



# IJRASET

International Journal For Research in  
Applied Science and Engineering Technology



---

# INTERNATIONAL JOURNAL FOR RESEARCH

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY

---

**Volume:** 2017 **Issue:** conference **Month of publication:** December 2017

**DOI:**

[www.ijraset.com](http://www.ijraset.com)

Call:  08813907089

E-mail ID: [ijraset@gmail.com](mailto:ijraset@gmail.com)

# Common Fixed-Point Theorems for Compatible Mappings of Type (E) in $\mathcal{M}$ - Fuzzy Metric Spaces

M. Jeyaraman <sup>1</sup>, S. Sowndrarajan <sup>2</sup>

<sup>1,2</sup> PG and Research Department of Mathematics, Raja Doraisingam Govt. Arts College, Sivaganga - 630561, Tamil Nadu,

**Abstract:** The aim of this paper is to obtain common fixed point theorems for self mappings in complete  $\mathcal{M}$  - fuzzy metric spaces by compatible of type (E). Our result generalizes and improves other similar results in Manandhar, Jha and Pathak [6].

**Keywords:** Common fixed point theorem, Generalized fuzzy metric spaces, Compatible mappings.

**AMS Mathematics Subject Classification (2010):** 47H10, 54H25

## I. INTRODUCTION

In 1965, the concept of fuzzy set was introduced by Zadeh [15]. Then in 1975, Kramosil and Michalek [5] introduced the fuzzy metric space as a generalization of a metric space. In 1994, George and Veeramani [1] modified the notion of fuzzy metric spaces with the help of continuous t-norms. In 1986, Jungck [2] introduced notion of compatible mappings in metric spaces. In 2000, Singh and Chouhan [11] introduced the concept of compatible mappings in fuzzy metric spaces. In 1993, Jungck, Murthy and Cho [3] gave a generalization of compatible mappings called compatible mappings of type (A) which is equivalent to the concept of compatible mappings under some conditions. In 1994, Pathak, Cho, Chang and Kang [8] introduced the concept of compatible mappings of type (P) and compared with compatible mappings of type (A) and compatible mappings. In 1998, Pant [7] introduced the notion of reciprocal continuity of mappings in metric spaces. In 1999, Vasuki [13] introduced the notion point wise R - weakly commuting mappings in  $\mathcal{M}$  - fuzzy metric spaces. Recently, in 2007, Singh and Singh [10] introduced the concept of compatible mappings of type (E) in metric spaces. Since then, many authors have obtained fixed point theorems in  $\mathcal{M}$  - fuzzy metric spaces using these compatible notions. The purpose of this paper is to establish common fixed point theorems for compatible mappings of type (E) in  $\mathcal{M}$  - fuzzy metric spaces with an example.

## II. PRELIMINARIES

### A. Definition 2.1.

A binary operation  $*$  :  $[0, 1] \times [0, 1] \rightarrow [0, 1]$  is a continuous t - norm if it satisfies the following conditions

- 1)  $*$  is associative and commutative
- 2)  $*$  is continuous
- 3)  $a * 1 = a$ , for all  $a \in [0, 1]$
- 4)  $a * b \leq c * d$  whenever  $a \leq c$  and  $b \leq d$ , for each  $a, b, c, d \in [0, 1]$

Examples for continuous t - norm are  $a * b = \min \{a, b\}$  and  $a * b = ab$

### B. Definition 2.2.

A 3 - tuple  $(X, \mathcal{M}, *)$  is called  $\mathcal{M}$  fuzzy metric space if X is an arbitrary non - empty set,  $*$  is a continuous t - norm, and  $\mathcal{M}$  is a fuzzy set on  $X^3 \times (0, \infty)$ , satisfying the following conditions for each  $x, y, z, a \in X$  and  $t, s > 0$

(FM - 1)  $\mathcal{M}(x, y, z, t) > 0$ ,

(FM - 2)  $\mathcal{M}(x, y, z, t) = 1$  if  $x = y = z$ ,

(FM - 3)  $\mathcal{M}(x, y, z, t) = \mathcal{M}(p\{x, y, z\}, t)$ , where  $p$  is a permutation function,

(FM - 4)  $\mathcal{M}(x, y, a, t) * \mathcal{M}(a, z, s) \leq \mathcal{M}(x, y, z, t + s)$ ,

(FM - 5)  $\mathcal{M}(x, y, z, \cdot) : (0, \infty) \rightarrow [0, 1]$  is continuous,

(FM - 6)  $\lim_{t \rightarrow \infty} \mathcal{M}(x, y, z, t) = 1$ .

### C. Example 2.3.

Let  $X$  be a non- empty set and  $D^*$  is the  $D^*$  - metric on  $X$ . Denote  $a * b = a.b$  for all  $a, b \in [0, 1]$ . For each  $t \in (0, \infty)$ , define  $\mathcal{M}(x, y, z, t) = \frac{t}{t+D^*(x,y,z)}$  for all  $x, y, z \in X$ , then  $(X, \mathcal{M}, *)$  is a  $\mathcal{M}$ - fuzzy metric space. We call this  $\mathcal{M}$ - fuzzy metric induced by  $D^*$  - metric space. Thus every  $D^*$  - metric induces a  $\mathcal{M}$ - fuzzy metric.

D. *Lemma 2.4.*

Let  $(X, \mathcal{M}, *)$  be a  $\mathcal{M}$ - fuzzy metric space. Then for every  $t > 0$  and for every  $x, y \in X$ . We have  $\mathcal{M}(x, x, y, t) = \mathcal{M}(x, y, y, t)$ .

E. *Lemma 2.5.*

Let  $(X, \mathcal{M}, *)$  be a  $\mathcal{M}$  - fuzzy metric space. Then  $\mathcal{M}(x, y, z, t)$  is non - decreasing with respect to  $t$ , for all  $x, y, z$  in  $X$ .

F. *Definition 2.6.*

Let  $(X, \mathcal{M}, *)$  be a  $\mathcal{M}$  - fuzzy metric space and  $\{x_n\}$  be a sequence in  $X$ .

- 1)  $\{x_n\}$  is said to be converges to a point  $x \in X$  if  $\lim_{n \rightarrow \infty} \mathcal{M}(x, x, x_n, t) = 1$  for all  $t > 0$
- 2)  $\{x_n\}$  is called Cauchy sequence, if  $\lim_{n \rightarrow \infty} \mathcal{M}(x_{n+p}, x_{n+p}, x_n, t) = 1$  for all  $t > 0$  and  $p > 0$ .
- 3) A  $\mathcal{M}$ - fuzzy metric space in which every Cauchy sequence is convergent is said to be complete.

G. *Definition 2.7.*

Let  $S$  and  $T$  be two self mappings of a  $\mathcal{M}$  – fuzzy metric space  $(X, \mathcal{M}, *)$ .

Then the mappings  $S$  and  $T$  are said to be weakly compatible if they commute at their coincidence points, that is,  $Sx = Tx$  for some  $x \in X$ , then  $STx = TSx$ .

H. *Definition 2.8.*

The self mappings  $A$  and  $S$  of a  $\mathcal{M}$ -fuzzy metric space  $(X, \mathcal{M}, *)$  are called point wise  $R$ - weakly commuting if there exists  $R > 0$  such that

$$\mathcal{M}(ASx, SAx, SAx, t) \geq \mathcal{M}(Ax, Sx, Sx, t/R) \text{ for all } x \text{ in } X \text{ and } t > 0.$$

I. *Definition 2.9.*

The self mappings  $A$  and  $S$  of a metric space  $(X, d)$  are said to be compatible of type (E), if  $\lim_{n \rightarrow \infty} AAx_n = \lim_{n \rightarrow \infty} ASx_n = S(t)$ , whenever  $\{x_n\}$  is a sequence in  $X$  such that  $\lim_{n \rightarrow \infty} Ax_n = t$  and  $\lim_{n \rightarrow \infty} Sx_n = t$ , for some  $t \in X$ .

J. *Definition 2.10.*

A self mappings  $A$  and  $S$  of a  $\mathcal{M}$  - fuzzy metric space  $(X, \mathcal{M}, *)$  are said to be compatible of type (E) iff  $\lim_{n \rightarrow \infty} \mathcal{M}(AAx_n, ASx_n,$

$$ASx_n, t) = 1,$$

$$\lim_{n \rightarrow \infty} \mathcal{M}(AAx_n, Sx_n, Sx_n, t) = 1, \lim_{n \rightarrow \infty} \mathcal{M}(ASx_n, Sx_n, Sx_n, t) = 1 \text{ and}$$

$$\lim_{n \rightarrow \infty} \mathcal{M}(SSx_n, SAx_n, SAx_n, t) = 1, \lim_{n \rightarrow \infty} \mathcal{M}(SSx_n, Ax_n, Ax_n, t) = 1,$$

$$\lim_{n \rightarrow \infty} \mathcal{M}(Sx_n, Ax_n, Ax_n, t) = 1, \text{ whenever } \{x_n\} \text{ is a sequence in } X \text{ such that}$$

$$\lim_{n \rightarrow \infty} Ax_n = \lim_{n \rightarrow \infty} Sx_n = x \text{ for some } x \text{ in } X \text{ and } t > 0.$$

K. *Lemma 2.11.*

Let  $\{y_n\}$  be a sequence in a  $\mathcal{M}$ - fuzzy metric space  $(X, \mathcal{M}, *)$  with the condition (FM- 6). If there exists  $k \in (0, 1)$  such that  $\mathcal{M}(y_n, y_{n+1}, y_{n+1}, kt) \geq \mathcal{M}(y_{n-1}, y_n, y_n, t)$  for all  $t > 0$  and  $n \in \mathbb{N}$ , then  $\{y_n\}$  is a Cauchy sequence in  $X$ .

We need the following proposition for the proof of our main result.

L. *Proposition 2.12*

If  $A$  and  $S$  are compatible mappings of type (E) on a  $\mathcal{M}$ - fuzzy metric space  $(X, \mathcal{M}, *)$  and if one of the function is continuous. Then, we have

$$A(x) = S(x) \text{ and } \lim_{n \rightarrow \infty} AAx_n = \lim_{n \rightarrow \infty} SSx_n = \lim_{n \rightarrow \infty} ASx_n = \lim_{n \rightarrow \infty} SAx_n$$

If these exist  $u \in X$  such that  $Au = Su = x$  then  $ASu = SAu$ .

Whenever  $\{x_n\}$  is a sequence in  $X$  such that  $\lim_{n \rightarrow \infty} Ax_n = \lim_{n \rightarrow \infty} Sx_n = x$  for some  $x$  in  $X$ .

1) *Proof*: Let  $\{x_n\}$  be a sequence of  $X$  such that  $\lim_{n \rightarrow \infty} Ax_n = \lim_{n \rightarrow \infty} Sx_n = x$  for some  $x$  in  $X$ .

Then by definition of compatible of type (E), we have  $\lim_{n \rightarrow \infty} AAx_n = ASx_n = S(x)$ .

If  $A$  is a continuous mapping, then we get

$$\lim_{n \rightarrow \infty} AAx_n = A(\lim_{n \rightarrow \infty} Ax_n) = A(x). \text{ This implies } A(x) = S(x). \text{ Also}$$

$$\lim_{n \rightarrow \infty} AAx_n = \lim_{n \rightarrow \infty} SSx_n = \lim_{n \rightarrow \infty} ASx_n = \lim_{n \rightarrow \infty} SAx_n.$$

Similarly, if  $S$  is continuous then, we get the same result. This is the proof of part (i). Again, suppose  $Au = Su = x$  for some  $u \in X$ . Then,  $ASu = A(Su) = Ax$  and

$$SAu = S(Au) = Sx. \text{ From (i), we have } Ax = Sx. \text{ Hence, } ASu = SAu.$$

This is the proof of part (ii).

### III. MAIN RESULTS

If  $P, Q, S, T, A$  and  $B$  are self mappings in  $\mathcal{M}$ -fuzzy metric space  $(X, \mathcal{M}, *)$ . We denote

$$\begin{aligned} \mathcal{M}_\alpha(x, y, t) &= \mathcal{M}(STx, Px, Px, t) * \mathcal{M}(ABx, Qy, Qy, t) * \mathcal{M}(STx, ABx, ABx, t) \\ &* \mathcal{M}(ABx, Px, Px, \alpha t) * \mathcal{M}(STx, Qy, Qy, (2-\alpha)t), \end{aligned}$$

for all  $x, y \in X, \alpha \in (0, 2)$  and  $t > 0$ .

#### A. Theorem 3.1

Let  $(X, \mathcal{M}, *)$  be a complete  $\mathcal{M}$ -fuzzy metric space with a  $* a \geq a$  for all  $a \in [0, 1]$  and with the condition (FM-6). Let one of the mapping of self mappings  $(P, ST)$  and  $(Q, AB)$  of  $X$  be continuous such that

$$PX \subset ABX, QX \subset STX;$$

There exists  $k \in (0, 1)$  such that  $\mathcal{M}(Px, Qy, Qy, kt) \geq \mathcal{M}_\alpha(x, y, y, t)$  for all  $x, y \in X, \alpha \in (0, 2)$  and  $t > 0$ .

If  $(P, ST)$  and  $(Q, AB)$  compatible of type of (E) then  $P, Q, ST$  and  $AB$  have a unique common fixed point. If the pair  $(A, B), (S, T), (Q, B)$  and  $(T, P)$  are commuting mappings then  $A, B, S, T, P$  and  $Q$  have a unique common fixed point.

1) *Proof*: Let  $x_0$  be any point in  $X$ . From the condition (i) there exists  $x_1, x_2 \in X$  such that  $Px_0 = ABx_1 = y_0$  and  $Qx_1 = STx_2 = y_1$ .

Inductively, we can construct sequences  $\{x_n\}$  and  $\{y_n\}$  in  $X$  such that  $Px_{2n} = ABx_{2n+1} = y_{2n}$  and  $Qx_{2n+1} = STx_{2n+2} = y_{2n+1}$  for  $n = 0, 1, 2, \dots$  for  $t > 0$  and  $\alpha = 1 - q$  with  $q \in (0, 1)$  in (ii) then, we have

$$\begin{aligned} \mathcal{M}(Px_{2n}, Qx_{2n+1}, Qx_{2n+1}, kt) &\geq \{ \mathcal{M}((STx_{2n}, Px_{2n}, Px_{2n}, t) \\ &* \mathcal{M}(ABx_{2n+1}, Qx_{2n+1}, Qx_{2n+1}, t) \\ &* \mathcal{M}(STx_{2n}, ABx_{2n+1}, ABx_{2n+1}, t) \\ &* \mathcal{M}(ABx_{2n+1}, Px_{2n}, Px_{2n}, (1-q)t) \\ &* \mathcal{M}(STx_{2n}, Qx_{2n+1}, Qx_{2n+1}, (1+q)t) \} \\ \mathcal{M}(y_{2n}, y_{2n+1}, y_{2n+1}, kt) &\geq \{ \mathcal{M}(y_{2n-1}, y_{2n}, y_{2n}, t) * \mathcal{M}(y_{2n}, y_{2n+1}, y_{2n+1}, t) \\ &* \mathcal{M}(y_{2n-1}, y_{2n}, y_{2n}, t) \\ &* \mathcal{M}(y_{2n}, y_{2n}, y_{2n}, (1-q)t) * \mathcal{M}(y_{2n-1}, y_{2n+1}, y_{2n+1}, (1+q)t) \} \\ &\geq \{ \mathcal{M}(y_{2n-1}, y_{2n}, y_{2n}, t) * \mathcal{M}(y_{2n}, y_{2n+1}, y_{2n+1}, t) \\ &* \mathcal{M}(y_{2n-1}, y_{2n}, y_{2n}, t) * \mathcal{M}(y_{2n}, y_{2n+1}, y_{2n+1}, qt) \}, \\ &\geq \{ \mathcal{M}(y_{2n-1}, y_{2n}, y_{2n}, t) * \mathcal{M}(y_{2n}, y_{2n+1}, y_{2n+1}, t) \\ &* \mathcal{M}(y_{2n}, y_{2n+1}, y_{2n+1}, qt) \}. \end{aligned}$$

Since  $t$ -norm  $*$  is continuous, letting  $q \rightarrow 1$ , we have

$$\begin{aligned} \mathcal{M}(y_{2n}, y_{2n+1}, y_{2n+1}, kt) &\geq \mathcal{M}(y_{2n-1}, y_{2n}, y_{2n}, t) * \mathcal{M}(y_{2n}, y_{2n+1}, y_{2n+1}, t) \\ &* \mathcal{M}(y_{2n}, y_{2n+1}, y_{2n+1}, t) \\ &\geq \mathcal{M}(y_{2n-1}, y_{2n}, y_{2n}, t) * \mathcal{M}(y_{2n}, y_{2n+1}, y_{2n+1}, t). \end{aligned}$$

It follows that  $\mathcal{M}(y_{2n}, y_{2n+1}, y_{2n+1}, k) \geq \mathcal{M}(y_{2n-1}, y_{2n}, y_{2n}, t) * \mathcal{M}(y_{2n}, y_{2n+1}, y_{2n+1}, t)$ .

Similarly,  $\mathcal{M}(y_{2n+1}, y_{2n+2}, y_{2n+2}, kt) \geq \mathcal{M}(y_{2n}, y_{2n+1}, y_{2n+1}, t) * \mathcal{M}(y_{2n+1}, y_{2n+2}, y_{2n+2}, t)$ .

Therefore, for all  $n$  even or odd, we have  $\mathcal{M}(y_n, y_{n+1}, y_{n+1}, kt) \geq \mathcal{M}(y_{n-1}, y_n, y_n, t) * \mathcal{M}(y_n, y_{n+1}, y_{n+1}, t)$ .

Consequently,  $\mathcal{M}(y_n, y_{n+1}, y_{n+1}, t) \geq \mathcal{M}(y_{n-1}, y_n, y_n, k^{-1} t) * \mathcal{M}(y_n, y_{n+1}, y_{n+1}, k^{-1} t)$  and hence  $\mathcal{M}(y_n, y_{n+1}, y_{n+1}, t) \geq \mathcal{M}(y_{n-1}, y_n, y_n, t) * \mathcal{M}(y_n, y_{n+1}, y_{n+1}, k^{-1} t)$ .

Since  $\mathcal{M}(y_n, y_{n+1}, y_{n+1}, k^{-m} t) \rightarrow 1$  as  $k \rightarrow 0$ , it follows that

$\mathcal{M}(y_n, y_{n+1}, y_{n+1}, kt) \geq \mathcal{M}(y_{n-1}, y_n, y_n, t)$  for all  $n \in \mathbb{N}$  and  $t > 0$ .

Therefore, by lemma (2.11),  $\{y_n\}$  is a Cauchy sequence.

Since  $X$  is complete, then there exists a point  $z$  in  $X$  such that  $y_n \rightarrow z$  as  $n \rightarrow \infty$ .

Moreover, we have  $y_{2n} = Px_{2n} = ABx_{2n+1} \rightarrow z$  and  $y_{2n+1} = Qx_{2n+1} = STx_{2n+2} \rightarrow z$ .

If  $P$  and  $ST$  are compatible of type(E) and one of mapping of the pair  $(P, ST)$  is continuous then by proposition (2.12), we have  $Pz = STz$ .

Since  $P(X) \subset AB(X)$ , there exists a point  $w$  in  $X$  such that  $Pz = ABw$ .

Using condition (ii), with  $\alpha = 1$ , we have

$$\begin{aligned} \mathcal{M}(Pz, Qw, Qw, kt) &\geq \{ \mathcal{M}(STz, Pz, Pz, t) * \mathcal{M}(ABw, Qw, Qw, t) \\ &\quad * \mathcal{M}(STz, ABw, ABw, t) \\ &\quad * \mathcal{M}(ABw, Pz, Pz, t) * \mathcal{M}(STz, Qw, Qw, t) \}. \\ &= \{ \mathcal{M}(Pz, Pz, Pz, t) * \mathcal{M}(Pz, Qw, Qw, t) \\ &\quad * \mathcal{M}(Pz, Pz, Pz, t) * \mathcal{M}(Pz, Pz, Pz, t) * \mathcal{M}(Pz, Qw, Qw, t) \} \\ &\geq \mathcal{M}(Pz, Qw, Qw, t). \end{aligned}$$

This implies  $Pz = Qw$ . Thus, we have  $Pz = STz = Qw = ABw$ . Also, we get

$$\begin{aligned} \mathcal{M}(Pz, Qx_{2n+1}, Qx_{2n+1}, kt) &\geq \{ \mathcal{M}(STz, Pz, Pz, t) * \mathcal{M}(ABx_{2n+1}, Qx_{2n+1}, Qx_{2n+1}, t) \\ &\quad * \mathcal{M}(STz, ABx_{2n+1}, ABx_{2n+1}, t) * \mathcal{M}(ABx_{2n+1}, Pz, Pz, t) \\ &\quad * \mathcal{M}(STz, Qx_{2n+1}, Qx_{2n+1}, t) \}. \end{aligned}$$

Letting  $n \rightarrow \infty$ , we get

$$\begin{aligned} \mathcal{M}(Pz, z, z, kt) &\geq \{ \mathcal{M}(Pz, Pz, Pz, t) * \mathcal{M}(Pz, z, z, t) * \mathcal{M}(Pz, Pz, Pz, t) \\ &\quad * \mathcal{M}(Pz, Pz, Pz, t) * \mathcal{M}(Pz, z, z, t) \} \\ &\geq \mathcal{M}(Pz, z, z, t). \end{aligned}$$

Hence, we get  $STz = Pz = z$ . Therefore,  $z$  is a common fixed point of  $P$  and  $ST$ .

Again, if  $Q$  and  $AB$  are compatible with the type (E) and one of the mapping of  $(Q, AB)$  is continuous. So, we get  $Qw = ABw = Pz = z$ .

By using proposition (2.12), we get  $QQw = QABw = ABQw = ABABw$ .

Thus, we get  $Qz = ABz$ . Also, using condition (ii) with  $\alpha = 1$ . We have,

$$\begin{aligned} \mathcal{M}(Px_{2n}, Qz, Qz, kt) &\geq \{ \mathcal{M}(STx_{2n}, Px_{2n}, Px_{2n}, t) * \mathcal{M}(ABz, Qz, Qz, t) \\ &\quad * \mathcal{M}(STx_{2n}, ABz, ABz, t) * \mathcal{M}(ABz, Px_{2n}, Px_{2n}, t) \\ &\quad * \mathcal{M}(STx_{2n}, Qz, Qz, t) \}. \end{aligned}$$

Letting  $n \rightarrow \infty$ , we get

$$\begin{aligned} \mathcal{M}(z, Qz, Qz, kt) &\geq \{ \mathcal{M}(z, z, z, t) * \mathcal{M}(ABz, Qz, Qz, t) * \mathcal{M}(z, ABz, ABz, t) \\ &\quad * \mathcal{M}(ABz, z, z, t) * \mathcal{M}(z, Qz, Qz, t) \} \\ &\geq \mathcal{M}(z, Qz, Qz, t). \end{aligned}$$

Hence, we have  $Qz = ABz = z$ . Therefore  $z$  is a common fixed point of  $Q$  and  $AB$ .

Hence  $z$  is a common fixed point of  $P, Q, ST$  and  $AB$ .

For uniqueness, suppose that  $(Pw \neq Pz = z)$  is another common fixed point of  $P, Q, ST$  and  $AB$ . Then, using condition (ii) with  $\alpha = 1$ . We have,

$$\begin{aligned} \mathcal{M}(PPz, QPw, QPw, kt) &= \mathcal{M}(Pz, Pw, Pw, kt) \\ &\geq \{ \mathcal{M}(STPz, PPz, PPz, t) * \mathcal{M}(ABPw, QPw, QPw, t) \\ &\quad * \mathcal{M}(STPz, ABPw, ABPw, t) * \mathcal{M}(ABPw, PPz, PPz, t) \\ &\quad * \mathcal{M}(STPz, QPw, QPw, t) \} \\ &= \mathcal{M}(Pz, Pz, Pz, t) * \mathcal{M}(Pw, Pw, Pw, t) \\ &\quad * \mathcal{M}(Pz, Pw, Pw, t) * \mathcal{M}(Pw, Pz, Pz, t) * \mathcal{M}(Pz, Pw, Pw, t) \end{aligned}$$

$\mathcal{M}(PPz, QPw, QPw, kt) \geq \mathcal{M}(Pz, Pw, Pw, t)$ .

That is,  $Pw = Pz = z$ . Thus,  $z$  is a unique common fixed point of  $P, Q, ST$  and  $AB$  by using the commutativity of the pairs  $(A, B), (S, T), (Q, B)$  and  $(T, P)$  we can easily prove that  $z$  is a unique common fixed point of  $A, B, S, T, P$  and  $Q$ .

If we take  $T = B = I_x$ , an identity mapping of  $X$ .

## B. Corollary 3.2

Let  $(X, \mathcal{M}, *)$  be a complete  $\mathcal{M}$ -fuzzy metric space with  $a * a \geq a$  for all  $a \in [0, 1]$  and with the condition (FM-6). If one of the mapping of self mappings  $(P, Q)$  and  $(Q, A)$  of  $X$  is continuous such that for  $k \in (0, 1)$  we have

$$\mathcal{M}(Px, Qy, Qy, kt) \geq \mathcal{M}_\alpha(x, y, y, t) \text{ for all } x, y \in X, \alpha \in (0, 2) \text{ and } t > 0, \text{ and if } (P, S) \text{ and } (Q, A) \text{ are compatible of type of (E)}$$

$$\mathcal{M}((x, y, y, t) = \mathcal{M}(Sx, Px, Px, t) * \mathcal{M}(Ay, Qy, Qy, t) * \mathcal{M}(Sx, Ay, Ay, t)$$

$$* \mathcal{M}(Ay, Px, Px, \alpha t) * \mathcal{M}(Sx, Qy, Qy, (2 - \alpha) t),$$

for all  $x, y \in X, \alpha \in (0, 2)$  and  $t > 0$ .

Similarly if we get the result for three self maps by taking  $S = A, T = B = I_x$  in the Theorem (3.1) and also by taking  $P = Q, T = B = I_x$  in Theorem (3.1) and obtain for two self maps by taking  $P = Q, A = S, B = T = I_x$  in Theorem (3.1) then  $P, A, S, Q$  have a unique common fixed point.

## C. Example 3.3

Let  $X = [2, 10]$  with the metric  $d$  defined by

$$D^*(x, y, z) = |x - y| + |y - z| + |z - x| \text{ and define } \mathcal{M}(x, y, z, t) = \frac{t}{t + D^*(x, y, z)}$$

for all  $x, y, z \in X, t > 0$ . Clearly  $(X, \mathcal{M}, *)$  is a complete  $\mathcal{M}$ -fuzzy metric space. Define  $P, Q, S, T, A$  and  $B: X \rightarrow X$  as follows;

$$Px = 2 \text{ for all } x, Qx = 2 \text{ if } x < 4 \text{ and } x \geq 5, Qx = 3+x \text{ if } 4 \leq x < 5$$

$$Sx = 2 \text{ if } x \leq 8, Sx = 8 \text{ if } x > 8;$$

$$Ax = 2 \text{ if } x < 4 \text{ or } x \geq 5, Ax = 5 + x \text{ if } 4 \leq x < 5$$

$$Bx = Tx = x, \text{ for all } x \in [2, 10].$$

Then  $P, Q, S, T, A$  and  $B$  satisfy all the conditions of the above theorem and have a unique common fixed point  $x = 2$ .

## REFERENCES

- [1] George. A and Veeramani. P : On some results in fuzzy metric spaces. Fuzzy sets and systems. 64 (1994), 395 – 399.
- [2] Jungck. G, Compatible mappings and common fixed points, Internat. Math. J. Maths. Sci., 9(1986), pp. 771 – 779.
- [3] Jungck. G, Murthy. P. P. and Cho. Y. J., compatible mappings of type (A) and common fixed points, Math. Japonica, 38(1993), pp. 381 – 390.
- [4] Jungck. G, Rhoades. B. E, Fixed point theorems for occasionally weakly compatible mappings, Fixed point theory, 7(2), 2006, 287 – 296.
- [5] Kramosil. I and Michalek. J: Fuzzy metric and statistical spaces, Kybernetika 11 (1975), 336 – 344.
- [6] Manandhar. K. B. Jha. K and Pathak. H. K, A common fixed point theorem for compatible mappings of type (E) in fuzzy metric space, Applied Mathematical Sciences, Vol. 8, 2014, no. 41, 2007 – 2014.
- [7] Pant. R. P, A common fixed point theorem under a new condition, Indian J. pure Appl. Math., 30(2) (1999) 147 – 152.
- [8] Pathak. H. K, Cho. Y. J, Chang. S. S. and Kang. S. M, Compatible mappings of type (P) and fixed point theorem in metric spaces and probabilistic metric spaces, Novi Sad J. Math., 26(2)(1996), pp. 87 – 109.
- [9] Sharma. S, Common fixed point theorems in fuzzy metric spaces, Fuzzy sets and systems, 127 (2002), 345 – 352.
- [10] Singh. M. R. and Singh. Y. M, Compatible mappings of type (E) and common fixed point theorems of Meir- Keeler type, Internat. J. Math. Sci and Engg. Appl., 1(2), (2007), 299 -315.
- [11] Singh. B, and Chouhan. M.S, Common fixed points of compatible maps in Fuzzy metric spaces, Fuzzy sets and systems, 115 (2000), 471-475.
- [12] Sintunavarat. W, Kuman, P, Common fixed point theorems for a pair of weakly compatible mappings in fuzzy metric space, J. Appl. Math, 14, 2011.
- [13] Sedghi. S and Shobe. N, “ Fixed point theorem in  $\mathcal{M}$ -fuzzy metric spaces with property (E)” ,Advances in Fuzzy Mathematics, 1(1) (2006), 55-65
- [14] Vasuki. R, Common fixed points for R - weakly commuting maps in fuzzy metric spaces, Indian, J. Pure Appl. Math., 30(1999), 419 – 423.
- [15] Zadeh L.A., “Fuzzy sets”, Inform. and Control, 8 (1965), 338- 353



10.22214/IJRASET



45.98



IMPACT FACTOR:  
7.129



IMPACT FACTOR:  
7.429



# INTERNATIONAL JOURNAL FOR RESEARCH

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY

Call : 08813907089  (24\*7 Support on Whatsapp)