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International Journal For Research in
Applied Science and Engineering Technology



INTERNATIONAL JOURNAL FOR RESEARCH

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY

Volume: 5 Issue: XII Month of publication: December 2017

DOI:

www.ijraset.com

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Melting and Magnetic Effect on Mixed Convective Flow from a Vertical Plate Embedded in Non-Darcy Porous Media with Aiding and Opposing External Flow and Variable Temperature

Purna Chandra Barman¹, Ashoke Das² and Md. Rabiul Islam¹, Mrityunjoy Ghosh⁴

^{1,2} Department of Physics, Raiganj University, Raiganj-733 134, West Bengal, India

³ Department of Mathematics, Dinhat College, Cooch Behar-736101, West Bengal, India

Abstract: The problem of mixed convective melting along a vertical plate in a Newtonian fluid saturated non-Darcy porous medium in the presence of magnetic effects for aiding and opposing external flows with variable temperature is presented. The Similarity solutions for this problem are used to reduce the governing partial differential equations into a system of nonlinear ordinary partial differential equations and then are solved numerically using the fourth order Runge-Kutta method. The effects of different controlling parameters on melting and heat transfer are investigated and presented graphically obtaining velocity and temperature variation in melting region. It is obvious that the rate of heat transfer decreases as the magnetic parameter and melting parameter increases.

Keywords: Melting effect, Newtonian fluids, non-Darcy porous medium, mixed convection

I. NOMENCLATURE

B_0	Magnetic field strength
C_s	specific heat of solid phase
f	Dimensionless stream function
g	Acceleration due to gravity
h_x	Local heat transfer coefficient
L	Latent heat of melting of solid
K	Permeability of the porous medium
F	dimensionless Inertia parameter
C	empirical constant
k	Thermal conductivity
Ha	the Hartmann number
M	Melting parameter
Nu	Local Nusselt
q_x	Wall heat flux
T	Temperature in thermal boundary layer
T_s	Temperature at the solid region
u	Velocity in x-direction
v	Velocity in y- direction
x	Coordinate along the melting plate
y	Coordinate normal to melting plate
η	dimensionless similarity variable

A. Greek symbols

α	Thermal diffusivity
β_T	Coefficient of thermal expansion

μ	Viscosity of fluid
ν	Kinematic viscosity
ρ	Density
f	Stream function
σ	Electrical conductivity
θ	Dimensionless temperature

B. Subscripts

m	Melting point
∞	Condition at infinity

II. INTRODUCTION

Convective heat transfer study in porous media in the presence of melting effect has gained some significant attention in recent years due to important application in permafrost melting, frozen ground thawing, casting and welding processes as well as phase change material. The mixed convection melting from a vertical plate embedded in porous medium studied by Gorla et al. (1999) [1]. He suggested that melting process analogous to mass injection or blowing processes. Epstein and Cho (1976) [2] examined melting rate over a vertical plate using Nusselt's method to conclude the negligence of transient effects is possible if the thickness of the thermal boundary layer is smaller than melting solid. Shateyi et al. (2010) [3] analyzed the problem of magnetic field on mixed convection from vertical surface in porous medium with various effects of thermal radiation, hall currents, Soret and dufour. Kazmierczak et al. (1986,1987) ([4], [5]) considered the velocity, temperature and Nusselt number from a flat plate embedded in porous medium in case of steady natural convection considering in the melting region. The investigated numerical solution to free convection over a vertical plate saturated in a power-law fluid magnetic and double dispersion effects is analyzed by Srinivasacharya et al. (2013) [6]. Hassanien and Bakier (1991) [7] studied melting with mixed convection flow from a horizontal flat plate embedded in a porous medium. Sparrow et al. (1977) [8] studied the heat transfer in the melting region of a non-porous medium furnishing the information regarding the velocity, temperature fields and the heat transfer rate by applying finite difference method. The effect of melting on mixed convection flow from a vertical plate embedded in a Non-Newtonian fluid saturated non-Darcy porous medium in the presence of thermal dispersion -radiation is analyzed in aiding and opposing flow cases by Prasad et al. (2014) [9]. Pozovonkov et al. (1968) [10] also calculated the heat transfer rate near a melting surface in the same medium using Karman-Pohlhausen scheme. Bakier (1997) [11] obtained the solution using analytical homotopy analysis for the case, aiding and opposing flow in a porous medium. He pointed out that in the liquid solid interface the melting phenomenon decreases the local Nusselt number. Obtaining similarity solution Cheng (1977) [12] studied the melting phenomenon for mixed convection over a flat plate. Epstein (1975) [13] analyzed the effect of melting of submerged bodies. HEMALATHA et al. (2015) [14] analyzed the effects of melting and solute dispersion on heat and mass transfer on non-Darcy mixed convective flow over a vertical plate. Madhavi et al. (2017) [15] presented melting phenomenon in the presence of heat absorption and heat generation coefficient with mixed convection flow and heat transfer in a saturated non-Darcy porous medium considering the effects of thermal dispersion, thermal radiation and applied magnetic field by taking Forcheimer extension in the flow equations. The effect of Magnetic and buoyancy on melting from a vertical plate embedded in saturated porous media is studied by Tashtoush [16]. He observed that the melting parameters increase with the decrease of temperature as well as Nusselt number at the solid liquid interface. The problems of melting effect on mixed convective heat transfer from a porous vertical plate with uniform wall temperature in the liquid-saturated porous medium with aiding and opposing external flows is numerically examined at steady state Cheng and Lin(2008) [17]. The main aim of the present investigation is to illustrate the melting effect with magnetic field from a vertical plate in a non Darcy porous medium for aiding and opposing external flows with variable temperature. The velocity and temperature profiles of various parameters under the influence of magnetic field with melting effect and the results so obtained are compared with relevant results in the existing literature and are found to be in good agreement in the absence of melting and magnetic effect.

III. MATHEMATICAL FORMULATION

Let us consider the free convection heat transfer from a vertical flat plate embedded in a non Darcyporous media saturated with Newtonian fluid. It is assumed that this plate constitutes the interface between the liquid and solid phases during melting inside the porous matrix. The plate temperature, T_m , is the melting temperature of the material occupying the porous matrix, which is regarded as constant. The temperature of the solid phase far from the interface is T_s and the liquid phase temperature is T_∞ ($T_\infty < T_m$). The origin

of the coordinate system is placed at the leading edge of the interface surface between the solid and liquid phases. x is the coordinate along the surface of the plate measured from the origin, and y is the coordinate normal to the surface. u and v are the components of the non-Darcy velocity in the x and y directions, respectively, is shown in the fig. 1.

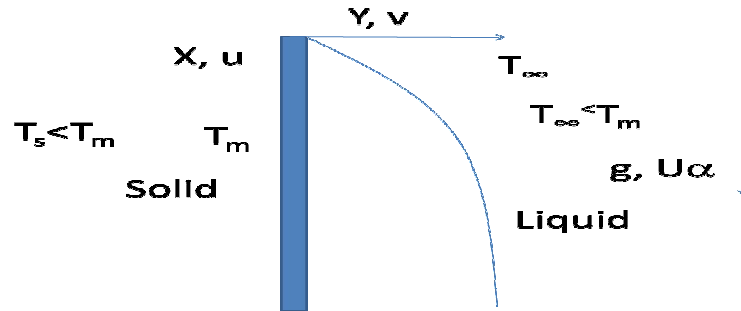


Fig.1. Schematic model and system of coordinates.

A magnetic field of strength B_0 is applied in the y direction which is normal to the flow direction. The flow is steady, laminar and two dimensional. The fluid and porous medium are in local thermal equilibrium and constant except density. The Boussinesq approximation is valid and the boundary layer approximation is applicable. The governing equation, namely the equation of continuity, momentum equation and energy equation for isotropic and homogeneous porous medium can be written as (see Bear [19]):

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$\frac{\partial u}{\partial y} + \frac{2C\sqrt{K}}{v} u \frac{\partial u}{\partial y} = \mp \frac{Kg\beta_T}{\mu} \left(\frac{\partial T}{\partial y} \right) \quad (2)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \frac{\sigma B_0^2 u^2}{\rho c_p} \quad (3)$$

$$\rho = \rho_\infty [1 - \beta_T (T - T_\infty)]$$

In the above equations, u and v are the component of darcian velocity along x and y direction, ρ_{eff} is the density, g and σ are the acceleration and electrical conductivity of the fluid respectively. $\alpha = \frac{k}{\rho c}$ is the equivalent thermal diffusivity

The boundary conditions necessary to complete the problem formulation are:

$$v = 0, T \rightarrow T_m = T_\infty + Ax^\lambda, k \frac{\partial T}{\partial y} = \rho [L + C_s (T_m - T_s)] v \quad \text{at } y = 0 \quad (4)$$

$$u = U_\infty, T \rightarrow T_\infty \text{ at } y \rightarrow \infty \quad (5)$$

Where L and C_s are the latent heat of solid and specific heat capacity of solid phase, respectively. The boundary condition (4) refers that the temperature on the plate is constant and thermal flux of heat conduction to the melting surface is equal to the sum of the melting heat and the heat required for raising the temperature of solid to its melting temperature [Epstein and Cho (1976)].

The stream function $\Psi(x, y)$ is chosen in such a way that the continuity equation automatically satisfied i.e.,

$$u = \frac{\partial \Psi}{\partial y} \quad \text{and} \quad v = - \frac{\partial \Psi}{\partial x} \quad (6)$$

Introducing the following similarity transformation equation, the above system of partial differential equations (1-3) are transferred into the non-linear ordinary differential equations

$$\eta = \left(\frac{U_\infty}{\alpha} \right)^{1/2} y x^{\frac{\lambda-1}{2}} \quad (7)$$

$$f(\eta) = \frac{\Psi}{(\alpha U_\infty)^{1/2} x^{\frac{\lambda-1}{2}}} \quad (8)$$

$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty} \quad (9)$$

Introducing Eqs. (7), (8), (9) into Eqs. (2) and (3), we obtain the following transformed governing equations:

$$(1 + Ff')f'' + \frac{Gr}{Re} = 0 \quad (10)$$

$$\theta'' + \frac{1}{2}(1 + \lambda)f\theta' - \lambda f'\theta + \frac{Ha^2 Ec}{Da} f'^2 = 0 \quad (11)$$

Where, $Ha = \sqrt{\frac{\sigma B_0^2}{\rho c_p}}$, Hartmann number, $Gr = \frac{kg\beta_T(T_\infty - T_m)x}{\nu^2}$ Grashof number, $M = \frac{C_p(T_\infty - T_m)}{L + C_s(T_m - T_\infty)}$, melting parameter, $F = \frac{2C\sqrt{K}}{\nu}$, inertia parameter, $Re = \frac{U_\infty x}{\nu}$, Reynolds number, $Ec = \frac{u_0^2}{C_p(T_m - T_\infty)}$, Eckert number, $Da = \frac{k}{x^2}$, Darcy number, buoyancy parameter, $\frac{Gr}{Re} = \frac{g\beta_T(T_\infty - T_m)x^3/\nu^2}{U_\infty x/\nu}$.

The forms of non linear differential are as follows:

$$f' + 2M\theta' = 0, \quad \theta = 0 \quad \text{at} \quad \eta = 0 \quad (12)$$

$$\text{And} \quad f' \rightarrow 1, \quad \theta \rightarrow 1 \quad \text{at} \quad \eta \rightarrow \infty \quad (13)$$

Physical quantities of interest is the Nusselt number Nu_x which are defined as

$$Nu_x = \left(\frac{q_w'' x}{k}\right) \quad (14)$$

Further, q_w'' is the heat transfer from the surface of the plate, and is given by

$$q_w'' = -k \left(\frac{\partial T}{\partial y}\right) \quad (15)$$

Using equation (13), equation (12) becomes

$$Nu_x = Ra^{1/2} \theta' \quad (16)$$

IV. SOLUTION PROCEDURE

The Eq. (10) together with Eq. (11) is split into system of first order ordinary differential equations. Using boundary conditions (12) and (13) they are solved numerically by means of the fourth order Runge- Kutta method coupled with a shooting technique and by giving appropriate initial guess values for $\theta'(0)$. The solution, thus, obtained is matched with the given values at $f'(\infty)$ and $\theta(0)$. An accuracy upto 4th decimal place is considered for convergence.

V. RESULTS AND DISCUSSION

In order to get clear insight on the physics of the problem, a parametric study is performed and the obtained numerical results are displayed with the help of graphical illustrations.

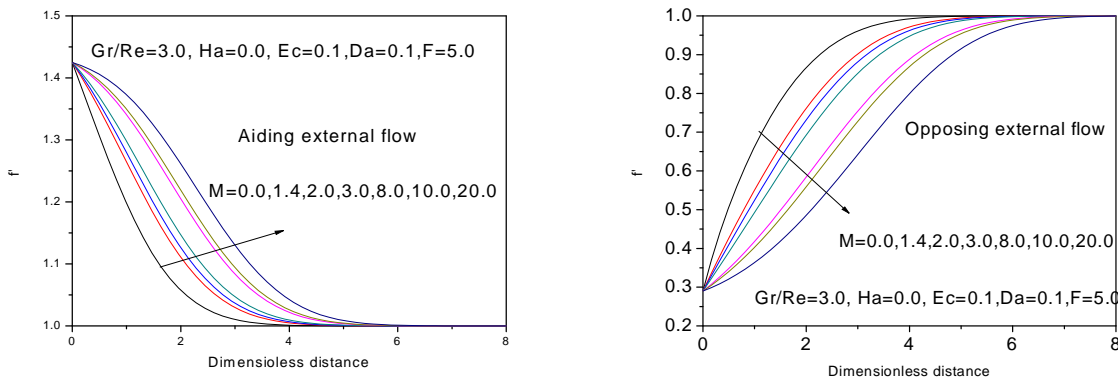


Fig.2. Velocity profiles for different values of melting parameter with aiding and opposing flow.

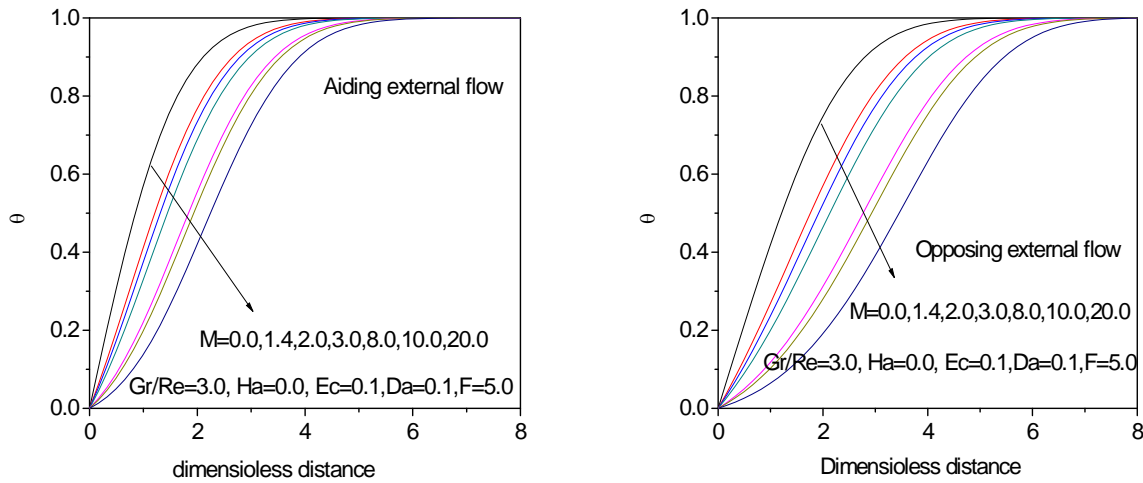


Fig.3. temperature profiles for different values of melting parameter with aiding and opposing flow.

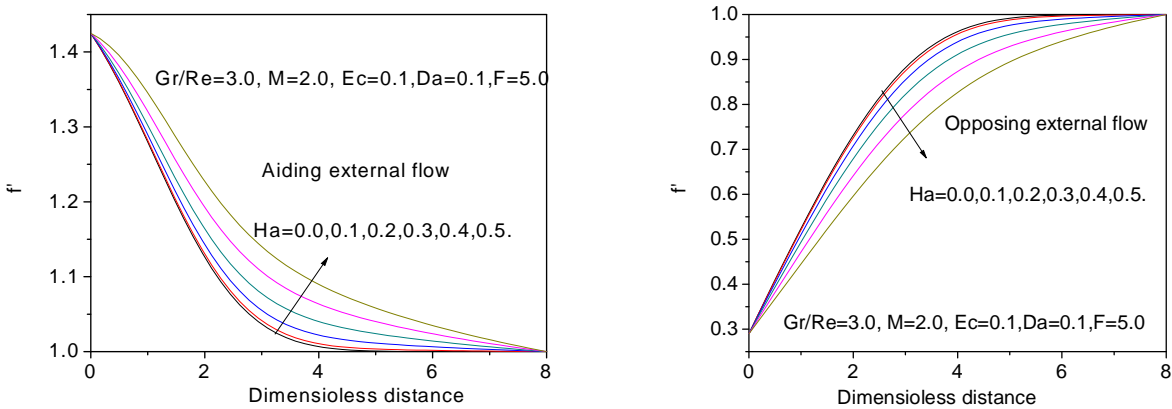


Fig.4. velocity profiles for different values of magnetic parameter with aiding and opposing flow.

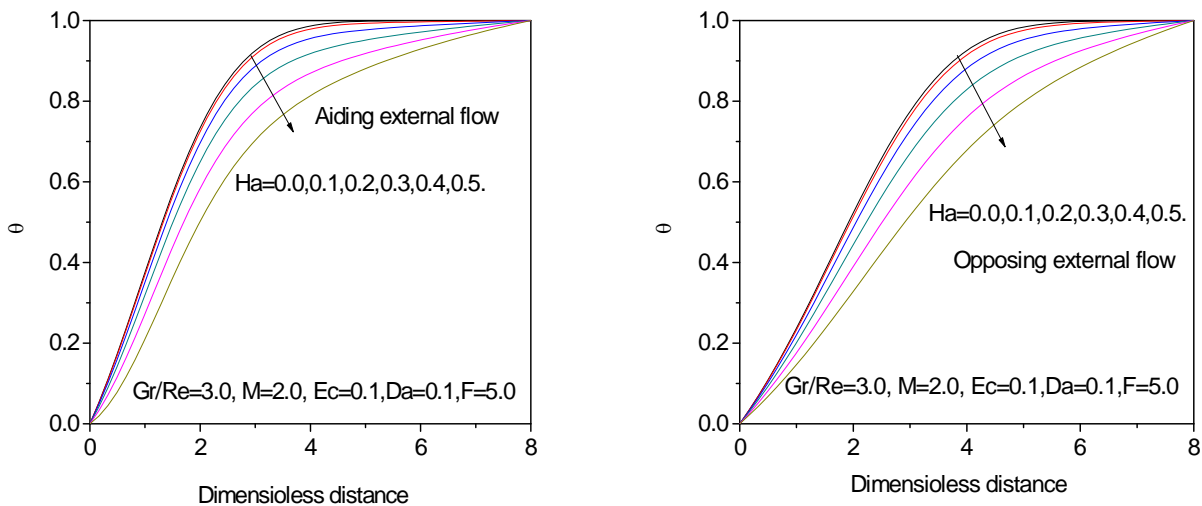


Fig.5. temperature profiles for different values of magnetic parameter with aiding and opposing flow.

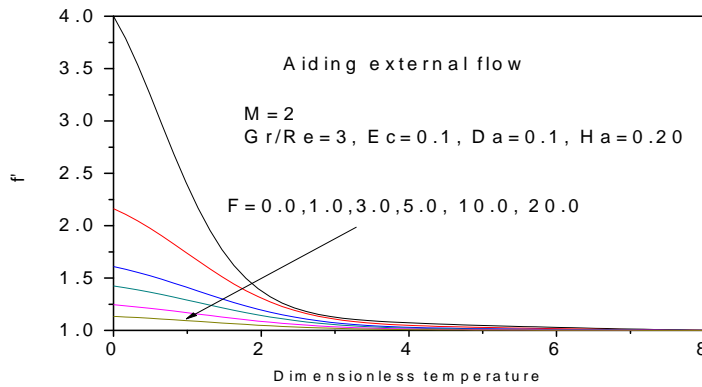


Fig.6. velocity profiles for different values inertia parameter with aiding and opposing flow.

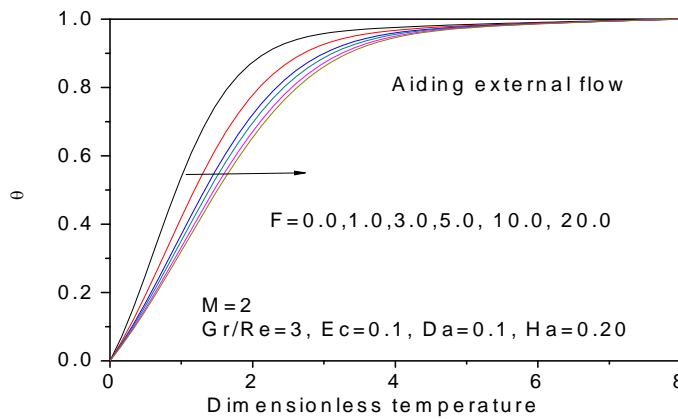


Fig.7. temperature profiles for different values of inertia parameter with aiding and opposing flow.

The effects of melting on velocity profiles for aiding and opposing external flow are shown in fig. 2. It is clear from the figure that an increase in melting parameter leads to increase the velocity profile and boundary layer thickness but this effect is found reverse for opposing external flow. From a physical point of view, this result may be attributed to the fact that convection heat transfer restrain from liquid saturated porous medium to the solid porous plate for aiding external flow. From fig.3 it is obvious that the temperature profiles decreases with increase of melting parameter both in aiding and opposing external flows without magnetic field, in the case of buoyancy parameter (Gr/Re)=3, Eckert number (Ec)=0.10, Darcy number (Da)=0.1 and inertia parameter $F=5$. Fig. 4 illustrates the influence of the magnetic field on velocity profiles. It is noted that the velocity profile increases with the increase of magnetic parameter's value in aiding flow cases and in the opposing flow case the velocity profile decreases with the increase of magnetic parameter. The temperature profiles for aiding and opposing flows are shown in Fig. 5 for different values of magnetic parameter. In aiding flow the increase in temperature is found for increases in magnetic parameter but in opposing flow case temperature decreases with an increase of magnetic parameter.

M	Gr/Re	Chamkha et.al (2010)	Cheng and Lin (2007)	Present Work
2.0	0.0	1.0000	1.0000	1.0000
	1.4	2.4000	2.4000	2.4000
	3.0	4.0000	4.0000	4.0000
	8.0	9.0000	9.0000	9.0000
	10.0	11.0000	11.0000	11.0000
	20.0	21.0000	21.0000	21.0000

TABLE 1. Comparison of $f_1(0)$ with values obtained by Chamkhaet *al.* (2010) and Cheng and Lin (2007) for Newtonian fluid ($n = 1.0$) with an aiding external flow for the case of $M = 2.0$

Fig. 6 depicts the effects inertia parameter on velocity profile. It is found that in aiding flow velocity decreases with increase of inertia parameter. From fig.7 it is obvious that temperature decreases with increase of inertia parameter.

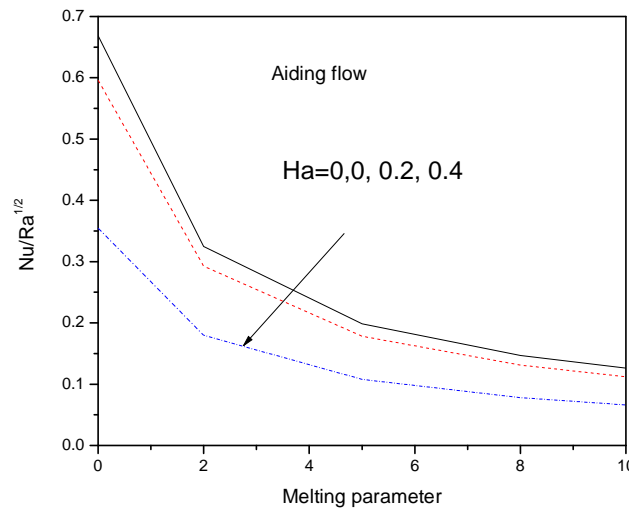


Fig.8. heat transfer rate for different values of magnetic parameter with aiding flow.

Fig. 8 illustrates the influence of the melting parameter and magnetic field on Nusselt number. It is noted that the heat transfer rate decreases with the increase of magnetic parameter as well as melting parameter in aiding flow cases.

VI. CONCLUSION

The problem of mixed convective melting along a vertical plate in a Newtonian fluid saturated non-Darcy porous medium in the presence of magnetic effects for aiding and opposing external flows with variable temperature is presented. The Similarity solutions for this problem are used to reduce the governing partial differential equations into a system of nonlinear ordinary partial differential equations and then are solved numerically using the fourth order Runge–Kutta method. A boundary condition to account for melting is used at the interface between the solid and liquid phases. Graphical representation of results regarding the velocity and temperature distributions as well as the Nusselt number were presented and discussed for different melting parameters. The increment of velocity is observed at aiding flow but reverse phenomenon are seen in opposing flow. In temperature distribution temperature decreases both aiding and opposing flow. The heat transfer rate decreases with increases of melting parameter as well as magnetic field.

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