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Bipolar Multi-Fuzzy Subalgebra Of Bg-Algebra

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Abstract: In this paper, we introduce the notion of bipolar multi-fuzzy subalgebra of BG-algebra by combining the two concepts of bipolar fuzzy sets and multi-fuzzy sets and discuss some of its properties. Also we define the level subsets positive t-cut and negative s-cut of bipolar multi-fuzzy BG-algebra and study some of its related properties.

Keywords : Bipolar fuzzy set, Multi-fuzzy set, BG-algebra, Fuzzy BG-subalgebra, Multi-fuzzy BG-subalgebra, Bipolar multi-fuzzy BG-subalgebra, positive t-cut, negative s-cut.

AMS Subject Classification (2010) : 06F35, 03G25, 08A72, 03E72.

I. INTRODUCTION

The notion of a fuzzy subset was initially introduced by Zadeh [8] in 1965, for representing uncertainty. In 2000, S.Sabu and T.V.Ramakrishnan [9,10] proposed the theory of multi-fuzzy sets in terms of multi-dimensional membership functions and investigated some properties of multilevel fuzziness. Theory of multi-fuzzy set is an extension of theory of fuzzy sets. Among these theories, a well-known extension of the classic fuzzy set is bipolar fuzzy set theory, which was pioneered by Zhang[11]. Bipolar fuzzy sets are an extension of fuzzy sets whose membership degree range is increased from the interval [0,1] to [-1,1]. In bipolar fuzzy set, the membership degree 0 means that elements are irrelevant to the corresponding property, the membership degrees on (0,1] denotes that elements somewhat satisfy the property and the membership degrees on [-1,0) means that elements somewhat satisfy the implicit counter-property. Although bipolar fuzzy sets and intuitionistic fuzzy sets look similar to each other, they are different sets.

Y.Imai and K.Iseki introduced two classes of abstract algebras: BCK algebras and BCI-algebras [1,2,3]. It is shown that the class of BCK-algebras is a proper subclass of the class of BCI-algebras. J.Negggers and H.S.Kim [4] introduced a new notion, called a B-algebra. In 2005, C.B.Kim and H.S.Kim [5] introduced the notion of a BG-algebra which is a generalization of B-algebras. With these ideas, fuzzy subalgebras of BG-algebra were developed by S.S.Ahn and H.D.Lee[6]. In 2015, T.Senapati[7] introduced the concepts of bipolar fuzzy subalgebra in BG-algebra. In this paper, we introduce the notion of bipolar multi-fuzzy BG-subalgebra and discuss some of its properties.

II. PRELIMINARIES

In this section we site the fundamental definitions that will be used in the sequel.

A. Definition 2.1

Let X be a non-empty set. A multi-fuzzy set A in X is defined as a set of ordered sequences:

$$A = \{ (x, \mu_1(x), \mu_2(x), \dots, \mu_k(x), \dots) : x \in X \}, \text{ where } \mu_i : X \rightarrow [0, 1] \text{ for all } i.$$

1) Remarks

- If the sequences of the membership functions have only k-terms (finite number of terms), k is called the dimension of A.
- The set of all multi-fuzzy sets in X of dimension k is denoted by $M^kFS(X)$
- The multi-fuzzy membership function μ_A is a function from X to $[0,1]^k$ such that for all $x \in X$, $\mu_A(x) = (\mu_1(x), \mu_2(x), \dots, \mu_k(x))$.
- For the sake of simplicity, we denote the multi-fuzzy set $A = \{(x, \mu_1(x), \mu_2(x), \dots, \mu_k(x)) : x \in X\}$ as $A = (\mu_1, \mu_2, \dots, \mu_k)$.

B. Definition 2.2

Let k be a positive integer and let A and B in $M^kFS(X)$, where $A = (\mu_1, \mu_2, \dots, \mu_k)$ and $B = (v_1, v_2, \dots, v_k)$, then we have the following relations and operations :

- $A \subseteq B$ if and only if $\mu_i \leq v_i$, for all $i=1,2,\dots,k$;
- $A = B$ if and only if $\mu_i = v_i$, for all $i=1,2,\dots,k$;
- $A \cup B = (\mu_1 \cup v_1, \dots, \mu_k \cup v_k) = \{(x, \max(\mu_1(x), v_1(x)), \dots, \max(\mu_k(x), v_k(x))) : x \in X\}$
- $A \cap B = ((\mu_1 \cap v_1, \dots, \mu_k \cap v_k) = \{(x, \min(\mu_1(x), v_1(x)), \dots, \min(\mu_k(x), v_k(x))) : x \in X\}$

C. Definition 2.3

Let X be a non-empty set. A bipolar fuzzy set ϕ in X is an object having the form $\phi = \{ \langle x, \phi^+(x), \phi^-(x) \rangle : x \in X \}$ where $\phi^+(x) : X \rightarrow [0,1]$ and $\phi^-(x) : X \rightarrow [-1,0]$ are the mappings.

We use the positive membership degree $\phi^+(x)$ to denote the satisfaction degree of an element x to the property corresponding to a bipolar fuzzy set ϕ and the negative membership degree $\phi^-(x)$ to denote the satisfaction degree of an element x to some implicit counter-property corresponding to a bipolar fuzzy set ϕ . If $\phi^+(x) \neq 0$ and $\phi^-(x) = 0$, it is the situation that x is regarded as having only positive satisfaction for ϕ . If $\phi^+(x) = 0$ and $\phi^-(x) \neq 0$, it is the situation that x does not satisfy the property of ϕ but somewhat satisfies the counter-property of ϕ . It is possible for an element x to be such that $\phi^+(x) \neq 0$ and $\phi^-(x) \neq 0$ when the membership function of the property overlaps that of its counter property over some portion of X .

D. Definition 2.4

Let X be a non-empty set. A bipolar multi-fuzzy set A in X is defined as an object of the form $A = \{ \langle x, A_i^+(x), A_i^-(x) \rangle : x \in X \}$ where $A_i^+(x) : X \rightarrow [0,1]$ and $A_i^-(x) : X \rightarrow [-1,0]$

The positive membership degree $A_i^+(x)$ denotes the satisfaction degree of an element x to the property corresponding to a bipolar fuzzy set A and the negative membership degree $A_i^-(x)$ denotes the satisfaction degree of an element x to some implicit counter-property corresponding to a bipolar fuzzy set A . If $A_i^+(x) \neq 0$ and $A_i^-(x) = 0$, it is the situation that x is regarded as having only positive satisfaction for A . If $A_i^+(x) = 0$ and $A_i^-(x) \neq 0$, it is the situation that x does not satisfy the property of A but somewhat satisfies the counter-property of A . It is possible for an element x to be such that $A_i^+(x) \neq 0$ and $A_i^-(x) \neq 0$ when the membership function of the property overlaps that of its counter property over some portion of X , where $i = 1,2,\dots,n$

E. Example 2.5

Let $X = \{ 1, m, n \}$ be a set. Then $A = \{ \langle 1, 0.5, 0.6, 0.4, -0.3, -0.5, -0.6 \rangle, \langle m, 0.8, 0.4, 0.2, -0.5, -0.8, -0.1 \rangle, \langle n, 0.3, 0.2, 0.1, -0.7, -0.6, -0.2 \rangle \}$ is a bipolar multi-fuzzy set of X .

F. Definition 2.6

Non-empty set X with a constant 0 and a binary operation “ $*$ ” is called a BG-algebra if it satisfies the following axioms:

- 1) $x * x = 0$
- 2) $x * 0 = x$
- 3) $(x * y) * (0 * y) = x, \forall x, y \in X$.

G. Example 2.7

Let $X = \{ 0, 1, 2 \}$ be a set with the following table

*	0	1	2
0	0	1	2
1	1	0	1
2	2	2	0

Then $(X; *, 0)$ is a BG-algebra.

H. Definition 2.8

Let S be a non-empty subset of a BG-algebra X , then S is called a subalgebra of X if $x * y \in S$ for all $x, y \in S$.

I. Definition 2.9

Let μ be a fuzzy set in BG-algebra. Then μ is called a fuzzy subalgebra of X if

$$\mu(x * y) \geq \min \{ \mu(x), \mu(y) \}, \forall x, y \in X.$$

II. BIPOLAR MULTI-FUZZY BG-SUBALGEBRA

In this section, the concept of bipolar multi-fuzzy subalgebra of BG-algebra is defined and their related properties are presented.

A. Definition 3.1

Let $A = \{ \langle x, A_i^+(x), A_i^-(x) \rangle / x \in X \}$ be a bipolar multi-fuzzy set in X , then the set A is bipolar multi-fuzzy BG-subalgebra over the binary operator $*$ if it satisfies the following conditions :

- 1) $A_i^+(x * y) \geq \min \{ A_i^+(x), A_i^+(y) \}$
- 2) $A_i^-(x * y) \leq \max \{ A_i^-(x), A_i^-(y) \}$

B. Example 3.2

Let $X = \{ 0,1,2,3 \}$ be a BG-algebra with the following cayley table

*	0	1	2	3
0	0	1	2	3
1	1	0	3	2
2	2	3	0	1
3	3	2	1	0

Let $A = \{ \langle x, A_i^+(x), A_i^-(x) \rangle / x \in X \}$ be a bipolar multi-fuzzy set defined by
 $A = \{ \langle 0, 0.6, 0.7, 0.3, -0.7, -0.5, -0.4 \rangle, \langle 1, 0.5, 0.4, 0.2, -0.5, -0.3, -0.2 \rangle, \langle 2, 0.4, 0.3, 0.1, -0.4, -0.2, -0.1 \rangle, \langle 3, 0.4, 0.3, 0.1, -0.4, -0.2, -0.1 \rangle \}$. Clearly, A is a bipolar multi-fuzzy BG-subalgebra in X .

C. Preposition 3.3

If $A = \{ \langle x, A_i^+(x), A_i^-(x) \rangle / x \in X \}$ is a bipolar multi-fuzzy subalgebra in X , then for all $x \in X$, $A_i^+(0) \geq A_i^+(x)$ and $A_i^-(0) \leq A_i^-(x)$.

1) *Proof:* Let $x \in X$.

$$\begin{aligned} \text{Then } A_i^+(0) &= A_i^+(x * x) \geq \min \{ A_i^+(x), A_i^+(x) \} = A_i^+(x) \\ A_i^-(0) &= A_i^-(x * x) \leq \max \{ A_i^-(x), A_i^-(x) \} = A_i^-(x) \end{aligned}$$

D. Theorem 3.4

If a bipolar multi-fuzzy set $A = \{ \langle x, A_i^+(x), A_i^-(x) \rangle / x \in X \}$ is a bipolar multi-fuzzy subalgebra, then for all $x \in X$, $A_i^+(0 * x) \geq A_i^+(x)$ and $A_i^-(0 * x) \leq A_i^-(x)$.

1) *Proof:* Let $x \in X$

$$\begin{aligned} \text{Then } A_i^+(0 * x) &\geq \min \{ A_i^+(0), A_i^+(x) \} \\ &= \min \{ A_i^+(x * x), A_i^+(x) \} \\ &\geq \min \{ \min \{ A_i^+(x), A_i^+(x) \}, A_i^+(x) \} \\ &= \min \{ A_i^+(x), A_i^+(x) \} \\ &= A_i^+(x) \\ A_i^-(0 * x) &\leq \max \{ A_i^-(0), A_i^-(x) \} \\ &= \max \{ A_i^-(x * x), A_i^-(x) \} \\ &\leq \max \{ \max \{ A_i^-(x), A_i^-(x) \}, A_i^-(x) \} \\ &= \max \{ A_i^-(x), A_i^-(x) \} \\ &= A_i^-(x) \end{aligned}$$

E. Theorem 3.5

Let $A = \{ \langle x, A_i^+(x), A_i^-(x) \rangle / x \in X \}$ and $B = \{ \langle x, B_i^+(x), B_i^-(x) \rangle / x \in X \}$ be two bipolar multi-fuzzy subalgebras of X . Then $A \cap B$ is a bipolar multi-fuzzy subalgebra in X .

1) *Proof:*

Let $x, y \in A \cap B$

Then $x, y \in A$ and B .

$$\begin{aligned} A_i^+ \cap B_i^+(x * y) &= \min \{ A_i^+(x * y), B_i^+(x * y) \} \\ &\geq \min \{ \min \{ A_i^+(x), A_i^+(y) \}, \min \{ B_i^+(x), B_i^+(y) \} \} \\ &= \min \{ \min \{ A_i^+(x), B_i^+(x) \}, \min \{ A_i^+(y), B_i^+(y) \} \} \\ &= \min \{ A_i^+ \cap B_i^+(x), A_i^+ \cap B_i^+(y) \} \end{aligned}$$

$$\begin{aligned} A_i^- \cap B_i^-(x * y) &= \max \{ A_i^-(x * y), B_i^-(x * y) \} \\ &\leq \max \{ \max \{ A_i^-(x), A_i^-(y) \}, \max \{ B_i^-(x), B_i^-(y) \} \} \\ &= \max \{ \max \{ A_i^-(x), B_i^-(x) \}, \max \{ A_i^-(y), B_i^-(y) \} \} \\ &= \max \{ A_i^- \cap B_i^-(x), A_i^- \cap B_i^-(y) \} \end{aligned}$$

F. Proposition 3.6

The union of any set of bipolar multi-fuzzy subalgebras need not be a bipolar multi-fuzzy subalgebra .

G. Theorem 3.7

If $A = \langle A_i^+, A_i^- \rangle$ is a bipolar multi-fuzzy subalgebra of X , then $H = \{ x \in X / A_i^+(x) = 1, A_i^-(x) = -1 \}$ is either empty or a subalgebra of X .

1) *Proof*: If no element satisfies the conditions of H , then the set H is empty. If x and $y \in H$ then $A_i^+(x) = 1, A_i^-(x) = -1, A_i^+(y) = 1, A_i^-(y) = -1$.

Since A is a bipolar multi-fuzzy subalgebra of $X, A_i^+(x * y) \geq \min \{ A_i^+(x), A_i^+(y) \} = \min \{ 1, 1 \} = 1$ and also $A_i^-(x * y) \leq 1$

Therefore $A_i^+(x * y) = 1$

$A_i^-(x * y) \leq \max \{ A_i^-(x), A_i^-(y) \} = \max \{ -1, -1 \} = -1$ and also $A_i^-(x * y) \geq -1$

Therefore $A_i^-(x * y) = -1$

Hence $x * y \in H$

Therefore H is a subalgebra of X .

H. Theorem 3.8

Let $A = \langle A_i^+, A_i^- \rangle$ be a bipolar multi-fuzzy subalgebra of X .

If $A_i^+(x * y) = 0$ then either $A_i^+(x) = 0$ or $A_i^+(y) = 0$ for x and $y \in X$

If $A_i^-(x * y) = 0$ then either $A_i^-(x) = 0$ or $A_i^-(y) = 0$ for x and $y \in X$

1) *Proof*: Let $x, y \in X$. Then $A_i^+(x * y) \geq \min \{ A_i^+(x), A_i^+(y) \}$ i.e., $0 \geq \min \{ A_i^+(x), A_i^+(y) \}$

This implies that either $A_i^+(x) = 0$ or $A_i^+(y) = 0$ or $A_i^-(x * y) \leq \max \{ A_i^-(x), A_i^-(y) \}$ i.e., $0 \leq \max \{ A_i^-(x), A_i^-(y) \}$

This implies that either $A_i^-(x) = 0$ or $A_i^-(y) = 0$

I. Theorem 3.9

If $A = \langle A_i^+, A_i^- \rangle$ be a bipolar multi-fuzzy subalgebra of X , then the set $H = \{ x \in X / A_i^+(x) = A_i^+(0) \text{ and } A_i^-(x) = A_i^-(0) \}$ is a subalgebra of X .

1) *Proof*: Let $x, y \in H$

Then $A_i^+(x) = A_i^+(y) = A_i^+(0)$ and $A_i^-(x) = A_i^-(y) = A_i^-(0)$

$A_i^+(x * y) \geq \min \{ A_i^+(x), A_i^+(y) \} = \min \{ A_i^+(0), A_i^+(0) \} = A_i^+(0)$

Also $A_i^+(x * y) \leq A_i^+(0)$

Therefore $A_i^+(x * y) = A_i^+(0)$

And $A_i^-(x * y) \leq \max \{ A_i^-(x), A_i^-(y) \} = \max \{ A_i^-(0), A_i^-(0) \} = A_i^-(0)$

Also $A_i^-(x * y) \geq A_i^-(0)$

This implies that $A_i^-(x * y) = A_i^-(0)$

Therefore $x * y \in H$

H is a subalgebra of X .

III. LEVEL SUBSETS OF A BIPOLAR MULTI-FUZZY SET

In this section, the positive t-cut and negative s-cut of a bipolar multi-fuzzy set is defined and some properties are discussed.

A. Definition 4.1

Let $A = \{ \langle x, A_i^+(x), A_i^-(x) \rangle / x \in X \}$ be a bipolar multi-fuzzy subalgebra of X . For $s \in [-1,0]$ and $t \in [0,1]$, the set $U(A_i^+; t) = \{ x \in X ; A_i^+(x) \geq t \}$ is called positive t-cut of A and the set $L(A_i^-; s) = \{ x \in X ; A_i^-(x) \leq s \}$ is called negative s-cut of A .

B. Theorem 4.2

If $A = \langle A_i^+, A_i^- \rangle$ is a bipolar multi-fuzzy subalgebra of X , then the positive t-cut and negative s-cut of A are subalgebras of X .

1) *Proof*: Let $x, y \in U(A_i^+; t)$

Then $A_i^+(x) \geq t$ and $A_i^+(y) \geq t$

$$A_i^+(x * y) \geq \min \{ A_i^+(x), A_i^+(y) \} \geq \min \{ t, t \} = t$$

Therefore $x * y \in U(A_i^+; t)$

Hence $U(A_i^+; t)$ is a subalgebra in X .

Let $x, y \in L(A_i^-; s)$

Then $A_i^-(x) \leq s$ and $A_i^-(y) \leq s$

$$A_i^-(x * y) \leq \max \{ A_i^-(x), A_i^-(y) \} \leq \max \{ s, s \} = s$$

Therefore $x * y \in L(A_i^-; s)$

Hence $L(A_i^-; s)$ is a subalgebra in X .

C. Theorem 4.3

Let $A = \langle A_i^+, A_i^- \rangle$ be a multi-fuzzy set in X , such that the level sets $U(A_i^+; t)$ and $L(A_i^-; s)$ are subalgebras of X for every $s \in [-1,0]$ and $t \in [0,1]$. Then A is a bipolar multi-fuzzy subalgebra in X .

1) *Proof*: Let $A = \langle A_i^+, A_i^- \rangle$ be a multi-fuzzy set in X , such that the level sets $U(A_i^+; t)$ and $L(A_i^-; s)$ are subalgebras of X for every $s \in [-1,0]$ and $t \in [0,1]$.

In contrary, let $x_0, y_0 \in X$ be such that $A_i^+(x_0 * y_0) < \min \{ A_i^+(x_0), A_i^+(y_0) \}$ and $A_i^-(x_0 * y_0) > \max \{ A_i^-(x_0), A_i^-(y_0) \}$

Let $A_i^+(x_0) = \alpha, A_i^+(y_0) = \beta, A_i^-(x_0) = \gamma, A_i^-(y_0) = \delta, A_i^+(x_0 * y_0) = t, A_i^-(x_0 * y_0) = s$

Then $t < \min \{ \alpha, \beta \}$ and $s > \max \{ \gamma, \delta \}$

$$\text{Put } t_1 = \frac{1}{2} [A_i^+(x_0 * y_0) + \min \{ A_i^+(x_0), A_i^+(y_0) \}]$$

$$\text{and } s_1 = \frac{1}{2} [A_i^-(x_0 * y_0) + \max \{ A_i^-(x_0), A_i^-(y_0) \}]$$

This implies, $t_1 = \frac{1}{2} [t + \min \{ \alpha, \beta \}]$ and $s_1 = \frac{1}{2} [s + \max \{ \gamma, \delta \}]$

Hence $\alpha > t_1 = \frac{1}{2} [t + \min \{ \alpha, \beta \}] > t, \beta > t_1 = \frac{1}{2} [t + \min \{ \alpha, \beta \}] > t$

and $\gamma < s_1 = \frac{1}{2} [s + \max \{ \gamma, \delta \}] < s, \delta < s_1 = \frac{1}{2} [s + \max \{ \gamma, \delta \}] < s$

$\Rightarrow \min \{ \alpha, \beta \} > t_1 > t = A_i^+(x_0 * y_0)$ and $\max \{ \gamma, \delta \} < s_1 < s = A_i^-(x_0 * y_0)$

So that $x_0 * y_0 \notin U(A_i^+; t)$ and $x_0 * y_0 \notin L(A_i^-; s)$ which is a contradiction, since

$A_i^+(x_0) = \alpha \geq \min \{ \alpha, \beta \} > t_1, A_i^+(y_0) = \beta \geq \min \{ \alpha, \beta \} > t_1$ and $A_i^-(x_0) = \gamma \leq \max \{ \gamma, \delta \} < s_1, A_i^-(y_0) = \delta \leq \max \{ \gamma, \delta \} < s_1$

This implies that $x_0, y_0 \in U(A_i^+; t)$ and $x_0, y_0 \in L(A_i^-; s)$

Thus $A_i^+(x * y) \geq \min \{ A_i^+(x), A_i^+(y) \}$ and $A_i^-(x * y) \leq \max \{ A_i^-(x), A_i^-(y) \}$, for $x, y \in X$.

Hence A is a bipolar multi-fuzzy subalgebra of X .

D. Theorem 4.4

Any BG-subalgebra of X can be realized as both the positive t-cut and negative s-cut of some bipolar multi-fuzzy subalgebra in X .

1) *Proof*: Let S be a subalgebra of a BG-algebra X and $A = \langle A_i^+, A_i^- \rangle$ be a bipolar multi-fuzzy set in X defined by

$$A_i^+(x) = \begin{cases} \lambda_i, & \text{if } x \in S \\ 0, & \text{otherwise} \end{cases} \quad A_i^-(x) = \begin{cases} \tau_i, & \text{if } x \in S \\ 0, & \text{otherwise} \end{cases} \quad \text{for all } \lambda_i \in [0,1], \tau_i \in [-1,0]$$

We consider the following four cases:

a) *Case(i)*: If $x, y \in S$, then $A_i^+(x) = \lambda_i, A_i^+(y) = \lambda_i, A_i^-(x) = \tau_i, A_i^-(y) = \tau_i$

Since S is a subalgebra of $X, x * y \in S$

$$A_i^+(x * y) = \lambda_i = \min \{ \lambda_i, \lambda_i \} = \min \{ A_i^+(x), A_i^+(y) \} \text{ and}$$

$$A_i^-(x * y) = \tau_i = \max \{ \tau_i, \tau_i \} = \max \{ A_i^-(x), A_i^-(y) \}$$

b) *Case(ii)*: If $x \in S$ and $y \notin S$, then $A_i^+(x) = \lambda_i, A_i^+(y) = 0, A_i^-(x) = \tau_i, A_i^-(y) = 0$

This implies that either $x * y \in S$ or $x * y \notin S$.

$$A_i^+(x * y) \geq 0 = \min \{ \lambda_i, 0 \} = \min \{ A_i^+(x), A_i^+(y) \} \text{ and}$$

$$A_i^-(x * y) \leq 0 = \max \{ \tau_i, 0 \} = \max \{ A_i^-(x), A_i^-(y) \}$$

c) *Case(iii)*: If $x \notin S$ and $y \in S$, then $A_i^+(x) = 0, A_i^+(y) = \lambda_i, A_i^-(x) = 0, A_i^-(y) = \tau_i$

This implies that either $x * y \in S$ or $x * y \notin S$.

$$A_i^+(x * y) \geq 0 = \min \{ 0, \lambda_i \} = \min \{ A_i^+(x), A_i^+(y) \} \text{ and}$$

$$A_i^-(x * y) \leq 0 = \max \{ 0, \tau_i \} = \max \{ A_i^-(x), A_i^-(y) \}$$

d) Case (iv) : $x \notin S$ and $y \notin S$, then $A_i^+(x) = 0$, $A_i^+(y) = 0$, $A_i^-(x) = 0$, $A_i^-(y) = 0$

This implies that either $x * y \in S$ or $\notin S$.

$$A_i^+(x * y) \geq 0 = \min \{ 0, 0 \} = \min \{ A_i^+(x), A_i^+(y) \} \text{ and}$$

$$A_i^-(x * y) \leq 0 = \max \{ 0, 0 \} = \max \{ A_i^-(x), A_i^-(y) \}$$

Thus, in all the cases, $A = \langle A_i^+, A_i^- \rangle$ is bipolar multi-fuzzy subalgebra in X .

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