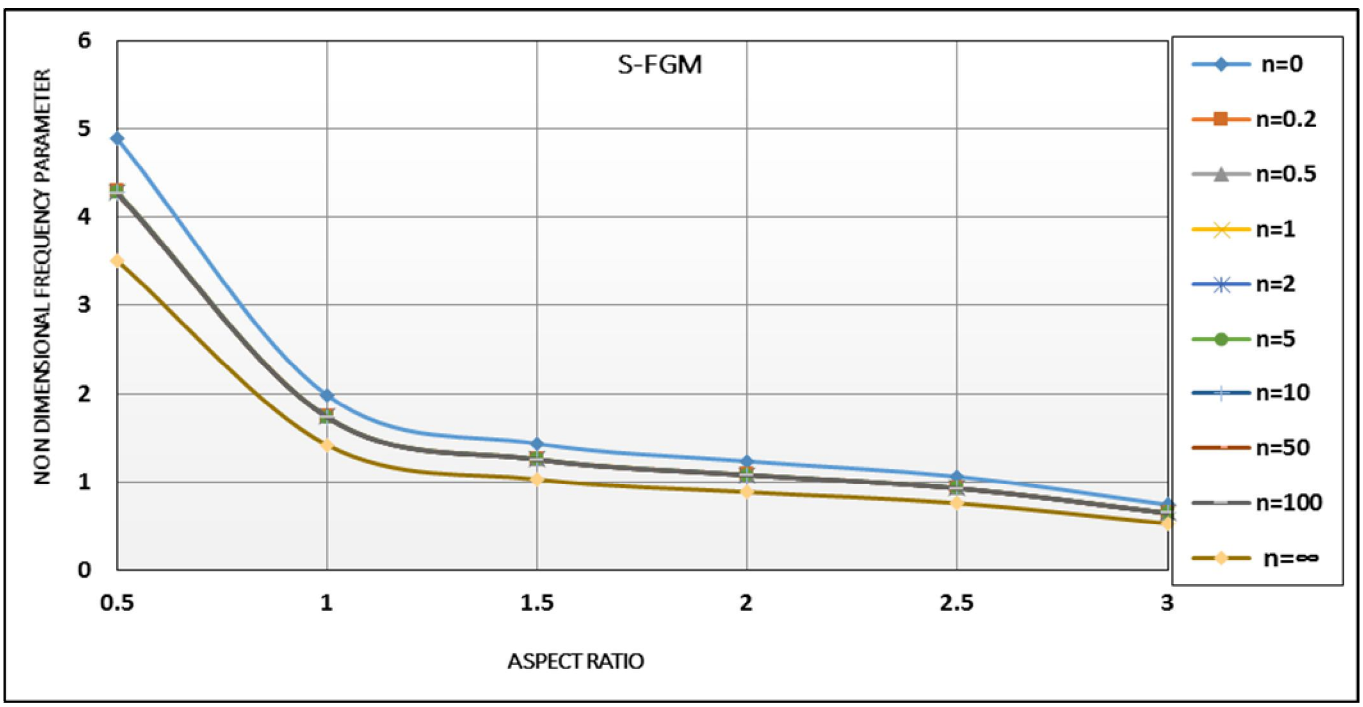


Figure 10: Variation of non-dimensional frequency parameter with material gradient index of S-FGM and aspect ratio (b/a=1.0) for SSSS Al/ZrO_2 plates.

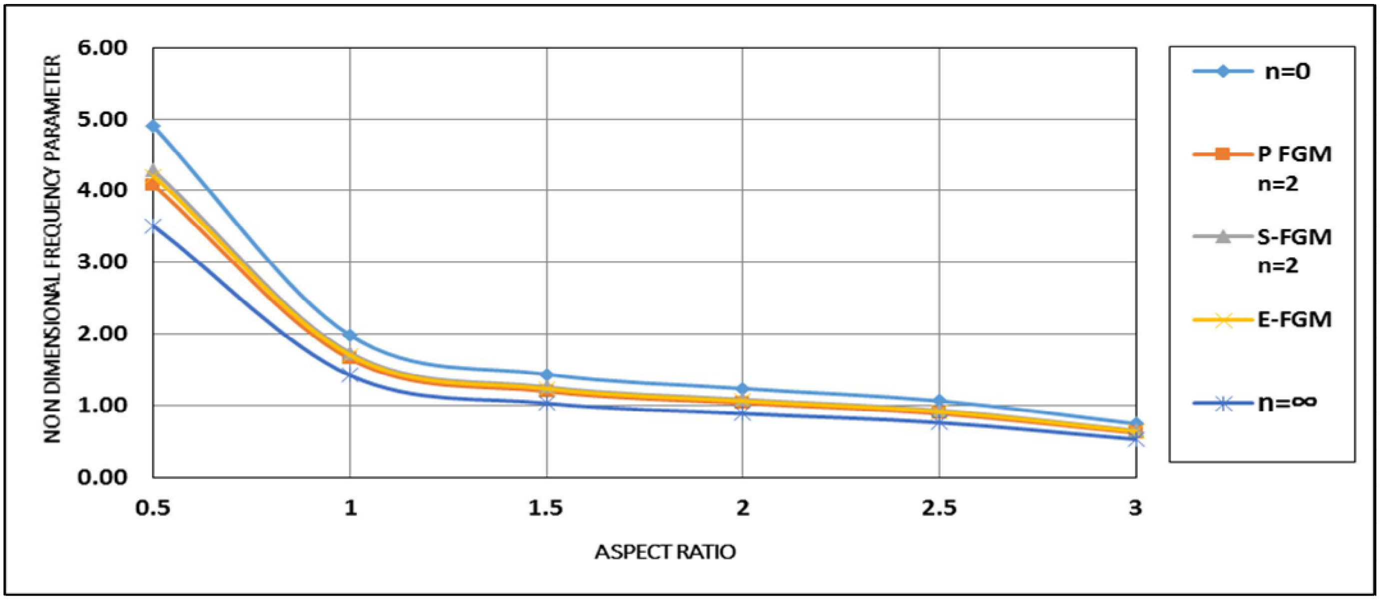


Figure 11: Comparison of non-dimensional frequency parameter with material gradient index n=2 of P-FGM, S-FGM & E-FGM with aspect ratio (b/a=1.0) for SSSS Al/ZrO_2 plates.

B. *Variation of Non-Dimensional Frequency Parameter with Aspect Ratio for SSCC FGM Plates*

Table 8: Variation of the Frequency Parameter $\overline{\omega} = \omega\sqrt{12(1-\nu^2)\rho_c a^2 b^2/\pi^4 E_c h^2}$ with the volume fraction index (n) for (SSCC) Square (b/a=1.0), (Al/ZrO_2) P-FGM plates (a/h=20)

Boundary condition	Mode	n							
		Ceramic(n=0)	0.5	2	5	10	50	100	Metal(n=∞)
SSCC	1	2.7034	2.4784	2.2521	2.1244	2.0594	2.0023	1.9999	1.9377
	2	5.9650	5.4686	4.9693	4.6873	4.5441	4.4180	4.4128	4.2755
	3	5.9935	5.4947	4.9931	4.7097	4.5658	4.4391	4.4338	4.2959
	4	9.0094	8.2597	7.5056	6.8663	6.8633	6.6729	6.6650	6.4576
	5	11.0815	10.6336	9.2317	8.7079	8.4417	8.2076	8.1978	7.9428

Table 9: Variation of the Frequency Parameter $\overline{\omega} = \omega\sqrt{12(1-\nu^2)\rho_c a^2 b^2/\pi^4 E_c h^2}$ with the volume fraction index (n) for (SSCC) Square (b/a=1.0), (Al/ZrO_2) S-FGM plates (a/h=20)

Boundary condition	Mode	n							
		Ceramic(n=0)	0.5	2	5	10	50	100	Metal(n=∞)
SSCC	1	2.7034	2.3703	2.3680	2.3649	2.3619	2.3588	2.3588	1.9377
	2	5.9650	5.2299	5.2249	5.2181	5.2115	5.2044	5.2047	4.2755
	3	5.9935	5.2549	5.2499	5.2430	5.2365	5.2296	5.2296	4.2959
	4	9.0094	7.8992	7.8917	7.8814	7.8714	7.8613	7.8612	6.4576
	5	11.0815	9.7159	9.7065	9.6939	9.6817	9.6691	9.6690	7.9428

Table 10: Variation of the Frequency Parameter $\overline{\omega} = \omega\sqrt{12\,(1-\nu^2)\,\rho_c a^2 b^2/\pi^4 E_c h^2}$ with the volume fraction index (n) for (SSCC) Square (b/a=1.0), (Al/ZrO_2) E-FGM plates (a/h=20)

Boundary condition	**Mode**	**Exponential FGM**
SSCC	**1**	2.3229
	2	5.1254
	3	5.1500
	4	7.7414
	5	9.5218

For functionally graded material plates with SSCC boundary conditions value of non-dimensional frequency parameter decreases by increasing material gradient index (n) and aspect ratio (b/a) of plate, which is shown in figure 12 & 13. SSCC FGM plates having variation in material property according to Sigmoid (S-FGM) law, non-dimensional frequency parameter does not have significant variation with increasing power law index (n) but variation in material property according to Power-law (P-FGM) distribution, non-dimensional frequency parameter shows substantial variation by increasing material gradient index between n=0 to ∞. Comparison of non-dimensional frequency parameter with Power-law, Sigmoid-law and Exponential FGM is shown in figure 14.

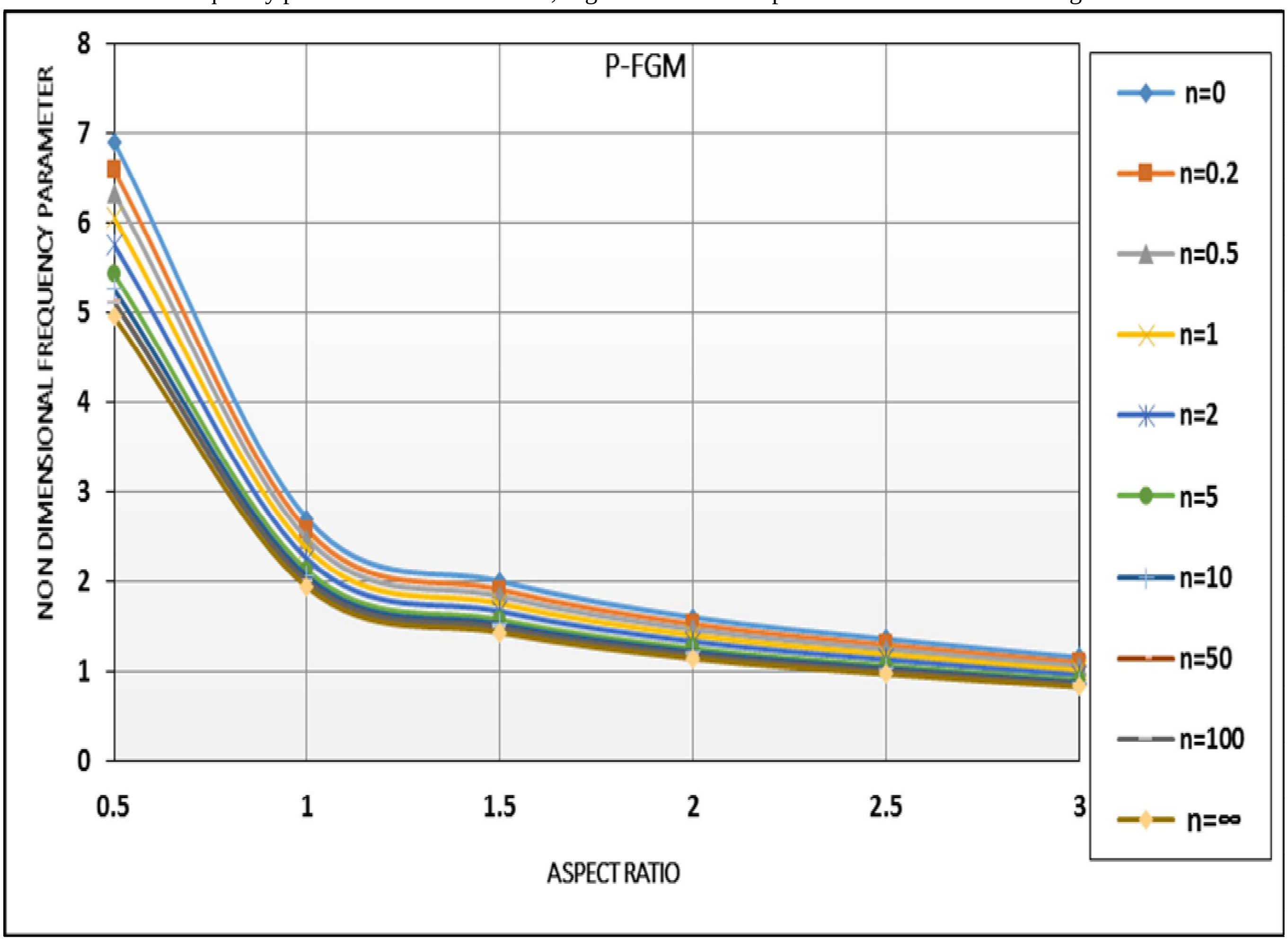


Figure 12: Variation of non-dimensional frequency parameter with material gradient index n of P-FGM and aspect ratio for SSCC Al/ZrO_2 plates.

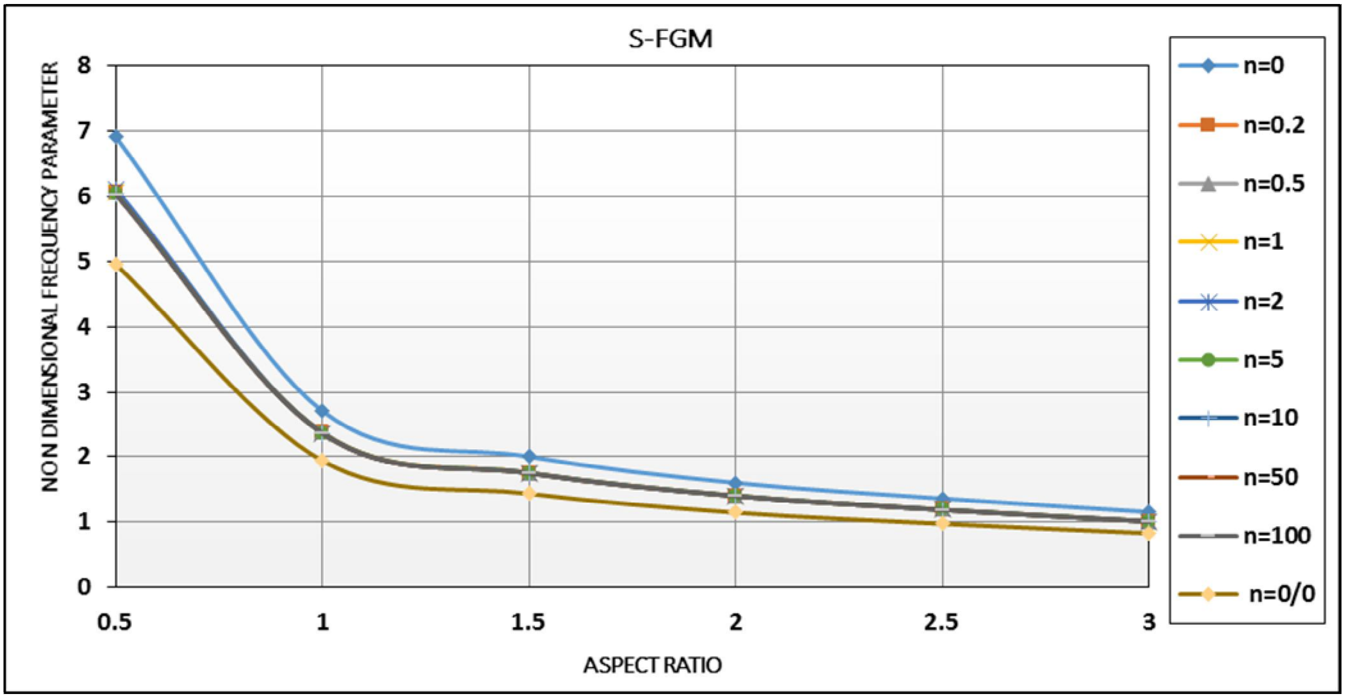


Figure 13: Variation of non-dimensional frequency parameter with material gradient index n of S-FGM and aspect ratio for SSCC Al/ZrO_2 plates.

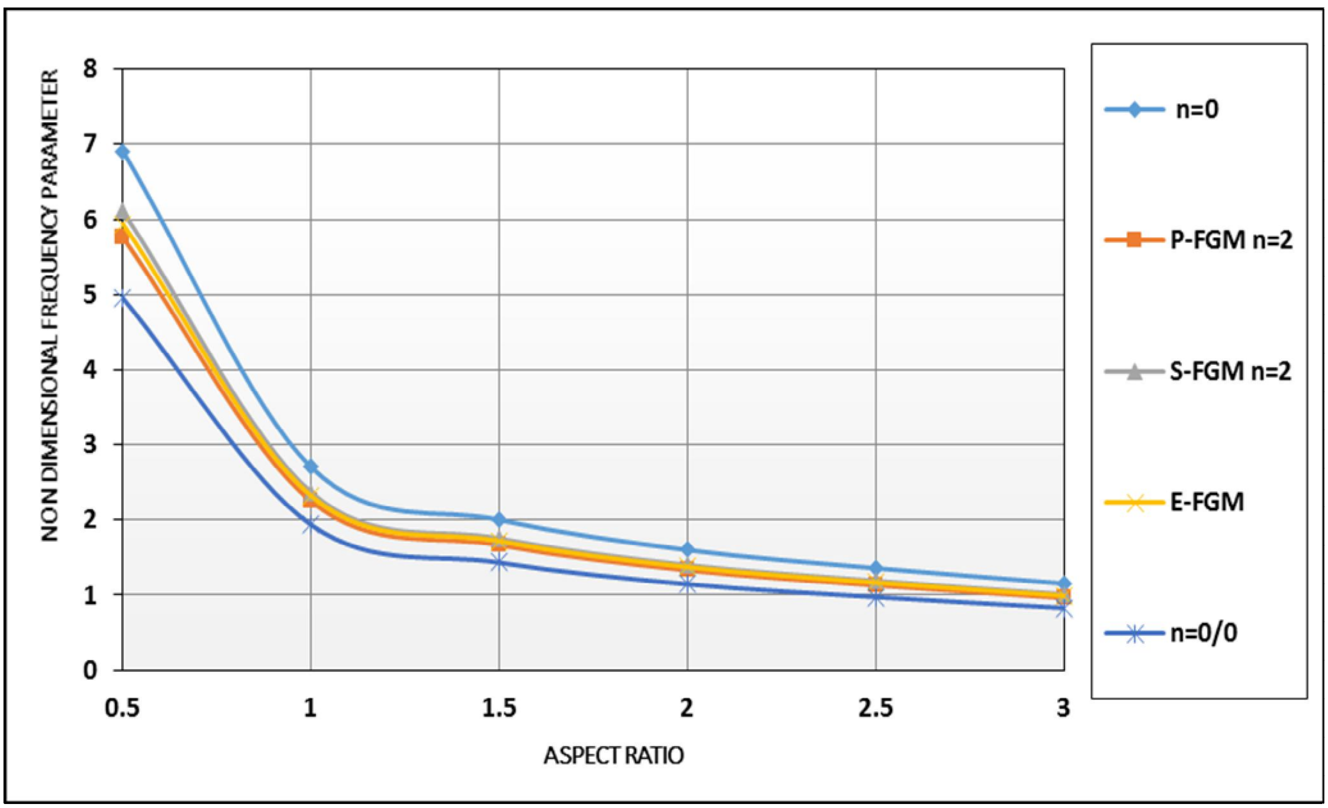


Figure 14: Comparison of non-dimensional frequency parameter with material gradient index n=2 of P-FGM, S-FGM & E-FGM and aspect ratio for SSCC Al/ZrO_2 plates.

C. Variation of Non-Dimensional Frequency Parameter with Material Gradient Index n for SSSS FGM Plates

Non-dimensional frequency parameter for SSSS FGM plates decreases as by increasing value of both volume fraction index (n) and plate aspect ratio for P-FGM as shown in figure 15. For S-FGM variation in non-dimensional frequency parameter is relatively very small for most values of volume fraction index (i.e. n>0 and n<∞), its value only changes for volume fraction index at n=0 and n=∞. So value of frequency parameter is almost constant for most values of volume fraction index. As plate aspect ratio increases non-dimensional frequency parameter reduces as shown in figure 15 to 17. Figure 17 shows variation of frequency parameter for E-FGM with all simply supported edges (SSSS).

Table 11: Variation of the First- Frequency Parameter $\overline{\omega} = \omega\sqrt{12\,(1-\nu^2)\,\rho_c a^2 b^2/\pi^4 E_c h^2}$ with the volume fraction index (n) for (SSSS) (Al/ZrO_2) P-FGM, E-FGM plates (a/h=20) with aspect ratio.

Aspect Ratio	n=0	n=0.2	n=0.5	n=1	n=2	n=5	n=10	n=50	n=100	n=∞	E-EGM
0.5	4.8972	4.6693	4.4817	4.2927	4.0804	3.8487	3.7309	3.6273	3.6229	3.5101	4.2085
1	1.9828	1.8906	1.7143	1.7379	1.6521	1.5583	1.5106	1.4687	1.4234	1.4212	1.7040
1.5	1.4361	1.3728	1.3167	1.2588	1.1965	1.1286	1.0941	1.0637	1.0624	1.0293	1.2341
2	1.2350	1.1806	1.1323	1.0826	1.0290	0.9706	0.9409	0.9148	0.9137	0.8852	1.0614
2.5	1.0621	1.0153	0.9738	0.9310	0.8850	0.8347	0.8092	0.7867	0.7858	0.7613	0.9128
3	0.7435	0.7107	0.6817	0.6517	0.6195	0.5843	0.5664	0.5507	0.5500	0.5329	0.6389

Table 12: Variation of the First- Frequency Parameter $\overline{\omega} = \omega\sqrt{12\,(1-\nu^2)\,\rho_c a^2 b^2/\pi^4 E_c h^2}$ with the volume fraction index (n) for (SSSS) (Al/ZrO_2) S-FGM, E-FGM plates (a/h=20) with aspect ratio.

Aspect Ratio	n=0	n=0.2	n=0.5	n=1	n=2	n=5	n=10	n=50	n=100	n=∞	E-EGM
0.5	4.8972	4.2942	4.2940	4.2910	4.2904	4.2850	4.2796	4.2741	4.2741	3.5101	4.2085
1	1.9828	1.7387	1.7386	1.7381	1.7371	1.7357	1.7335	1.7313	1.7313	1.4212	1.7040
1.5	1.4361	1.2593	1.2592	1.2584	1.2581	1.2565	1.2550	1.2534	1.2533	1.0293	1.2341
2	1.2350	1.0830	1.0829	1.0822	1.0820	1.0806	1.0793	1.0779	1.0779	0.8852	1.0614
2.5	1.0621	0.9314	0.9313	0.9307	0.9305	0.9293	0.9282	0.9270	0.9270	0.7613	0.9128
3	0.7435	0.6519	0.6519	0.6515	0.6514	0.6505	0.6497	0.6489	0.6489	0.5329	0.6389

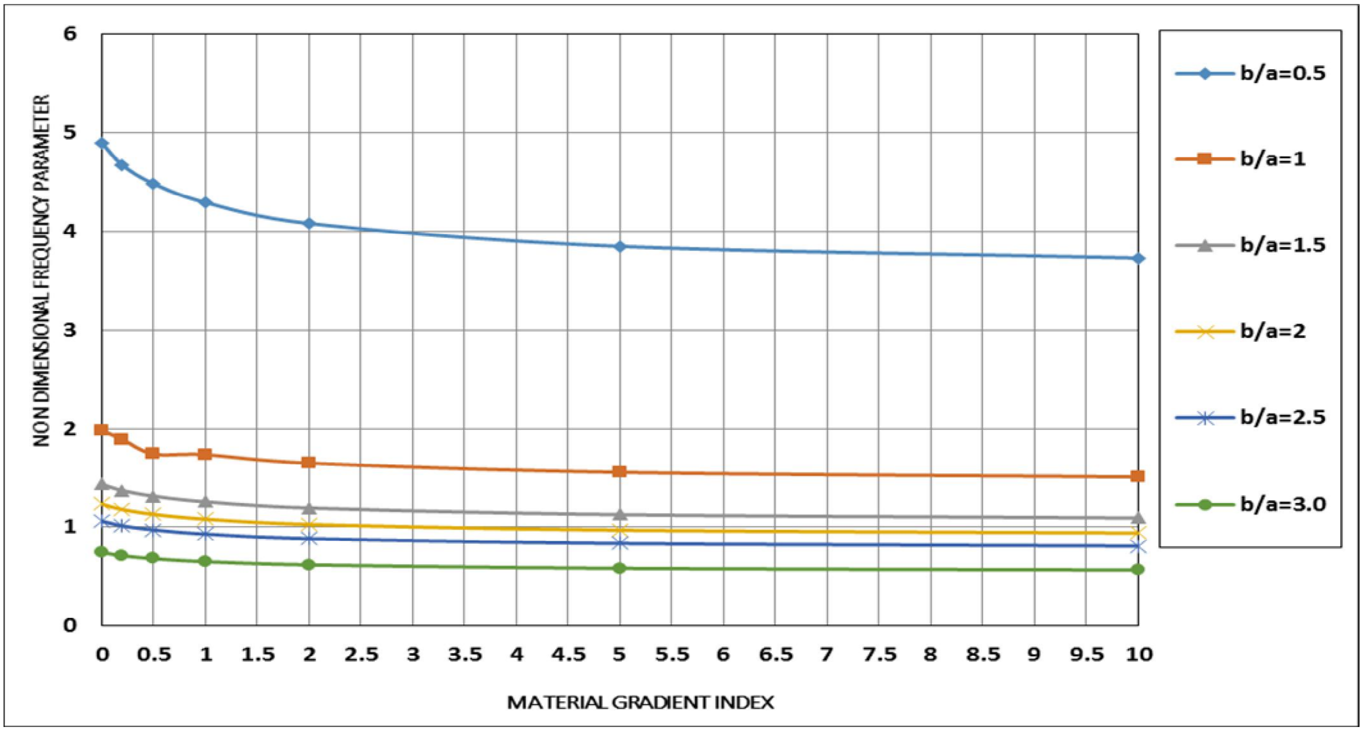


Figure 15: Non-dimensional frequency parameter v/s material index n of P-FGM for SSSS Al/ZrO_2 rectangular plate.

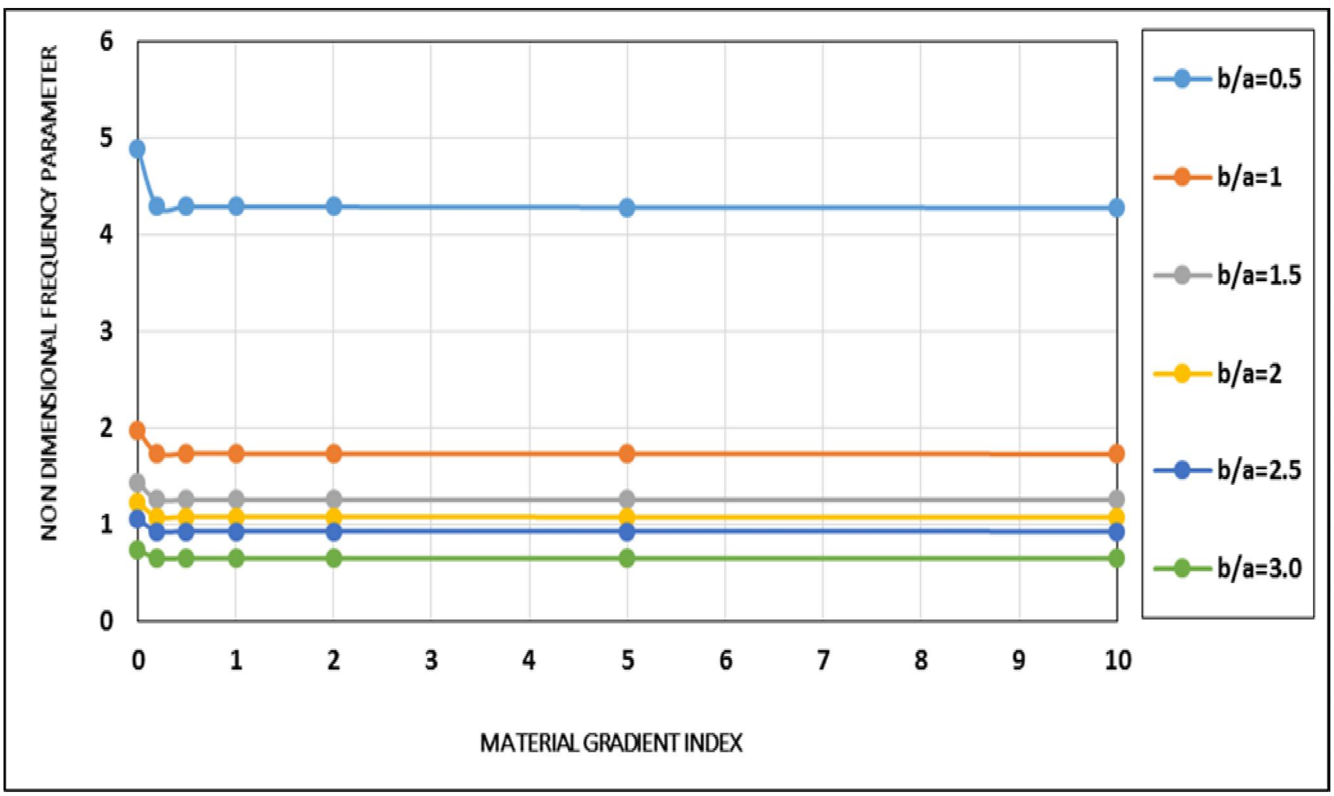


Figure 16: Non-dimensional frequency parameter v/s material gradient index n of S-FGM for SSSS Al/ZrO_2 rectangular plate.

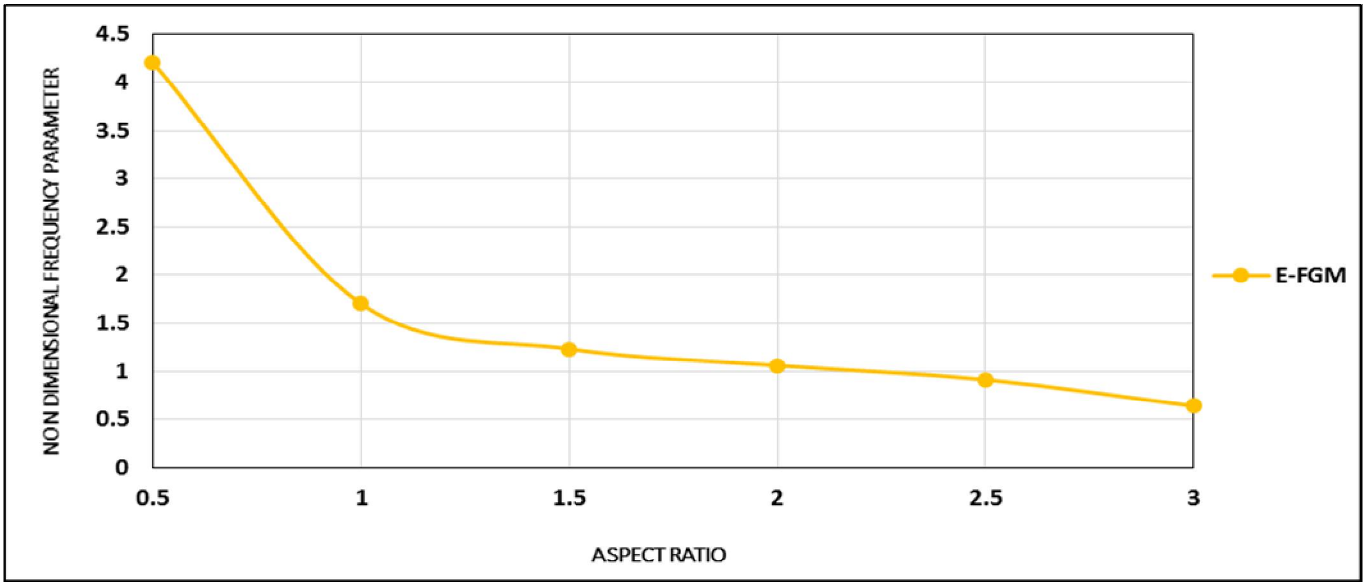


Figure 17: Non-dimensional frequency parameter v/s aspect ratio for Exponential FGM SSSS Al/ZrO_2 rectangular plate.

D. Variation of Non-Dimensional Frequency Parameter with Material Gradient Index n for SSCC FGM Plates

Table 13: Variation of the First- Frequency Parameter $\overline{\omega} = \omega\sqrt{12\,(1-\nu^2)\,\rho_c a^2 b^2/\pi^4 E_c h^2}$ with the volume fraction index (n) for (SSCC) (Al/ZrO_2) P-FGM, E-FGM plates (a/h=20) with aspect ratio.

Aspect Ratio	n=0	n=0.2	n=0.5	n=1	n=2	n=5	n=10	n=50	n=100	n=∞	E-EGM
0.5	6.9110	6.5892	6.3359	6.0572	5.7574	5.4307	5.2642	5.1182	5.1126	4.9535	5.9383
1.0	2.7034	2.5775	2.4784	2.3693	2.2521	2.1244	2.0594	2.0023	1.9999	1.9377	2.3229
1.5	2.0027	1.9094	1.8360	1.7553	1.6684	1.5737	1.5256	1.4833	1.4816	1.4355	1.7208
2.0	1.6021	1.5275	1.4688	1.4042	1.3347	1.2590	1.2205	1.1867	1.1852	1.1484	1.3767
2.5	1.3618	1.2984	1.2485	1.1936	1.1345	1.0701	1.0374	1.0087	1.0075	0.9761	1.1702
3.0	1.1575	1.1037	1.0612	1.0145	0.9643	0.9096	0.8818	0.8574	0.8563	0.8297	0.9946

Table 14 Variation of the First- Frequency Parameter $\overline{\omega} = \omega\sqrt{12\,(1-\nu^2)\,\rho_c a^2 b^2/\pi^4 E_c h^2}$ with the volume fraction index (n) for (SSCC) (Al/ZrO_2) S-FGM, E-FGM plates (a/h=20) with aspect ratio.

Aspect Ratio	N=0	S-FGM N=0.2	S-FGM N=0.5	S-FGM N=1	S-FGM N=2	S-FGM N=5	S-FGM N=10	S-FGM N=50	S-FGM N=100	N=∞	E-EGM
0.5	6.9110	6.0599	6.0594	6.0547	6.1069	6.0456	6.0381	6.0302	6.0301	4.9535	5.9383
1.0	2.7034	2.3704	2.3703	2.3685	2.3680	2.3649	2.3619	2.3588	2.3588	1.9377	2.3229
1.5	2.0027	1.7560	1.7559	1.7546	1.7542	1.7519	1.7497	1.7474	1.7474	1.4355	1.7208
2.0	1.6021	1.4048	1.4047	1.4037	1.4034	1.4015	1.3998	1.3980	1.3980	1.1484	1.3767
2.5	1.3618	1.1941	1.1940	1.1931	1.1929	1.1913	1.1898	1.1883	1.1883	0.9761	1.1702
3.0	1.1575	1.0150	1.0149	1.0141	1.0139	1.0126	1.0113	1.0100	1.0100	0.8297	0.9946

For SSCC FGM plate value of non-dimensional frequency parameter reduces by increasing value of material gradient index for P-FGM. Its value is maximum for n=0 and minimum for n=∞, as shown in figure 18.

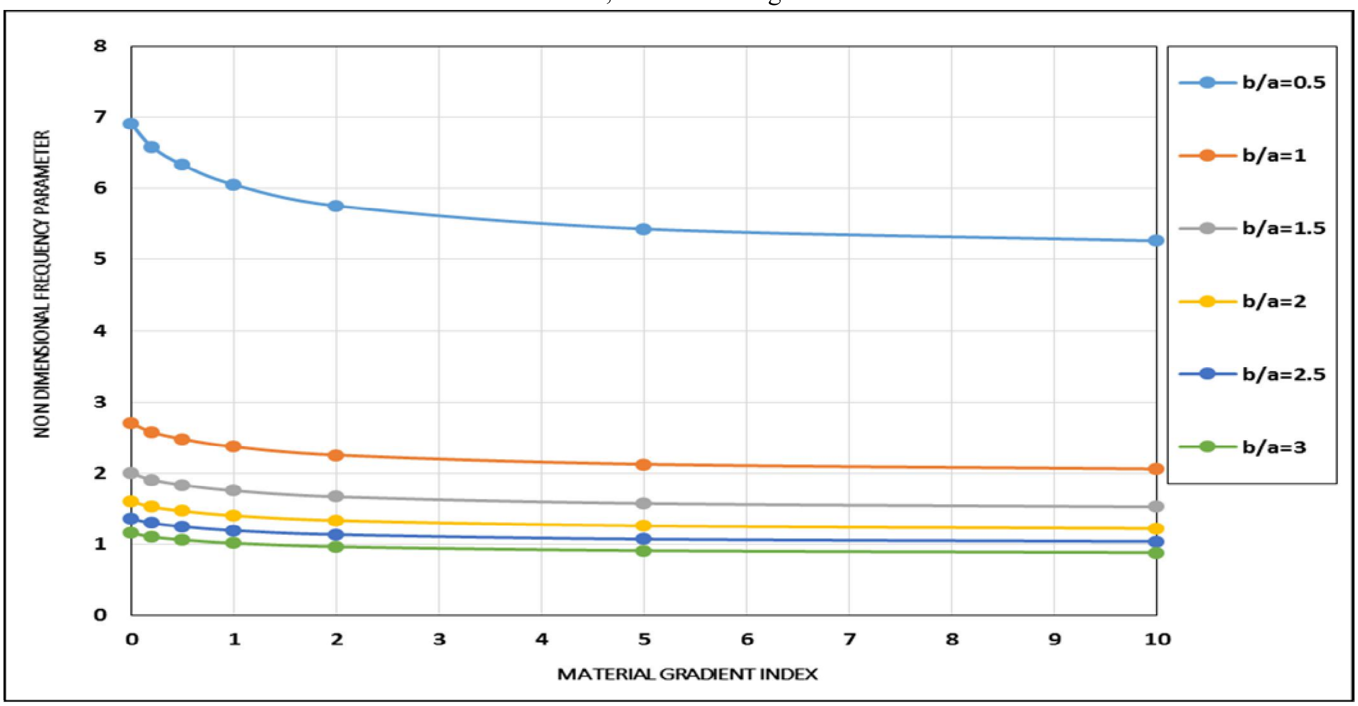


Figure 18: Non-dimensional frequency parameter v/s material gradient index n of P-FGM for SSCC Al/ZrO_2 rectangular plate.

International Journal for Research in Applied Science & Engineering Technology (IJRASET)
ISSN: 2321-9653; IC Value: 45.98; SJ Impact Factor:6.887
Volume 5 Issue XII December 2017- Available at www.ijraset.com

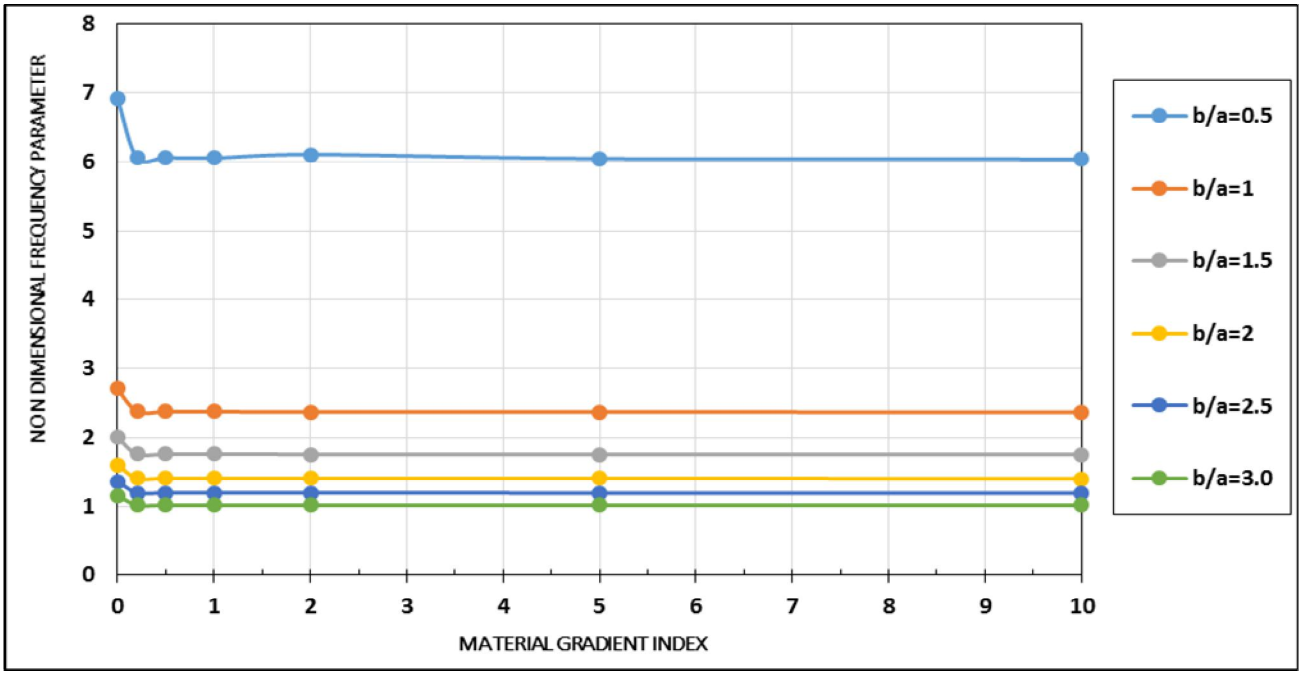


Figure 19: Non-dimensional frequency parameter v/s material gradient index n of S-FGM for SSCC Al/ZrO_2 rectangular plate.

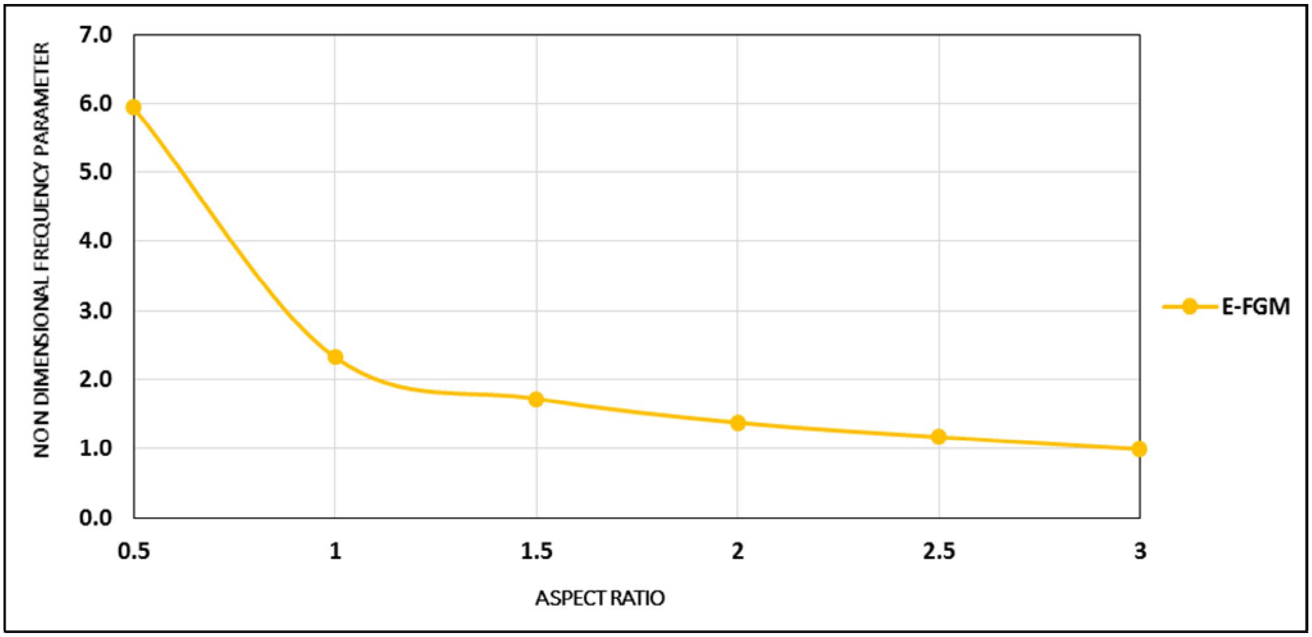


Figure 20: Non-dimensional frequency parameter v/s aspect ratio for Exponential FGM SSCC Al/ZrO_2 rectangular plate.

In case of Sigmoid SSCC FGM plate there is small variation in material properties i.e. elastic constant, so variation in non-dimensional frequency parameter is very small and its value is constant for most values of material gradation index. Non-dimensional frequency parameter changes for only values of n=0 and n=∞. Frequency parameter is maximum for n=0 and minimum at n=∞ as shown in figure 19. Figure 20 shows variation of non-dimensional frequency parameter for SSCC E-FGM plates. From figure 18 to 20 it is clear that by increasing value of aspect ratio non-dimensional frequency parameter reduces.

IX. CONCLUSION

An extensive study of free vibration analysis of square and rectangular functionally graded material plates subjected to both uniformly distributed load and Thermo mechanical loading I presented. Analysis is performed using ANSYS 15.0 APDL software. A Solid-Shell element (SOLSH190) based on first-order shear deformation theory is used for above analysis. Effective material properties of functionally graded material plate are calculated for each layer according to Simple Power law (P-FGM), Sigmoid law (S-FGM) and Exponential law (E-FGM). Accuracy of present methodology i.e. number of FGM layers, element type is justified by performing convergence and validation studies. Results obtained from above analysis is in good agreement with those available in literature. The following important conclusions are observed from the result obtained from free vibration analysis considering various parameters viz. different material gradient index i.e. P-FGM, S-FGM, E-FGM laws, plate aspect ratio thermal loadings and various combination of boundary conditions.

A. It is concluded that free vibration frequency of FGM plate is maximum for pure ceramic (n=0) and minimum for metal (n=∞) & decreases gradually as volume fraction index n increases. By increasing value of material gradient index n, sharing of ceramic(ZrO_2) phase in FGM plate decreases and sharing of Aluminium phase increases which tends t reduce the stiffness of FGM plate and thereby reducing the natural frequency, because th Young's Modulus of Aluminium is 151GPa and Young's Modulus of Zirconia is 70GPa.

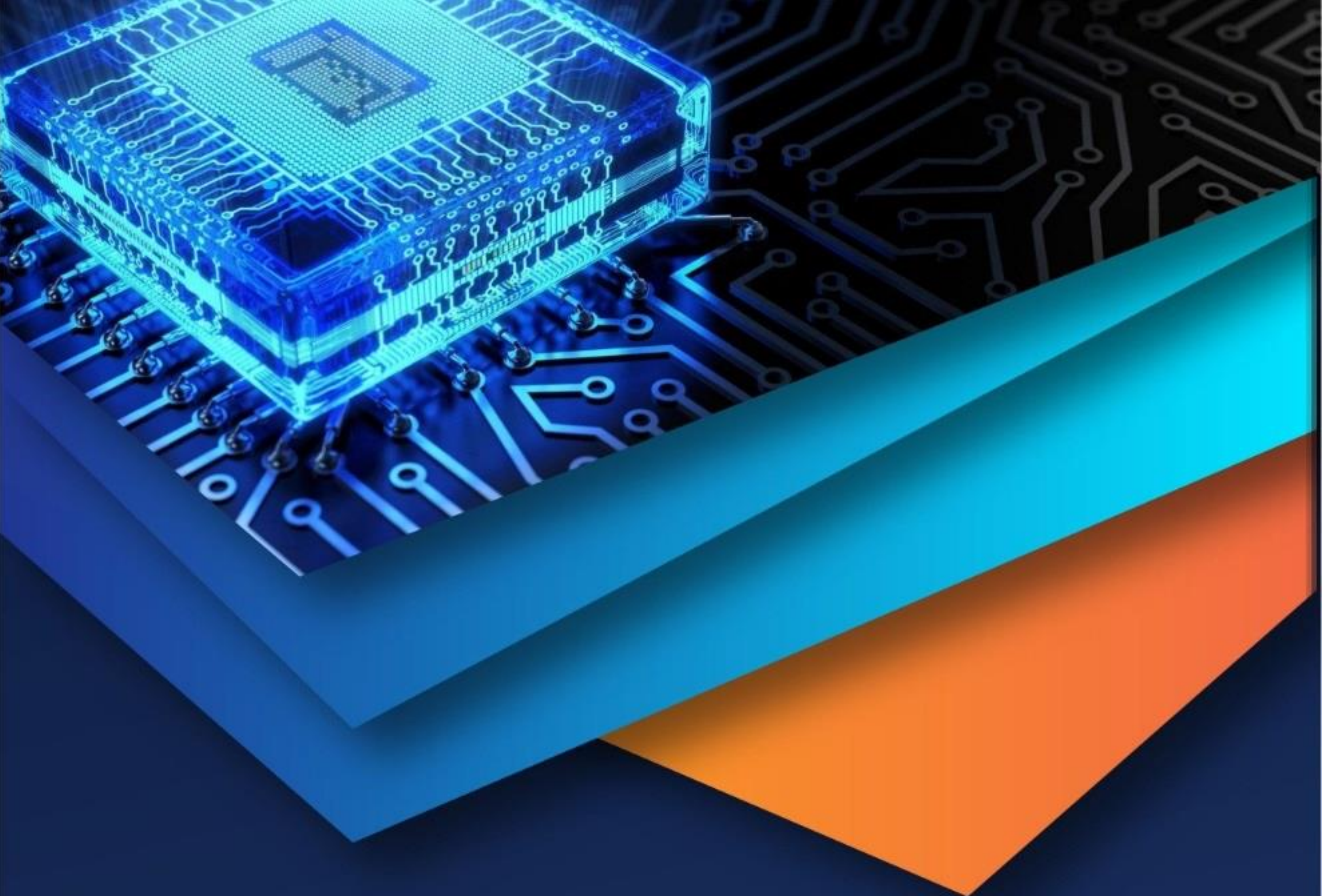
B. It is concluded that an increase in plate aspect ratio, leads to decrease in stiffness of plate, and consequently the value of non-dimensional frequency parameter decreases.

C. To study influence of boundary conditions of plate, a set of two types of boundary conditions i.e., SSSS, and SSCC are considered here. It is expected that stiffer boundaries yield higher value of non-dimensional frequency parameter whereas less stiff boundaries correspond to lower values of non-dimensional frequency parameter. In above analysis, order of non-dimensional frequency parameter is SSCC>SSSS. It means that Simply Supported and Clamped (SSCC) boundary condition is stiffer than Simply Supported (SSSS) boundaries are very less stiff.

REFERENCES

[1] G. N. PRAVEEN and J. N. REDDY, "Nonlinear Transient Thermo-elastic Analysis of Functionally Graded Ceramic-Metal Plates" Int. Journal of Solids Structures, 1998 Vol. 35. No. 3, pp. 4457-447
[2] J. N. Reddy, "Analysis of functionally graded plates" INTERNATIONAL JOURNAL FOR NUMERICAL METHODS IN ENGINEERING, 2000 Vol. 47, pp. 663 -68
[3] Z.-Q. CHENG and R. C. BATRA, "Exact Correspondence between Eigenvalues of Membranes and Functionally Graded Simply Supported Polygonal Plates" Journal of Sound and Vibration, 2000 Vol. 229(4) pp. 879-89
[4] A. Chakraborty, S. Gopalakrishnana, J.N. Reddy, "A New Beam Finite Element for th Analysis of Functionally Graded Materials" International Journal of Mechanical Sciences 200 Vol.45 pp.519-53
[5] L.F.Qian and R.C.Batra, "Transient Thermoelastic Deformation of a Thick Functionally Graded Plate" International Journal of Thermal Stresses 2004 Vol. 27 pp 705–74
[6] A.J.M. Ferreira, R.C. Batra, C.M.C. Roque, L.F. Qian, P.A.L.S. Martins "Static Analysis of Functionally Graded Plates using Third-order Shear Deformation Theory And a Meshless Method" International Journal of Composite Structures 2005, Vol. 69 pp.449-457
[7] A.M. Zenkour, "A Comprehensive Analysis of Functionally Graded Sandwich Plates: Pa 2-Buckling and Free Vibration" International Journal of Solids and Structures 2005, Vol. 42 pp. 5243–525
[8] R.C. Batra, J.Jin, "Natural Frequencies of a Functionally Graded Anisotropic Rectangula plate" International Journal of Sound and Vibration 2005, Vol. 282 pp. 509-516
[9] J. Woo, S.A. Meguid, L.S. Ong, "Nonlinear Free Vibration Behaviour of Functionally Graded Plates" International Journal of Sound and Vibration 2006, Vol. 289 pp.595-611
[10] Shyang-Ho Chi, Yen-Ling Chung, "Mechanical Behavior of Functionally Graded Material
[11] Plates Under Transverse Load —Part I: Analysis" International Journal of Solids and Structures2006, Vol. 43 pp.3657-367
[12] Shyang-Ho Chi, Yen-Ling Chung, "Mechanical Behavior of Functionally Ggraded Material
[13] Plates under Transverse Load—Part II: Numerical results" International Journal of Solids and Structures 2006, Vol. 43 pp. 3675–36
[14] Serge Abrate, "Free vibration, Buckling, and Static Deflections of Functionally Graded Plates" Composites Science and Technology 2006, Vol.66 pp.2383-239
[15] T. Prakash, M. Ganapathi, "Asymmetric Flexural Vibration and Thermoelastic Stability o
[16] FGM Circular Plates using Finite Element Method" International Journal of Composites 2006: Part B Vol 37 pp.642-64
[17] C.M.C. Roque, A.J.M. Ferreira, R.M.N. Jorge, "A Radial Basis Function Approach For th
[18] Free Vibration Analysis of Functionally Graded Plates using a Refined Theory" Journal of Soun and Vibration 2007, Vol.300 pp.1048-107
[19] Hiroyuki Matsunaga, "Free Vibration and Stability of Functionally Graded Plates According to a 2-D Higher-Order Deformation Theory" Journal of Composite Structures 2008 Vol. 82 pp. 499-51
[20] Trung-Kien Nguyen, Karam Sab, Guy Bonnet, "First-Order Shear Deformation Plate Models for Functionally Graded Materials" Journal of Composite Structures 2008, Vol.83 pp.25-3

[21] Farzad Ebrahimi, Abbas Rastgo, "An Analytical Study on The Free Vibration of Smart Circular Thin FGM Plate Based on Classical Plate Theory" Journal of Thin-Walled Structures 2008, Vol. 46 pp. 1402– 140
[22] X.Zhao, Y.Y.Lee, K.M. Liew, "Free Vibration Analysis of Functionally Graded Platesn Using The Element-Free kp-Ritz Method" Journal of Sound and Vibration 2009, Vol.319 pp 918-93
[23] Wei Zhang, Jie Yang, Yuxin Hao, "Chaotic Vibrations of An Orthotropic FG Rectangular Plate based on Third-Order Shear Deformation Theory" Journal of Nonlinear Dynamics 2010 Vol. 59 pp.619-66
[24] Sh. Hosseini, Hashemi , M.Es'haghi , M.Karimi, "Closed-form Vibration Analysis of Thick Annular Functionally Graded Plates With Integrated Piezoelectric Layers" International Journal of Mechanical Sciences 2010, Vol.52 pp.410-42
[25] D.Y. Liu , C.Y. Wang, W.Q. Chen, "Free vibration of FGM plates with in-plane material inhomogeneity" Journal of Composite Structures 2010, Vol. 92 pp. 1047–10
[26] Sh. Hosseini-Hashemi , H. Rokni Damavandi Taher , H. Akhavan , M. Omidi , "Fre Vibration of Functionally Graded Rectangular Plates using First-order Shear Deformation Plate Theory" Journal of Applied Mathematical Modelling 2010, Vol. 34 pp. 1276–129
[27] Mohammad Talha, B.N. Singh, "Static response and free vibration analysis of FGM plate using higher order shear deformation theory" Journal of Applied Mathematical Modelling 2010, Vol. 34 pp. 3991–40
[28] Song Xiang, Yao-xing Jin, Ze-yang Bi, Shao-xi Jiang, Ming-sui Yang, "A n-order Shear
[29] Deformation Theory for Free Vibration of Functionally Graded and Composite Sandwich Plates Journal of Composite Structures 2011 Vol.93 pp. 2826–283
[30] Abdelkader Benachour , Hassaine Daouadji Tahar, Hassen Ait Atmane, Abdelouahe
[31] Tounsi, Meftah Sid Ahmed, "A Four Variable Refined Plate Theory for Free Vibrations of Functionally Graded Plates with Arbitrary Gradient" International journal of Composites 2011 Vol. Part B (42) pp. 1386–139
[32] J. Suresh Kumar, B. Sidda Reddy, C. Eswara Reddy and K. Vijaya Kumar Reddy, "Higher
[33] Order Theory for Free Vibration Analysis of Functionally Graded Material Plates" ARP Journal of Engineering and Applied Sciences October 2011, Vol.6 No 10 pp.105
[34] Y.X. Hao, W. Zhang, J. Yang, "Nonlinear Oscillation of a Cantilever FGM Rectangular Plate Based on Third-Order Plate Theory and Asymptotic Perturbation Method" Journal of Composites 2011 Part B Vol.42 pp. 402-4
[35] D.K. Jha, Tarun Kant, R.K. Singh, "A Critical Review of Recent Research on Functionally Graded Plates" Journal of Composite Structures 2013, Vol. 96 pp. 833–84
[36] T. Prakash, M.K.Singha, M.Ganapathi, "A Finite Element Study on The Large Amplitude Flexural Vibration Characteristics of FGM Plates under Aerodynamic Load" International Journal of Non-Linear Mechanics 2012, Vol. 47 pp. 439–447
[37] Hui-Shen Shen, Zhen-Xin Wang, "Assessment of Voigt and Mori–Tanaka models fo Vibration Analysis of Functionally Graded Plates" Journal of Composite Structures 2012 Vol. 94 pp. 2197–220
[38] K. Sanjay Anandrao, R.K. Gupta, P. Ramachandran, and G. Venkateswara Rao, "Free Vibration Analysis of Functionally Graded Beams" Defence Science Journal May 2012, Vol. 62, No. 30, pp. 139-146
[39] Koteswara Rao D, Blessington P. J, Tarapada Roy, "Finite Element Modeling and Analysis of Functionally Graded (FG) Composite Shell Structures" ICMOC Procedia Engineering 2012, Vol. 38 pp. 3192 – 319
[40] D.K. Jha, Tarun Kant, R.K. Singh, "Higher order shear and normal deformation theory for natural frequency of functionally graded rectangular plates" journal of Nuclear Engineering an Design 2012, Vol. 250 pp. 8– 1
[41] Mohammad Talha and B N Singh, "Thermo-Mechanical Deformation Behavior o Functionally Graded Rectangular Plates Subjected to Various Boundary Conditions and Loadings" International Journal of Aerospace and Mechanical Engineering 2012, Vol. 6:1 pp.
[42] K. Swaminathan, D. T. Naveenkumar, "Assessment of First Order Computational Model for Free Vibration Analysis of FGM Plates" International Journal of Scientific & Engineering Research May-2013 Volume 4, Issue 5 pp. 115-11
[43] Shuohui Yin, Tiantang Yu, and Peng Liu, "Free Vibration Analyses of FGM Thin Plates n Isogeometric Analysis Based on Classical Plate TheorNeutral
[44] Surface" Hindawi Publishing Corporation Advances in Mechanical Engineering Volume 2013, Article ID 634584, 10 page Z.X. Lei, K.M. Liew, J.L. Yu, "Free Vibration Analysis of Functionally Graded Carbon
[45] Nanotube-Reinforced Composite Plates using the Element-Free kp-Ritz Method in Thermal Environment" Journal of Composite Structures 2013, Vol. 106 pp. 128-13
[46] Huu-Tai Thai, Dong-Ho Choi, "A simple first-order shear deformation theory for the bending and free vibration analysis of functionally graded plates" Journal of Composit Structures 2013 Vol. 101 pp.332-340.



10.22214/IJRASET

INDEX COPERNICUS
45.98

IMPACT FACTOR:
7.129

TOGETHER WE REACH THE GOAL

IMPACT FACTOR:
7.429

INTERNATIONAL JOURNAL FOR RESEARCH

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY