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Performance measures of $M^{(x)}$ / G / 1 Queue with Balking

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Abstract: Queuing systems with batch arrivals / or batch services are common in a number of real situations. Baha (1986) has studied a batch arrival queue with server vacations in which the server takes a sequence of identically distributed vacations. The vacation sequence starts immediately when the system becomes empty and ends at the instant when the server finds at least one customer in the system at the end of a vacation. The Laplace Stieltje's Transform (LST) of the distribution of queue waiting time has been obtained using the method of supplementary variables. An $M^{(X)}$ / G / 1 queue with server vacations has been studied in Lee in which the vacations are identically distributed and the main objective is the determination of optimal N-policy in addition to the determination of the system size distribution.
Keyword: Balking, Server vacation, Busy period, Waiting Time, System Size

I. INTRODUCTION

Customers arrive in batches according to a poisson process with rate λ upon arrival, each customer decides independently to join the queue with probability P or balks with probability $1 - P$ $(0 \le P \le 1)$. The single server in the system serves the customers one at a time and the service times are independent and identically distributed with a common distribution function. As soon as the system becomes empty the server takes a vacation of duration V_1. At the end of the vacation, the server returns to the system and starts servicing the customers, provided the system is non-empty. On the other hand if the system is empty, the server takes the next vacation of duration V_2 and so on ... .

We assume that the sequence of random variables $\{V_j, j = 1, 2, 3, ...\}$ are independent.

Let X = random variable representing batch size

$\alpha(Z)$ = probability generating function of X

$$= \sum_{k=1}^{\infty} P(X = k) Z^k$$

S = random variable representing service times

S(X) = probability density function of S

S′(X) = LST of the distribution function of S

$$= \int_0^{\infty} e^{-\theta x} \left(\int_0^{x} S(u)du \right) du$$

V_j = random variable representing j-th vacation

$V_j(X)$ = probability density function of V_j

$V_j'(\theta)$ = LST of the distribution function of V_j

S' = remaining service time of a customer in service

V_j' = remaining vacation time when the server is on the j-th vacation

$$Y = \begin{cases} 0, & \text{if the server is busy} \\ j, & \text{if the server is on the j-th vacation for } j = 1,2,3,... \end{cases}$$

N(t) = system size at time t

$\overline{X}$ = random variable representing the number of customers who decide to join the queue

$\psi(z)$ = the PGF of $\overline{X}$

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$$\rho = \lambda\, E(\overline{X})\, E(S)$$
$$= \lambda\, \rho\, E(X) E(S)$$

$P_n(x,t) = P\{N(t) = n,\ X \le S' \le x + \Delta x,\ y = 0\},\ n = 1, 2, 3, \ldots$

$Q_{n_j}(x,t) = P\{N(t) = n,\ X \le V' \le x+\Delta x,\ y = j\},\ n = 0, 1, 2, \ldots$ and $j = 1, 2, 3, \ldots$

A. Model Analysis

$\overline{X} = X_1 + X_2 + \cdots + X_n$ Where each X_i is a binomial random variable with parameters (1, P) and i = 1, 2, 3, ...

$$P(\overline{X} = i) = \sum_{k=1}^{\infty} P(\overline{X} = i/X = k) P(X = k)$$
$$= \sum_{k=1}^{\infty} kC_i P^i (1-P)^{k-i} P(X = k)$$

The PGF of $\overline{X}$ is

$$\psi(z) = \sum_{i=0}^{\infty} P(\overline{X} = i) z^i$$
$$= \sum_{i=0}^{\infty} \sum_{k=1}^{\infty} kC_i P^i (1-P)^{k-i} P(X = k) z^i$$
$$= \sum_{k=1}^{\infty} \sum_{i=0}^{\infty} kC_i (Pz)^i (1-P)^{k-i} P(X = k)$$
$$= \sum_{k=1}^{\infty} (1 - P + Pz)^k P(X = k)$$
$$= X(1 - P + Pz)$$

$$P_1(X - \Delta t, t + \Delta t) = P_1(X,t)(1 - \lambda\, \Delta t) + P_1(X,t)\lambda\, P(\overline{X} = 0)\Delta) + P_2(0,t)S(X)\Delta) + \sum_{j=1}^{\infty} Q_{1j}(0,t)S(X)\Delta)$$

$$\frac{P_1(X - \Delta t, t + \Delta t) - P_1(X,t)}{\Delta t} = -\lambda P_1(X,t) + \lambda P_1(X,t) P(\overline{X} = 0) + P_2(0,t)S(X) + \sum_{j=1}^{\infty} Q_{1j}(0,t)S(X)$$

As $\Delta t \to 0$

$$\frac{d}{dX} P_1(X) = \lambda P_1(X) - \lambda P(\overline{X} = 0) P_1(X) + P_2(0)S(X) + \sum_{j=1}^{\infty} Q_{1j}(0)S(X) \qquad (1)$$

Similarly

$$\frac{d}{dX} P_n(X) = \lambda P_n(X) - \lambda \sum_{k=0}^{n-1} P_{n-k}(X) P(\overline{X} = k) - P_{n+1}(0)S(X) - \sum_{j=1}^{\infty} Q_{nj}(0,t)S(X), \quad n \ge 2 \qquad (2)$$

$$\frac{d}{dx}Q_{01}(x) = \lambda Q_{01}(X) - \lambda P(\overline{X}=0)Q_{01}(X) - P_1(0)V_1(X) \quad (3)$$

$$\frac{d}{dx}Q_{0j}(x) = \lambda Q_{0j}(x) - \lambda P(\overline{X}=0)Q_{0j}(X) - Q_{0j-1}(0)V_j(X), \qquad j \geq 2 \quad (4)$$

$$\frac{d}{dx}Q_{n1}(x) = \lambda Q_{n1}(X) - \lambda \sum_{k=0}^{n} Q_{n\text{-}k,1}(X)P(\overline{X}=k) \quad (5)$$

$$\frac{d}{dx}Q_{nj}(x) = \lambda Q_{nj}(X) - \lambda \sum_{k=0}^{n} Q_{n\text{-}k,j}(X)P(\overline{X}=k), \quad j \geq 2, n \geq 1 \quad (6)$$

Taking LST of equations (1) to (6)

$$\theta P_1^1(\theta) - P_1(0) = \lambda P_1^1(0) - \lambda P_1^1(0)P(\overline{X}=0) - P_2(0)S'(0) - \sum_{j=1}^{\infty} Q_{1j}(0)S'(0) \quad (7)$$

$$\theta P_n^1(\theta) - P_n(0) = \lambda P_n^1(0) - \lambda \sum_{k=0}^{n-1} P_{n\text{-}k}^1(0)P(\overline{X}=k) - P_{n+1}(0)S'(0) - \sum_{j=1}^{\infty} Q_{nj}(0)S'(0) \quad (8)$$

$$\theta Q_{01}(\theta) - Q_{01}(0) = \lambda Q_{01}^1(\theta) - \lambda Q_{01}^1(\theta)P(\overline{X}=0) - P_1(0)V_1^1(0) \quad (9)$$

$$\theta Q_{0j}^1(\theta) - Q_{0j}(0) = \lambda Q_{0j}^1(\theta) - \lambda Q_{0j}^1(\theta)P(\overline{X}=0) - Q_{0j-1}(0)V_j^1(0) \quad (10)$$

$$\theta Q_{n1}^1(\theta) - Q_{n1}(0) = \lambda Q_{n1}^1(\theta) - \lambda \sum_{k=0}^{n} Q_{n\text{-}k,1}^1(\theta)P(\overline{X}=k) \quad (11)$$

$$\theta Q_{nj}^1(\theta) - Q_{nj}(0) = \lambda Q_{nj}^1(\theta) - \lambda \sum_{k=0}^{n} Q_{n\text{-}k,1}^1(\theta)P(\overline{X}=k) \quad (12)$$

Define the generating function

$$P'(z,0) = \sum_{n=1}^{\infty} P_n^1(0)z^n$$

$$P(z,0) = \sum_{n=1}^{\infty} P_n(0)z^n$$

$$Q_j^1(z,0) = \sum_{n=0}^{\infty} Q_{nj}^1(0)z^n$$

$$Q(z,0) = \sum_{n=0}^{\infty} Q_{nj}(0)z^n$$

After simplify

$$Q_1(z,0) = P_1(0)V_1^1(\lambda - \lambda\psi(z))$$

$$\text{Let } \overline{\lambda} = \lambda - \lambda\psi(0)$$

at z = 0

$$Q_1(0,0) = \left[\sum_{n=0}^{\infty} Q_{n1}(0) z^n\right]_{z=0}$$
$$= Q_{n1}(0)$$
$$= P_1(0)\, V_1^1(\bar{\lambda})$$

Similarly

$$Q_j(0,0) = Q_{0j}(0)$$
$$= Q_{0j-1}(0)\, V_1^1(\bar{\lambda})$$

Let

$$R(z) = P_1(0)\left\{\sum_{j=1}^{\infty} [V_1^1(\lambda - \lambda\,\psi(z) - V_1^1(\bar{\lambda})] \prod_{k=1}^{j-1} V_k^1(\bar{\lambda}) - 1\right\}$$

$$P(z,0) = \frac{zR(z)S'(\lambda - \lambda\,\psi(z))}{z - S'(\lambda - \lambda\,\psi(z))}$$

$$P(z,0) = \frac{zR(z)[S'(\lambda - \lambda\,\psi(z)) - S'(0)]}{[\theta - \lambda + \lambda\,\psi(z)][z - S'(\lambda - \lambda\,\psi(z))]}$$

$$Q_j^1(z,0) = \frac{P_1(0)[V_1^1(\lambda - \lambda\,\psi(z)) - V_j^1(0)]\prod_{k=1}^{j-1} V_k^1(\bar{\lambda})}{[\theta - \lambda + \lambda\,\psi(z))]}$$

$$\text{Since } P'(1,0) + \sum_{j=1}^{\infty} Q_j^1(1,0) = 1$$

$$\therefore\ P_1(0) = \frac{1-\rho}{\sum_{j=1}^{\infty} E(V_j) \prod_{k1}^{j-1} V_k^1(\bar{\lambda})}$$

where

$$\rho = \lambda\ E(\bar{X})\ E(S)$$
$$= \lambda\ E(X)\ E(S)$$

B. *The LST of the Waiting Time Distribution*

The waiting time W of a random customer has two components namely

(i) Waiting time w_q of the first customer in the batch to which the random customer

(ii) Waiting time w_s due to service times of those customers who are ahead of the random customer in his batch.

Let $\bar{\omega}(\theta)$ $\bar{\omega}_q(\theta)$ and $\bar{\omega}_s(\theta)$ denote the LSTs of ω, ω_q and ω_s

$$\bar{\omega}_q(\theta) = \frac{P_1(0)\left\{\sum_{j=1}^{\infty} V_1^1(\bar{\lambda}) - V_j^1(0) \prod_{k=1}^{j-1} V_k^1(\bar{\lambda})\right\} + 1}{[\theta - \theta + \theta\,\psi\, S'(0)]}$$

System performance

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$$E(\omega)=\frac{\rho E(S^2)}{2E(S)(1-\rho)}+\frac{\lambda E(\overline{X}^2)E^2(S)}{2\rho(1-\rho)}+\frac{\sum_{j=1}^{\infty}E(V_j^2)\prod_{k=1}^{j-1}V_k^1(\overline{\lambda})}{2\sum_{j=1}^{\infty}E(V_j)\prod_{k=1}^{j-1}V_k^1(\overline{\lambda})}$$

$$E(\omega^2)=\left\{\frac{\rho E(S^2)}{2E(S)(1-\rho)}+\frac{\lambda E(\overline{X}^2)E^2(S)}{(1-\rho)}\right\}E(\omega)$$

$$+\frac{(1+\rho)E(\overline{X}^2)E(S^2)}{2E(\overline{X})(1-\rho)}$$

$$+\left[\frac{\lambda}{3(1-\rho)}\right]\{E(\overline{X}^2)E^3(S)+E(\overline{X})E(S^3)\}$$

$$+\frac{E(\overline{X}^3)E^2(S)}{3E(\overline{X})}+\left[\frac{E(\overline{X}^2)E(S)}{2E(\overline{X})}\right]\left[\frac{\sigma_2}{\sigma_1}\right]+\frac{\sigma_3}{3\sigma_1}$$

$$\sigma_k=\sum_{j=1}^{\infty}E(V_j^k)\prod_{k=1}^{j-1}V_k^1(\overline{\lambda})$$

$$V(\omega)=E(\omega^2)-(E(\omega))^2$$

when the vacation times are identically distributed

$$E(V_j)=E(V)$$

$$E(V_j^2)=E(V^2)$$

We note that $\rho=1$, $\overline{\lambda}=\lambda$

$$E(\omega)=\frac{\rho E(S^2)}{2E(S)(1-\rho)}+\frac{\lambda E(\overline{X}^2)E^2(S)}{2\rho(1-\rho)}+\frac{E(V^2)}{2E(V)}$$

$$=\frac{\lambda E^2(X)E^2(S)+E(X^2)E(S)}{2E(X)(1-\rho)}+\frac{E(V^2)}{2E(V)}$$

$$E(\overline{X})=E(X)=1 \quad \text{and} \quad E(\overline{X}^2)=0$$

$$E(\omega)=\frac{\lambda E(S^2)}{2(1-\rho)}+\frac{E(V^2)}{2E(V)}$$

II. CONCLUSION

In this paper we analyzed the vacation model of an $M^{(X)}/G/1$ queue with balking and variable vacations. This type of modeling and analysis can be applied in many situations. Queuing model can be used to the employment services system and its design and to optimize the actual system according to the specific requirements of the system. Queuing system is suitable for analyzing and studying random phenomenon such as the employment service system services.

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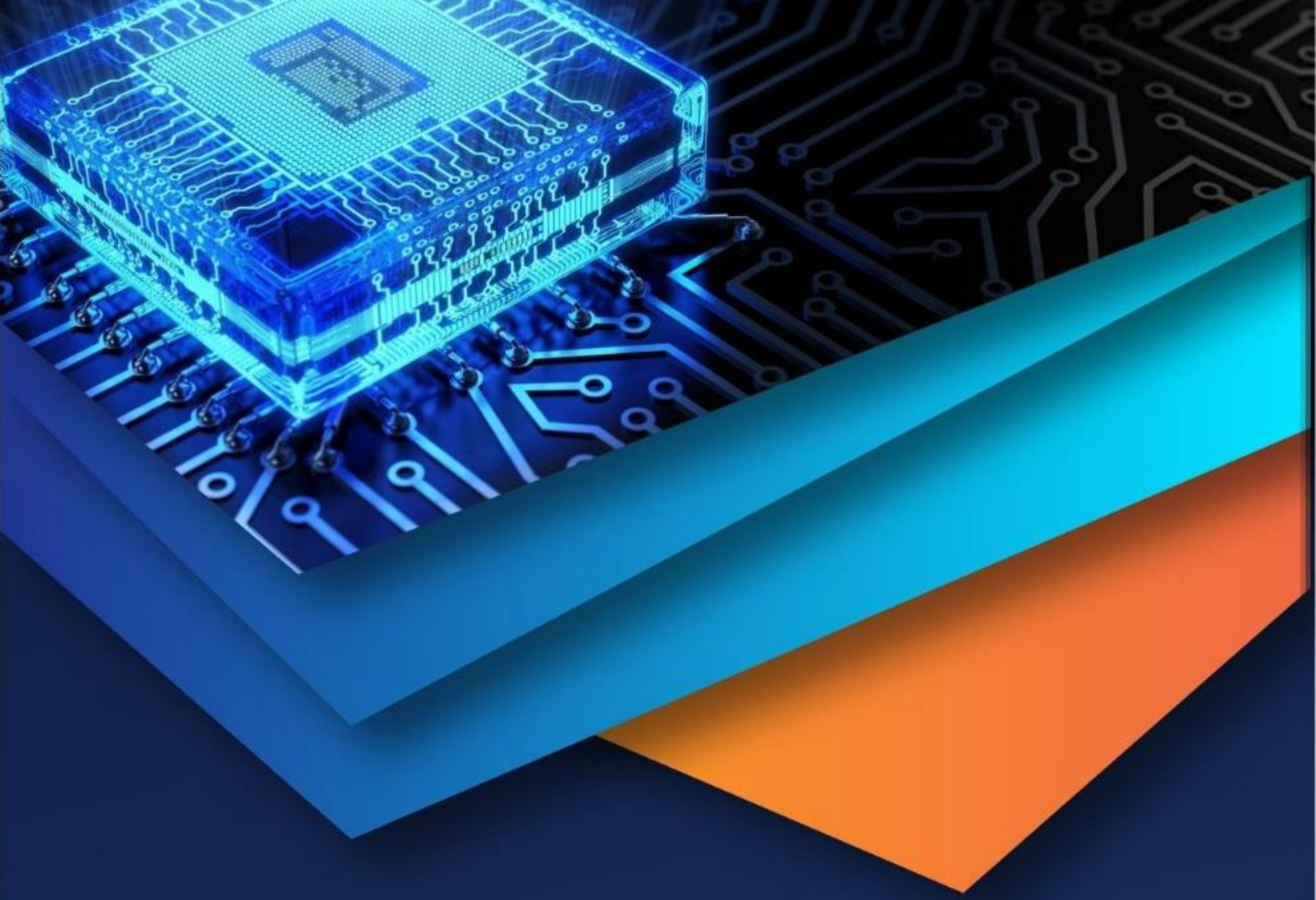

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