Common Fixed-Point Theorems for Compatible Mappings of Type (E) in $\mathcal{M}$- Fuzzy Metric Spaces

M. Jeyaraman [1,] S. Sowndrarajan [2]

[1,2] *PG and Research Department of Mathematics, Raja Doraisingam Govt. Arts College, Sivaganga - 630561, Tamil Nadu,*

Abstract: The aim of this paper is to obtain common fixed point theorems for self mappings in complete $\mathcal{M}$ - fuzzy metric spaces by compatible of type (E). Our result generalizes and improves other similar results in Manandhar, Jha and Pathak [6].

Keywords: Common fixed point theorem, Generalized fuzzy metric spaces, Compatible mappings.

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I. INTRODUCTION

In 1965, the concept of fuzzy set was introduced by Zadeh [15]. Then in 1975, Kramosil and Michalek [5] introduced the fuzzy metric space as a generalization of a metric space. In 1994, George and Veeramani [1] modified the notion of fuzzy metric spaces with the help of continuous t-norms. In 1986, Jungck [2] introduced notion of compatible mappings in metric spaces. In 2000, Singh and Chouhan [11] introduced the concept of compatible mappings in fuzzy metric spaces. In 1993, Jungck, Murthy and Cho [3] gave a generalization of compatible mappings called compatible mappings of type (A) which is equivalent to the concept of compatible mappings under some conditions. In 1994, Pathak, Cho, Chang and Kang [8] introduced the concept of compatible mappings of type (P) and compared with compatible mappings of type (A) and compatible mappings. In 1998, Pant [7] introduced the notion of reciprocal continuity of mappings in metric spaces. In 1999, Vasuki [13] introduced the notion point wise R - weakly commuting mappings in $\mathcal{M}$ - fuzzy metric spaces. Recently, in 2007, Singh and Singh [10] introduced the concept of compatible mappings of type (E) in metric spaces. Since then, many authors have obtained fixed point theorems in $\mathcal{M}$ - fuzzy metric spaces using these compatible notions. The purpose of this paper is to establish common fixed point theorems for compatible mappings of type (E) in $\mathcal{M}$ - fuzzy metric spaces with an example.

II. PRELIMINARIES

A. *Definition 2.1.*

A binary operation $*$: [0, 1] x [0,1] → [0, 1] is a continuous t – norm if it satisfies the following conditions

1) $*$ is associative and commutative
2) $*$ is continuous
3) $a * 1 = a$, for all $a \in [0, 1]$
4) $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$, for each $a, b, c, d \in [0, 1]$

Examples for continuous t – norm are $a * b = \min\{a, b\}$ and $a * b = ab$

B. *Definition 2.2.*

A 3 – tuple $(X, \mathcal{M}, *)$ is called $\mathcal{M}$ fuzzy metric space if X is an arbitrary non – empty set, $*$ is a continuous t – norm, and $\mathcal{M}$ is a fuzzy set on $X^3 \times (0, \infty)$, satisfying the following conditions for each x, y, z, a ∈ X and t, s > 0

(FM – 1) $\mathcal{M}(x, y, z, t) > 0$,

(FM – 2) $\mathcal{M}(x, y, z, t) = 1$ if $x = y = z$,

(FM – 3) $\mathcal{M}(x, y, z, t) = \mathcal{M}(p\{x, y, z\}, t)$, where p is a permutation function,

(FM – 4) $\mathcal{M}(x, y, a, t) * \mathcal{M}(a, z, z, s) \leq \mathcal{M}(x, y, z, t+s)$,

(FM – 5) $\mathcal{M}(x, y, z, .): (0, \infty) \to [0, 1]$ is continuous,

(FM – 6) $\lim_{t\to\infty}\mathcal{M}(x, y, z, t) = 1$.

C. *Example 2.3.*

Let X be a non- empty set and D* is the D* - metric on X. Denote a $*$ b = a.b for all a, b ∈ [0, 1]. For each t ∈ (0, ∞), define $\mathcal{M}$(x, y, z, t) = $\frac{t}{t+D^*(x,y,z)}$
for all x, y, z ∈ X, then (X, $\mathcal{M}$, $*$) is a $\mathcal{M}$- fuzzy metric space. We call this $\mathcal{M}$- fuzzy metric induced by D* - metric space. Thus every D* - metric induces a $\mathcal{M}$- fuzzy metric.

D. *Lemma 2.4.*
Let (X, $\mathcal{M}$, $*$) be a $\mathcal{M}$- fuzzy metric space. Then for every t > 0 and for every
x, y ∈ X. We have $\mathcal{M}$(x, x, y, t) = $\mathcal{M}$(x, y, y, t).

E. *Lemma 2.5.*
Let (X, $\mathcal{M}$, $*$) be a $\mathcal{M}$ - fuzzy metric space. Then $\mathcal{M}$(x, y, z, t) is non - decreasing with respect to t, for all x, y, z in X.

F. *Definition 2.6.*
Let (X, $\mathcal{M}$, *) be a $\mathcal{M}$ - fuzzy metric space and $\{x_n\}$ be a sequence in X.
1) $\{x_n\}$ is said to be converges to a point x ∈ X if $\lim_{n\to\infty}\mathcal{M}(x, x, x_n, t) = 1$ for all t > 0
2) $\{x_n\}$ is called Cauchy sequence, if $\lim_{n\to\infty} \mathcal{M}(x_{n+p}, x_{n+p}, x_n, t) = 1$ for all t > 0 and p > 0.
3) A $\mathcal{M}$- fuzzy metric space in which every Cauchy sequence is convergent is said to be complete.

G. *Definition 2.7.*
Let S and T be two self mappings of a $\mathcal{M}$ – fuzzy metric space (X, $\mathcal{M}$, $*$).
Then the mappings S and T are said to be weakly compatible if they commute at their coincidence points, that is, Sx = Tx for some x ∈ X, then STx = TSx.

H. *Definition 2.8.*
The self mappings A and S of a $\mathcal{M}$-fuzzy metric space (X, $\mathcal{M}$, $*$) are called point wise R- weakly commuting if there exists R > 0 such that
$\mathcal{M}$(ASx, SAx, SAx, t) ≥ $\mathcal{M}$ (Ax, Sx, Sx, t/R) for all x in X and t > 0.

I. *Definition 2.9.*
The self mappings A and S of a metric space (X, d) are said to be compatible of type (E), if $\lim_{n\to\infty} AAx_n = \lim_{n\to\infty} ASx_n = S(t)$, whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} Ax_n = t$ and $\lim_{n\to\infty} Sx_n = t$, for some t ∈ X.

J. *Definition 2.10.*
A self mappings A and S of a $\mathcal{M}$ - fuzzy metric space(X, $\mathcal{M}$, $*$) are said to be compatible of type (E) iff $\lim_{n\to\infty} \mathcal{M}(AAx_n, ASx_n, ASx_n, t) = 1$,
$\lim_{n\to\infty}\mathcal{M}(AAx_n, Sx_n, Sx_n, t) = 1$, $\lim_{n\to\infty} \mathcal{M}(ASx_n, Sx_n, Sx_n, t) = 1$ and
$\lim_{n\to\infty}\mathcal{M}(SSx_n, SAx_n, SAx_n, t) = 1$, $\lim_{n\to\infty}\mathcal{M}(SSx_n, Ax_n, Ax_n, t) = 1$,
$\lim_{n\to\infty}\mathcal{M}(Sx_n, Ax_n, Ax_n, t) = 1$, whenever $\{x_n\}$ is a sequence in X such that
$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = x$ for some x in X and t > 0.

K. *Lemma 2.11.*
Let $\{y_n\}$ be a sequence in a $\mathcal{M}$- fuzzy metric space (X, $\mathcal{M}$, $*$) with the condition (FM- 6). If there exists k ∈ (0, 1) such that $\mathcal{M}(y_n, y_{n+1}, y_{n+1}, kt) \geq \mathcal{M}(y_{n-1}, y_n, y_n, t)$ for all t > 0 and n ∈ N, then $\{y_n\}$ is a Cauchy sequence in X.
We need the following proposition for the proof of our main result.

L. *Proposition 2.12*
If A and S are compatible mappings of type (E) on a $\mathcal{M}$- fuzzy metric space
(X, $\mathcal{M}$, $*$) and if one of the function is continuous. Then, we have

$A(x) = S(x)$ and $\lim_{n\to\infty} AAx_n = \lim_{n\to\infty} SSx_n = \lim_{n\to\infty} ASx_n = \lim_{n\to\infty} SAx_n$

If these exist $u \in X$ such that $Au = Su = x$ then $ASu = SAu$.

Whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = x$ for some x in X.

1) *Proof :* Let $\{x_n\}$ be a sequence of X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = x$ for some x in X.

Then by definition of compatible of type (E), we have $\lim_{n\to\infty} AAx_n = ASx_n = S(x)$.

If A is a continuous mapping, then we get

$\lim_{n\to\infty} AAx_n = A(\lim_{n\to\infty} Ax_n) = A(x)$. This implies $A(x) = S(x)$. Also

$\lim_{n\to\infty} AAx_n = \lim_{n\to\infty} SSx_n = \lim_{n\to\infty} ASx_n = \lim_{n\to\infty} SAx_n$.

Similarly, if S is continuous then, we get the same result. This is the proof of part (i). Again, suppose $Au = Su = x$ for some $u \in X$. Then, $ASu = A(Su) = Ax$ and

$SAu = S(Au) = Sx$. From (i), we have $Ax = Sx$. Hence, $ASu = SAu$.

This is the proof of part (ii).

III. MAIN RESULTS

If P, Q, S, T, A and B are self mappings in $\mathcal{M}$ - fuzzy metric space $(X, \mathcal{M}, *)$. We denote

$$\mathcal{M}_\alpha(x, y, y, t) = \mathcal{M}(STx, Px, Px, t) * \mathcal{M}(ABy, Qy, Qy, t) * \mathcal{M}(STx, ABy, ABy, t)$$
$$* \mathcal{M}(ABy, Px, Px, \alpha t) * \mathcal{M}(STx, Qy, Qy, (2-\alpha)t),$$

for all $x, y \in X$, $\alpha \in (0, 2)$ and $t > 0$.

A. *Theorem 3.1*

Let $(X, \mathcal{M}, *)$ be a complete $\mathcal{M}$- fuzzy metric space with $a * a \geq a$ for all $a \in [0, 1]$ and with the condition (FM -6). Let one of the mapping of self mappings (P, ST) and (Q, AB) of X be continuous such that

$PX \subset ABX$, $QX \subset STX$;

There exists $k \in (0,1)$ such that $\mathcal{M}(Px, Qy, Qy, kt) \geq \mathcal{M}_\alpha(x, y, y, t)$ for all $x, y \in X$, $\alpha \in (0, 2)$ and $t > 0$.

If (P, ST) and (Q, AB) compatible of type of (E) then P, Q, ST and AB have a unique common fixed point. If the pair (A, B), (S, T), (Q, B) and (T, P) are commuting mappings then A, B, S, T, P and Q have a unique common fixed point.

1) *Proof:* Let x_0 be any point in X. From the condition (i) there exists $x_1, x_2 \in X$ such that $Px_0 = ABx_1 = y_0$ and $Qx_1 = STx_2 = y_1$. Inductively, we can construct sequences $\{x_n\}$ and $\{y_n\}$ in X such that $Px_{2n} = ABx_{2n+1} = y_{2n}$ and $Qx_{2n+1} = STx_{2n+2} = y_{2n+1}$ for $n = 0, 1, 2 \ldots$ for $t > 0$ and $\alpha = 1 - q$ with $q \in (0, 1)$ in (ii) then, we have

$$\mathcal{M}(Px_{2n}, Qx_{2n+1}, Qx_{2n+1}, kt) \geq \{ \mathcal{M}((STx_{2n}, Px_{2n}, Px_{2n}, t)$$
$$* \mathcal{M}(ABx_{2n+1}, Qx_{2n+1}, Qx_{2n+1}, t)$$
$$* \mathcal{M}(STx_{2n}, ABx_{2n+1}, ABx_{2n+1}, t)$$
$$* \mathcal{M}(ABx_{2n+1}, Px_{2n}, Px_{2n}, (1-q) t)$$
$$* \mathcal{M}(STx_{2n}, Qx_{2n+1}, Qx_{2n+1}, (1+q) t)\}$$
$$\mathcal{M}(y_{2n}, y_{2n+1}, y_{2n+1}, kt) \geq \{\mathcal{M}(y_{2n-1}, y_{2n}, y_{2n}, t) * \mathcal{M}(y_{2n}, y_{2n+1}, y_{2n+1}, t)$$
$$* \mathcal{M}(y_{2n-1}, y_{2n}, y_{2n}, t)$$
$$* \mathcal{M}(y_{2n}, y_{2n}, y_{2n}, (1-q)t) * \mathcal{M}(y_{2n-1}, y_{2n+1}, y_{2n+1}, (1+q) t)\}$$
$$\geq\{ \mathcal{M}(y_{2n-1}, y_{2n}, y_{2n}, t) * \mathcal{M}(y_{2n}, y_{2n+1}, y_{2n+1}, t)$$
$$* \mathcal{M}(y_{2n-1}, y_{2n}, y_{2n}, t) * \mathcal{M}(y_{2n}, y_{2n+1}, y_{2n+1}, qt)\},$$
$$\geq\{ \mathcal{M}(y_{2n-1}, y_{2n}, y_{2n}, t) * \mathcal{M}(y_{2n}, y_{2n+1}, y_{2n+1}, t)$$
$$* \mathcal{M}(y_{2n}, y_{2n+1}, y_{2n+1}, qt)\}.$$

Since t - norm $*$ is continuous, letting $q \to 1$, we have

$$\mathcal{M}(y_{2n}, y_{2n+1}, y_{2n+1}, kt) \geq \mathcal{M}(y_{2n-1}, y_{2n}, y_{2n}, t) * \mathcal{M}(y_{2n}, y_{2n+1}, y_{2n+1}, t)$$
$$* \mathcal{M}(y_{2n}, y_{2n+1}, y_{2n+1}, t)$$
$$\geq \mathcal{M}(y_{2n-1}, y_{2n}, y_{2n}, t) * \mathcal{M}(y_{2n}, y_{2n+1}, y_{2n+1}, t).$$

It follows that $\mathcal{M}(y_{2n}, y_{2n+1}, y_{2n+1}, k t) \geq \mathcal{M}(y_{2n-1}, y_{2n}, y_{2n}, t) * \mathcal{M}(y_{2n}, y_{2n+1}, y_{2n+1}, t)$.

Similarly, $\mathcal{M}(y_{2n+1}, y_{2n+2}, y_{2n+2}, kt) \geq \mathcal{M}(y_{2n}, y_{2n+1}, y_{2n+1}, t) * \mathcal{M}(y_{2n+1}, y_{2n+2}, y_{2n+2}, t)$.

Therefore, for all n even or add, we have $\mathcal{M}(y_n, y_{n+1}, y_{n+1}, kt) \geq \mathcal{M}(y_{n-1}, y_n, y_n, t) * \mathcal{M}(y_n, y_{n+1}, y_{n+1}, t)$.

Consequently, $\mathcal{M}(y_n, y_{n+1}, y_{n+1}, t) \geq \mathcal{M}(y_{n-1}, y_n, y_n, k^{-1} t) * \mathcal{M}(y_n, y_{n+1}, y_{n+1}, k^{-1} t)$ and hence $\mathcal{M}(y_n, y_{n+1}, y_{n+1}, t) \geq \mathcal{M}(y_{n-1}, y_n, y_n, t) * \mathcal{M}(y_n, y_{n+1}, y_{n+1}, k^{-1}t)$.

Since $\mathcal{M}(y_n, y_{n+1}, y_{n+1}, k^{-m} t) \to 1$ as $k \to 0$, it follows that

$\mathcal{M}(y_n, y_{n+1}, y_{n+1}, kt) \geq \mathcal{M}(y_{n-1}, y_n, y_n, t)$ for all $n \in N$ and $t > 0$.

Therefore, by lemma (2.11), $\{y_n\}$ is a Cauchy sequence.

Since X is complete, then there exists a point z in X such that $y_n \to z$ as $n \to \infty$.

Moreover, we have $y_{2n} = Px_{2n} = ABx_{2n+1} \to z$ and $y_{2n+1} = Qx_{2n+1} = STx_{2n+2} \to z$.

If P and ST are compatible of type(E) and one of mapping of the pair (P, ST) is continuous then by proposition (2.12), we have Pz = STz.

Since $P(X) \subset AB(X)$, there exists a point w in X such that Pz = ABw.

Using condition (ii), with $\alpha = 1$, we have

$$
\begin{aligned}
\mathcal{M}(Pz, Qw, Qw, kt) &\geq \{ \mathcal{M}(STz, Pz, Pz, t) * \mathcal{M}(ABw, Qw, Qw, t) \\
&\quad * \mathcal{M}(STz, ABw, ABw, t) \\
&\quad * \mathcal{M}(ABw, Pz, Pz, t) * \mathcal{M}(STz, Qw, Qw, t)\}. \\
&= \{\mathcal{M}(Pz, Pz, Pz, t) * \mathcal{M}(Pz, Qw, Qw, t) \\
&\quad * \mathcal{M}(Pz, Pz, Pz, t) * \mathcal{M}(Pz, Pz, Pz, t) * \mathcal{M}(Pz, Qw, Qw, t)\} \\
&\geq \mathcal{M}(pz, Qw, Qw, t).
\end{aligned}
$$

This implies Pz = Qw. Thus, we have Pz = STz = Qw = ABw. Also, we get

$$
\begin{aligned}
\mathcal{M}(Pz, Qx_{2n+1}, Qx_{2n+1}, kt) &\geq \{\mathcal{M}(STz, Pz, Pz, t) * \mathcal{M}(ABx_{2n+1}, Qx_{2n+1}, Qx_{2n+1}, t) \\
&\quad * \mathcal{M}(STz, ABx_{2n+1}, ABx_{2n+1}, t) * \mathcal{M}(ABx_{2n+1}, Pz, Pz, t) \\
&\quad * \mathcal{M}(STz, Qx_{2n+1}, Qx_{2n+1}, t)\}.
\end{aligned}
$$

Letting $n \to \infty$, we get

$$
\begin{aligned}
\mathcal{M}(Pz, z, z, kt) &\geq \{\mathcal{M}(Pz, Pz, Pz, t) * \mathcal{M}(Pz, z, z, t) * \mathcal{M}(Pz, Pz, Pz, t) \\
&\quad * \mathcal{M}(Pz, Pz, Pz, t) * \mathcal{M}(Pz, z, z, t)\} \\
&\geq \mathcal{M}(Pz, z, z, t).
\end{aligned}
$$

Hence, we get STz = Pz = z. Therefore, z is a common fixed point of P and ST.

Again, if Q and AB are compatible with the type (E) and one of the mapping of (Q, AB) is continuous. So, we get Qw = ABw = Pz =z.

By using proposition (2.12), we get QQw = QABw = ABQw = ABABw.

Thus, we get Qz = ABz . Also, using condition (ii) with $\alpha = 1$. We have,

$$
\begin{aligned}
\mathcal{M}(Px_{2n}, Qz, Qz, kt) &\geq \{\mathcal{M}(STx_{2n}, Px_{2n}, Px_{2n}, t) * \mathcal{M}(ABz, Qz, Qz, t) \\
&\quad * \mathcal{M}(STx_{2n}, ABz, ABz, t) * \mathcal{M}(ABz, Px_{2n}, Px_{2n}, t) \\
&\quad * \mathcal{M}(STx_{2n}, Qz, Qz, t)\}.
\end{aligned}
$$

Letting $n \to \infty$, we get

$$
\begin{aligned}
\mathcal{M}(z, Qz, Qz, kt) &\geq \{\mathcal{M}(z, z, z, t) * \mathcal{M}(ABz, Qz, Qz, t) * \mathcal{M}(z, ABz, ABz, t) \\
&\quad * \mathcal{M}(ABz, z, z, t) * \mathcal{M}(z, Qz, Qz, t)\} \\
&\geq \mathcal{M}(z, Qz, Qz, t).
\end{aligned}
$$

Hence, we have Qz = ABz = z. Therefore z is a common fixed point of Q and AB.

Hence z is a common fixed point of P, Q, ST and AB.

For uniqueness, suppose that ($Pw \neq Pz = z$) is another common fixed point of P, Q, ST and AB. Then, using condition (ii) with $\alpha = 1$. We have,

$$
\begin{aligned}
\mathcal{M}(PPz, QPw, QPw, kt) &= \mathcal{M}(Pz, Pw, Pw, kt) \\
&\geq \{ \mathcal{M}(STPz, PPz, PPz, t) * \mathcal{M}(ABPw, QPw, QPw, t) \\
&\quad * \mathcal{M}(STPz, ABPw, ABPw, t) * \mathcal{M}(ABPw, PPz, PPz, t) \\
&\quad * \mathcal{M}(STPz, QPw, QPw, t)\} \\
&= \mathcal{M}(Pz, Pz, Pz, t) * \mathcal{M}(Pw, Pw, Pw, t) \\
&\quad * \mathcal{M}(Pz, Pw, Pw, t) * \mathcal{M}(Pw, Pz, Pz, t) * \mathcal{M}(Pz, Pw, Pw, t)
\end{aligned}
$$

$\mathcal{M}(PPz, QPw, QPw, kt) \geq \mathcal{M}(Pz, Pw, Pw, t)$.

That is, Pw = Pz =z. Thus, z is a unique common fixed point of P, Q, ST and AB by using the commutatively of the pairs (A, B), (S, T), (Q, B) and (T, P) we can easily prove that z is a unique common fixed point of A, B, S, T, P and Q.

If we take $T = B = I_x$, an identity mapping of X.

B. Corollary 3.2

Let (X, $\mathcal{M}$, $*$) be a complete $\mathcal{M}$ - fuzzy metric space with a $*$ a $\geq$ a for all a $\in$ [0, 1] and with the condition (FM-6). If one of the mapping of self mappings (P, Q) and (Q, A) of X is continuous such that for k $\in$ (0, 1) we have

$\mathcal{M}$(Px, Qy, Qy, kt) $\geq \mathcal{M}_\alpha$(x, y, y, t) for all x, y $\in$ X, $\alpha \in$ (0, 2) and t > 0, and if (P, S) and (Q, A) are compatible of type of (E)

$$\mathcal{M}((x, y, y, t) = \mathcal{M}(Sx, Px, Px, t) * \mathcal{M}(Ay, Qy, Qy, t) * \mathcal{M}(Sx, Ay, Ay, t) * \mathcal{M}(Ay, Px, Px, \alpha t) * \mathcal{M}(Sx, Qy, Qy, (2-\alpha) t),$$

for all x, y $\in$X, $\alpha \in$ (0, 2) and t > 0.

Similarly if we get the result for three self maps by taking S = A, T = B = I_x in the Theorem (3.1) and also by taking P = Q, T = B = I_x in Theorem (3.1) and obtain for two self maps by taking P = Q, A = S, B = T= I_x in Theorem (3.1) then P, A, S, Q have a unique common fixed point.

C. Example 3.3

Let X = [2, 10] with the metric d defined by

D*(x, y, z) = |x – y| +|y – z| + |z –x| and define $\mathcal{M}(x, y, z, t) = \frac{t}{t+D^*(x,y,z)}$

for all x, y, z $\in$ X, t > 0. Clearly (X, $\mathcal{M}$, $*$) is a complete $\mathcal{M}$- fuzzy metric space. Define P, Q, S, T, A and B: X→ X as follows;

Px = 2 for all x, Qx = 2 if x < 4 and x $\geq$ 5, Qx = 3+x if 4 $\leq$ x < 5

Sx = 2 if x $\leq$ 8, Sx = 8 if x > 8;

Ax = 2 if x < 4 or x $\geq$ 5, Ax = 5 + x if 4 $\leq$ x < 5

Bx = Tx= x , for all x $\in$ [2, 10] .

Then P, Q, S, T, A and B satisfy all the conditions of the above theorem and have a unique common fixed point x = 2.

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