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Lehmer-3 Mean Number of Graphs

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I. INTRODUCTION

A graph considered here are finite, undirected and simple. The vertex set and edge set of a graph are denoted by V(G) and E(G) respectively. A path of length n is denoted by P_n . For standerd terminology and notations we follow Harrary[1]. S Somasundaram & S.S Sandhya introduced the concept of Harmonic Mean Labeling of Graphs in [2] .We will provide a brief summary of other information's which are necessary for our present investigation.

A. Definition 1.1

A graph G=(V,E) with P vertices and q edges is called Lehmer -3 mean graph. If it is possible to label vertices x ЄV with distinct labels f(x) from 1,2,3,…….q+1 in such a way that when each edge e=uv is labeled with f(e=uv)=$\left\lceil \frac{f(u)^3+f(v)^3}{f(u)^2+f(v)^2} \right\rceil$ (or) $\left\lfloor \frac{f(u)^3+f(v)^3}{f(u)^2+f(v)^2} \right\rfloor$, then the edge labels are distinct. In this case "f " is called Lehmer -3 mean labeling of G.

B. Definition 1.2

Let G be a graph and f:V(G)→{1,2,….n} be a function such that the label of the edge f (e=uv) is Labeled with $\left\lceil \frac{f(u)^3+f(v)^3}{f(u)^2+f(v)^2} \right\rceil$ (or) $\left\lfloor \frac{f(u)^3+f(v)^3}{f(u)^2+f(v)^2} \right\rfloor$, where {(f(e);eЄE(G)}⊆{1,2,…..n}. If n is the smallest positive integer satisfying this condition together with the condition that there is no edges in common. Then n is called the Lehmer-3 mean number of a graph G and is denoted as L_{3m} (G).

II. MAIN RESULTS

A. Theorem :2.1

L_{3m} (P_n) = n.

1) Proof : Let $u_1,u_2,\ldots u_n$ be the vertices of the path P_n. Define a function f:V(P_n)→{1,2,….n} by $f(u_1)=1$, $f(u_i)= i + 1$; $2 \le i \le n$. Then the edge labels are $f(u_i u_{i+1}) = i$; $1 \le i \le n-1$, $f(u_{n-1} u_n) = n$. Thus L_{3m} (P_n) = n.

B. Example 2.2

Lehmer-3 mean number of P_7 is
L_{3m} (P_n) = n = 7 is given below.

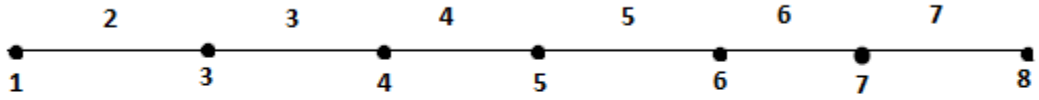


Figure-1

C. Theorem :2.3

L_{3m} (C_n) = n +1

1) Proof : Let C_n be a cycle of n vertices $u_1, u_2, \ldots u_n, u_1$. We define a function f:V(C_n)→{1,2,….n} by $f(u_1)=1$, $f(u_i)= i + 1$; $2 \le i \le n$. Then the distinct edge labels are $f(u_i u_{i+1}) = i+1$; $1 \le i \le n-1$, $f(u_n u_1) = n+1$. Thus L_{3m} (C_n) = n+1.

D. Example 2.4

L_{3m} (C_8) = n+1 = 8+1 = 9 is displayed below.

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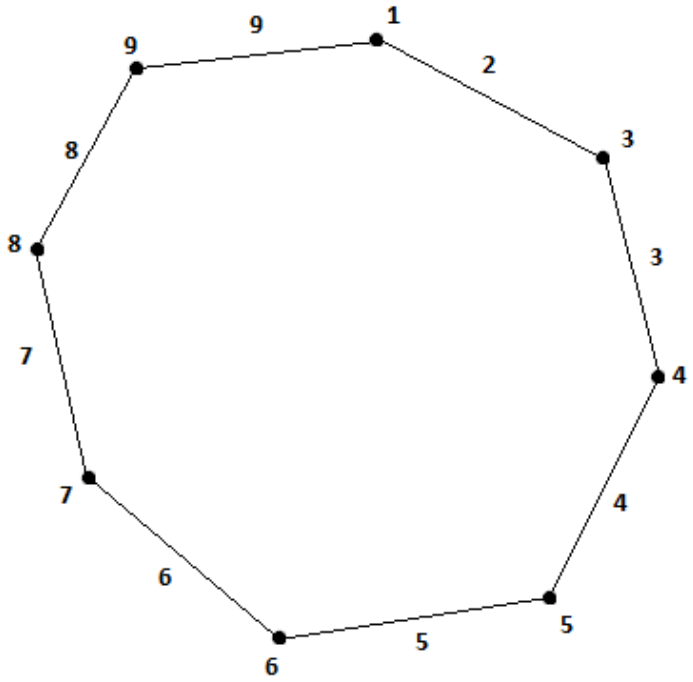


Figure-2

E. Theorem :2.5

L_{3m} (P_n ΘK_1) = 2n.

1) Proof : Let G be a P_n ΘK_1, graph and its vertices $u_1, u_2, \ldots u_n, v_1, v_2, \ldots v_n$. respectively. Let us define a function $f:V(G)\rightarrow\{1,2,\ldots.n\}$ by $f(u_1)=1$, $f(u_i)= 2i$; $2 \leq i \leq n$, $f(v_i)= 2i+1$; $1 \leq i \leq n$. and the obtained distinct edge labels are $f(u_i, u_{i+1})=2i+1$; $1 \leq i \leq n-1$, $f(u_i v_i)= 2i$; $1 \leq i \leq n-1$, $f(u_n v_n)= 2n$

Hence L_{3m} (P_n ΘK_1) = 2n

F. Example 2.6

L_{3m} (P_6 ΘK_1) = 2n = 2x6 = 12 is given below

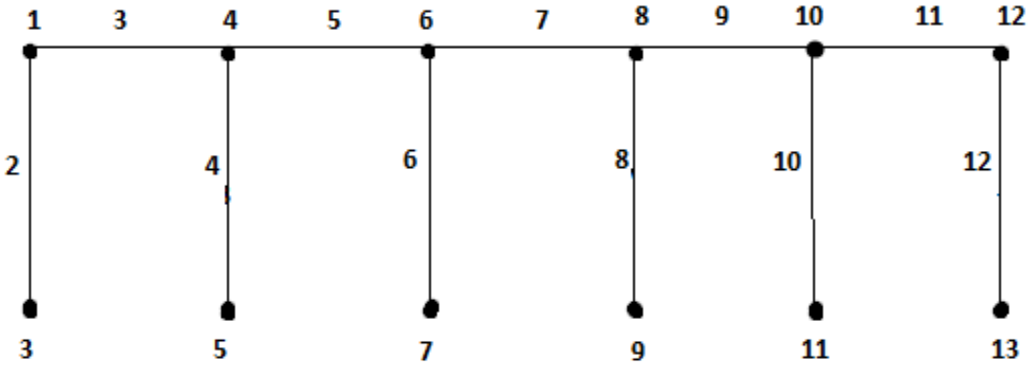


Figure-3

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G. Theorem :2.7

L_{3m} (P_n ʘK_1) ʘK_1) = 3n.

1) Proof : Let G be a graph of n vertices $u_1, u_2, \ldots u_n, v_1, v_2, \ldots v_n, w_1, w_2, \ldots w_n$ respectively. Define a function f:V(G)→{1,2,….n} by $f(u_1)=1$, $f(u_i)= 3i-1$; $2 \le i \le n$, $f(v_i)= 3i$; $1 \le i \le n$, $f(w_i)= 3i+1$; $1 \le i \le n$. then the distinct edge labels are $f(u_i u_{i+1})=3i+1$; $1 \le i \le n-1$, $f(u_i v_i)= 3i-1$; $1 \le i \le n$, $f(v_i w_i)= 3i$; $1 \le i \le n-1$ and $f(v_n w_n)= 3n$. Therefore L_{3m} ((P_n ʘK_1) ʘK_1) = 3n

H. Example 2.8

Lehmer-3 mean number of graph with n=5 is given as

L_{3m} (G) = 3n = 3x5 = 15

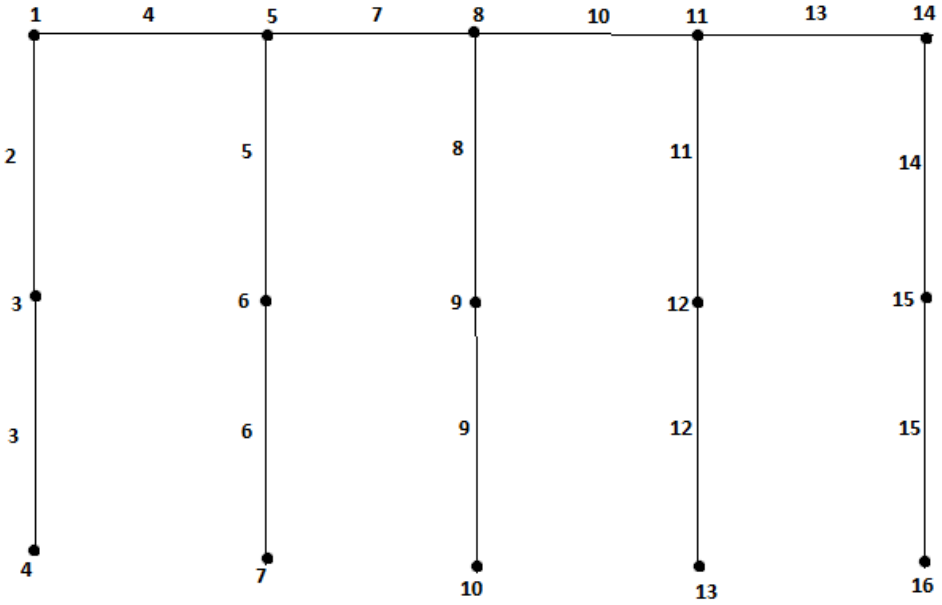


Figure-4

I. Theorem:2.9

L_{3m} (P_n ʘ$K_{1,2}$) = 3n.

1) Proof : Let P_n be a path of n vertices $u_1, u_2, \ldots u_n$, and the vertices of $K_{1,2}$ is denoted as $v_1, v_2, \ldots v_n, w_1, w_2, \ldots w_n$. Define a function f:V(G)→{1,2,….n} by $f(u_1)=1$, $f(u_i)= 3i-1$; $2 \le i \le n$, $f(v_1)= 2$, $f(v_i)= 3i$; $2 \le i \le n$, $f(w_1)= 3$; $f(w_i)= 3i+1$; $2 \le i \le n$. The distinct edge labels are $f(u_i u_{i+1})=3i+1$; $1 \le i \le n-1$, $f(u_1 v_1)= 1$, $f(u_i v_i)= 3i-1$; $2 \le i \le n$, $f(u_1 w_1)= 2$, $f(u_i w_i)= 3i$, $2 \le i \le n-1$, $f(u_n w_n)= 3n$ Thus L_{3m} (P_n ʘ$K_{1,2}$) = 3n

J. Example 2.10

Lehmer-3 mean number of (P_5 ʘ$K_{1,2}$) is

L_{3m} (P_5 ʘ$K_{1,2}$) = 3n = 3x5 = 15

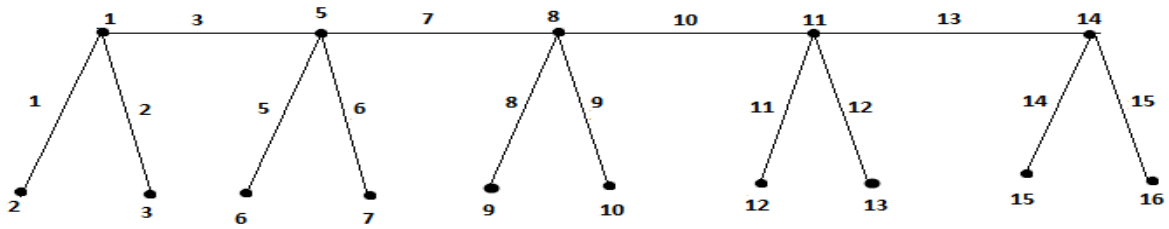


Figure-5

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K. Theorem :2.11

L_{3m} (P_n ϴ$K_{1,3}$) = 4n.

1) *Proof :* Let P_n be a path of n vertices $u_1, u_2, \ldots u_n$, Let $K_{1,3}$ be a complete graph with vertices v_i, w_i, x_i, such that $1 \le i \le n$ attached to the vertices of the path respectively. Define a function f:V(G)→{1,2,….n} by $f(u_{1)}=1$, $f(u_i)= 4i-2$; $2 \le i \le n$, $f(v_1)= 2$, $f(v_i)= 4i-1$; $2 \le i \le n$, $f(w_i)= 4n$; $1 \le i \le n$, $f(x_i)= 4i +1$; $1 \le i \le n$, Then we obtain distinct edge labels as $f(u_{i,}\, u_{i+1})=4i+1$; $1 \le i \le n-1$, $f(u_1v_1)= 1$, $f(u_iv_i)= 4i-2$; $2 \le i \le n$, $f(u_iw_i)= 4i-1$; $1 \le i \le n$, $f(u_ix_i)= 4i$, $1 \le i \le n-1$and $f(u_nx_n)= 4n$ Thus L_{3m} (P_n ϴ$K_{1,3}$) = 4n

L. Example 2.12

L_{3m} (P_6 ϴ$K_{1,3}$) = 4n = 4x6 = 24 is given below

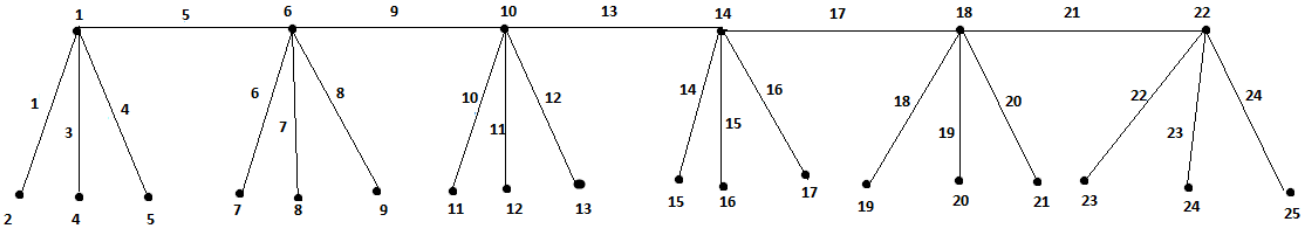


Figure-6

M. Theorem:2.13

L_{3m} (P_n ϴK_3) = 4n.

1) *Proof :* Let P_n be a path of n vertices and Let $K_{1,3}$ be a complete graph of n vertices attached to each vertices of the path. We define a function f:V(G)→{1,2,….n} by $f(u_{1)}=1$, $f(u_i)= 4i-2$; $2 \le i \le n$, $f(v_1)= 2$, $f(v_i)= 4i-1$; $2 \le i \le n$, $f(w_i)= 4i$; $1 \le i \le n$. Then the edge labels as $f(u_{i,}\, u_{i+1})=4i+1$; $1 \le i \le n-1$, $f(u_1v_1)= 1$, $f(u_iv_i)= 4i-2$; $2 \le i \le n$, $f(v_1w_1)= 3$; $f(v_iw_i)= 4i - 1$; $2 \le i \le n$ and $f(u_iw_i)= 4i$; $1 \le i \le n-1$ and $f(u_nw_n)= 4n$ Hence L_{3m} (P_n ϴK_3) = 4n.

N. Example 2.14

Lehmer-3 mean number of (P_5 ϴK_3) is given below

L_{3m} (P_5 ϴK_3) = 4n = 4x5 = 20

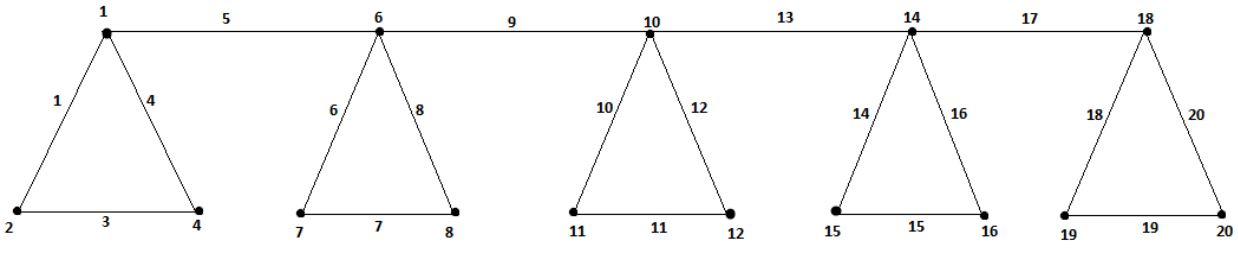


Figure-7

O. Theorem:2.15

L_{3m} (P_n ϴK_1) ϴ$K_{1,2}$) = 4n

1) *Proof :* Let (P_n ϴK_1) ϴ$K_{1,2}$) be a graph with vertices $u_{i,}$ v_i, w_i, x_i, ; $1 \le i \le n$, where P_n ϴK_1 be a comb graph in which $K_{1,2}$ is attached to each vertex of the comb. A function defined on G by f:V(G)→{1,2,….n} by $f(u_{1)}=1$, $f(u_i)= 4i-2$; $2 \le i \le n$, $f(v_i)= 2$; $f(v_i)= 4i -1$; $2 \le i \le n$, $f(w_{i)}=4i$, $1 \le i \le n$, $f(x_i) = 4i+1$; $1 \le i \le n$ then the distinct edge labels are $f(u_{i,}\, u_{i+1})=4i+1$; $1 \le i \le n-1$, $f(u_1 v_1)= 1$; $f(u_iv_i)= 4i-2$; $2 \le i \le n$, $f(v_iw_i)= 4i-1$; $1 \le i \le n$, $f(v_ix_i)= 4i$; $1 \le i \le n-$, $f(v_nx_n)= 4n$. Thus the Lehmer -3 mean number of (P_n ϴK_1) ϴ$K_{1,2}$) is 4n

P. Example 2.16

L_{3m} (P_5 ϴK_1) ϴ$K_{1,2}$) = 4n = 4x5 = 20 is given below

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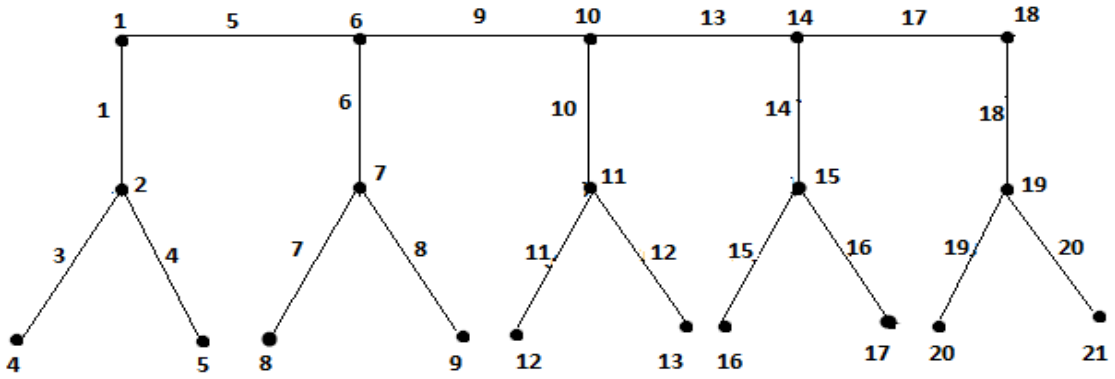


Figure-8

Q. Theorem:2.17

L_{3m} (C_n ʘK_1) = 2n+1

1) Proof : Let C_n be a cycle with vertices $u_1, u_2, \ldots u_n$, Let K_1 be a graph attached to each vertex of the cycle. Such that its vertices be $v_1, v_2, \ldots v_n$, Define a function $f:V(G)\rightarrow\{1,2,\ldots.n\}$ by $f(u_{1)}=1$, $f(u_i)= 2i$; $2 \leq i \leq n$, $f(v_i)= 2i+1$, $1 \leq i \leq n$. The edge labels as $f(u_{i,}\, u_{i+1})=2i+1$; $1 \leq i \leq n-1$, $f(u_nu_1)= 2n$, $f(u_iv_i)= 2i$; $1 \leq i \leq n-1$, $f(u_nv_n)= 2n+1$. Thus L_{3m} (C_n ʘK_1) = 2n+1

R. Example 2.18

Lehmer -3 mean number of C_6 ʘK_1 = 2n+1 = 2x6+1 = 13 is given below

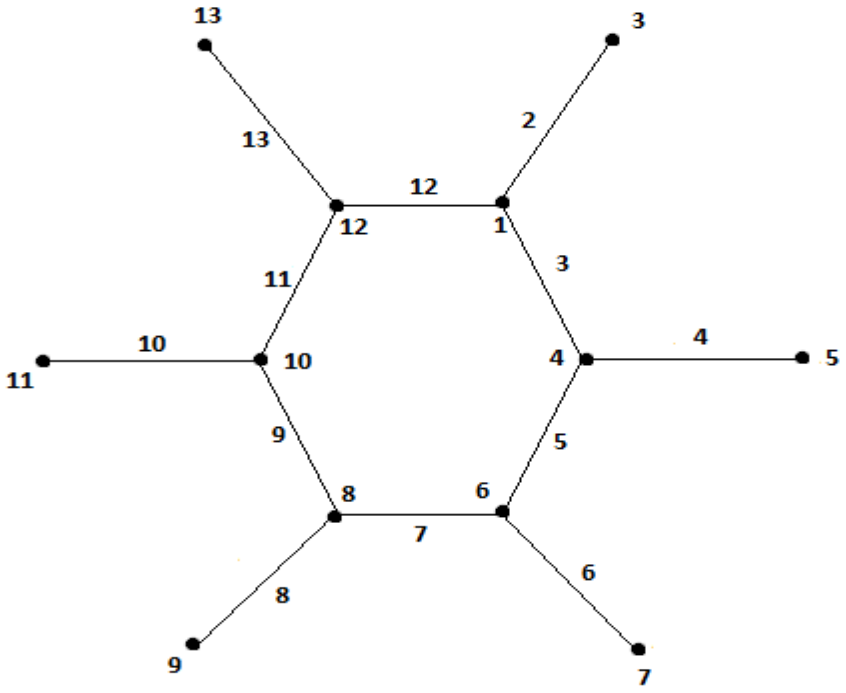


Figure-9

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S. Theorem :2.19

$L_{3m}(C_n \odot K_{1,2}) = 3n+1$

1) Proof : Let $C_n \odot K_{1,2}$ be a graph of n vertices $u_1, u_2, \ldots u_n, v_1, v_2, \ldots v_n, w_1, w_2, .. w_n$ respectively. A function defined on $(C_n \odot K_{1,2})$ as $f:V(C_n \odot K_{1,2}) \rightarrow \{1,2,\ldots.n\}$ by $f(u_1)=1$, $f(u_i)= 3i-1$; $2 \le i \le n$, $f(v_i)= 3i$, $1 \le i \le n$, $f(w_i)= 3i+1$; $1 \le i \le n$. The edge labels as $f(u_i, u_{i+1})=3i+1$; $1 \le i \le n-1$, $f(u_nu_1)= 3n-1$, $f(u_iv_i)= 3i-1$; $1 \le i \le n-1$, $f(u_nv_n)= 3n$, $f(u_iw_i)= 3i$; $1 \le i \le n-1$ and $f(u_nw_n)= 3n+1n$ Therefore $L_{3m}(C_n \odot K_{12}) = 3n+1$ Hence the proof

T. Example 2.20

$L_{3m}(C_5 \odot K_{1,2}) = 3n+1 = 3x5+1 = 16$ is shown below

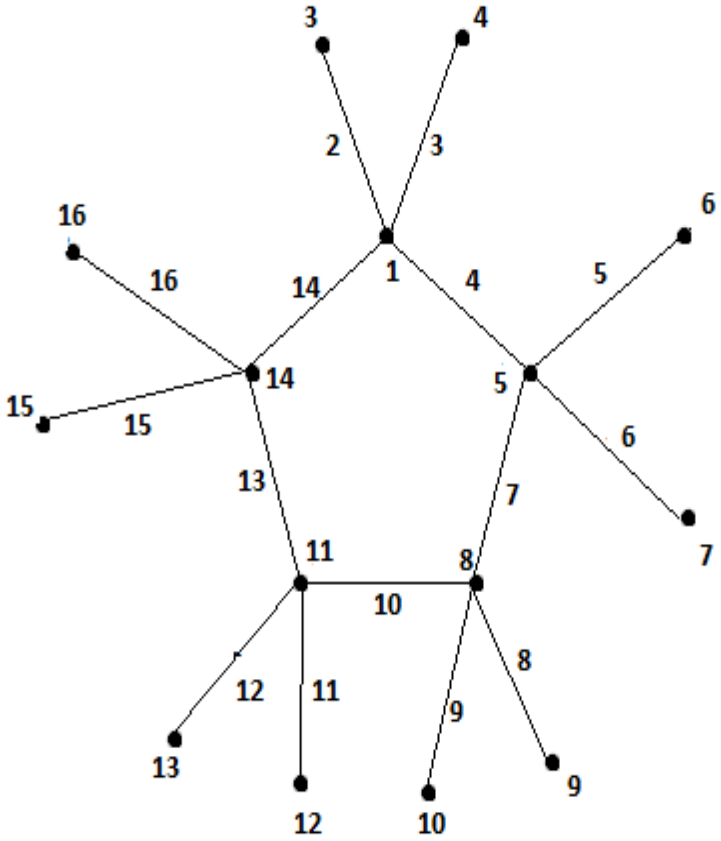


Figure-10

U. Theorem :2.21

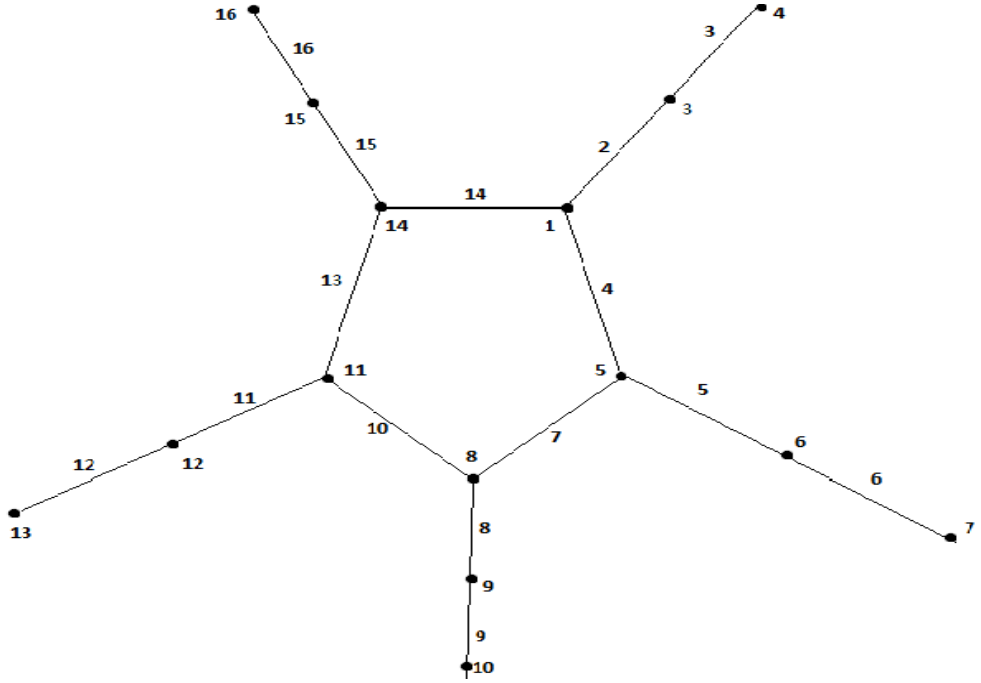
$L_{3m}(C_n \odot K_1) \odot K_1) = 3n+1$

1) Proof : Let G be a $(C_n \odot K_1) \odot K_1)$ graph the vertices be denoted u_i, v_i, w_i, where $1 \le i \le n$ respectively, w_i be the vertices attached t each pendant vertices of the crown. Define a function $f:V(G) \rightarrow \{1,2,\ldots.n\}$ by $f(u_1)=1$, $f(u_i)= 3i-1$; $2 \le i \le n$, $f(v_i)= 3i$; $i=1,2\ldots n$, $f(w_i)= 3i+1$; $1 \le i \le n$, The edge labels are $f(u_i, u_{i+1})=3i+1$; $1 \le i \le n-1$, $f(u_nu_1)= 3n-1$; $f(u_iv_i)= 3i-1$; $2 \le i \le n-1$, $f(u_nv_n)= 3n$, $f(v_iw_i)= 3i$; $1 \le i \le n-1$, $f(v_nw_n)= 3n+1$. Thus $L_{3m}(G) = 3n+1$ hence the proof

V. Example 2.22

$L_{3m}(C_5 \odot K_1) \odot K_1) = 3n+1 = 3x5+1 = 16$ diagram pattern is given below

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W. Theorem :2.23

L_{3m} (C_n ΘK$_1$) ΘK$_{1,2}$) = 4n+1

1) Proof : Let G be a graph of vertices $u_1, u_2, \ldots u_n, v_1, v_2, \ldots v_n, w_1, w_2, \ldots w_n, x_1, x_2, \ldots x_n$ respectively. This is a crown attached with $K_{1,2}$ at each pendant vertex of a crown. Define a function f:V(G)→{1,2,3….n} by $f(u_1)=1$, $f(u_i)= 4i-2$; $2 \le i \le n$, $f(v_i)= 4i-1$; $1 \le i \le n$, $f(w_i)= 4i$; $1 \le i \le n$, $f(x_i)= 4i+1$; $1 \le i \le n$ The distinct edge labels are $f(u_i u_{i+1})=4i+1$; $1 \le i \le n-1$, $f(u_n u_1)= 4n-2$; $f(u_i v_i)= 4i-1$; $1 \le i \le n-1$, $f(u_n v_n)= 4n-1$, $f(v_i w_i)= 4i-1$; $1 \le i \le n-1$, $f(v_n w_n)= 4n$, $f(v_i x_i)= 4i$; $1 \le i \le n-1$, $f(v_n x_n)= 4n+1$ Thus L_{3m}(C_n ΘK$_1$) ΘK$_{1,2}$) = 4n+1

X. Example 2.24

Lehmer -3 mean number of (C_4 ΘK$_1$) ΘK$_{1,2}$) is 4n+1 = 4x4+1 = 16+1 = 17 is given below

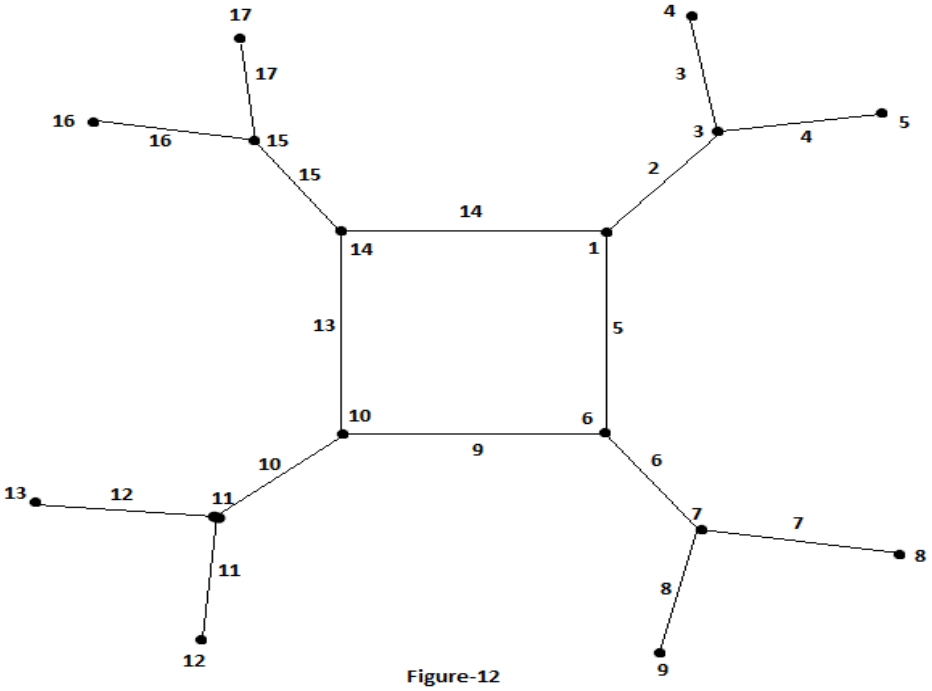


Figure-12

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Y. Theorem :2.25

$L_{3m}(L_n) = 3n - 1$

1) Proof : Let L_n be a ladder graph of n vertices $u_1, u_2, \ldots u_n, v_1, v_2, \ldots v_n$, respectively. Define a function $f:V(L_n)\rightarrow\{1,2,\ldots.n\}$ by $f(u_i)=3i-2 ; 1 \le i \le n$, $f(v_i)= 3i-1 ; 1 \le i \le n$. The edge are labeled as $f(u_iu_{i+1}) = 3i ; 1 \le i \le n-1$, $f(v_iv_{i+1}) = 3i+1 ; 1 \le i \le n-1$, $f(u_1v_1) = 1$, $f(u_iv_i) = 3i-1 ; 2 \le i \le n-1$, $f(u_nv_n) = 3n-1$. Hence $L_{3m}(L_n) = 3n-1$. Hence ladder satisfies lehmer -3 mean number

Z. Example 2.26

Lehmer -3 mean number of L7 is given as $L_{3m}(L7) = 3x7-1 = 21-1 = 20$.

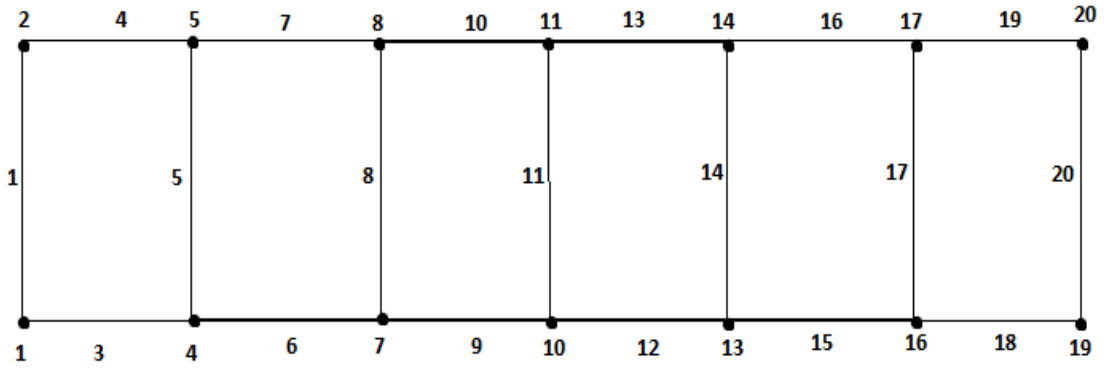


Figure-13

AA. Theorem :2.27

$L_{3m}(D_n) = n + m$

1) Proof : Let D_n be a dragon graph where the head of the dragon has n vertices $u_1, u_2, \ldots u_n$, and the path attached to it be $v_1, v_2, \ldots v_m$. Define a function of $f:V(D_n)\rightarrow\{1,2,\ldots.n\}$ by $f(u_1=v_1) = 1$, $f(u_i) = i+1 ; 2 \le i \le n$, $f(v_i) = n+j ; 2 \le j \le m$. The distinct edge labels are $f(u_iu_{i+1}) = i+1 ; 1 \le i \le n$, $f(v_jv_{j+1}) = (n+1)+j ; 1 \le j \le m-2$, $f(v_{n-1}v_n) = n + m$.

Thus $L_{3m}(D_n) = n+m$.

BB. Example 2.28

Lehmer -3 mean number of D_n is given below. $L_{3m}(D_n) = m+n$

Hence m=5 and n=6; n+m = 5+6 = 11 is drawn as

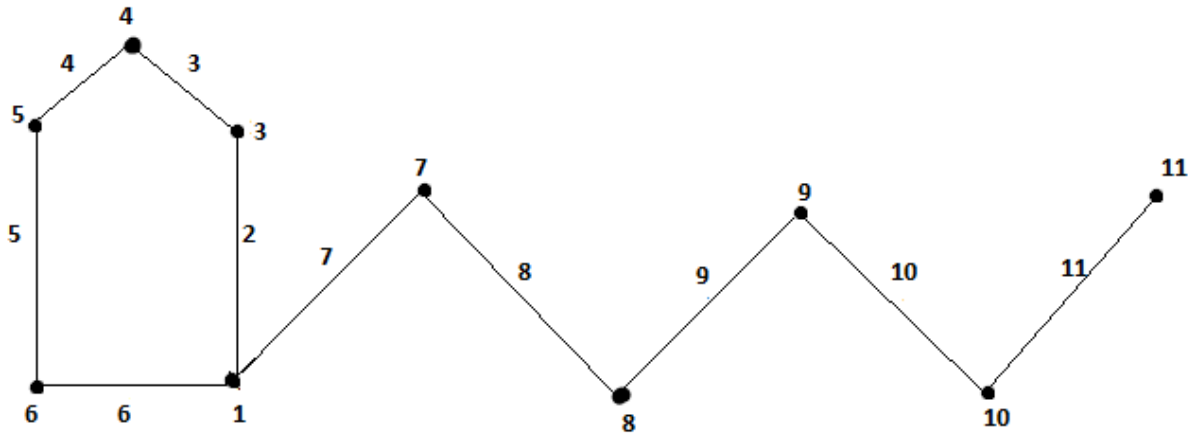


Figure-15

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CC. Theorem :2.29

Lehmer -3 mean number of caterpillar is 3n

1) *Proof :* Let G be a graph of n vertices where each vertex of path is attached to pendant vertex on its both side. Let the vertices be denoted as $u_1, u_2, \ldots u_n, v_1, v_2, \ldots v_n, w_1, w_2, \ldots w_n$. Let $f:V(G) \rightarrow \{1,2,\ldots.n\}$ be a function defined by $f(u_1) = 1$, $f(u_i) = 3i-1$; $2 \leq i \leq n$, $f(v_i) = 3i$; $1 \leq i \leq n$, $f(w_i) = 3i+1$; $1 \leq i \leq n$ and The edges are labeled as $f(u_iu_{i+1}) = 3i+1$; $1 \leq i \leq n-1$, $f(u_iv_i) = 3i-1$; $1 \leq i \leq n$, $f(u_iw_i) = 3i$; $1 \leq i \leq n-1$, and $f(u_nw_n) = 3n$. hence the Lehmer -3 mean number of a caterpillar is 3n

DD. Example 2.30

Lehmer -3 mean number of a caterpillar of 5 vertices is 3n = 3x5 is given below

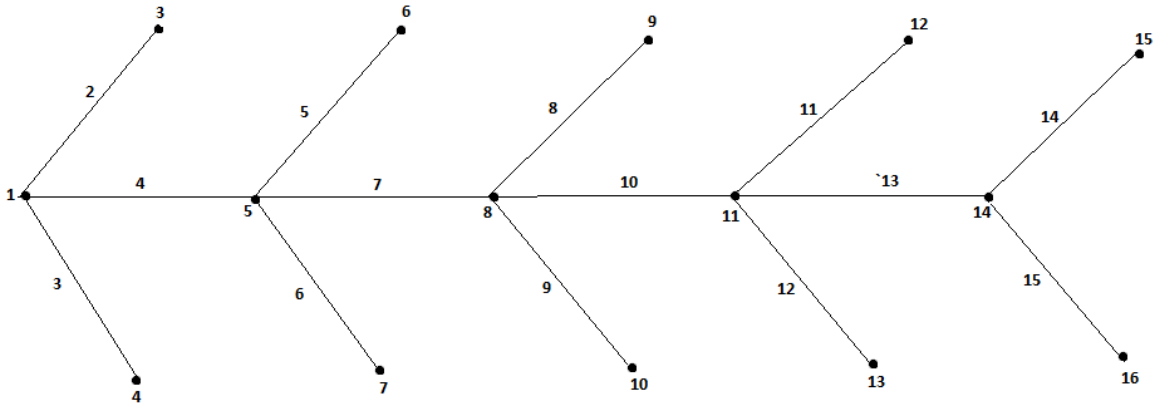


Figure-16

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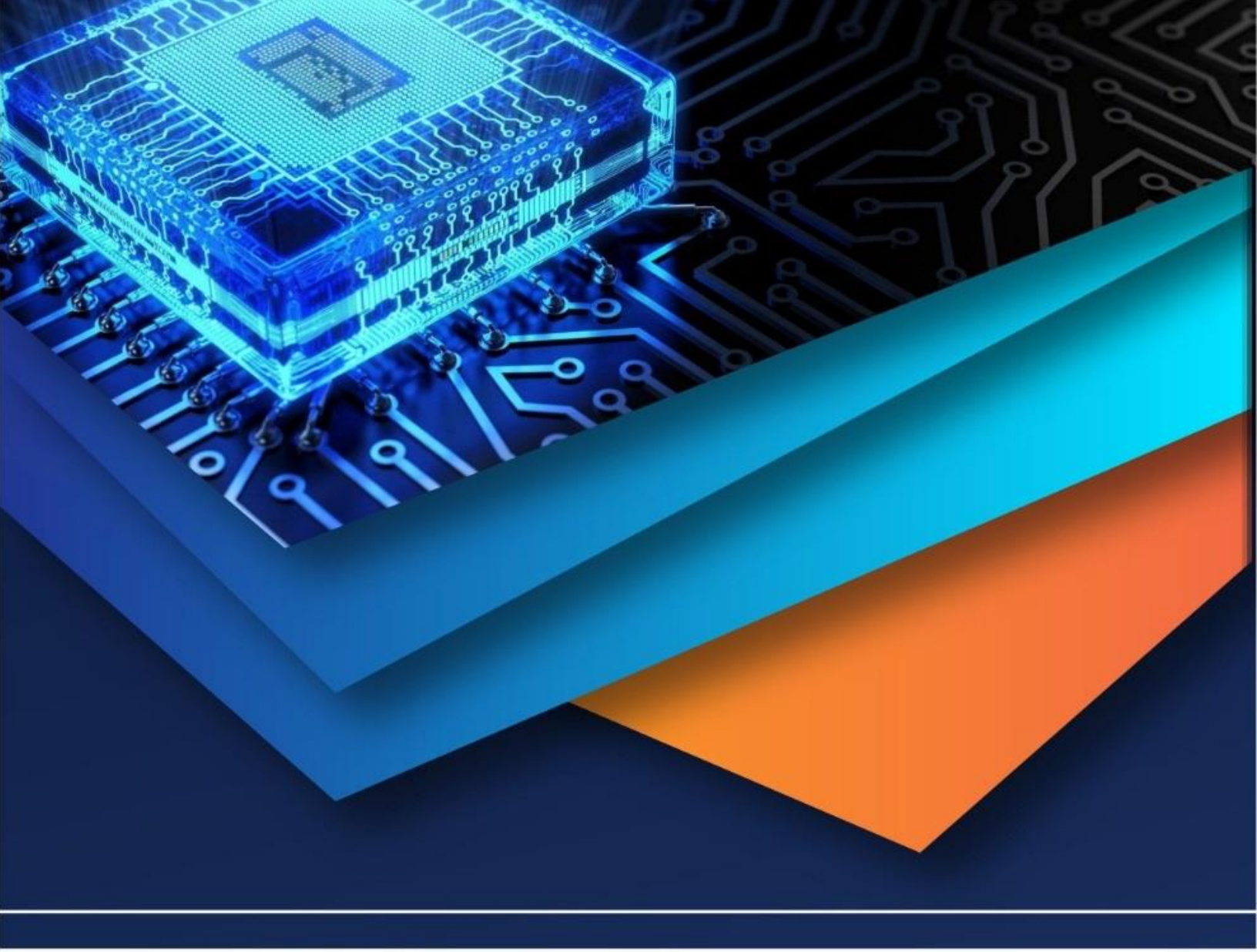
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