Lehmer-3 Mean Number of Graphs

S. Somasundaram[1], T.S. Pavithra[2]
[1]Professor in Mathematics, Manonmaniam Sundaranar University, Tirunelveli-627012
[2]Research Scholar, Manonmaniam Sundaranar University, Tirunelveli-627012.

I. INTRODUCTION

A graph considered here are finite, undirected and simple. The vertex set and edge set of a graph are denoted by V(G) and E(G) respectively. A path of length n is denoted by P_n . For standerd terminology and notations we follow Harrary[1]. S Somasundaram & S.S Sandhya introduced the concept of Harmonic Mean Labeling of Graphs in [2] .We will provide a brief summary of other information's which are necessary for our present investigation.

A. Definition 1.1

A graph G=(V,E) with P vertices and q edges is called Lehmer -3 mean graph. If it is possible to label vertices x ЄV with distinct labels f(x) from 1,2,3,………….q+1 in such a way that when each edge e=uv is labeled with f(e=uv)=$\left\lceil \frac{f(u)^3+f(v)^3}{f(u)^2+f(v)^2} \right\rceil$ (or) $\left\lfloor \frac{f(u)^3+f(v)^3}{f(u)^2+f(v)^2} \right\rfloor$, then the edge labels are distinct. In this case "f " is called Lehmer -3 mean labeling of G.

B. Definition 1.2

Let G be a graph and f:V(G)→{1,2,….n} be a function such that the label of the edge f (e=uv) is Labeled with $\left\lceil \frac{f(u)^3+f(v)^3}{f(u)^2+f(v)^2} \right\rceil$ (or) $\left\lfloor \frac{f(u)^3+f(v)^3}{f(u)^2+f(v)^2} \right\rfloor$, where {(f(e);eЄE(G)}⊆{1,2,…..n}. If n is the smallest positive integer satisfying this condition together with the condition that there is no edges in common. Then n is called the Lehmer-3 mean number of a graph G and is denoted as L_{3m} (G).

II. MAIN RESULTS

A. Theorem :2.1

L_{3m} (P_n) = n.

1) *Proof :* Let $u_1,u_2,\ldots u_n$ be the vertices of the path P_n. Define a function f:V(P_n)→{1,2,….n} by f(u_1)=1, f(u_i)= i + 1; 2 ≤ i ≤ n. Then the edge labels are f(u_i u_{i+1}) = i ; 1 ≤ i ≤ n-1, f(u_{n-1} u_n) = n. Thus L_{3m} (P_n) = n.

B. Example 2.2

Lehmer-3 mean number of P_7 is
L_{3m} (P_n) = n = 7 is given below.

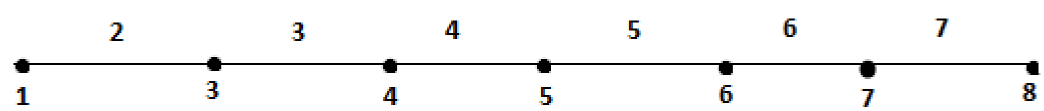


Figure-1

C. Theorem :2.3

L_{3m} (C_n) = n +1

1) *Proof :* Let C_n be a cycle of n vertices u_1, u_2, …u_n, u_1. We define a function f:V(C_n)→{1,2,….n} by f(u_1)=1, f(u_i)= i + 1; 2 ≤ i ≤ n. Then the distinct edge labels are f(u_i u_{i+1}) = i+1 ; 1 ≤ i ≤ n-1, f(u_n u_1) = n+1. Thus L_{3m} (C_n) = n+1.

D. Example 2.4

L_{3m} (C_8) = n+1 = 8+1 = 9 is displayed below.

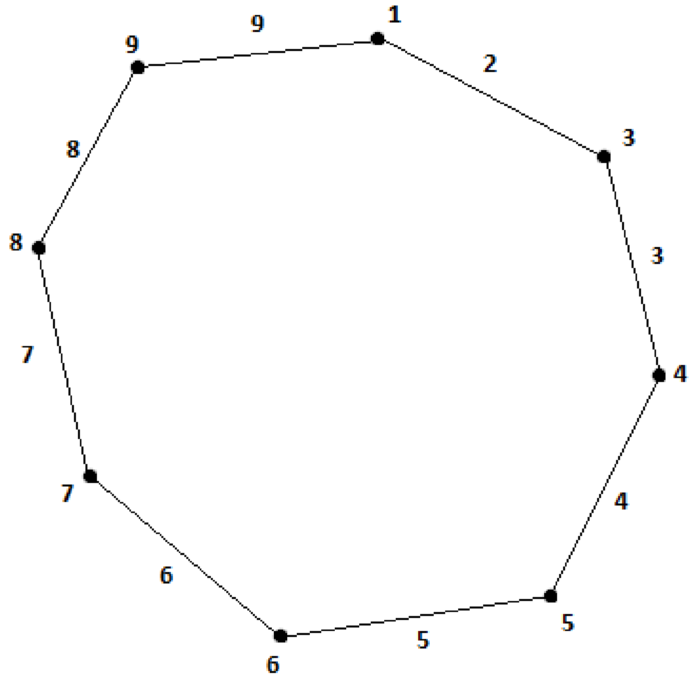


Figure-2

E. Theorem :2.5

$L_{3m}(P_n \Theta K_1) = 2n$.

1) *Proof :* Let G be a $P_n \Theta K_1$, graph and its vertices $u_1, u_2, \ldots u_n, v_1, v_2, \ldots v_n$. respectively. Let us define a function $f:V(G)\rightarrow\{1,2,\ldots.n\}$ by $f(u_1)=1$, $f(u_i)= 2i$; $2 \le i \le n$, $f(v_i)= 2i+1$; $1 \le i \le n$. and the obtained distinct edge labels are $f(u_i, u_{i+1})=2i+1$; $1 \le i \le n-1$, $f(u_i v_i)= 2i$; $1 \le i \le n-1$, $f(u_n v_n)= 2n$

Hence $L_{3m}(P_n \Theta K_1) = 2n$

F. Example 2.6

$L_{3m}(P_6 \Theta K_1) = 2n = 2x6 = 12$ is given below

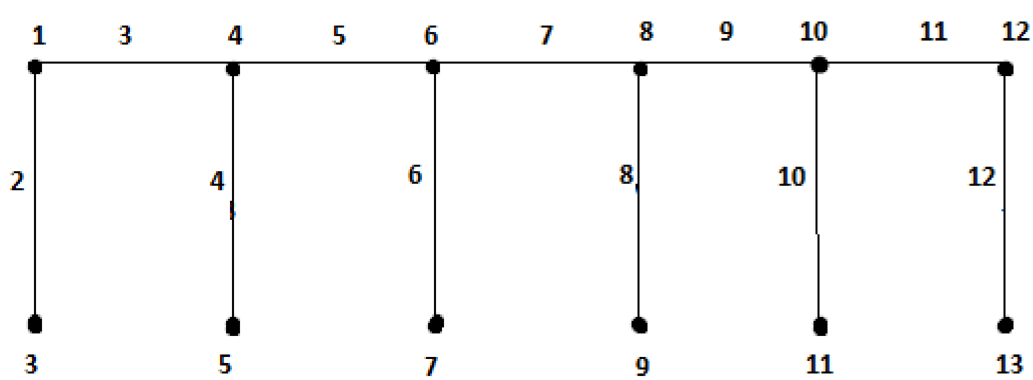


Figure-3

G. Theorem :2.7

L_{3m} (P_n ΘK_1) ΘK_1) = 3n.

1) *Proof :* Let G be a graph of n vertices $u_1, u_2, \ldots u_n, v_1, v_2, \ldots v_n, w_1, w_2, \ldots w_n$ respectively. Define a function $f:V(G)\rightarrow\{1,2,\ldots.n\}$ by $f(u_1)=1$, $f(u_i)= 3i-1$; $2 \le i \le n$, $f(v_i)= 3i$; $1 \le i \le n$, $f(w_i)= 3i +1$; $1 \le i \le n$. then the distinct edge labels are $f(u_{i,} u_{i+1})=3i+1$; $1 \le i \le n-1$, $f(u_i v_i)= 3i-1$; $1 \le i \le n$, $f(v_iw_i)= 3i$; $1 \le i \le n-1$ and $f(v_nw_n)= 3n$. Therefore L_{3m} ((P_n ΘK_1) ΘK_1) = 3n

H. Example 2.8

Lehmer-3 mean number of graph with n=5 is given as

L_{3m} (G) = 3n = 3x5 = 15

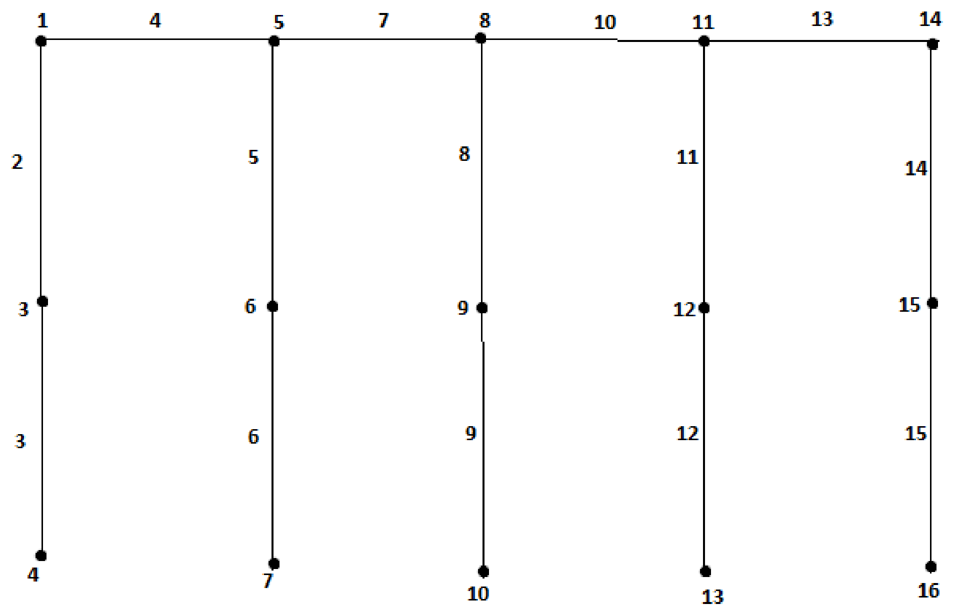


Figure-4

I. Theorem:2.9

L_{3m} (P_n Θ$K_{1,2}$) = 3n.

1) *Proof :* Let P_n be a path of n vertices $u_1, u_2, \ldots u_n$, and the vertices of $K_{1,2}$ is denoted as $v_1, v_2, \ldots v_n, w_1, w_2, \ldots w_n$. Define a function $f:V(G)\rightarrow\{1,2,\ldots.n\}$ by $f(u_1)=1$, $f(u_i)= 3i-1$; $2 \le i \le n$, $f(v_1)= 2$, $f(v_i)= 3i$; $2 \le i \le n$, $f(w_1)= 3$; $f(w_i)= 3i +1$; $2 \le i \le n$. The distinct edge labels are $f(u_{i,} u_{i+1})=3i+1$; $1 \le i \le n-1$, $f(u_1v_1)= 1$, $f(u_iv_i)= 3i-1$; $2 \le i \le n$, $f(u_1w_1)= 2$, $f(u_iw_i)= 3i$, $2 \le i \le n-1$, $f(u_nw_n)= 3n$ Thus L_{3m} (P_n Θ$K_{1,2}$) = 3n

J. Example 2.10

Lehmer-3 mean number of (P_5 Θ$K_{1,2}$) is

L_{3m} (P_5 Θ$K_{1,2}$) = 3n = 3x5 = 15

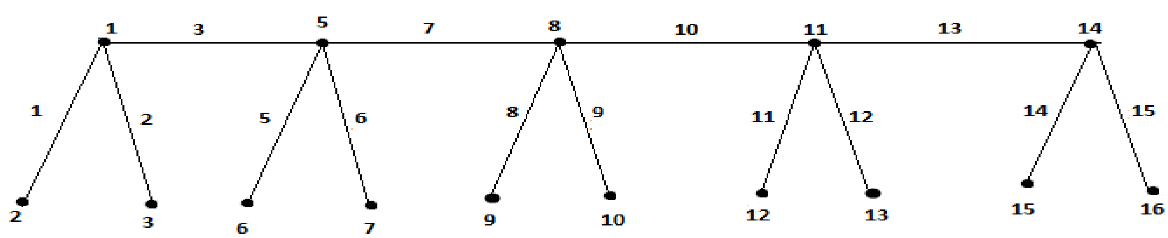


Figure-5

K. Theorem :2.11

L_{3m} (P_n $\Theta K_{1,3}$) = 4n.

1) *Proof :* Let P_n be a path of n vertices u_1, u_2, …u_n, Let $K_{1,3}$ be a complete graph with vertices v_i, w_i, x_i, such that $1 \le i \le n$ attached to the vertices of the path respectively. Define a function f:V(G)→{1,2,….n} by $f(u_1)=1$, $f(u_i)= 4i-2$; $2 \le i \le n$, $f(v_1)= 2$, $f(v_i)= 4i-1$; $2 \le i \le n$, $f(w_i)= 4n$; $1 \le i \le n$, $f(x_i)= 4i +1$; $1 \le i \le n$, Then we obtain distinct edge labels as $f(u_i, u_{i+1})=4i+1$; $1 \le i \le n-1$, $f(u_1v_1)= 1$, $f(u_iv_i)= 4i-2$; $2 \le i \le n$, $f(u_iw_i)= 4i-1$; $1 \le i \le n$, $f(u_ix_i)= 4i$, $1 \le i \le n-1$and $f(u_nx_n)= 4n$ Thus L_{3m} (P_n $\Theta K_{1,3}$) = 4n

L. Example 2.12

L_{3m} (P_6 $\Theta K_{1,3}$) = 4n = 4x6 = 24 is given below

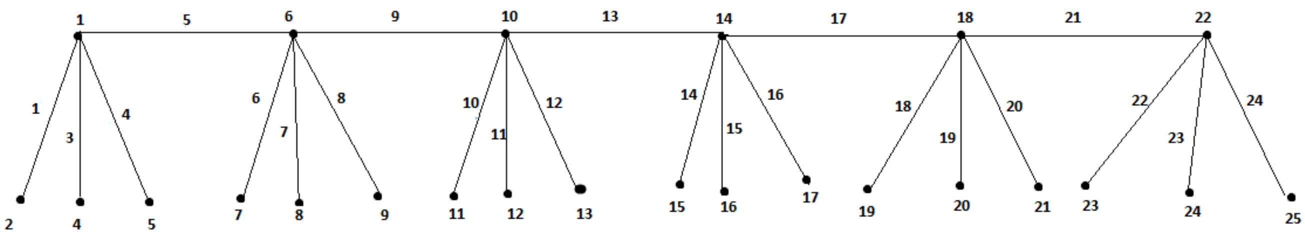


Figure-6

M. Theorem:2.13

L_{3m} (P_n ΘK_3) = 4n.

1) *Proof :* Let P_n be a path of n vertices and Let $K_{1,3}$ be a complete graph of n vertices attached to each vertices of the path. We define a function f:V(G)→{1,2,….n} by $f(u_1)=1$, $f(u_i)= 4i-2$; $2 \le i \le n$, $f(v_1)= 2$, $f(v_i)= 4i-1$; $2 \le i \le n$, $f(w_i)= 4i$; $1 \le i \le n$. Then the edge labels as $f(u_i, u_{i+1})=4i+1$; $1 \le i \le n-1$, $f(u_1v_1)= 1$, $f(u_iv_i)= 4i-2$; $2 \le i \le n$, $f(v_1w_1)= 3$; $f(v_iw_i)= 4i – 1$; $2 \le i \le n$ and $f(u_iw_i)= 4i$; $1 \le i \le n-1$ and $f(u_nw_n)= 4n$ Hence L_{3m} (P_n ΘK_3) = 4n.

N. Example 2.14

Lehmer-3 mean number of (P_5 ΘK_3) is given below

L_{3m} (P_5 ΘK_3) = 4n = 4x5 = 20

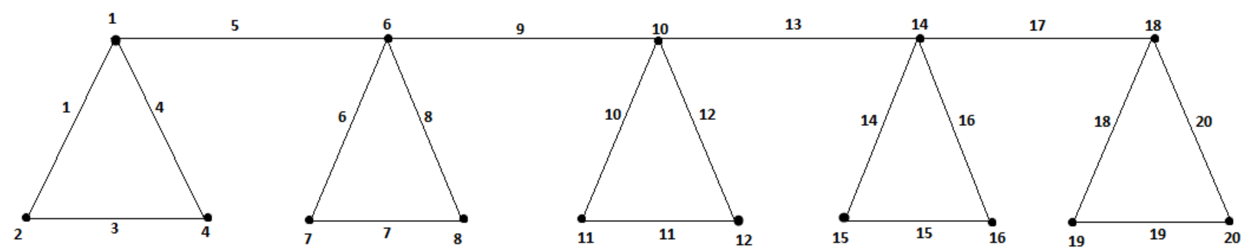


Figure-7

O. Theorem:2.15

L_{3m} (P_n ΘK_1) $\Theta K_{1,2}$) = 4n

1) *Proof :* Let (P_n ΘK_1) $\Theta K_{1,2}$) be a graph with vertices u_i, v_i, w_i, x_i, ; $1 \le i \le n$, where P_n ΘK_1 be a comb graph in which $K_{1,2}$ is attached to each vertex of the comb. A function defined on G by f:V(G)→{1,2,….n} by $f(u_1)=1$, $f(u_i)= 4i-2$; $2 \le i \le n$, $f(v_i)= 2$; $f(v_i)= 4i -1$; $2 \le i \le n$, $f(w_i)=4i$, $1 \le i \le n$, $f(x_i) = 4i+1$; $1 \le i \le n$ then the distinct edge labels are $f(u_i, u_{i+1})=4i+1$; $1 \le i \le n-1$, $f(u_1 v_1)= 1$; $f(u_iv_i)= 4i-2$; $2 \le i \le n$, $f(v_iw_i)= 4i-1$; $1 \le i \le n$, $f(v_ix_i)= 4i$; $1 \le i \le n$-, $f(v_nx_n)= 4n$. Thus the Lehmer -3 mean number of (P_n ΘK_1) $\Theta K_{1,2}$) is 4n

P. Example 2.16

L_{3m} (P_5 ΘK_1) $\Theta K_{1,2}$) = 4n = 4x5 = 20 is given below

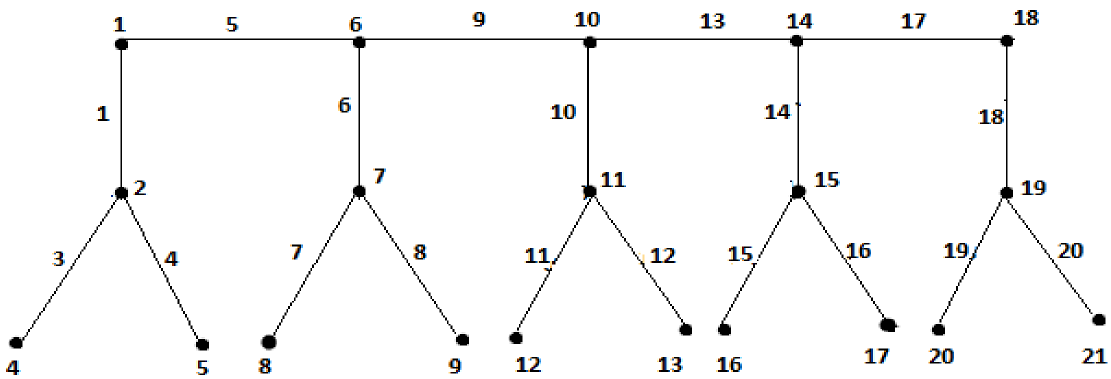


Figure-8

Q. Theorem:2.17

L_{3m} (C_n $ΘK_1$) = 2n+1

1) *Proof :* Let C_n be a cycle with vertices $u_1, u_2, \ldots u_n$, Let K_1 be a graph attached to each vertex of the cycle. Such that its vertices be $v_1, v_2, \ldots v_n$, Define a function $f:V(G)\rightarrow\{1,2,\ldots.n\}$ by $f(u_{1)}=1$, $f(u_i)= 2i$; $2 \le i \le n$, $f(v_i)= 2i+1$, $1 \le i \le n$. The edge labels as $f(u_{i,}\ u_{i+1})=2i+1$; $1 \le i \le n-1$, $f(u_nu_1)= 2n$, $f(u_iv_i)= 2i$; $1 \le i \le n-1$, $f(u_nv_n)= 2n+1$. Thus L_{3m} (C_n $ΘK_1$) = 2n+1

R. Example 2.18

Lehmer -3 mean number of C_6 $ΘK_1$ = 2n+1 = 2x6+1 = 13 is given below

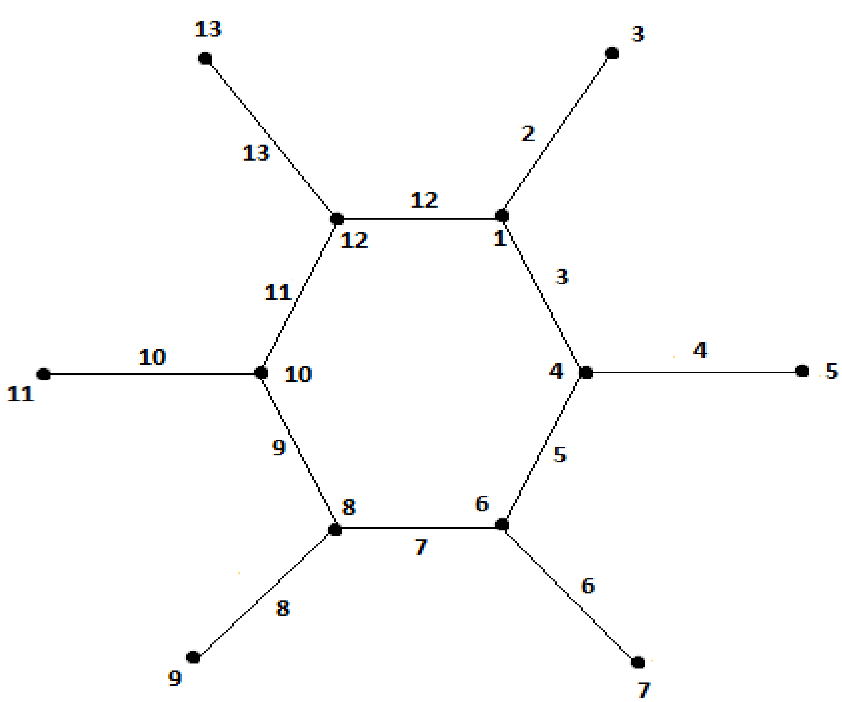


Figure-9

S. Theorem :2.19

L_{3m} (C_n Θ$K_{1,2}$) = 3n+1

1) *Proof :* Let C_n Θ$K_{1,2}$ be a graph of n vertices u_1, u_2, …u_n, v_1, v_2,…v_n, w_1, w_2,.. w_n respectively. A function defined on (C_n Θ$K_{1,2}$) as f:V(C_n Θ$K_{1,2}$)→{1,2,….n} by f(u_1)=1, f(u_i)= 3i-1 ; 2 ≤ i ≤ n, f(v_i)= 3i, 1 ≤ i ≤ n, f(w_i)= 3i+1 ; 1 ≤ i ≤ n. The edge labels as f(u_i, u_{i+1})=3i+1 ; 1 ≤ i ≤ n-1, f(u_nu_1)= 3n-1, f(u_iv_i)= 3i-1 ; 1 ≤ i ≤ n-1, f(u_nv_n)= 3n, f(u_iw_i)= 3i ; 1 ≤ i ≤ n-1 and f(u_nw_n)= 3n+1n Therefore L_{3m} (C_n ΘK_{12}) = 3n+1 Hence the proof

T. Example 2.20

L_{3m} (C_5 Θ$K_{1,2}$) = 3n+1 = 3x5+1 = 16 is shown below

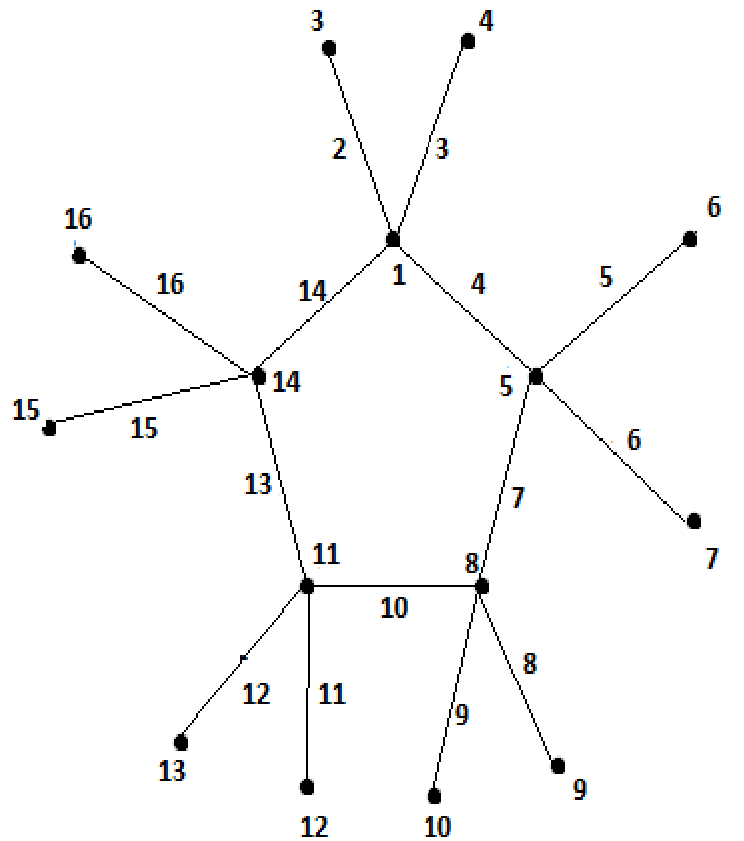


Figure-10

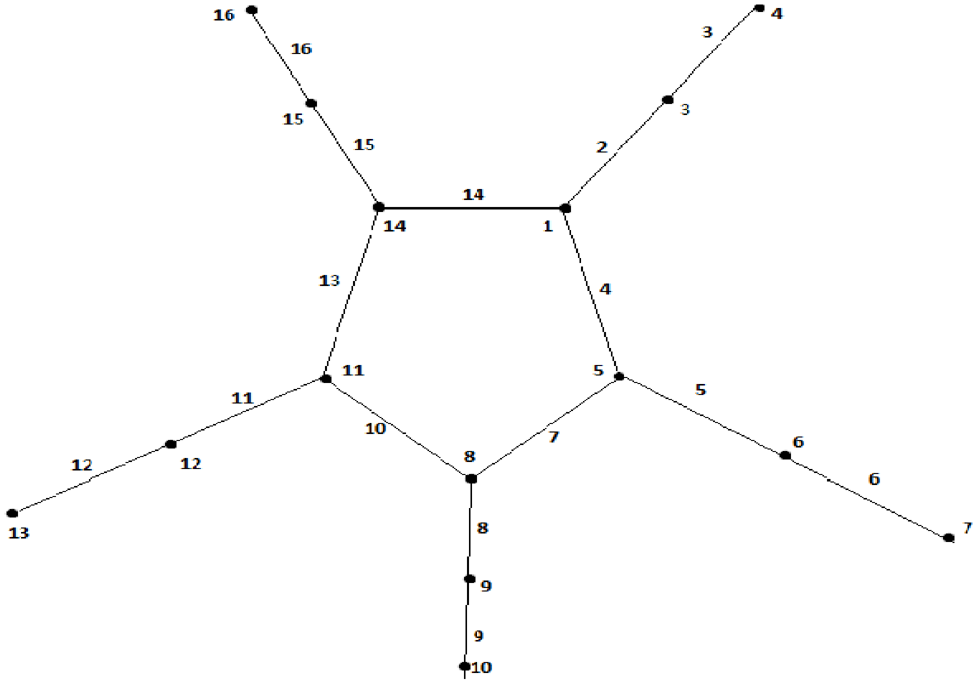
U. Theorem :2.21

L_{3m} (C_n ΘK_1) ΘK_1) = 3n+1

1) *Proof :* Let G be a (C_n ΘK_1) ΘK_1) graph the vertices be denoted u_i, v_i, w_i, where 1 ≤ i ≤ n respectively, w_i be the vertices attached t each pendant vertices of the crown. Define a function f:V(G)→{1,2,….n} by f(u_1)=1, f(u_i)= 3i-1 ; 2 ≤ i ≤ n, f(v_i)= 3i ; i=1,2…n, f(w_i)= 3i +1; 1 ≤ i ≤ n, The edge labels are f(u_i, u_{i+1})=3i+1 ; 1 ≤ i ≤ n-1, f(u_nu_1)= 3n-1; f(u_iv_i)= 3i-1 ; 2 ≤ i ≤ n-1, f(u_nv_n)= 3n, f(v_iw_i)= 3i ; 1 ≤ i ≤ n-1, f(v_nw_n)= 3n+1. Thus L_{3m}(G) = 3n+1 hence the proof

V. Example 2.22

L_{3m} (C_5 ΘK_1) ΘK_1) = 3n+1 = 3x5+1 = 16 diagram pattern is given below



W. Theorem :2.23

L_{3m} (C_n ΘK_1) $\Theta K_{1,2}$) = 4n+1

1) *Proof :* Let G be a graph of vertices $u_1, u_2, \ldots u_n, v_1, v_2, \ldots v_n, w_1, w_2, \ldots w_n, x_1, x_2, \ldots x_n$ respectively. This is a crown attached with $K_{1,2}$ at each pendant vertex of a crown. Define a function $f:V(G)\rightarrow\{1,2,3\ldots.n\}$ by $f(u_1)=1$, $f(u_i)= 4i-2$; $2 \le i \le n$, $f(v_i)= 4i-1$; $1 \le i \le n$, $f(w_i)= 4i$; $1 \le i \le n$, $f(x_i)= 4i+1$; $1 \le i \le n$ The distinct edge labels are $f(u_i, u_{i+1})=4i+1$; $1 \le i \le n-1$, $f(u_nu_1)= 4n-2$; $f(u_iv_i)= 4i-1$; $1 \le i \le n-1$, $f(u_nv_n)= 4n-1$, $f(v_iw_i)= 4i-1$; $1 \le i \le n-1$, $f(v_nw_n)= 4n$, $f(v_ix_i)= 4i$; $1 \le i \le n-1$, $f(v_nx_n)= 4n+1$ Thus $L_{3m}(C_n \Theta K_1)$ $\Theta K_{1,2}$) = 4n+1

X. Example 2.24

Lehmer -3 mean number of (C_4 ΘK_1) $\Theta K_{1,2}$) is 4n+1 = 4x4+1 = 16+1 = 17 is given below

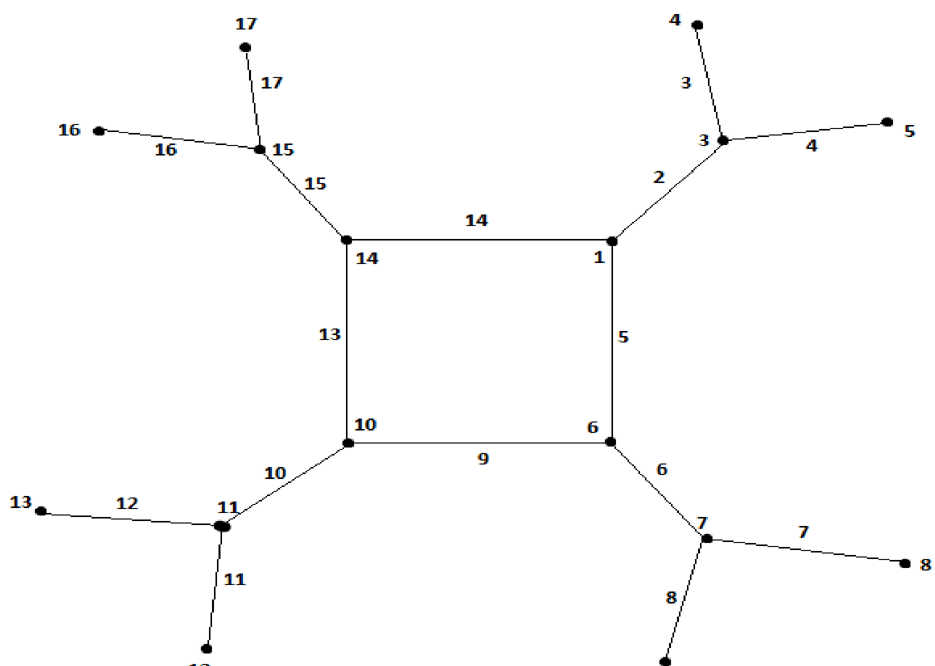


Figure-12

Y. Theorem :2.25

L_{3m} $(L_n) = 3n - 1$

1) *Proof :* Let L_n be a ladder graph of n vertices $u_1, u_2, \ldots u_n, v_1, v_2, \ldots v_n$, respectively. Define a function $f:V(L_n)\rightarrow\{1,2,\ldots.n\}$ by $f(u_i)=3i-2$; $1 \leq i \leq n$, $f(v_i)= 3i-1$; $1 \leq i \leq n$. The edge are labeled as $f(u_iu_{i+1}) = 3i$; $1 \leq i \leq n-1$, $f(v_iv_{i+1}) = 3i+1$; $1 \leq i \leq n-1$, $f(u_1v_1) = 1$, $f(u_iv_i) = 3i-1$; $2 \leq i \leq n-1$, $f(u_nv_n) = 3n-1$. Hence L_{3m} $(L_n) = 3n-1$. Hence ladder satisfies lehmer -3 mean number

Z. Example 2.26

Lehmer -3 mean number of L7 is given as L_{3m} (L7) = 3x7-1 = 21-1 = 20.

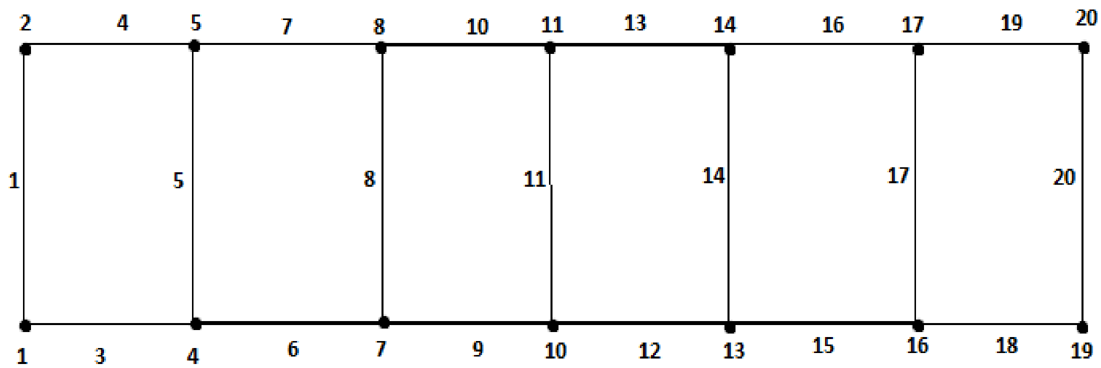


Figure-13

AA. Theorem :2.27

L_{3m} $(D_n) = n + m$

1) *Proof :* Let D_n be a dragon graph where the head of the dragon has n vertices $u_1, u_2, \ldots u_n$, and the path attached to it be $v_1, v_2, \ldots v_m$. Define a function of $f:V(D_n)\rightarrow\{1,2,\ldots.n\}$ by $f(u_1=v_1) = 1$, $f(u_i) = i+1$; $2 \leq i \leq n$, $f(v_i) = n+j$; $2 \leq j \leq m$. The distinct edge labels are $f(u_iu_{i+1}) = i+1$; $1 \leq i \leq n$, $f(v_jv_{j+1}) = (n+1)+j$; $1 \leq j \leq m-2$, $f(v_{n-1}v_n) = n + m$.

Thus L_{3m} $(D_n) = n+m$.

BB. Example 2.28

Lehmer -3 mean number of D_n is given below. $L_{3m}(D_n) = m+n$

Hence m=5 and n=6; n+m = 5+6 = 11 is drawn as

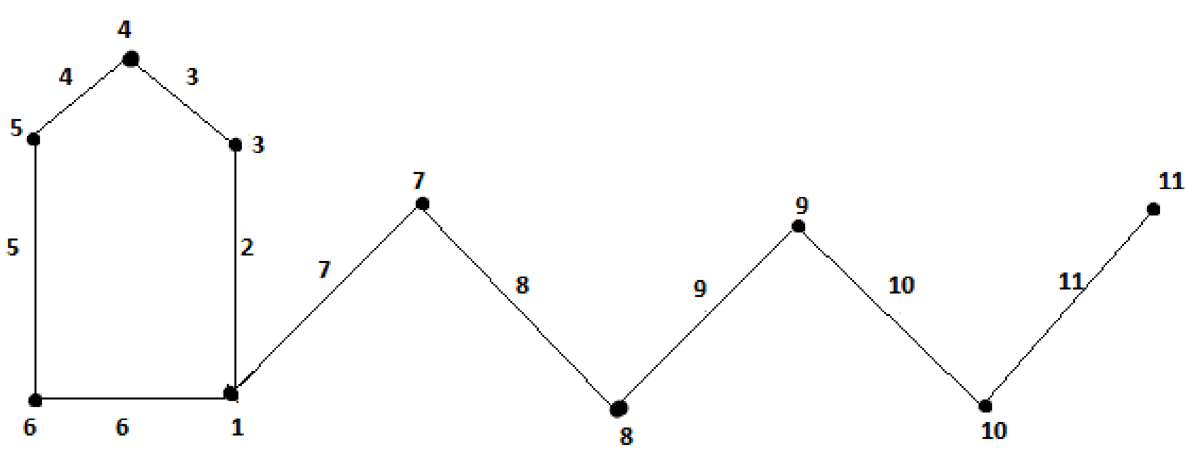


Figure-15

CC. Theorem :2.29

Lehmer -3 mean number of caterpillar is 3n

1) *Proof :* Let G be a graph of n vertices where each vertex of path is attached to pendant vertex on its both side. Let the vertices be denoted as $u_1, u_2, \ldots u_n, v_1, v_2, \ldots v_n, w_1, w_2, \ldots w_n$. Let $f:V(G) \rightarrow \{1,2,\ldots.n\}$ be a function defined by $f(u_1) = 1$, $f(u_i) = 3i-1$; $2 \leq i \leq n$, $f(v_i) = 3i$; $1 \leq i \leq n$, $f(w_i) = 3i+1$; $1 \leq i \leq n$ and The edges are labeled as $f(u_iu_{i+1}) = 3i+1$; $1 \leq i \leq n-1$, $f(u_iv_i) = 3i-1$; $1 \leq i \leq n$, $f(u_iw_i) = 3i$; $1 \leq i \leq n-1$, and $f(u_nw_n) = 3n$. hence the Lehmer -3 mean number of a caterpillar is 3n

DD. Example 2.30

Lehmer -3 mean number of a caterpillar of 5 vertices is 3n = 3x5 is given below

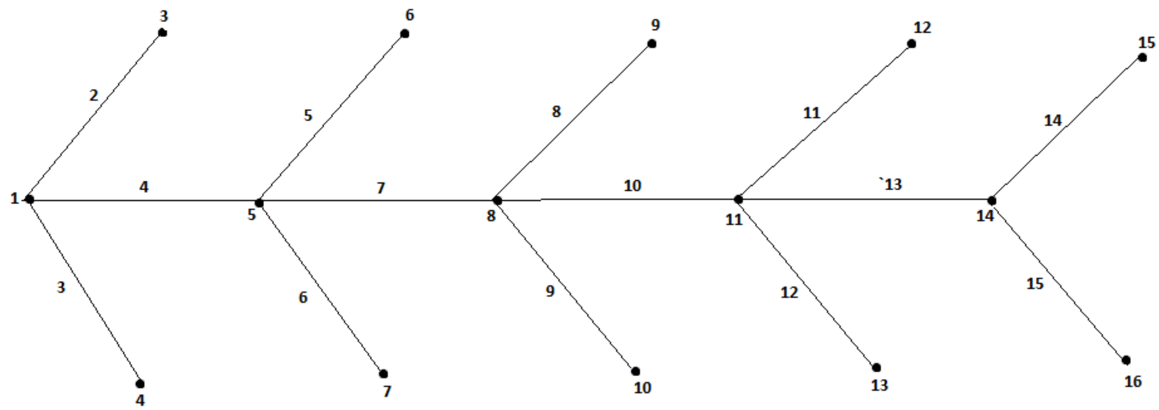


Figure-16

REFERENCES

[1] Harary.F 1988 Graph theory, Narosa Publication House reading, New Delhi

[2] S Somasundaram and R Ponraj and S S Sandhya 'Harmonic mean labeling of graphs' ommunicated to journal of combinatorial mathematics and combinatorial computing.

[3] S.Somasundaram, S.S.Sandhya and T.S.Pavithra, "Lehmer-3 Mean Labeling of graphs" "International Journal of Mathematical Forum"

[4] S.Somasundaram, S.S.Sandhya and T.S.Pavithra, "Lehmer -3 Mean Labeling of Disconnected Graphs" in "Asia Pacific Journal of Research ''ISSN 2320-5504 ,vol:I. issue XL, June 2016.

[5] S.Somasundaram, S.S.Sandhya and T.S.Pavithra, "Lehmer -3 Mean Labeling of Some New Disconnected Graphs" in "International Journal of Mathematics Trends and Technology '' ISSN:2231-5373 ,vol:35. Number 1, July 2016.

[6] S.Somasundaram, S.S.Sandhya and T.S.Pavithra, "Lehmer -3 Mean Labeling of Some Disconnected Graphs" in "International Journal of Mathematics Research'' ISSN:0976- 5840, vol 8 ,Number 2(2016) .