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An Efficient Product-Type Estimator of Finite Population Mean

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Abstract: In this paper, a new product-type estimator is proposed and is compared with the Bahl and Tuteja's product-type exponential estimator and the customary product estimator from the standpoint of bias and mean square error with large sample approximations. While the newly proposed estimator is found to be equally efficient with the Bahl and Tuteja's product type exponential estimator to the first degree of approximation, the same, to the second degree of approximation, fares better than the usual product estimator and Bahl and Tuteja's estimator under certain conditions.
Keywords: simple random sampling, product type estimator, bias, mean square error.

I. INTRODUCTION

The use of information on auxiliary character to improve estimates of population parameters of the study variable is a common phenomenon in sample surveys, especially when there is a linear relationship between the study and auxiliary variables. In survey sampling, when the study variable y is negetively correlated with the auxiliary variable x and complete information on x is available, the product method of estimation is followed to estimate the population mean ($\bar{Y}$) or the population total (Y) .The conventional product estimator given by Murthy (1967) is defined as

$$\bar{y}_p = \frac{\bar{y}\bar{x}}{\bar{X}},$$

where $\bar{y}$ and $\bar{x}$ are sample means of y and x ,respectively, and $\bar{X}$, the population mean of x. The product estimator $\bar{y}_P$ envisages advanced knowledge of $\bar{X}$. The bias and mean square error of $\bar{y}_P$ to $O(1/n)$ are given by

$$B(\bar{y}_P = \theta\bar{Y}C_{11})$$

$$MSE(\bar{y}_P) = \theta\bar{Y}^2(C_{20} + C_{02} + 2C_{11}),$$

where $\theta = \frac{N-n}{(N-1)n}$, $C_{rs} = \frac{\sum_{i=1}^{N}(y_i-\bar{Y})^r(x_i-\bar{X})^s}{N\bar{Y}^r\bar{X}^s}$; $r, s = 0,1,2$

The product estimator $\bar{y}_P$ is biased and the bias decreases with increase in sample size. In large samples $\bar{y}_P$ is more efficient than the mean per unit estimator $\bar{y}$ if $k = \rho\sqrt{\frac{C_{20}}{C_{02}}} < -\frac{1}{2}$ or if $\rho < -\frac{1}{2}$ when $C_{20} = C_{02}$, ρ being the correlation coefficient between y and x.

Bahl and Tuteja (1991) have suggested a product-type exponential estimator given by

$$\bar{y}_{BTP} = \bar{y}\exp\left(\frac{\bar{x}-\bar{X}}{\bar{x}+\bar{X}}\right).$$

To $O(1/n)$, the bias and mean square error of $\bar{y}_{BTP}$ are given by

$$B(\bar{y}_{BTP}) = \theta\bar{Y}\left(\frac{1}{2}C_{11} - \frac{1}{8}C_{02}\right)$$

and $MSE(\bar{y}_{BTP}) = \theta\bar{Y}^2\left(C_{20} + \frac{1}{4}C_{02} + C_{11}\right)$.

Thus, $\bar{y}_{BTP}$ is more efficient than $\bar{y}_P$ and $\bar{y}$ if $-\frac{3}{4} < k < -\frac{1}{4}$.

II. A NEW PRODUCT-TYPE ESTIMATOR, ITS BIAS AND MEAN SQUARE ERROR

Following Swain (2014), we propose a new product-type estimator using square root transformation as

$$\bar{y}_{SQP} = \bar{y}\left(\frac{\bar{x}}{\bar{X}}\right)^{1/2}. \tag{2.1}$$

For the purpose of obtaining the bias and mean square error of $\bar{y}_{SQP}$, we write that $\bar{y} = \bar{Y}(1+e_0)$ and $\bar{x} = \bar{X}(1+e_1)$ with $E(e_0) = E(e_1) = 0, V(e_0) = \theta C_{20}, V(e_1) = \theta C_{02}$ and $Cov(e_0, e_1) = \theta C_{11}$. Expanding $\bar{y}_{SQP}$ in power series with the assumption that $|e_1| < 1$ for all possible samples and retaining terms up to and including degree four in e_0 and e_1, we have arrived at the expressions for bias and mean square error as

$$B(\bar{y}_{SQP}) = \bar{Y}\frac{N-n}{N-1}\frac{1}{n}\left[\left(\frac{1}{2}C_{11} - \frac{1}{8}C_{02}\right)\right] + \bar{Y}\frac{N-n}{N-1}\frac{1}{n}\left[\frac{A}{n}\left(\frac{1}{16}C_{03} - \frac{1}{8}C_{12}\right) + \frac{B}{n^2}\left(\frac{1}{16}C_{13} - \frac{5}{128}C_{04}\right) + \frac{3D}{n^2}\left(\frac{1}{16}C_{11}C_{02} - \frac{5}{128}{C_{02}}^2\right)\right] \tag{2.2}$$

and

$$MSE(\bar{y}_{SQP}) = \frac{N-n}{N-1}\frac{1}{n}\left[\left(C_{20} + \frac{1}{4}C_{02} + C_{11}\right)\right] + \bar{Y}^2\frac{N-n}{N-1}\frac{1}{n}\left[\frac{A}{n}\left(\frac{1}{4}C_{12} + C_{21} - \frac{1}{8}C_{03}\right) + \frac{B}{n^2}\left(\frac{5}{64}C_{04} - \frac{1}{8}C_{13}\right) + \frac{D}{n^2}\left(\frac{15}{64}C_{02}^2 - \frac{3}{8}C_{11}C_{02}\right)\right], \tag{2.3}$$

where $A = \frac{N-2n}{N-2}$, $B = \frac{N^2+N-6nN+6n^2}{(N-2)(N-3)}$ and $D = \frac{N(N-n-1)(n-1)}{(N-2)(N-3)}$.

III. BIAS AND MEAN SQUARE ERROR OF $\bar{y}_{BTP}$ AND $\bar{y}_P$

Expanding $\bar{y}_{BTP}$ in power series with the same assumptions used in the expansion of $\bar{y}_{SQP}$ and retaining terms up to and including degree four in e_0 and e_1, we have

$$B(\bar{y}_{BTP}) = \bar{Y}\frac{N-n}{N-1}\frac{1}{n}\left[\left(\frac{1}{2}C_{11} - \frac{1}{8}C_{02}\right)\right] + \bar{Y}\frac{N-n}{N-1}\frac{1}{n}\left[\frac{A}{n}\left(\frac{1}{48}C_{03} - \frac{1}{8}C_{12}\right) + \frac{B}{n^2}\left(\frac{1}{48}C_{13} + \frac{1}{384}C_{04}\right) + \frac{3D}{n^2}\left(\frac{1}{48}C_{11}C_{02} + \frac{1}{384}{C_{02}}^2\right)\right] \tag{3.1}$$

and
$$MSE(\bar{y}_{BTP}) = \bar{Y}^2\frac{N-n}{N-1}\frac{1}{n}\left[\left(C_{20} + \frac{1}{4}C_{02} + C_{11}\right)\right] + \bar{Y}^2\frac{N-n}{N-1}\frac{1}{n}\left[\frac{A}{n}\left(\frac{1}{4}C_{12} + C_{21} - \frac{1}{8}C_{03}\right) + \frac{B}{n^2}\left(\frac{7}{192}C_{04} - \frac{5}{24}C_{13}\right) + \frac{D}{n^2}\left(\frac{7}{64}C_{02}^2 - \frac{5}{8}C_{11}C_{02}\right)\right]. \tag{3.2}$$

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Again

$$B(\bar{y}_P) = \bar{Y}\frac{N-n}{N-1}\frac{1}{n}C_{11} = \theta\bar{Y}C_{11} \tag{3.3}$$

And $$MSE(\bar{y}_P) = \bar{Y}^2\frac{N-n}{N-1}\frac{1}{n}\left[(C_{20} + C_{02} + 2C_{11}) + \frac{2A}{n}(C_{12} + C_{21}) + \frac{B}{n^2}C_{22} + \frac{D}{n^2}(C_{20}C_{02} + 2C_{11}^2)\right]. \tag{3.4}$$

Under assumption of bivariate normality of (y, x) and equality of coefficients of variation of y and x, i.e, $C_{20} = C_{02}$, it may be shown that

$$MSE(\bar{y}_P) = \bar{Y}^2\frac{C_{02}}{n}\left[2(1+\rho) + \frac{C_{02}}{n}(1+2\rho^2)\right] \tag{3.5}$$

$$MSE(\bar{y}_{BTP}) = \bar{Y}^2\frac{C_{02}}{n}\left[\left(\frac{5}{4}+\rho\right) + \frac{C_{02}}{n}\left(\frac{7}{64} - \frac{5}{8}\rho\right)\right] \tag{3.6}$$

$$MSE(\bar{y}_{SQP}) = \bar{Y}^2\frac{C_{02}}{n}\left[\left(\frac{5}{4}+\rho\right) + \frac{C_{02}}{n}\left(\frac{15}{64} - \frac{3}{8}\rho\right)\right] \tag{3.7}$$

IV. COMPARISON OF EFFICIENCY

(1) Considering terms up to $O(1/n)$,

$$MSE(\bar{y}_P) = \theta\bar{Y}^2(C_{20} + C_{02} + 2C_{11})$$

$$MSE(\bar{y}_{SQP}) = MSE(\bar{y}_{BTP}) = \theta\bar{Y}^2\left(C_{20} + \frac{1}{4}C_{02} + C_{11}\right)$$

Now,
$$MSE(\bar{y}_P) - MSE(\bar{y}_{SQP}) = \theta\bar{Y}^2\left(\frac{3}{4}C_{02} + C_{11}\right).$$

It is thus evident that, to $O(1/n)$, both the estimators $\bar{y}_{BTP}$ and $\bar{y}_{SQP}$, being equally efficient, are more efficient than $\bar{y}_P$

$$\text{if } \frac{C_{11}}{C_{02}} > -\frac{3}{4}$$

$$\text{or } k > -\frac{3}{4},$$

and more efficient than $\bar{y}$ if

$$MSE(\bar{y}_{SQP}) < MSE(\bar{y})$$

$$\text{or } k < -\tfrac{1}{4}.$$

Hence, $\bar{y}_{SQP}$ and $\bar{y}_{BTP}$ are more efficient than both $\bar{y}_P$ and $\bar{y}$ if

$$-\tfrac{3}{4} < k < -\tfrac{1}{4} \tag{4.1}$$

(2) To $O(1/n^2)$ and for large N, the difference

$$MSE(\bar{y}_{SQP}) - MSE(\bar{y}_{BTP}) = \bar{Y}^2 \frac{1}{4n^2}\left(\frac{1}{2}{C_{02}}^2 + C_{11}C_{02}\right)$$

Indicates that $\bar{y}_{SQP}$ will be more efficient than $\bar{y}_{BTP}$ if

$$\frac{C_{11}}{C_{02}} < -\frac{1}{2}$$

$$\text{or } k < -\tfrac{1}{2}, \text{ when } C_{20} = C_{02}. \tag{4.2}$$

Under the assumptions of bivariate normality of y and x and $C_{20} = C_{02}$, we have

$$MSE(\bar{y}_{BTP}) - MSE(\bar{y}_P) = \bar{Y}^2 \frac{1}{n} C_{02}\left[\left(-\frac{3}{4} - \rho\right) + \frac{C_{02}}{n}\left(-\frac{57}{64} - \frac{5}{8}\rho - 2\rho^2\right)\right] \tag{4.3}$$

$$MSE(\bar{y}_{SQP}) - MSE(\bar{y}_P) = \bar{Y}^2 \frac{1}{n} C_{02}\left[\left(-\frac{3}{4} - \rho\right) + \frac{C_{02}}{n}\left(-\frac{49}{64} - \frac{3}{8}\rho - 2\rho^2\right)\right] \tag{4.4}$$

So, $\bar{y}_{SQP}$ will be more efficient than $\bar{y}_P$ if

$$\rho > -\frac{3}{4} \tag{4.5}$$

Hence, $\bar{y}_{SQP}$ and $\bar{y}_{BTP}$ will be more efficient than $\bar{y}_P$ iff

$$-\frac{3}{4} < \rho < 0 \tag{4.6}$$

Further, $\bar{y}_{SQP}$ will be more efficient than $\bar{y}_{BTP}$ if

$$\rho < -\frac{1}{2} \tag{4.7}$$

Thus, from the condition (4.6) and (4.7), $\bar{y}_{SQP}$ will be more efficient than both $\bar{y}_{BTP}$ and $\bar{y}_P$ if

$$-\frac{3}{4} < \rho < -\frac{1}{2}. \tag{4.8}$$

V. NUMERICAL ILLUSTRATION

For the purpose of numerical illustration, we refer to Weisberg (1980) wherein the following information in respect of a real population is given:
The variables are

y: PA=1 if principal arterial highway, 0 otherwise.
x: MA=1 if minor arterial highway, 0 otherwise.

and $N = 39,\ n = 10,\ \bar{Y} = 0.4872,\ C_y^2 = 1.0802,\ C_x^2 = 1.0533,\ \rho = -0.69.$

Comparison of mean square error and their relative efficiency

Pop. No.	$\dfrac{MSE(\bar{y}_{SQP})}{\left(\bar{Y}^2 \frac{1}{n} C_{02}\right)}$	$\dfrac{MSE(\bar{y}_{BTP})}{\left(\bar{Y}^2 \frac{1}{n} C_{02}\right)}$	$\dfrac{MSE(\bar{y}_P)}{\left(\bar{Y}^2 \frac{1}{n} C_{02}\right)}$	$\dfrac{\left\{MSE(\bar{y}_P)/\left(\bar{Y}^2 \frac{1}{n} C_{02}\right)\right\}}{\left\{MSE(\bar{y}_{SQP})/\left(\bar{Y}^2 \frac{1}{n} C_{02}\right)\right\}} \times 100$	$\dfrac{\left\{MSE(\bar{y}_P)/\left(\bar{Y}^2 \frac{1}{n} C_{02}\right)\right\}}{\left\{MSE(\bar{y}_{BTP})/\left(\bar{Y}^2 \frac{1}{n} C_{02}\right)\right\}} \times 100$
1	0.6612	0.6709	1.0208	154.3860	152.1538

VI. CONCLUSION

To $O\left(\frac{1}{n}\right)$, $\bar{y}_{SQP}$ and $\bar{y}_{BTP}$ are equally efficient and more efficient than both $\bar{y}_P$ and $\bar{y}$ if $-\frac{3}{4} < k < -\frac{1}{4}$. Again under the assumptions of bivariate normality of (y, x) and equality of coefficients of variation of y and x, $\bar{y}_{SQP}$ is, to $O\left(\frac{1}{n^2}\right)$, more efficient than both $\bar{y}_{BTP}$ and $\bar{y}_P$ if $-\frac{3}{4} < \rho < -\frac{1}{2}$.

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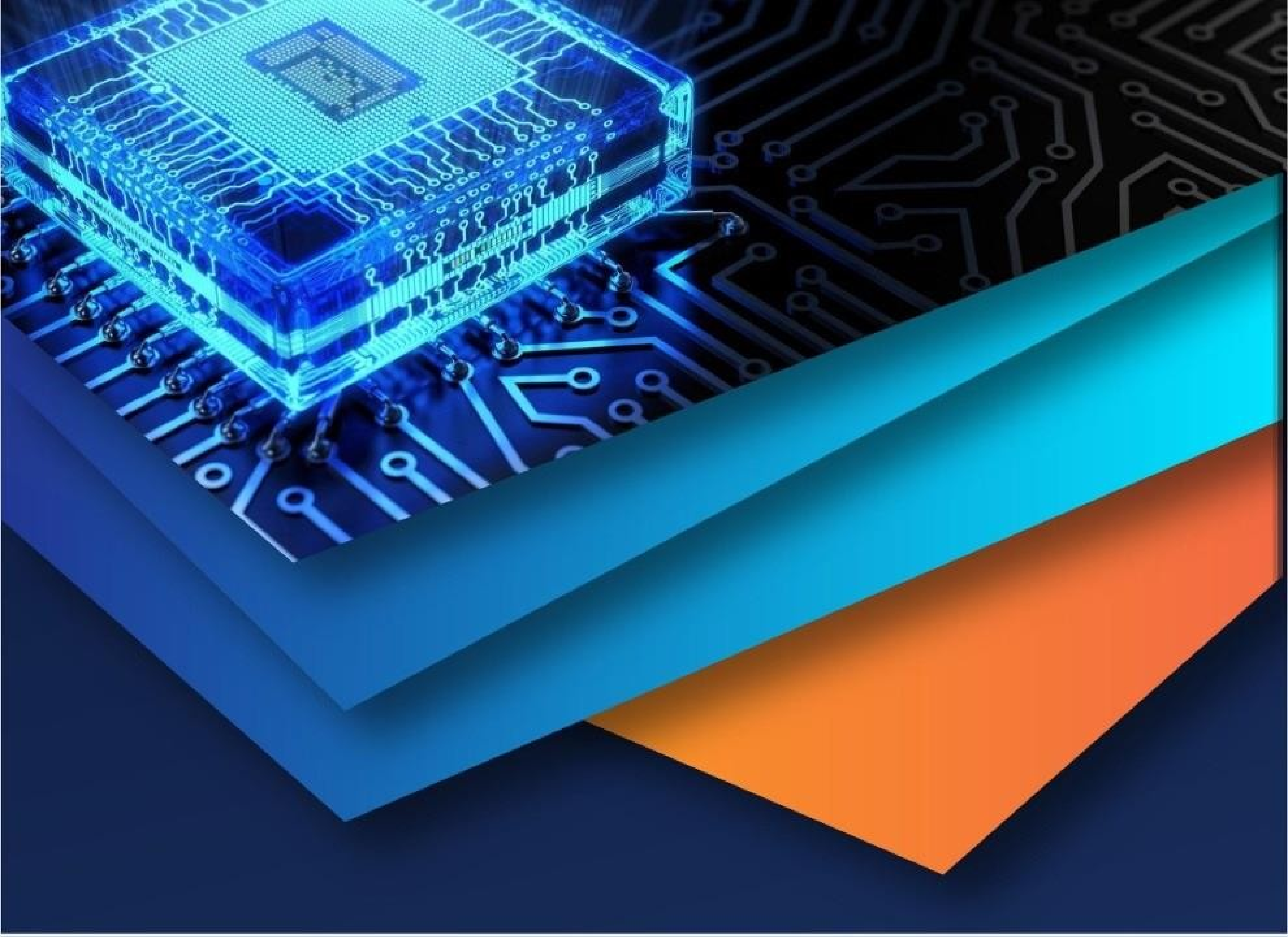

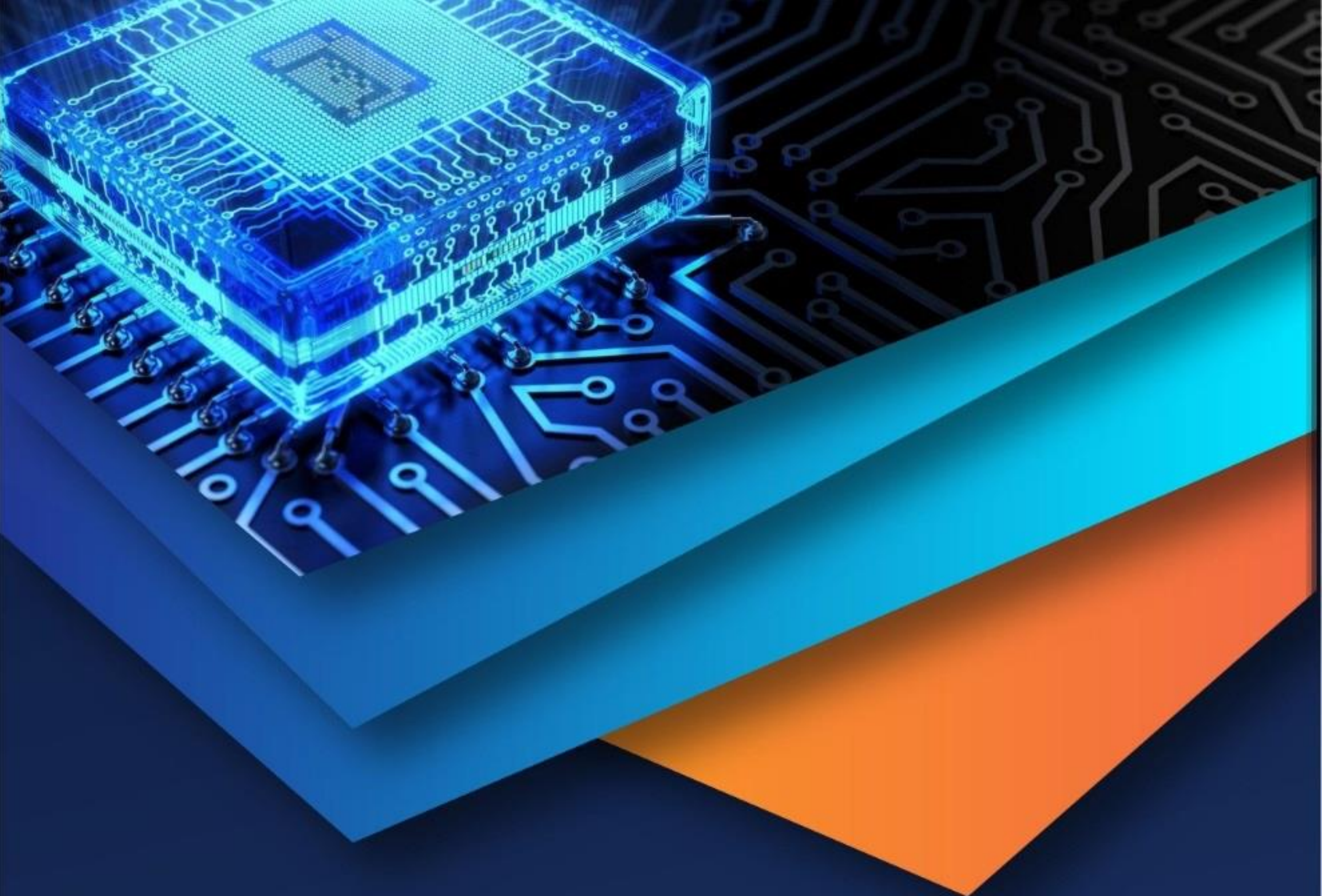

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