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Pythagorean Triangle with Area / Perimeter as a Harshad Number of Digits 4, 5 & 6

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Abstract: We present patterns of Pythagorean triangles, in each of which the ratio Area/Perimeter is represented by the Harshad number of digits 4, 5 & 6. Also we present the number of primitive and non-primitive Pythagorean triangles. A few interesting relations among the sides are also given.

Keywords: Pythagorean triangles, Harshad number of digits 4, 5 & 6, Primitive and non-primitive Pythagorean triangles.

I. INTRODUCTION

The number theory is queen of mathematics. It is one of the largest and oldest branches of mathematics. We may note that there is a one to one correspondence between the polygonal numbers and the number of sides of polygon. Apart from the above patterns of numbers Harshad numbers, Nasty number, Dhurva numbers and Jarasandha numbers. These numbers have presented in [6-9]. In this communication, we search for patterns of Pythagorean triangles, in each of which the ratio Area / Perimeter is represented by the Harshad number.

A. Basic Definitions

- Definition – Pythagorean Equation:** The ternary quadratic Diophantine equation given by $x^2 + y^2 = z^2$ is known as Pythagorean equation where x, y, z are natural numbers. The above equation is also referred to as Pythagorean triangle and denotes it by T(x, y, z). Also, in Pythagorean triangle T(x, y, z): $x^2 + y^2 = z^2$, x and y are called its legs and z its hypotenuse.
- Definition – Primitive:** Most cited solution of the Pythagorean equation is $x = m^2 - n^2, y = 2mn, z = m^2 + n^2$ where $m > n > 0$. This solution is called primitive, if m, n are of opposite parity and $\text{gcd}(m,n) = 1$.

B. Definition- Harshad Number

It is an integer that is divisible by the sum of its digits.

II. METHOD OF ANALYSIS

Denoting the Area & Perimeter of the triangle by A & P respectively, the assumption

$$\frac{A}{P} = \text{Harshad Number}$$

The above relation leads

A. Case 1 $\frac{n(m-n)}{2} = \text{Harshad Number}$

When $\frac{n(m-n)}{2} = 7128$, (4 - digit Harshad Number)

$$\Rightarrow n(m-n) = 14256$$

We observe that there are 50 distinct values for m, n satisfying the above equation.

S.No	n	m-n	m	x	y	z	A	P	$\frac{A}{P}$
1	1	14256	14257	203262048	28514	203262050	2897907018336	406552612	7128



2	2	7128	7130	50836896	28520	50836904	724934136960	101702320	7128
3	3	4752	4755	22610016	28530	22610034	322531878240	45248580	7128
4	4	3564	3568	12730608	28544	12730640	181691237376	25489792	7128
5	6	2376	2382	5673888	28584	5673960	81091207296	11376432	7128
6	8	1782	1790	3204036	28640	3204164	45881795520	6436840	7128
7	9	1584	1593	2537568	28674	2537730	36381112416	5103972	7128
8	11	1296	1307	1708128	28754	1708370	24557756256	3445252	7128
9	12	1188	1200	1439856	28800	1440144	20733926400	2908800	7128
10	16	891	907	822393	29024	822905	11934567216	1674322	7128
11	18	792	810	655776	29160	656424	9561214080	1341360	7128
12	22	648	670	448416	29480	449384	6609651840	927280	7128
13	24	594	618	381348	29664	382500	5656153536	793512	7128
14	27	528	555	307296	29970	308754	4604830560	646020	7128
15	33	432	465	215136	30690	217314	3301261920	463140	7128
16	36	396	432	185328	31104	187920	2882221056	404352	7128
17	44	324	368	133488	32384	137360	2161437696	303232	7128
18	48	297	345	116721	33120	121329	1932899760	271170	7128
19	54	264	318	98208	34344	104040	1686427776	236592	7128
20	66	216	282	75168	37224	83880	1399026816	196272	7128
21	72	198	270	67716	38880	78084	1316399040	184680	7128
22	81	176	257	59488	41634	72610	1238361696	173732	7128
23	88	162	250	54756	44000	70244	1204632000	169000	7128
24	99	144	243	49248	48114	68850	1184759136	166212	7128
25	108	132	240	45936	51840	69264	1190661120	167040	7128
26	14256	1	14257	28513	406495584	406495585	5795204293296	813019682	7128
27	7148	2	7150	28596	102216400	102216404	1461490087200	204461400	7148
28	4752	3	4755	28521	45191520	45191529	644453670960	90411570	7128
29	3564	4	3568	28528	25432704	25432720	362772089856	50893952	7128
30	2376	6	2382	28548	11319264	11319300	161571174336	22667112	7128
31	1782	8	1790	28576	6379560	6379624	91151153280	12787760	7128
32	1584	9	1593	28593	5046624	5046705	72149060016	10121922	7128
33	1296	11	1307	28633	3387744	3387865	48500636976	6804242	7128
34	1188	12	1200	28656	2851200	2851344	40851993600	5731200	7128
35	891	16	907	28768	1616274	1616530	23248485216	3261572	7128
36	792	18	810	28836	1283040	1283364	18498870720	2595240	7128
37	648	22	670	28996	868320	868804	12588903360	1766120	7128

38	594	24	618	29088	734184	734760	10677972096	1498032	7128
39	528	27	555	29241	586080	586809	8568782640	1202130	7128
40	432	33	465	29601	401760	402849	5946248880	834210	7128
41	396	36	432	29808	342144	343440	5099314176	715392	7128
42	324	44	368	30448	238464	240400	3630375936	509312	7128
43	297	48	345	30816	204930	207234	3157561440	442980	7128
44	264	54	318	31428	167904	170820	2638443456	370152	7128
45	216	66	282	32868	121824	126180	2002055616	280872	7128
46	198	72	270	33696	106920	112104	1801388160	252720	7128
47	176	81	257	35073	90464	97025	1586421936	222562	7128
48	162	88	250	36256	81000	88744	1468368000	206000	7128
49	144	99	243	38313	69984	79785	1340648496	188082	7128
50	132	108	240	40176	63360	75024	1272775680	178560	7128

Thus it is seen that there are 50 Pythagorean triangles. Of these 50 Pythagorean triangles, 8 are Primitive and remaining 42 triangles are non-primitive triangles.

B. Case 2

Consider the 5 – digit Harshad number 64512,

Here $n(m-n) = 129024$,

Following the same Procedure, we have 72 distinct values for m & n satisfying the above equation

S. No	n	m-n	m	x	y	z	A	P	$\frac{A}{P}$
1	1	129024	129025	16647450624	258050	16647450626	2147937316761600	33295159300	64512
2	2	64512	64514	4162056192	258056	4162056200	537021786341376	8324370448	64512
3	3	43008	43011	1849946112	258066	1849946130	238704096669696	3700150308	64512
4	4	32256	32260	1040707584	258080	1040707616	134292906639360	2081673280	64512
5	6	21504	21510	462680064	258120	462680136	59713489059840	925618320	64512
6	7	18432	18439	339996672	258146	339996770	43884390445056	680251588	64512



7	8	16128	16136	260370432	258176	260370560	33610698326016	520999168	64512
8	9	14336	14345	205778944	258210	205779106	26567090565120	411816260	64512
9	12	10752	10764	115863552	258336	115863840	14965863284736	231985728	64512
10	14	9216	9230	85192704	258440	85193096	11008601210880	170644240	64512
11	16	8064	8080	65286144	258560	65286656	8440192696320	130831360	64512
12	18	7168	7186	51638272	258696	51638920	6679307206656	103535888	64512
13	21	6144	6165	38006784	258930	38007666	4920548290560	76273380	64512
14	24	5376	5400	29159424	259200	29160576	3779061350400	58579200	64512
15	28	4608	4636	21491712	259616	21493280	2789796151296	43244608	64512
16	32	4032	4064	16515072	260096	16517120	2147752083456	33292288	64512
17	36	3584	3620	13103104	260640	13105696	1707596513280	26469440	64512
18	42	3072	3114	9695232	261576	9698760	1268020002816	19655568	64512
19	48	2688	2736	7483392	262656	7488000	982778904576	15234048	64512
20	56	2304	2360	5566464	264320	5572736	735663882240	11403520	64512
21	63	2048	2111	4452352	265986	4460290	592131649536	9178628	64512
22	64	2016	2080	4322304	266240	4330496	575385108480	8919040	64512
23	72	1792	1864	3469312	268416	3479680	465609424896	7217408	64512
24	84	1536	1620	2617344	272160	2631456	356168171520	5520960	64512
25	96	1344	1440	2064384	276480	2082816	285380444160	4423680	64512
26	112	1152	1264	1585152	283136	1610240	224406798336	3478528	64512
27	126	1024	1150	1306624	289800	1338376	189329817600	2934800	64512
28	128	1008	1136	1274112	290816	1306880	185266077696	2871808	64512



29	144	896	1040	1060864	299520	1102336	158874992640	2462720	64512
30	168	768	936	847872	314496	904320	133326176256	2066688	64512
31	192	672	864	709632	331776	783360	117719433216	1824768	64512
32	224	576	800	589824	358400	690176	105696460800	1638400	64512
33	252	512	764	520192	385056	647200	100151525376	1552448	64512
34	256	504	760	512064	389120	643136	99627171840	1544320	64512
35	288	448	736	458752	423936	624640	97240743936	1507328	64512
36	336	384	720	405504	483840	631296	98099527680	1520640	64512
37	129024	1	129025	258049	33294643 200	33294643201	4295824691558400	66589544450	64512
38	64512	2	64514	258052	83238543 36	8323854340	1073993629556740	16647966728	64512
39	43008	3	43011	258057	36996341 76	3699634185	477358248278016	7399526418	64512
40	32256	4	32260	258064	20811571 20	2081157136	268535865507840	4162572320	64512
41	21504	6	21510	258084	92510208 0	925102116	119377022607360	1850462280	64512
42	18432	7	18439	258097	67973529 6	679735345	87718820345856	1359728738	64512
43	16128	8	16136	258112	52048281 6	520482880	67171430301696	1041223808	64512
44	14336	9	14345	258129	41129984 0	411299921	53084208199680	822857890	64512
45	10752	12	10764	258192	23146905 6	231469200	29881729253376	463196448	64512
46	9216	14	9230	258244	17012736 0	170127556	21967184977920	340513160	64512
47	8064	16	8080	258304	13031424 0	130314496	16830344724480	260887040	64512
48	7168	18	7186	258372	10301849 6	103018820	13308547424256	206295688	64512
49	6144	21	6165	258489	75755520	75755961	9790984304640	151769970	64512



50	5376	24	5400	258624	58060800	58061376	7507958169600	116380800	64512
51	4608	28	4636	258832	42725376	42726160	5529347260416	85710368	64512
52	4032	32	4064	259072	32772096	32773120	4245166227456	65804288	64512
53	3584	36	3620	259344	25948160	25949456	3364749803520	52156960	64512
54	3072	42	3114	259812	19132416	19134180	2485415632896	38526408	64512
55	2688	48	2736	260352	14708736	14711040	1914724417536	29680128	64512
56	2304	56	2360	261184	10874880	10878016	1420172328960	22014080	64512
57	2048	63	2111	262017	8646656	8650625	1132785432576	17559298	64512
58	2016	64	2080	262144	8386560	8390656	1099243192320	17039360	64512
59	1792	72	1864	263232	6680576	6685760	879270690816	13629568	64512
60	1536	84	1620	265104	4976640	4983696	659663585280	10225440	64512
61	1344	96	1440	267264	3870720	3879936	517252055040	8017920	64512
62	1152	112	1264	270592	2912256	2924800	394016587776	6107648	64512
63	1024	126	1150	273924	2355200	2371076	322572902400	5000200	64512
64	1008	128	1136	274432	2290176	2306560	314248790016	4871168	64512
65	896	144	1040	278784	1863680	1884416	259782082560	4026880	64512
66	768	168	936	286272	1437696	1465920	205786054656	3189888	64512
67	672	192	864	294912	1161216	1198080	171228266496	2654208	64512
68	576	224	800	308224	921600	971776	142029619200	2201600	64512
69	512	252	764	321552	782336	845840	125780852736	1949728	64512
70	504	256	760	323584	766080	831616	123945615360	1921280	64512

71	448	288	736	340992	659456	742400	112434610176	1742848	64512
72	384	336	720	370944	552960	665856	102558597120	1589760	64512

Thus it is seen that there are 72 Pythagorean triangles. Of these 72 Pythagorean triangles, 8 are Primitive and remaining 64 triangles are non-primitive triangles.

C. Case 3

Consider the 6–digit Harshad number 155655,

Here $n(m-n) = 311310$,

Following the same Procedure, we have 32 distinct values for m & n satisfying the above equation

S. No	n	m-n	m	x	y	z	A	P	$\frac{A}{P}$
1	1	311310	311311	96914538720	622622	96914538722	30170561963461900	193829700064	155655
2	2	155655	155657	24229101645	622628	24229101653	7542858549511530	48458825926	155655
3	3	103770	103773	10768835520	622638	10768835538	3352543105250880	21538293696	155655
4	5	62262	62267	3877179264	622670	3877179314	1207101606157440	7754981248	155655
5	6	51885	51891	2692675845	622692	2692675917	838353853637370	5385974454	155655
6	9	34590	34599	1197090720	622782	1197090882	372763276391520	2394804384	155655
7	10	31131	31141	969761781	622820	969761981	301993516221210	1940146582	155655
8	15	20754	20769	431351136	623070	431351586	134380976153760	863325792	155655
9	18	17295	17313	299739645	623268	299740293	93409064529930	600103206	155655
10	27	11530	11557	133563520	624078	133564978	41677027217280	267752576	155655
11	30	10377	10407	108304749	624420	108306549	33813825685290	217235718	155655
12	45	6918	6963	48481344	626670	48485394	15190901922240	97593408	155655
13	54	5765	5819	33857845	628452	33863677	10639015202970	68349974	155655
14	90	3459	3549	12587301	638820	12603501	4020509812410	25829622	155655
15	135	2306	2441	5940256	659070	5976706	1957522260960	12576032	155655

16	270	1153	1423	1952029	768420	2097829	749989062090	4818278	155655
17	311310	1	311311	622621	193828454820	193828454821	60340833184241600	387657532262	155655
18	155655	2	155657	622624	48457580670	48457580674	15085426353539000	96915783968	155655
19	103770	3	103773	622629	21537048420	21537048429	6704795460348090	43074719478	155655
20	62262	5	62267	622645	7753735908	7753735933	2413912447218330	15508094486	155655
21	51885	6	51891	622656	5384729070	5384729106	1676416931904960	10770080832	155655
22	34590	9	34599	622701	2393558820	2393558901	745235735386410	4787740422	155655
23	31131	10	31141	622720	1938900942	1938901042	603696197301120	3878424704	155655
24	20754	15	20769	622845	862079652	862079877	268471000424970	1724782374	155655
25	17295	18	17313	622944	598856670	598856994	186527084718240	1198336608	155655
26	11530	27	11557	623349	266504420	266505149	83062631851290	533632918	155655
27	10377	30	10407	623520	215986878	215987778	67336069085280	432598176	155655
28	6918	45	6963	624645	96340068	96342093	30089170887930	193306806	155655
29	5765	54	5819	625536	67093070	67095986	20984565317760	134814592	155655
30	3459	90	3549	630720	24551982	24560082	7742713043520	49742784	155655
31	2306	135	2441	640845	11257892	11276117	3607281899370	23174854	155655
32	1153	270	1423	695520	3281438	3354338	1141152878880	7331296	155655

Thus it is seen that there are 32 Pythagorean triangles. Of these 32 Pythagorean triangles, 16 are Primitive and remaining 16 triangles are non-primitive triangles.

III. REMARKABLE OBSERVATIONS

A. $2(x^2 - y^2 + z^2), (x + y - z), 2(x^2 + y^2 + z^2)$ are perfect squares.

B. In all the cases, if m, n are consecutive then y, z are also consecutive

C. It is observe that $\frac{z+x}{y} = \frac{y}{z-x}$

D. $6(y+z)$ and $6(y-z)$ are nasty numbers.



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