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Bipolar Multi-Fuzzy Subalgebra Of Bg-Algebra

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Abstract: In this paper, we introduce the notion of bipolar multi-fuzzy subalgebra of BG-algebra by combining the two concepts of bipolar fuzzy sets and multi-fuzzy sets and discuss some of its properties. Also we define the level subsets positive t-cut and negative s-cut of bipolar multi –fuzzy BG-algebra and study some of its related properties.
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I. INTRODUCTION

The notion of a fuzzy subset was initially introduced by Zadeh [8] in 1965, for representing uncertainity. In 2000, S.Sabu and T.V.Ramakrishnan [9,10] proposed the theory of multi-fuzzy sets in terms of multi-dimensional membership functions and investigated some properties of multilevel fuzziness. Theory of multi-fuzzy set is an extension of theory of fuzzy sets. Among these theories, a well-known extension of the classic fuzzy set is bipolar fuzzy set theory, which was pioneered by Zhang[11]. Bipolar fuzzy sets are an extension of fuzzy sets whose membership degree range is increased from the interval [0,1] to [-1,1]. In bipolar fuzzy set, the membership degree 0 means that elements are irrelevant to the corresponding property, the membership degrees on (0,1] denotes that elements somewhat satisfy the property and the membership degrees on [-1,0) means that elements somewhat satisfy the implicit counter-property. Although bipolar fuzzy sets and intuitionistic fuzzy sets look similar to each other, they are different sets.

Y.Imai and K.Iseki introduced two classes of abstract algebras: BCK algebras and BCI-algebras [1,2,3]. It is shown that the class of BCK-algebras is a proper subclass of the class of BCI-algebras. J.Neggers and H.S.Kim [4] introduced a new notion, called a B-algebra. In 2005, C.B.Kim and H.S.Kim [5] introduced the notion of a BG-algebra which is a generalization of B-algebras. With these ideas, fuzzy subalgebras of BG-algebra were developed by S.S.Ahn and H.D.Lee[6]. In 2015, T.Senapati[7] introduced the concepts of bipolar fuzzy subalgebra in BG-algebra. In this paper, we introduce the notion of bipolar multi-fuzzy BG-subalgebra and discuss some of its properties.

II. PRELIMINARIES

In this section we site the fundamental definitions that will be used in the sequel.

A. Definition 2.1

Let X be a non-empty set. A multi-fuzzy set A in X is defined as as a set of ordered sequences:

$A = \{ (x , \mu_1(x), \mu_2(x),......,\mu_i(x),.....) : x \in X\}$, where $\mu_i : X \to [0 , 1]$ for all i.

1) Remarks

a) If the sequences of the membership functions have only k-terms (finite number of terms), k is called the dimension of A.

b) The set of all multi-fuzzy sets in X of dimension k is denoted by $M^kFS(X)$

c) The multi-fuzzy membership function μ_A is a function from X to $[0,1]^k$ such that for all x X , $\mu_A(x) = (\mu_1(x), \mu_2(x),......,\mu_k(x))$.

d) For the sake of simplicity, we denote the multi-fuzzy set $A = \{(x, \mu_1(x), \mu_2(x),......,\mu_k(x)): x \in X\}$ as $A = (\mu_1, \mu_2,......,\mu_k)$.

B. Definition 2.2

Let k be a positive integer and let A and B in $M^kFS(X)$, where $A = (\mu_1, \mu_2,......,\mu_k)$ and $B = (\nu_1, \nu_2 ,........ \nu_k)$, then we have the following relations and operations :

1) $A \subseteq B$ if and only if $\mu_i \le \nu_i$, for all i=1,2,………..,k;

2) $A = B$ if and only if $\mu_i = \nu_i$, for all i=1,2,………..,k;

3) $A \cup B = (\mu_1 \cup \nu_1,.....\mu_k \cup \nu_k) = \{(x , \max(\mu_1(x),\nu_1(x)),...\max(\mu_k(x),\nu_k(x)): x\in X\}$

4) $A \cap B = ((\mu_1 \cap \nu_1,.....\mu_k \cap \nu_k) = \{(x , \min(\mu_1(x),\nu_1(x)),....\min(\mu_k(x),\nu_k(x)) : x\in X\}$

C. Definition 2.3

Let X be a non-empty set. A bipolar fuzzy set φ in X is an object having the form φ = { < x , $φ^+(x)$, $φ^-(x)$ > : x ∈ X } where $φ^+(x)$: X ⟶[0,1] and $φ^-(x)$: X⟶[-1,0] are the mappings.

We use the positive membership degree $φ^+(x)$ to denote the satisfaction degree of an element x to the property corresponding to a bipolar fuzzy set φ and the negative membership degree $φ^-(x)$ to denote the satisfaction degree of an element x to some implicit counter-property corresponding to a bipolar fuzzy set φ . If $φ^+(x) \neq 0$ and $φ^-(x) = 0$, it is the situation that x is regarded as having only positive satisfaction for φ. If $φ^+(x) = 0$ and $φ^-(x) \neq 0$, it is the situation that x does not satisfy the property of φ but somewhat satisfies the counter-property of φ. It is possible for an element x to be such that $φ^+(x) \neq 0$ and $φ^-(x) \neq 0$ when the membership function of the property overlaps that of its counter property over some portion of X.

D. *Definition 2.4*

Let X be a non-empty set. A bipolar multi-fuzzy set A in X is defined as an object of the form A = { < x , $A_i^+(x)$, $A_i^-(x)$ > : x ∈ X } where $A_i^+(x)$: X⟶[0,1] and $A_i^-(x)$: X⟶[-1,0]

The positive membership degree $A_i^+(x)$ denotes the satisfaction degree of an element x to the property corresponding to a bipolar fuzzy set A and the negative membership degree $A_i^-(x)$ denotes the satisfaction degree of an element x to some implicit counter-property corresponding to a bipolar fuzzy set A. If $A_i^+(x) \neq 0$ and $A_i^-(x) = 0$, it is the situation that x is regarded as having only positive satisfaction for A. If $A_i^+(x) = 0$ and $A_i^-(x) \neq 0$, it is the situation that x does not satisfy the property of A but somewhat satisfies the counter-property of A. It is possible for an element x to be such that $A_i^+(x) \neq 0$ and $A_i^-(x) \neq 0$ when the membership function of the property overlaps that of its counter property over some portion of X , where i = 1,2,…..n

E. *Example 2.5*

Let X = { l , m , n } be a set. Then A = { < l ,0.5,0.6, 0.4, -0.3,-0.5,-0.6 > , < m , 0.8 , 0.4, 0.2 , -0.5,-0.8, -0.1 > , < n , 0.3 , 0.2 , 0.1 , -0.7, -0.6,-0.2 > } is a bipolar multi-fuzzy set of X.

F. *Definition 2.6*

Non-empty set X with a constant 0 and a binary operation "∗ " is called a BG-algebra if it satisfies the following axioms:

1) $x * x = 0$

2) $x * 0 = x$

3) $(x * y) * (0 * y) = x$, ∀ x , y ∈ X.

G. *Example 2.7*

Let X = { 0, 1, 2 } be a set with the following table

∗	0	1	2
0	0	1	2
1	1	0	1
2	2	2	0

Then (X; ∗ , 0) is a BG-algebra.

H. *Definition 2.8*

Let S be a non-empty subset of a BG-algebra X, then S is called a subalgebra of X if x ∗ y ∈ S for all x , y ∈ S.

I. *Definition 2.9*

Let μ be a fuzzy set in BG-algebra . Then μ is called a fuzzy subalgebra of X if

$$\mu(x * y) \geq \min\{\mu(x), \mu(y)\}, \forall x, y \in X.$$

II. BIPOLAR MULTI-FUZZY BG-SUBALGEBRA

In this section, the concept of bipolar multi-fuzzy subalgebra of BG-algebra is defined and their related properties are presented.

A. Definition 3.1

Let A = { < x , $A_i^+(x)$, $A_i^-(x)$ > / x ∈ X } be a bipolar multi-fuzzy set in X , then the set A is bipolar multi-fuzzy BG-subalgebra over the binary operator ∗ if it satisfies the following conditions :

1) $A_i^+(x * y) \geq \min \{ A_i^+(x), A_i^+(y) \}$
2) $A_i^-(x * y) \leq \max \{ A_i^-(x), A_i^-(y) \}$

B. Example 3.2

Let X = { 0,1,2,3 } be a BG-algebra with the following cayley table

∗	0	1	2	3
0	0	1	2	3
1	1	0	3	2
2	2	3	0	1
3	3	2	1	0

Let A = { < x , $A_i^+(x)$, $A_i^-(x)$ > / x ∈ X } be a bipolar multi-fuzzy set defined by
A = { < 0 , 0.6, 0.7, 0.3, - 0.7,- 0.5, - 0.4 > , < 1, 0.5 , 0.4, 0.2 ,-0.5, -0.3, -0.2 >, < 2, 0.4, 0.3, 0.1, -0.4, -0.2, -0.1 >, < 3, 0.4, 0.3, 0.1, -0.4, -0.2, -0.1 > }. Clearly, A is a bipolar multi-fuzzy BG-subalgebra in X.

C. Preposition 3.3

If A = { < x , $A_i^+(x)$, $A_i^-(x)$ > / x ∈ X } is a bipolar multi-fuzzy subalgebra in X, then for all x ∈ X , $A_i^+(0) \geq A_i^+(x)$ and $A_i^-(0) \leq A_i^-(x)$.

1) *Proof:* Let x ∈ X .

Then $A_i^+(0) = A_i^+(x * x) \geq \min \{ A_i^+(x), A_i^+(x) \} = A_i^+(x)$

$A_i^-(0) = A_i^-(x * x) \leq \max \{ A_i^-(x), A_i^-(x) \} = A_i^-(x)$

D. Theorem 3.4

If a bipolar multi-fuzzy set A = { < x , $A_i^+(x)$, $A_i^-(x)$ > / x ∈ X } is a bipolar multi-fuzzy subalgebra , then for all x ∈ X , $A_i^+(0 * x) \geq A_i^+(x)$ and $A_i^-(0 * x) \leq A_i^-(x)$.

1) *Proof :* Let x ∈ X

Then $A_i^+(0 * x) \geq \min \{ A_i^+(0), A_i^+(x) \}$
$= \min \{ A_i^+(x * x), A_i^+(x) \}$
$\geq \min \{ \min \{ A_i^+(x), A_i^+(x) \}, A_i^+(x) \}$
$= \min \{ A_i^+(x), A_i^+(x) \}$
$= A_i^+(x)$

$A_i^-(0 * x) \leq \max \{ A_i^-(0), A_i^-(x) \}$
$= \max \{ A_i^-(x * x), A_i^-(x) \}$
$\leq \max \{ \max \{ A_i^-(x), A_i^-(x) \}, A_i^-(x) \}$
$= \max \{ A_i^-(x), A_i^-(x) \}$
$= A_i^-(x)$

E. Theorem 3.5

Let A = { < x , $A_i^+(x)$, $A_i^-(x)$ > / x ∈ X } and B = { < x , $B_i^+(x)$, $B_i^-(x)$ > / x ∈ X } be two bipolar multi-fuzzy subalgebras of X. Then A⋂B is a bipolar multi-fuzzy subalgebra in X.

1) *Proof* **:**

Let x , y ∈ A⋂B

Then x , y ∈ A and B.

$A_i^+ \cap B_i^+(x * y) = \min \{ A_i^+(x * y), B_i^+(x * y) \}$
$\geq \min \{ \min \{ A_i^+(x), A_i^+(y) \}, \min \{ B_i^+(x), B_i^+(y) \} \}$
$= \min \{ \min \{ A_i^+(x), B_i^+(x) \}, \min \{ A_i^+(y), B_i^+(y) \} \}$
$= \min \{ A_i^+ \cap B_i^+(x), A_i^+ \cap B_i^+(y) \}$

$A_i^- \cap B_i^- (x * y) = \max \{ A_i^- (x * y) , B_i^- (x * y) \}$
$\leq \max \{ \max \{ A_i^-(x) , A_i^- (y) \} , \max \{ B_i^- (x) , B_i^- (y) \} \}$
$= \max \{ \max \{ A_i^- (x) , B_i^- (x) \} , \max \{ A_i^- (y) , B_i^- (y) \} \}$
$= \max \{ A_i^- \cap B_i^- (x) , A_i^- \cap B_i^- (y) \}$

F. Preposition 3.6

The union of any set of bipolar multi-fuzzy subalgebras need not be a bipolar multi-fuzzy subalgebra .

G. Theorem 3.7

If A = < A_i^+ , A_i^- > is a bipolar multi-fuzzy subalgebra of X , then H = { x ∈ X / $A_i^+(x) = 1$, $A_i^-(x) = -1$ } is either empty or a subalgebra of X.

1) Proof : If no element satisfies the conditions of H , then the set H is empty. If x and y ∈ H then $A_i^+(x) = 1$, $A_i^-(x) = -1$, $A_i^+(y) = 1$, $A_i^-(y) = -1$.

Since A is a bipolar multi-fuzzy subalgebra of X , $A_i^+ (x * y) \geq \min \{ A_i^+ (x) , A_i^+ (y) \} = \min \{ 1 ,1 \} = 1$ and also $A_i^+ (x * y) \leq 1$

Therefore $A_i^+ (x * y) = 1$

$A_i^- (x * y) \leq \max \{ A_i^- (x) , A_i^-(y) \} = \max \{ -1 ,-1 \} = -1$ and also $A_i^- (x * y) \geq -1$

Therefore $A_i^- (x * y) = -1$

Hence $x * y \in H$

Therefore H is a subalgebra of X.

H. Theorem 3.8

Let A = < A_i^+ , A_i^- > be a bipolar multi-fuzzy subalgebra of X.

If $A_i^+ (x * y) = 0$ then either $A_i^+ (x) = 0$ or $A_i^+ (y) = 0$ for x and y ∈ X

If $A_i^- (x * y) = 0$ then either $A_i^- (x) = 0$ or $A_i^- (y) = 0$ for x and y ∈ X

1) Proof : Let x , y ∈ X . Then $A_i^+ (x * y) \geq \min \{ A_i^+ (x) , A_i^+ (y) \}$ i.e., $0 \geq \min \{ A_i^+ (x) , A_i^+ (y) \}$

This implies that either $A_i^+ (x) = 0$ or $A_i^+ (y) = 0 A_i^- (x * y) \leq \max \{ A_i^- (x) , A_i^- (y) \}$ i.e., $0 \leq \max \{ A_i^- (x) , A_i^- (y) \}$

This implies that either $A_i^- (x) = 0$ or $A_i^- (y) = 0$

I. Theorem 3.9

If A = < A_i^+ , A_i^- > be a bipolar multi-fuzzy subalgebra of X , then the set H = { x ∈ X / $A_i^+(x) = A_i^+ (0)$ and $A_i^-(x) = A_i^-(0)$ } is a subalgebra of X.

*1) Proof :*Let x , y ∈ H

Then $A_i^+(x) = A_i^+(y) = A_i^+ (0)$ and $A_i^-(x) = A_i^-(y) = A_i^-(0)$

$A_i^+ (x * y) \geq \min \{ A_i^+ (x) , A_i^+ (y) \} = \min \{ A_i^+ (0) , A_i^+ (0) \} = A_i^+ (0)$

Also $A_i^+ (x * y) \leq A_i^+ (0)$

Therefore $A_i^+ (x * y) = A_i^+ (0)$

And $A_i^- (x * y) \leq \max \{ A_i^- (x) , A_i^- (y) \} = \max \{ A_i^- (0) , A_i^- (0) \} = A_i^- (0)$

Also $A_i^- (x * y) \geq A_i^- (0)$

This implies that $A_i^- (x * y) = A_i^- (0)$

Therefore $x * y \in H$

H is a subalgebra of X.

III. LEVEL SUBSETS OF A BIPOLAR MULTI-FUZZY SET

In this section, the positive t-cut and negative s-cut of a bipolar multi-fuzzy set is defined and some properties are discussed.

A. Definition 4.1

Let A = { < x , $A_i^+ (x)$, $A_i^- (x)$ > / x ∈ X } be a bipolar multi-fuzzy subalgebra of X. For s ∈ [-1,0] and t ∈ [0,1] , the set U (A_i^+ ; t) = { x ∈ X ; $A_i^+(x) \geq t$ } is called positive t-cut of A and the set L(A_i^- ; t) = { x ∈ X ; $A_i^-(x) \leq s$ } is called negative s-cut of A.

B. Theorem 4.2

If A = < A_i^+ , A_i^- > is a bipolar multi-fuzzy subalgebra of X , then the positive t-cut and negative s-cut of A are subalgebras of X.

1) *Proof :* Let $x , y \in U(A_i^+ ; t)$

Then $A_i^+(x) \geq t$ and $A_i^+(y) \geq t$

$A_i^+(x * y) \geq \min\{A_i^+(x), A_i^+(y)\} \geq \min\{t, t\} = t$

Therefore $x * y \in U(A_i^+ ; t)$

Hence $U(A_i^+ ; t)$ is a subalgebra in X.

Let $x , y \in L(A_i^- ; s)$

Then $A_i^-(x) \leq s$ and $A_i^-(y) \leq s$

$A_i^-(x * y) \leq \max\{A_i^-(x), A_i^-(y)\} \leq \max\{s, s\} = s$

Therefore $x * y \in L(A_i^- ; s)$

Hence $L(A_i^- ; s)$ is a subalgebra in X.

C. Theorem 4.3

Let $A = \langle A_i^+, A_i^- \rangle$ be a multi-fuzzy set in X, such that the level sets $U(A_i^+ ; t)$ and $L(A_i^- ; s)$ are subalgebras of X for every $s \in [-1,0]$ and $t \in [0,1]$. Then A is a bipolar multi-fuzzy subalgebra in X.

1) *Proof :* Let $A = \langle A_i^+, A_i^- \rangle$ be a multi-fuzzy set in X, such that the level sets $U(A_i^+ ; t)$ and $L(A_i^- ; s)$ are subalgebras of X for every $s \in [-1,0]$ and $t \in [0,1]$.

In contrary , let $x_0 , y_0 \in X$ be such that $A_i^+(x_0 * y_0) < \min\{A_i^+(x_0), A_i^+(y_0)\}$ and $A_i^-(x_0 * y_0) > \max\{A_i^-(x_0), A_i^-(y_0)\}$

Let $A_i^+(x_0) =$, $A_i^+(y_0) = \beta$, $A_i^-(x_0) = \gamma$, $A_i^-(y_0) = \delta$, $A_i^+(x_0 * y_0) = t$, $A_i^-(x_0 * y_0) = s$

Then $t < \min\{ \ , \beta\}$ and $s > \max\{\gamma, \delta\}$

Put $t_1 = \frac{1}{2}[A_i^+(x_0 * y_0) + \min\{A_i^+(x_0), A_i^+(y_0)\}]$

and $s_1 = \frac{1}{2}[A_i^-(x_0 * y_0) + \max\{A_i^-(x_0), A_i^-(y_0)\}]$

This implies, $t_1 = \frac{1}{2}[t + \min\{\alpha, \beta\}]$ and $s_1 = \frac{1}{2}[s + \max\{\gamma, \delta\}]$

Hence $\alpha > t_1 = \frac{1}{2}[t + \min\{\alpha, \beta\}] > t$, $\beta > t_1 = \frac{1}{2}[t + \min\{\alpha, \beta\}] > t$

and $\gamma < s_1 = \frac{1}{2}[s + \max\{\gamma, \delta\}] < s$, $\delta < s_1 = \frac{1}{2}[s + \max\{\gamma, \delta\}] < s$

$\Rightarrow \min\{\alpha, \beta\} > t_1 > t = A_i^+(x_0 * y_0)$ and $\max\{\gamma, \delta\} < s_1 < s = A_i^-(x_0 * y_0)$

So that $x_0 * y_0 \notin U(A_i^+ ; t)$ and $x_0 * y_0 \notin L(A_i^- ; s)$ which is a contradiction , since

$A_i^+(x_0) = \alpha \geq \min\{\alpha,\beta\} > t_1$, $A_i^+(y_0) = \beta \geq \min\{\alpha,\beta\} > t_1$ and $A_i^-(x_0) = \gamma \leq \max\{\gamma,\delta\} < s_1$, $A_i^-(y_0) = \delta \leq \max\{\gamma, \delta\} < s_1$

This implies that $x_0 , y_0 \in U(A_i^+ ; t)$ and $x_0 , y_0 \in L(A_i^- ; s)$

Thus $A_i^+(x * y) \geq \min\{A_i^+(x), A_i^+(y)\}$ and $A_i^-(x * y) \leq \max\{A_i^-(x), A_i^-(y)\}$,for $x,y \in X$.

Hence A is a bipolar multi-fuzzy subalgebra of X.

D. Theorem 4.4

Any BG-subalgebra of X can be realized as both the positive t-cut and negative s-cut of some bipolar multi-fuzzy subalgebra in X.

1) *Proof :* Let S be a subalgebra of a BG-algebra X and $A = \langle A_i^+, A_i^- \rangle$ be a bipolar multi-fuzzy set in X defined by

$$A_i^+(x) = \begin{cases} \lambda_i & , \text{if } x \in S \\ 0 & , \text{otherwise} \end{cases} \qquad A_i^-(x) = \begin{cases} \tau_i & , \text{if } x \in S \\ 0 & , \text{otherwise} \end{cases} \quad \text{for all } \lambda_i \in [0,1] , \tau_i \in [-1,0]$$

We consider the following four cases:

a) *Case(i) :* If $x , y \in S$, then $A_i^+(x) = \lambda_i$, $A_i^+(y) = \lambda_i$, $A_i^-(x) = \tau_i$, $A_i^-(y) = \tau_i$

Since S is a subalgebra of X , $x * y \in S$

$A_i^+(x * y) = \lambda_i = \min\{\lambda_i, \lambda_i\} = \min\{A_i^+(x), A_i^+(y)\}$ and

$A_i^-(x * y) = \tau_i = \max\{\tau_i, \tau_i\} = \max\{A_i^-(x), A_i^-(y)\}$

b) *Case (ii)* **:** If $x \in S$ and $y \notin S$, then $A_i^+(x) = \lambda_i$, $A_i^+(y) = 0$, $A_i^-(x) = \tau_i$, $A_i^-(y) = 0$

This implies that either $x * y \in S$ or $\notin S$.

$A_i^+(x * y) \geq 0 = \min\{\lambda_i, 0\} = \min\{A_i^+(x), A_i^+(y)\}$ and

$A_i^-(x * y) \leq 0 = \max\{\tau_i, 0\} = \max\{A_i^-(x), A_i^-(y)\}$

c) *Case (iii) :* If $x \notin S$ and $y \in S$, then $A_i^+(x) = 0$, $A_i^+(y) = \lambda_i$, $A_i^-(x) = 0$, $A_i^-(y) = \tau_i$

This implies that either $x * y \in S$ or $\notin S$.

$A_i^+(x * y) \geq 0 = \min\{0, \lambda_i\} = \min\{A_i^+(x), A_i^+(y)\}$ and

$A_i^-(x * y) \leq 0 = \max \{ 0 , \tau_i \} = \max \{ A_i^-(x) , A_i^-(y) \}$

d) Case (iv) : $x \notin S$ and $y \notin S$, then $A_i^+(x) = 0$, $A_i^+(y) = 0$, $A_i^-(x) = 0$, $A_i^-(y) = 0$

This implies that either $x * y \in S$ or $\notin S$.

$A_i^+(x * y) \geq 0 = \min \{ 0 , 0 \} = \min \{ A_i^+(x) , A_i^+(y)$ and

$A_i^-(x * y) \leq 0 = \max \{ 0 , 0 \} = \max \{ A_i^-(x) , A_i^-(y) \}$

Thus, in all the cases, $A = < A_i^+ , A_i^- >$ is bipolar multi-fuzzy subalgebra in X.

REFERENCES

[1] Y.Imai and K.Iseki , On axiom system of propositional calculi, 15 Proc,Japan Academy,42(1966), pp: 19-22.
[2] K.Iseki and S.Tanaka, An introduction to theory of BCK- algebras, Math. Japonica, 23 (1978), pp: 1-26.
[3] K.Iseki, On BCI-algebras, Math. Seminor Notes 8 (1980), pp:125-130.
[4] J.Neggers and H.S.Kim, On B-algebras, Mat. Vesnik 54 (2002), 21-29.
[5] C.B.Kim and H.S.Kim, On BG-algebras, Demonstratio Mathematica, 41(2008), pp:497-505.
[6] S.S.Ahn and D.Lee , Fuzzy subalgebras of BG algebras, Commun. Korean Math.Soc, 19 (2004) , No 2, pp: 243-251.
[7] T.Senapati , Bipolar Fuzzy Structure of BG-subalgebras , The Journal of Fuzzy Mathematics , Vol.23 , No.1, (2015) , pp: 209-220
[8] L.A.Zadeh, Fuzzy Sets , Information and Control, 8(1965), pp: 338-353
[9] S.Sabu and T.V.Ramakrishnan, Multi-fuzzy sets, International Mathematical Forum, 50(2010) ,pp: 2471-2476.
[10] S.Sabu and T.V.Ramakrishnan, Multi-fuzzy Topology, International Journal of Applied Mathematics, Vol 24, No.1(2011), pp:117-129.
[11] W.R.Zhang , Bipolar fuzzy sets and relations, A computational framework for cognitive modeling and multiagent analysis, Proceedings of IEEE Conf., (1994) ,pp:305-309.
[12] K.M.Lee , Bipolar-valued fuzzy sets and their operations , Proc.Int.Conf. On Intelligent Technologies, Bangkok, Thailand, (2000) , pp: 307-312
[13] W.R.Zhang and L.Zhang, Yin Yang bipolar logic and bipolar fuzzy logic, Inform.Sci., 165(3-4), (2004), pp: 265-287
[14] K.J.Lee, Bipolar fuzzy subalgebras and bipolar fuzzy ideals of BCK/BCI-algebras, Bull Malays. Math.Sci.Soc.,32(3) (2009), pp:361-373.