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Behavior in Cylindrical Liquid Retaining Tank of Varying Thickness

Sandeep Singh Namad[1], Dr. Vikas Garg[2], Dr.Vijay Raj[3]
[1]Reserarch scholar M.Tech Structures, Civil Engineering Department University of Petroleum and Energy studies, Dehradun, India
[2]Professor and Head, Civil Engineering Department University of Petroleum and Energy Studies, Dehradun, India.
[3] Adjunct Professors, Civil Engineering Department University of Petroleum and Energy Studies, Dehradun, India

Abstract: Liquid retaining tank are always subjected to the hydrostatic pressure. In the past there was no provision of designing tank wall for varying thickness, they were designed on the basis of their own experience. These storage tanks were either design on the basis of Timoshenko classical theory for cylindrical shell or on the basis of different authenticated code of structural design. The design method proposed by I.S: 3370 deals with cylindrical wall liquid retaining structure, having uniform thickness and there is no provision for designing varying wall thickness and also recommend that the minimum thickness for liquid storage be 150mm throughout the height which may lead to extra reinforcement, which is uneconomical. This paper deals with the hydrostatic behavior of liquid retaining structure along with bending moment and shear forces in varying wall thickness making use of the above codes and theory.
Keywords: Cylindrical tank, flexural rigidity, Bessel function, deflection, stresses, bending moment.

I. INTRODUCTION

The behavior of tank varying wall thickness has been studied as follows,

A. Static behavior

According to Timoshenko theory and IS code

B. Static behavior

The classical theory of design for liquid retaining structure as proposed by Timoshenko has been applied to study the behavior of liquid retaining structures of varying wall thickness.,

C. Timoshenko theory of cylindrical shell,

Timoshenko theory deal with the practical applications of a circular cylindrical shell which are subjected to the action of steam pressure , stresses in container having vertical axis along with internal liquid pressure.

This theory says that the solution for varying thickness required the integration of following equation(1) where considering the flexural rigidity D and the thickness h as no longer constant but as functions of x. While solving the general equation we have to deal with the linear differential equation of fourth order with variable coefficients

$$\frac{d^2}{dx^2}\left(\frac{Dd^2w}{dx^2}\right)+\frac{Eh}{a^2}w=z \quad \text{……………….(1)}$$

$h = \alpha x$, $D = \frac{E\alpha^3 x^{\wedge}3}{12(1-\mu^2)}$,

$w_1 = -\frac{Ya^2*}{E\alpha}\left(\frac{x-x_*}{x}\right)$

above expression also represent the internal pressure of $Y(x-x_0)$ which may lead to generation of certain amount of the bending moment

$$M_x = \frac{-Dd^2w1}{dx^2} = \frac{Ya^2*a^2*(x_*)}{6(1-\mu^2)} \text{……………………..(2)}$$

The general solution of homogenous equation be

$$\frac{d^2}{dx^2}\left(\frac{x^3 d^2 w}{dx^2}\right) + \frac{12(1-\mu^2)}{\alpha^2 a^2} xw = 0$$ ....(3)

After solving the general and homogenous equation we have fourth order differential equation as follow

$$\frac{d^2}{xdx^2}\left(\frac{x^3 d^2 w}{dx^2}\right) + \frac{12(1-\mu^2)}{\alpha^2 a^2} w = 0$$ ....(4)

And equation (4) can also be written as

$$\frac{d^2}{xdx^2}\left(\frac{x^3 d^2 w}{dx^2}\right) = \frac{d}{dx}\left(\frac{x^2 d}{dx}\left(\frac{1}{x}\frac{d}{dx}\left(x^2\frac{dw}{dx}\right)\right)\right)$$ ....(5)

$\text{Ш}_1(2\rho\sqrt{x}) = 1 - \frac{1}{(2*4)^2}(2\rho\sqrt{x})^4 + \frac{1}{(2*4*6*8)^2}(2\rho\sqrt{x})^8$

$\text{Ш}_2(2\rho\sqrt{x}) = -\frac{1}{(2)^2}(2\rho\sqrt{x})^2 + \frac{1}{(2*4*6)^2}(2\rho\sqrt{x})^6$

$\text{Ш}_3(2\rho\sqrt{x}) = \text{Ш}_1(2\rho\sqrt{x})\frac{1}{2}$-

Which further give the following general equation,

$$W = \frac{\zeta}{\sqrt{x}} = \frac{[c1*\text{Ш}1'(2\rho\sqrt{x}) + c2*\text{Ш}2'(2\rho\sqrt{x}) + c3*\text{Ш}3'(2\rho\sqrt{x}) + c4\text{Ш}4'(2\rho\sqrt{x})]}{\sqrt{x}}$$

$$M_x = \frac{-Dd^2w}{dx^2} \quad \frac{E\alpha^3\sqrt{x}}{48(1-\mu^2)} * \{c1 * [\zeta^2\text{Ш}2'(\zeta) - 4\zeta\text{Ш}2(\zeta) + 8 * \text{Ш}1'(\zeta)] -$$
$$c2*[\zeta^2\text{Ш}1'(\zeta) - 4\zeta\text{Ш}1(\zeta) + 8 * \text{Ш}2'(\zeta)] + c3*[\zeta^2\text{Ш}4'(\zeta) - 4\zeta\text{Ш}4(\zeta) + 8 * \text{Ш}3'(\zeta)] -$$
$$c4*[\zeta^2\text{Ш}3'(\zeta) - 4\zeta\text{Ш}3(\zeta) + 8 * \text{Ш}4'(\zeta)]$$ ....(6)

$$Q_x = \frac{E\alpha^3\rho^3}{24(1-\mu^2)} * \{c1[\zeta\text{Ш}1(\zeta)\ 2\text{Ш}2'(\zeta)] + c2*[\zeta\text{Ш}2(\zeta) - 2\text{Ш}1'(\zeta)] + c3*[\zeta\text{Ш}3(\zeta) + 2\text{Ш}4'(\zeta)] -$$
$$c4*[\zeta\text{Ш}4(\zeta) + 2\text{Ш}3'(\zeta)$$ ....(7)

D. As per I.S code

In India generally two codes for design of liquid retaining structure, the IS: 3370(part1-4) and IS: 1893are followed. IS: 3370 (1) emphasis the requirements of the design and construction of the concrete structures for storing the liquids especially the water. Part II focus on the requirements applicable to reinforced concrete structures for the storage of liquids at the normal temperature which have no detrimental action on concrete and steel. Part III also focus on the requirements applicable to the prestressed concrete structures for the storage of liquids Part IV recommends design tables, which are intended as an aid for the design of reinforced or prestressed concrete structures for storage of liquid,

According to IS 3370 to determine the bending moment the general parameter $\frac{H^2}{Dt} < 16$ is used, from where we can find the moment coefficient according to given ratio, and place them in equation

B.M $= C_E * w * H^3$

Whereas for shear force,

S.F $= C_E * w * H$

II. GENERAL EQUATION GOVERNED BY MODIFIED TIMOSHENKO

A. Calculation of bending moment

The general equation proposed by Timoshenko has been modified to consider the varying thickness of tank wall and is proposed as follows

$$W = \frac{\zeta}{\sqrt{x}} = \frac{[c1*\text{Ш}1'(2\rho\sqrt{x}) + c2*\text{Ш}2'(2\rho\sqrt{x}) + c3*\text{Ш}3'(2\rho\sqrt{x}) + c4\text{Ш}4'(2\rho\sqrt{x})]}{\sqrt{x}}$$

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$M_x = \frac{-Dd^2w}{dx^2}$

$= \frac{E\alpha^3\sqrt{x}}{48(1-\mu^2)} * \{c1 * [\zeta^2\psi2'(\zeta) - 4\zeta\psi2(\zeta) + 8 * \psi1'(\zeta)]$ -

$c2*[\zeta^2\psi1'(\zeta) - 4\zeta\psi1(\zeta) + 8 * \psi2'(\zeta)] + c3*[\zeta^2\psi4'(\zeta) - 4\zeta\psi4(\zeta) + 8 * \psi3'(\zeta)] -$

$c4*[\zeta^2\psi3'(\zeta) - 4\zeta\psi3(\zeta) + 8 * \psi4'(\zeta)]$

$Q_x = \frac{E\alpha^3\rho^3}{24(1-\mu^2)} * \{c1 * [\zeta\psi1(\zeta)\ 2\psi2'(\zeta)] + c2*[\zeta\psi2(\zeta) - 2\psi1'(\zeta)] + c3*[\zeta\psi3(\zeta) + 2\psi4'(\zeta) - c4*[\zeta\psi4(\zeta) + 2\psi3'(\zeta)$

As we know that the equation (6&7) from the Timoshenko theory of cylindrical shell have four unknown coefficient and another unknown variable which were directly solved after neglecting the certain unknown parameter and there value were derived from the Bessel function which have directly given value for four unknown parameter and its derivative and also proposed the general formula for the equation as we go beyond the parameter value of six .But in this paper we have solved those eight unknown variable and finally cluster them into following general coefficient

$$C_e = \frac{0.733\rho + \frac{(2\rho\sqrt{x})*(-2.05-2.96\rho)}{\sqrt{x}} + (2\rho\sqrt{x})^3 * (1.91 + \frac{0.17}{\sqrt{x}}) + \frac{0.183*(2\rho\sqrt{x})^2*\left(1+\frac{2\rho-2}{\sqrt{x}}\right)}{\sqrt{x}}}{-\frac{(2\rho\sqrt{x})^2}{8} + \frac{2\log\beta(2\rho\sqrt{x})}{\pi*2}}$$

Above coefficient has been substituted in the general equation which was proposed by the Timoshenko in his plate and shell paper,[1]

So bending moment for non-uniform thickness cylindrical tank is obtained as follows,

$$B.M = -\frac{\gamma\alpha^2\alpha^2(x\sqrt{x}+d)}{48\rho x(1-\mu^2)} Ce \text{..(8)}$$

B: calculation of shear force

After solving Timoshenko shear force equation the shear force is obtain as follows,

$$Ce_1 = 1.15\rho + (\rho\sqrt{x}) * \left(1 - \frac{1}{\sqrt{(x)}}\right) + (2\rho\sqrt{x})^2 \left(\frac{-0.184}{(2\sqrt{(x}} + 0.184\left(\frac{\rho}{2\sqrt{5x}} - 1\right) + 0.23 * \left(\frac{0.771+0.2*\rho}{\sqrt{x}} - 0.27722\right) + (2\rho\sqrt{x})^3 (-0.125 + 0.32 + (1 - \frac{1}{2\sqrt{x}}) + \frac{(2\rho\sqrt{x})^3}{\sqrt{x}}(0.0625 + (((0.5 - 2.96 * \rho) * \frac{0.2266}{\sqrt{(x)}} - 0.09 - 0.184(0.5 - 2.96\rho))$$

$$Ce_2 = (-\frac{(2*\rho*\sqrt{x})^2}{8} + (0.1838 * (2\rho\sqrt{x})))$$

Now dividing the ce_{1} with ce_2

$$S.F = -1 * \frac{Ce1*t^3*\rho^2*E}{Ce2*24*(1-\mu^2)} \text{.........................(9)}$$

The above concept have been illustrated with help of following problem

III. PROBLEM OF LIQUID RETAINING STRUCTURE WITH VARYING WALL THICKNESS HAS BEEN SOLVED BY USING THIS CONCEPT

Considering a cylindrical tank of wall thickness varying from 230 mm at bottom to 100 mm at the top for a height of 4.75m. Assume internal diameter as16m and free head of 0.25m and total water depth be 4.5m. ($Y=9800\ N/m^3$)

The bending moment and shear force are calculated with modified Timoshenko formula and IS code for varying height of tank as shown in table 1 &2 and have been shown graphically figure 1 &2..

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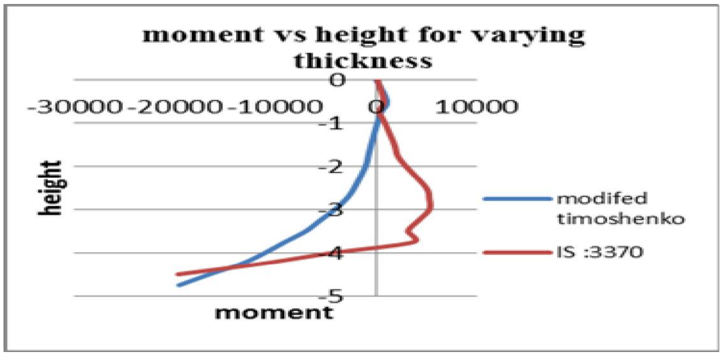

Figure 1: height v/s moment for varying thickness,

Table 1: bending moment derived from

Height(M) (Top to bottom)	B.M with modified Timoshenko formula(N-M)	Coefficient derived as per $\frac{H^2}{D*t}$	B.M according to I.S code(N-M)
0.5	1048.07	0.000107	111.8553
0.75	498.056	0.000693	727.8471
1	109.318	0.0003	315.0853
1.5	-510.35	0.0008	840.2275
1.75	-791.58	0.0017	1785.483
2	-1066.87	0.002	2100.569
2.5	-2180.03	0.0028	2940.796
2.75	-2946.69	0.0047	4936.337
3	-4112.05	0.005	5251.422
41. 3.25	-5567.33	0.00502	5272.428
3.5	-6996.21	0.004	4201.138
3.75	-9076.59	0.003	3150.853
4	-11062.1	0.0035	3675.995
4.25	-13329	-0.0042	-4411.19
4.5	-16555.3	-0.0108	-11343.1
4.75	-19541.5	-0.0187	-19640.3

Timoshenko

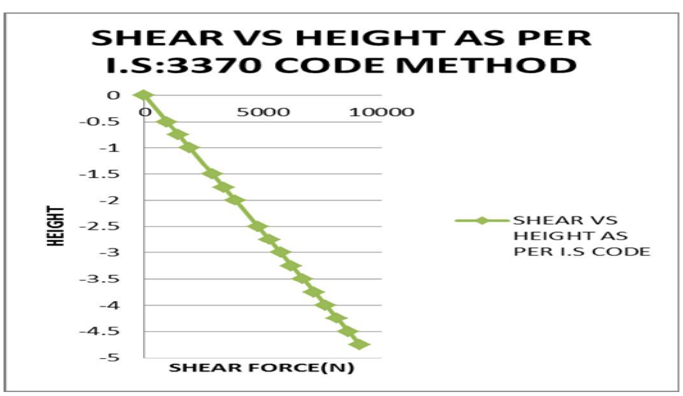

Figure 2: shear force v/s height fas per IS :3370

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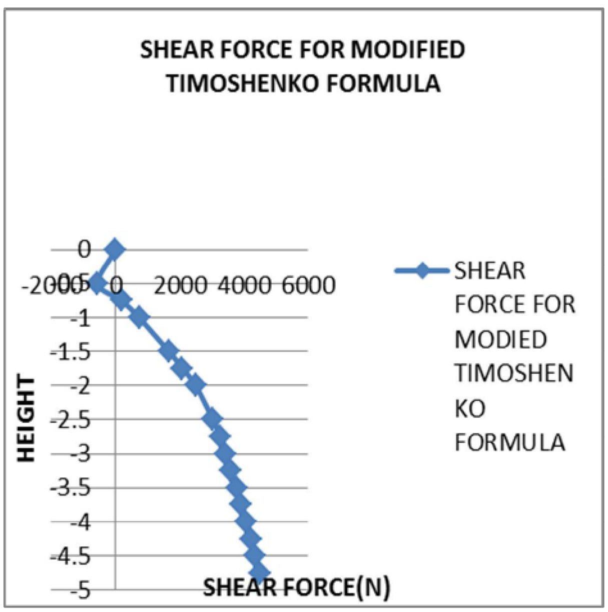
From the figure 1 and table (1&2) it is observed that from Timoshenko formula the bending moment is linearly varying with changing thickness along at respective height which may help in optimizing the reinforcement of tank. In contrast IS:3370 recommends only uniform thickness and if the same thickness is used then as shown in figure

no.1 the nature of bending moment changes drastically as we move down from the top towards the bottom of tank and reinforcement may become heavier which may become uneconomical .The comparison of base shear as obtained from the table no 2 and figure (2&3) shows that base shear as per Timoshenko 4.5kN were as Per IS:3370 base shear come out to be about 9.2kN which may lead to heavier wall thickness/ reinforce at the base

Table 2: shear force derived by modified Timoshenko

HEIGHT (M)	SHEAR FORCE BY MODIFIED TIMOSHENKO(N)	SHEAR FORCE BY I.S CODE (N)
.5	-555.198	955.5
0.75	178.1527	1433.25
1	730.541	1911
1.5	1660.866	2866.5
1.75	2085.931	3344.25
2	2496.014	3822
2.5	3039.336	4777.5
2.75	3272.212	5255.25
3	3435.357	5733
3.25	3587.004	6210.75
3.5	3774.652	6688.5
3.75	3907.343	7166.25
4	4076.454	7644
4.25	4238.016	8121.75
4.5	4351.513	8599.5
4.75	4501.059	9077.25



2: height v/s shear force for varying thickness

VI. CONCLUSIONS

After doing the comparative study of bending moment and shear force for proposed modified equation for Timoshenko non uniform thickness and IS code formula, it is observed that the value obtained are close agreement to each other. So these value are recommend for design purpose of liquid retaining structure of varying thickness and relevant code need to be modified to that extend .

A. Notations

R, a = radius of tank
H =overall height of tank
E = modulus of elasticity=5700$\sqrt{f_{ck}}$
d= height of liquid retained in tank
D= flexural rigidity
M_x= bending moment
x = free fall height
w=9800N/m^2
μ = Poisson ratio
α= varying thickness
ρ= $\left(\frac{12*(1-\mu^2)}{\alpha^2*r^2}\right)^4$
ζ=2 ρ$\sqrt{x}$
$Ш_1$= $Ш_2$= $Ш_3$= $Ш_4$= parameter
t = thickness of wall (for I.S code)
S.F = shear force
B.M = bending moment
Q_x = shear force
Logβ=0.57722

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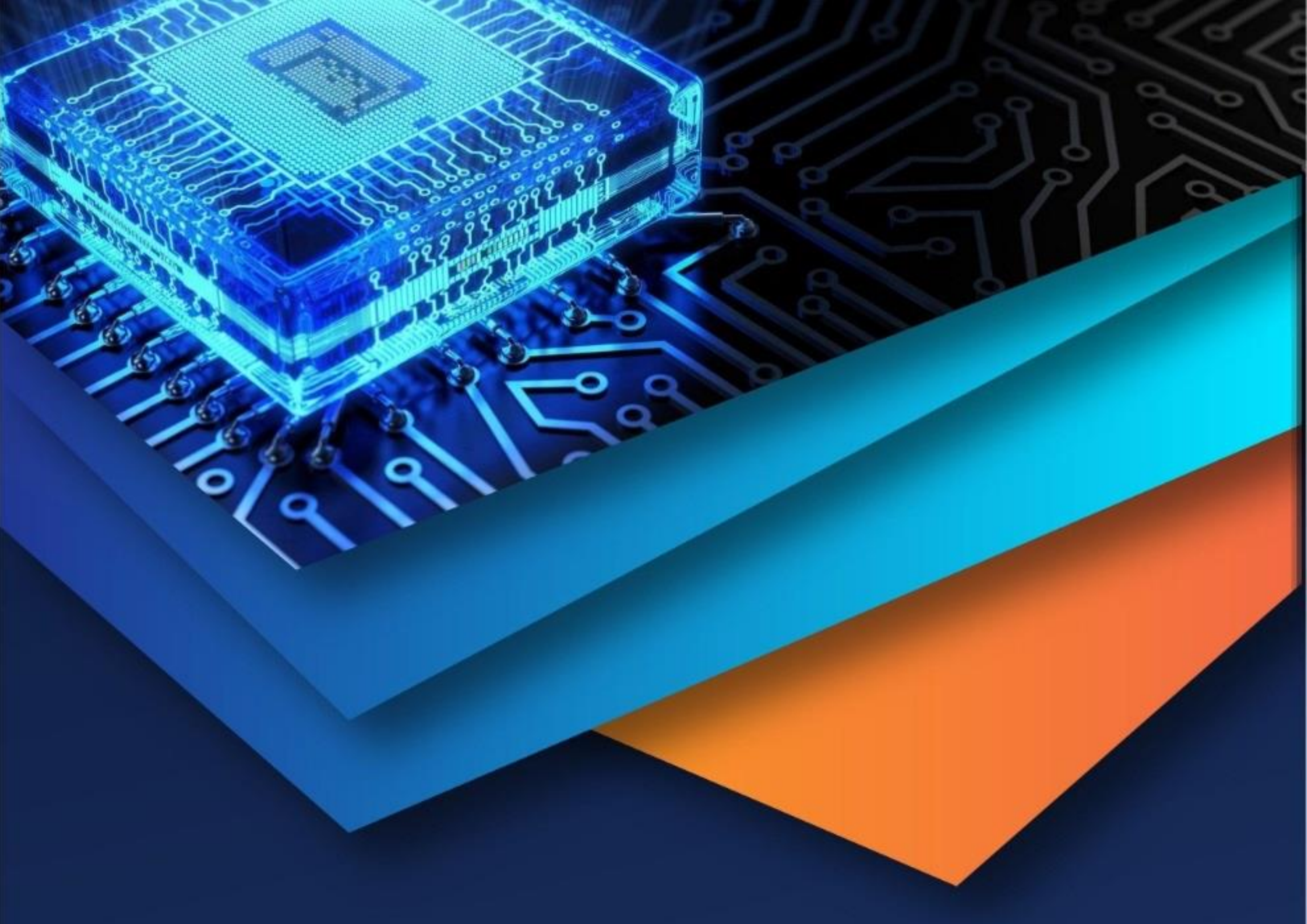

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