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# Lehmer-3 Mean Number of Graphs

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## I. INTRODUCTION

A graph considered here are finite, undirected and simple. The vertex set and edge set of a graph are denoted by  $V(G)$  and  $E(G)$  respectively. A path of length  $n$  is denoted by  $P_n$ . For standard terminology and notations we follow Harary[1]. S Somasundaram & S.S Sandhya introduced the concept of Harmonic Mean Labeling of Graphs in [2]. We will provide a brief summary of other information's which are necessary for our present investigation.

### A. Definition 1.1

A graph  $G=(V,E)$  with  $P$  vertices and  $q$  edges is called Lehmer -3 mean graph. If it is possible to label vertices  $x \in V$  with distinct labels  $f(x)$  from  $1,2,3,\dots,q+1$  in such a way that when each edge  $e=uv$  is labeled with  $f(e=uv)=\left[\frac{f(u)^3+f(v)^3}{f(u)^2+f(v)^2}\right]$  (or)  $\left\lfloor\frac{f(u)^3+f(v)^3}{f(u)^2+f(v)^2}\right\rfloor$ , then the edge labels are distinct. In this case "f" is called Lehmer -3 mean labeling of  $G$ .

### B. Definition 1.2

Let  $G$  be a graph and  $f:V(G)\rightarrow\{1,2,\dots,n\}$  be a function such that the label of the edge  $f(e=uv)$  is Labeled with  $\left[\frac{f(u)^3+f(v)^3}{f(u)^2+f(v)^2}\right]$  (or)  $\left\lfloor\frac{f(u)^3+f(v)^3}{f(u)^2+f(v)^2}\right\rfloor$ , where  $\{f(e);e\in E(G)\}\subseteq\{1,2,\dots,n\}$ . If  $n$  is the smallest positive integer satisfying this condition together with the condition that there is no edges in common. Then  $n$  is called the Lehmer-3 mean number of a graph  $G$  and is denoted as  $L_{3m}(G)$ .

## II. MAIN RESULTS

### A. Theorem :2.1

$L_{3m}(P_n) = n$ .

1) Proof : Let  $u_1, u_2, \dots, u_n$  be the vertices of the path  $P_n$ . Define a function  $f:V(P_n)\rightarrow\{1,2,\dots,n\}$  by  $f(u_1)=1, f(u_i)=i+1; 2 \leq i \leq n$ . Then the edge labels are  $f(u_i u_{i+1}) = i; 1 \leq i \leq n-1, f(u_{n-1} u_n) = n$ . Thus  $L_{3m}(P_n) = n$ .

### B. Example 2.2

Lehmer-3 mean number of  $P_7$  is

$L_{3m}(P_n) = n = 7$  is given below.

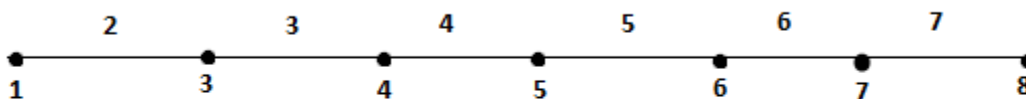


Figure-1

### C. Theorem :2.3

$L_{3m}(C_n) = n + 1$

1) Proof : Let  $C_n$  be a cycle of  $n$  vertices  $u_1, u_2, \dots, u_n, u_1$ . We define a function  $f:V(C_n)\rightarrow\{1,2,\dots,n\}$  by  $f(u_1)=1, f(u_i)=i+1; 2 \leq i \leq n$ . Then the distinct edge labels are  $f(u_i u_{i+1}) = i+1; 1 \leq i \leq n-1, f(u_n u_1) = n+1$ . Thus  $L_{3m}(C_n) = n+1$ .

### D. Example 2.4

$L_{3m}(C_8) = n+1 = 8+1 = 9$  is displayed below.

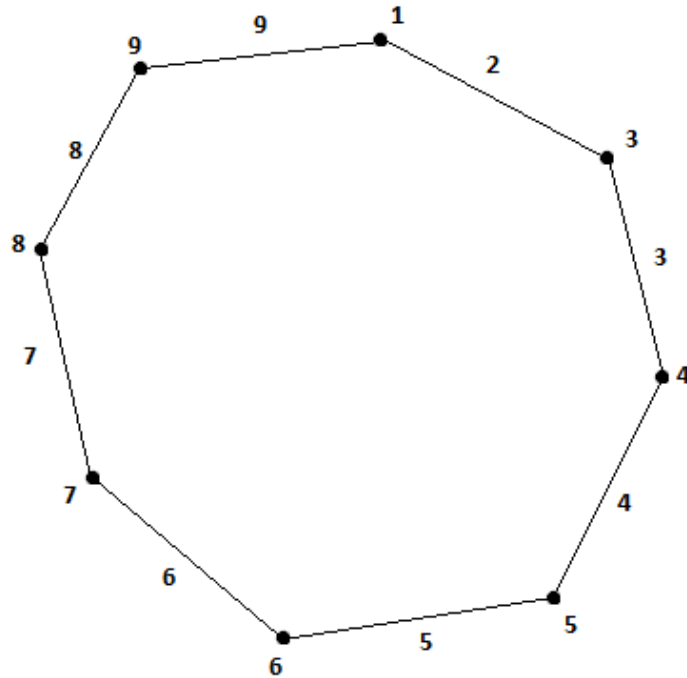


Figure-2

E. Theorem :2.5

$$L_{3m}(P_n \circ K_1) = 2n.$$

1) Proof : Let  $G$  be a  $P_n \circ K_1$  graph and its vertices  $u_1, u_2, \dots, u_n, v_1, v_2, \dots, v_n$ . respectively. Let us define a function  $f:V(G) \rightarrow \{1,2,\dots,n\}$  by  $f(u_1)=1, f(u_i)= 2i ; 2 \leq i \leq n, f(v_i)= 2i+1 ; 1 \leq i \leq n$ . and the obtained distinct edge labels are  $f(u_i, u_{i+1})=2i+1 ; 1 \leq i \leq n-1, f(u_i, v_i)= 2i ; 1 \leq i \leq n-1, f(u_n, v_n)= 2n$

Hence  $L_{3m}(P_n \circ K_1) = 2n$

F. Example 2.6

$L_{3m}(P_6 \circ K_1) = 2n = 2 \times 6 = 12$  is given below

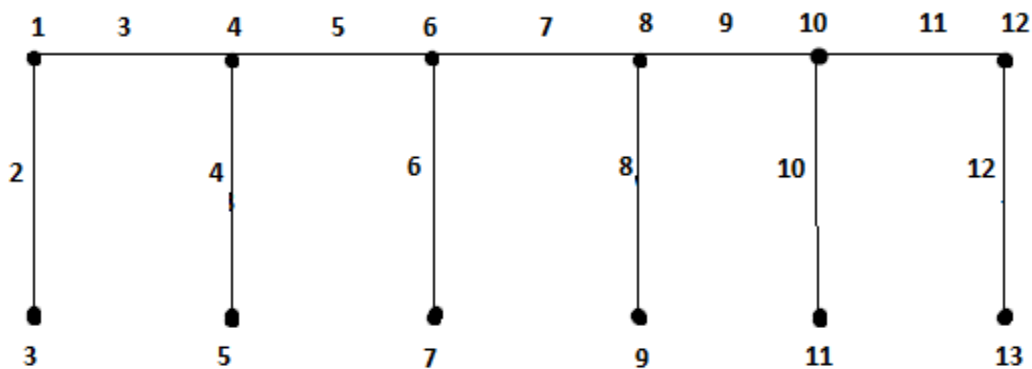


Figure-3

G. Theorem :2.7

$$L_{3m}(P_n \circ K_1) \circ K_1 = 3n.$$

1) Proof : Let G be a graph of n vertices  $u_1, u_2, \dots, u_n, v_1, v_2, \dots, v_n, w_1, w_2, \dots, w_n$  respectively. Define a function  $f:V(G) \rightarrow \{1,2,\dots,n\}$  by  $f(u_1)=1, f(u_i)=3i-1; 2 \leq i \leq n, f(v_i)=3i; 1 \leq i \leq n, f(w_i)=3i+1; 1 \leq i \leq n$ . then the distinct edge labels are  $f(u_i, u_{i+1})=3i+1; 1 \leq i \leq n-1, f(u_i, v_i)=3i-1; 1 \leq i \leq n, f(v_i, w_i)=3i; 1 \leq i \leq n-1$  and  $f(v_n, w_n)=3n$ . Therefore  $L_{3m}((P_n \circ K_1) \circ K_1) = 3n$

H. Example 2.8

Lehmer-3 mean number of graph with n=5 is given as

$$L_{3m}(G) = 3n = 3 \times 5 = 15$$

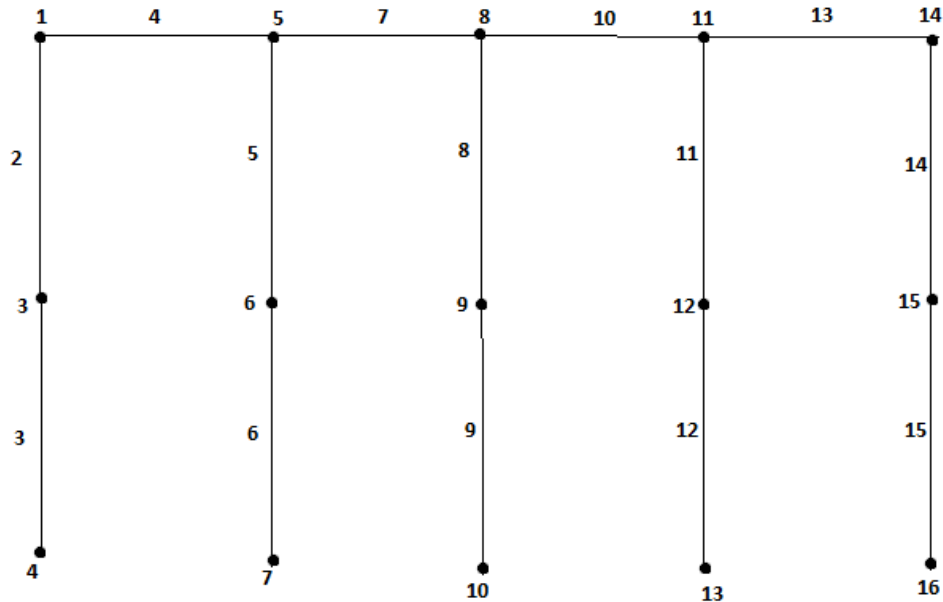


Figure-4

I. Theorem:2.9

$$L_{3m}(P_n \circ K_{1,2}) = 3n.$$

1) Proof : Let  $P_n$  be a path of n vertices  $u_1, u_2, \dots, u_n$ , and the vertices of  $K_{1,2}$  is denoted as  $v_1, v_2, \dots, v_n, w_1, w_2, \dots, w_n$ . Define a function  $f:V(G) \rightarrow \{1,2,\dots,n\}$  by  $f(u_1)=1, f(u_i)=3i-1; 2 \leq i \leq n, f(v_i)=2, f(v_i)=3i; 2 \leq i \leq n, f(w_i)=3; f(w_i)=3i+1; 2 \leq i \leq n$ . The distinct edge labels are  $f(u_i, u_{i+1})=3i+1; 1 \leq i \leq n-1, f(u_i, v_i)=1, f(u_i, v_i)=3i-1; 2 \leq i \leq n, f(u_i, w_i)=2, f(u_i, w_i)=3i, 2 \leq i \leq n-1, f(u_n, w_n)=3n$  Thus  $L_{3m}(P_n \circ K_{1,2}) = 3n$

J. Example 2.10

Lehmer-3 mean number of  $(P_5 \circ K_{1,2})$  is

$$L_{3m}(P_5 \circ K_{1,2}) = 3n = 3 \times 5 = 15$$

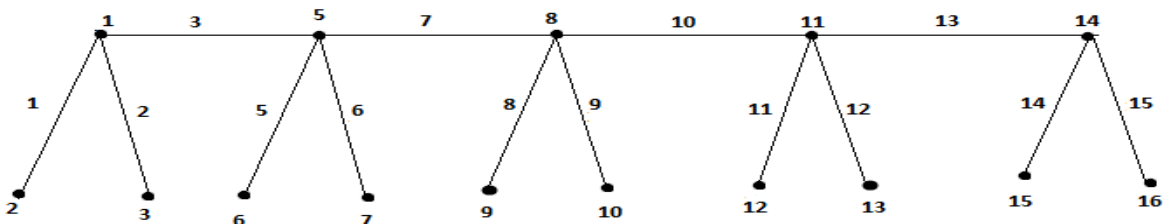


Figure-5

**K. Theorem :2.11**

$$L_{3m}(P_n \circ K_{1,3}) = 4n.$$

1) *Proof* : Let  $P_n$  be a path of  $n$  vertices  $u_1, u_2, \dots, u_n$ , Let  $K_{1,3}$  be a complete graph with vertices  $v_i, w_i, x_i$ , such that  $1 \leq i \leq n$  attached to the vertices of the path respectively. Define a function  $f:V(G) \rightarrow \{1,2,\dots,n\}$  by  $f(u_1)=1, f(u_i)= 4i-2 ; 2 \leq i \leq n, f(v_1)= 2, f(v_i)= 4i-1 ; 2 \leq i \leq n, f(w_i)= 4n ; 1 \leq i \leq n, f(x_i)= 4i + 1 ; 1 \leq i \leq n$ , Then we obtain distinct edge labels as  $f(u_i, u_{i+1})=4i+1 ; 1 \leq i \leq n-1, f(u_1v_1)= 1, f(u_iv_i)= 4i-2 ; 2 \leq i \leq n, f(u_iw_i)= 4i-1 ; 1 \leq i \leq n, f(u_ix_i)= 4i, 1 \leq i \leq n-1$  and  $f(u_nx_n)= 4n$  Thus  $L_{3m}(P_n \circ K_{1,3}) = 4n$

**L. Example 2.12**

$L_{3m}(P_6 \circ K_{1,3}) = 4n = 4 \times 6 = 24$  is given below

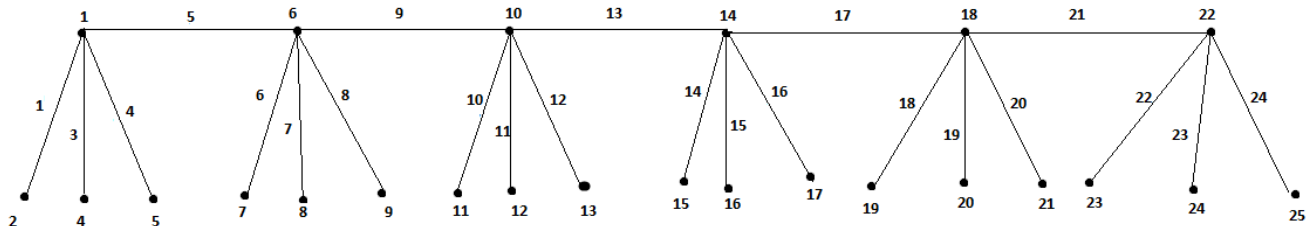


Figure-6

**M. Theorem:2.13**

$$L_{3m}(P_n \circ K_3) = 4n.$$

1) *Proof* : Let  $P_n$  be a path of  $n$  vertices and Let  $K_{1,3}$  be a complete graph of  $n$  vertices attached to each vertices of the path. We define a function  $f:V(G) \rightarrow \{1,2,\dots,n\}$  by  $f(u_1)=1, f(u_i)= 4i-2 ; 2 \leq i \leq n, f(v_1)= 2, f(v_i)= 4i-1 ; 2 \leq i \leq n, f(w_i)= 4i ; 1 \leq i \leq n$ . Then the edge labels as  $f(u_i, u_{i+1})=4i+1 ; 1 \leq i \leq n-1, f(u_1v_1)= 1, f(u_iv_i)= 4i-2 ; 2 \leq i \leq n, f(v_1w_1)= 3; f(v_iw_i)= 4i - 1 ; 2 \leq i \leq n$  and  $f(u_iw_i)= 4i ; 1 \leq i \leq n-1$  and  $f(u_nw_n)= 4n$  Hence  $L_{3m}(P_n \circ K_3) = 4n$ .

**N. Example 2.14**

Lehmer-3 mean number of  $(P_5 \circ K_3)$  is given below

$$L_{3m}(P_5 \circ K_3) = 4n = 4 \times 5 = 20$$

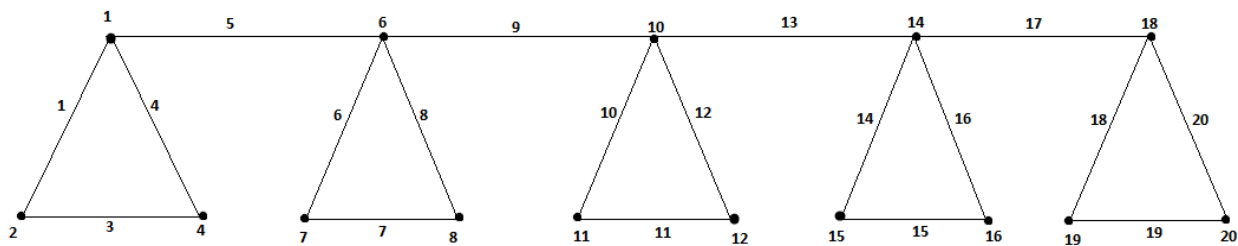


Figure-7

**O. Theorem:2.15**

$$L_{3m}(P_n \circ K_1) \circ K_{1,2} = 4n$$

1) *Proof* : Let  $(P_n \circ K_1) \circ K_{1,2}$  be a graph with vertices  $u_i, v_i, w_i, x_i ; 1 \leq i \leq n$ , where  $P_n \circ K_1$  be a comb graph in which  $K_{1,2}$  is attached to each vertex of the comb. A function defined on  $G$  by  $f:V(G) \rightarrow \{1,2,\dots,n\}$  by  $f(u_1)=1, f(u_i)= 4i-2 ; 2 \leq i \leq n, f(v_i)= 2 ; f(v_i)= 4i - 1 ; 2 \leq i \leq n, f(w_i)=4i, 1 \leq i \leq n, f(x_i) = 4i+1 ; 1 \leq i \leq n$  then the distinct edge labels are  $f(u_i, u_{i+1})=4i+1 ; 1 \leq i \leq n-1, f(u_1v_1)= 1; f(u_iv_i)= 4i-2 ; 2 \leq i \leq n, f(v_iw_i)= 4i-1 ; 1 \leq i \leq n, f(v_ix_i)= 4i ; 1 \leq i \leq n-$ ,  $f(v_nx_n)= 4n$ . Thus the Lehmer -3 mean number of  $(P_n \circ K_1) \circ K_{1,2}$  is  $4n$

**P. Example 2.16**

$L_{3m}(P_5 \circ K_1) \circ K_{1,2} = 4n = 4 \times 5 = 20$  is given below

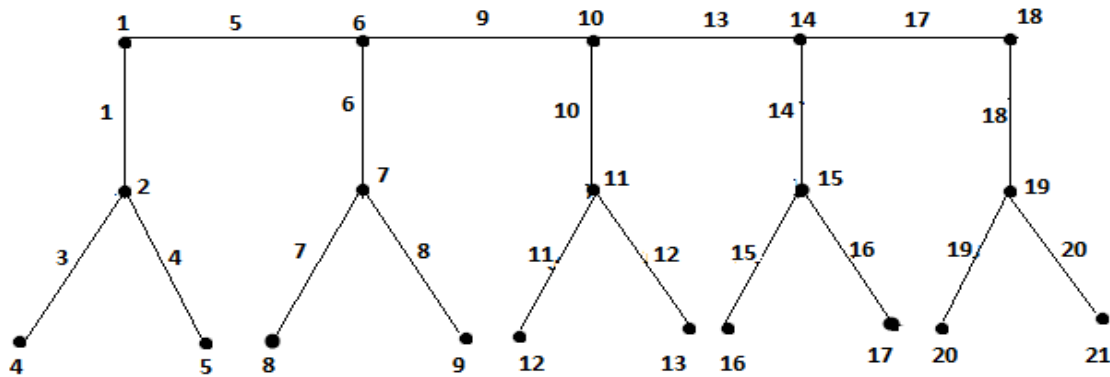


Figure-8

Q. Theorem:2.17

$$L_{3m}(C_n \circledast K_1) = 2n+1$$

1) Proof : Let  $C_n$  be a cycle with vertices  $u_1, u_2, \dots, u_n$ , Let  $K_1$  be a graph attached to each vertex of the cycle. Such that its vertices be  $v_1, v_2, \dots, v_n$ , Define a function  $f:V(G) \rightarrow \{1,2,\dots,n\}$  by  $f(u_1)=1, f(u_i)=2i; 2 \leq i \leq n, f(v_i)=2i+1, 1 \leq i \leq n$ . The edge labels as  $f(u_i, u_{i+1})=2i+1; 1 \leq i \leq n-1, f(u_n u_1)=2n, f(u_i v_i)=2i; 1 \leq i \leq n-1, f(u_n v_n)=2n+1$ . Thus  $L_{3m}(C_n \circledast K_1) = 2n+1$

R. Example 2.18

Lehmer -3 mean number of  $C_6 \circledast K_1 = 2n+1 = 2 \times 6+1 = 13$  is given below

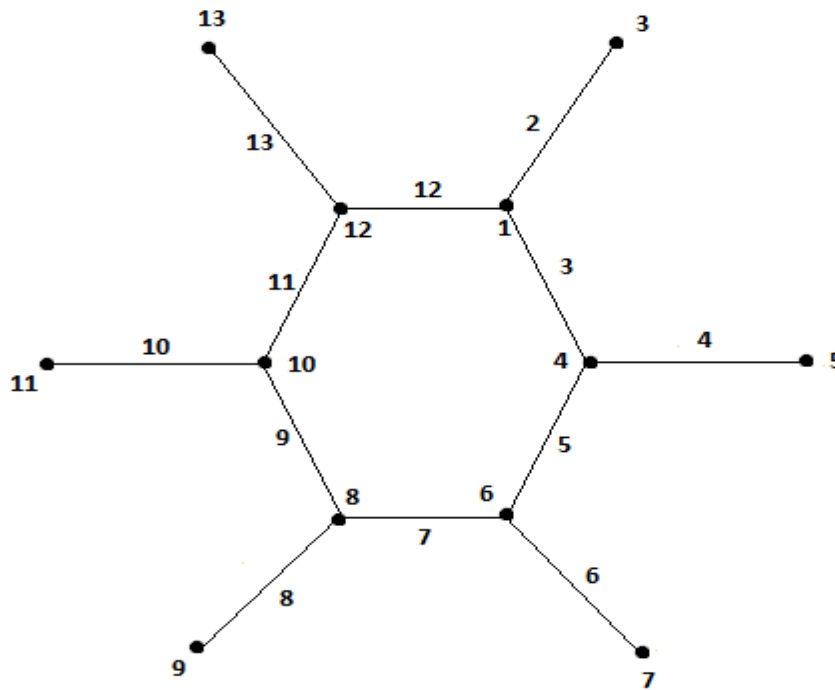


Figure-9

S. *Theorem :2.19*

$$L_{3m}(C_n \circ K_{1,2}) = 3n+1$$

1) *Proof* : Let  $C_n \circ K_{1,2}$  be a graph of  $n$  vertices  $u_1, u_2, \dots, u_n, v_1, v_2, \dots, v_n, w_1, w_2, \dots, w_n$  respectively. A function defined on  $(C_n \circ K_{1,2})$  as  $f:V(C_n \circ K_{1,2}) \rightarrow \{1,2,\dots,n\}$  by  $f(u_1)=1, f(u_i)= 3i-1 ; 2 \leq i \leq n, f(v_i)= 3i, 1 \leq i \leq n, f(w_i)= 3i+1 ; 1 \leq i \leq n$ . The edge labels as  $f(u_i, u_{i+1})=3i+1 ; 1 \leq i \leq n-1, f(u_n u_1)= 3n-1, f(u_i v_i)= 3i-1 ; 1 \leq i \leq n-1, f(u_n v_n)= 3n, f(u_i w_i)= 3i ; 1 \leq i \leq n-1$  and  $f(u_n w_n)= 3n+1n$  Therefore  $L_{3m}(C_n \circ K_{1,2}) = 3n+1$  Hence the proof

T. *Example 2.20*

$L_{3m}(C_5 \circ K_{1,2}) = 3n+1 = 3 \times 5+1 = 16$  is shown below

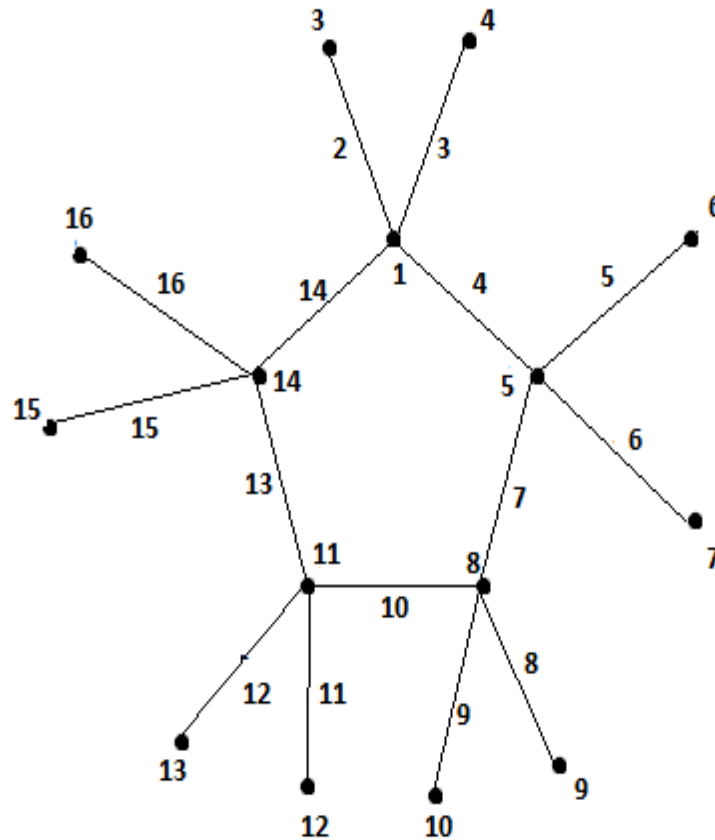


Figure-10

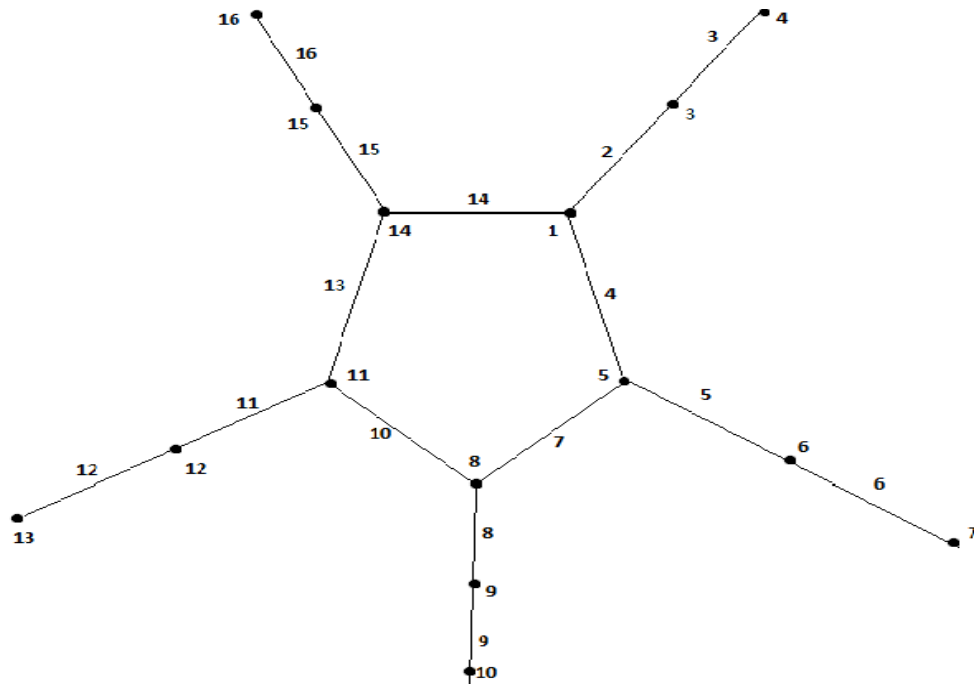
U. *Theorem :2.21*

$$L_{3m}(C_n \circ K_1) \circ K_1 = 3n+1$$

1) *Proof* : Let  $G$  be a  $(C_n \circ K_1) \circ K_1$  graph the vertices be denoted  $u_i, v_i, w_i$ , where  $1 \leq i \leq n$  respectively,  $w_i$  be the vertices attached to each pendant vertices of the crown. Define a function  $f:V(G) \rightarrow \{1,2,\dots,n\}$  by  $f(u_1)=1, f(u_i)= 3i-1 ; 2 \leq i \leq n, f(v_i)= 3i ; i=1,2,\dots,n, f(w_i)= 3i+1 ; 1 \leq i \leq n$ , The edge labels are  $f(u_i, u_{i+1})=3i+1 ; 1 \leq i \leq n-1, f(u_n u_1)= 3n-1 ; f(u_i v_i)= 3i-1 ; 2 \leq i \leq n-1, f(u_n v_n)= 3n, f(v_i w_i)= 3i ; 1 \leq i \leq n-1, f(v_n w_n)= 3n+1$ . Thus  $L_{3m}(G) = 3n+1$  hence the proof

V. *Example 2.22*

$L_{3m}(C_5 \circ K_1) \circ K_1 = 3n+1 = 3 \times 5+1 = 16$  diagram pattern is given below



W. Theorem :2.23

$$L_{3m}(C_n \circ K_1) \circ K_{1,2} = 4n+1$$

1) Proof : Let G be a graph of vertices  $u_1, u_2, \dots, u_n, v_1, v_2, \dots, v_n, w_1, w_2, \dots, w_n, x_1, x_2, \dots, x_n$  respectively. This is a crown attached with  $K_{1,2}$  at each pendant vertex of a crown. Define a function  $f:V(G) \rightarrow \{1,2,3,\dots,n\}$  by  $f(u_1)=1, f(u_i)= 4i-2 ; 2 \leq i \leq n, f(v_i)= 4i-1 ; 1 \leq i \leq n, f(w_i)= 4i ; 1 \leq i \leq n, f(x_i)= 4i+1 ; 1 \leq i \leq n$  The distinct edge labels are  $f(u_i, u_{i+1})=4i+1 ; 1 \leq i \leq n-1, f(u_n u_1)= 4n-2; f(u_i v_i)= 4i-1 ; 1 \leq i \leq n-1, f(u_n v_n)= 4n-1, f(v_i w_i)= 4i-1 ; 1 \leq i \leq n-1, f(v_n w_n)= 4n, f(v_i x_i)= 4i ; 1 \leq i \leq n-1, f(v_n x_n)= 4n+1$  Thus  $L_{3m}(C_n \circ K_1) \circ K_{1,2} = 4n+1$

X. Example 2.24

Lehmer -3 mean number of  $(C_4 \circ K_1) \circ K_{1,2}$  is  $4n+1 = 4 \times 4+1 = 16+1 = 17$  is given below

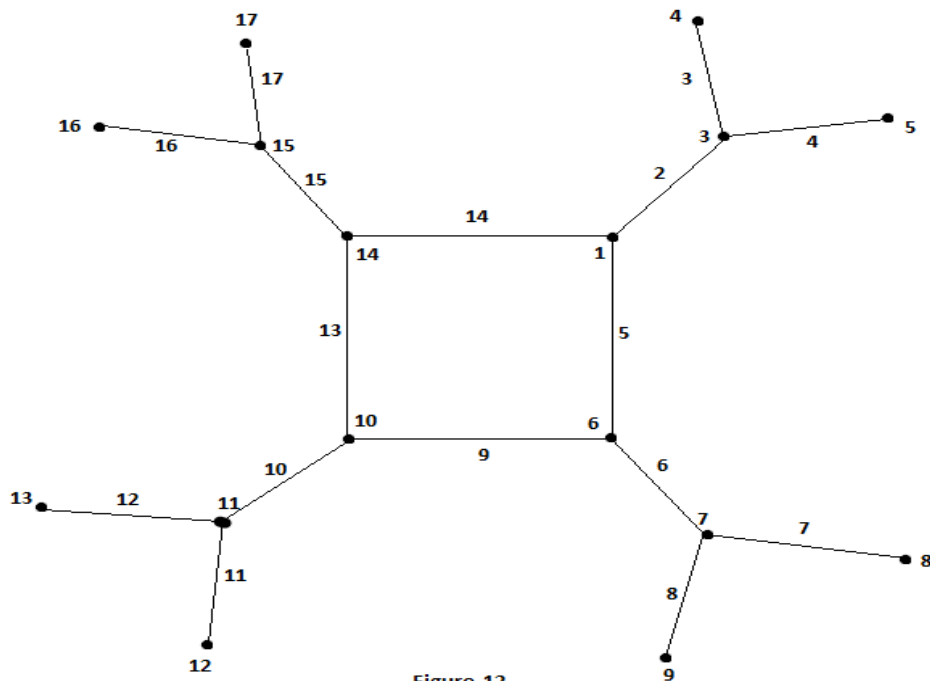


Figure-12



Y. Theorem :2.25

$$L_{3m}(L_n) = 3n - 1$$

1) Proof : Let  $L_n$  be a ladder graph of  $n$  vertices  $u_1, u_2, \dots, u_n, v_1, v_2, \dots, v_n$ , respectively. Define a function  $f:V(L_n) \rightarrow \{1,2,\dots,n\}$  by  $f(u_i)=3i-2 ; 1 \leq i \leq n, f(v_i)= 3i-1 ; 1 \leq i \leq n$ . The edge are labeled as  $f(u_i u_{i+1}) = 3i ; 1 \leq i \leq n-1, f(v_i v_{i+1}) = 3i+1 ; 1 \leq i \leq n-1, f(u_i v_i) = 1, f(u_i v_i) = 3i-1 ; 2 \leq i \leq n-1, f(u_n v_n) = 3n-1$ . Hence  $L_{3m}(L_n) = 3n-1$ . Hence ladder satisfies lehmer -3 mean number

Z. Example 2.26

Lehmer -3 mean number of  $L_7$  is given as  $L_{3m}(L_7) = 3 \times 7 - 1 = 21 - 1 = 20$ .

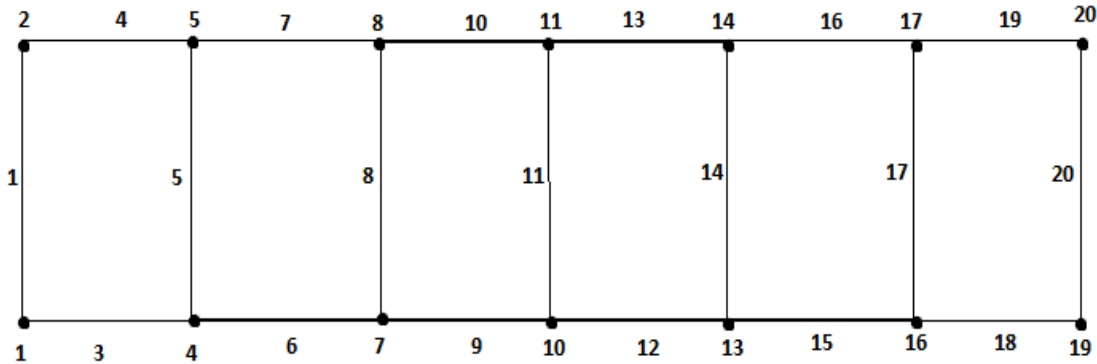


Figure-13

AA. Theorem :2.27

$$L_{3m}(D_n) = n + m$$

1) Proof : Let  $D_n$  be a dragon graph where the head of the dragon has  $n$  vertices  $u_1, u_2, \dots, u_n$ , and the path attached to it be  $v_1, v_2, \dots, v_m$ . Define a function  $f:V(D_n) \rightarrow \{1,2,\dots,n\}$  by  $f(u_1=v_1) = 1, f(u_i) = i+1 ; 2 \leq i \leq n, f(v_i) = n+j ; 2 \leq j \leq m$ . The distinct edge labels are  $f(u_i u_{i+1}) = i+1 ; 1 \leq i \leq n, f(v_j v_{j+1}) = (n+1)+j ; 1 \leq j \leq m-2, f(v_{n-1} v_n) = n + m$ .

Thus  $L_{3m}(D_n) = n+m$ .

BB. Example 2.28

Lehmer -3 mean number of  $D_n$  is given below.  $L_{3m}(D_n) = m+n$

Hence  $m=5$  and  $n=6$ ;  $n+m = 5+6 = 11$  is drawn as

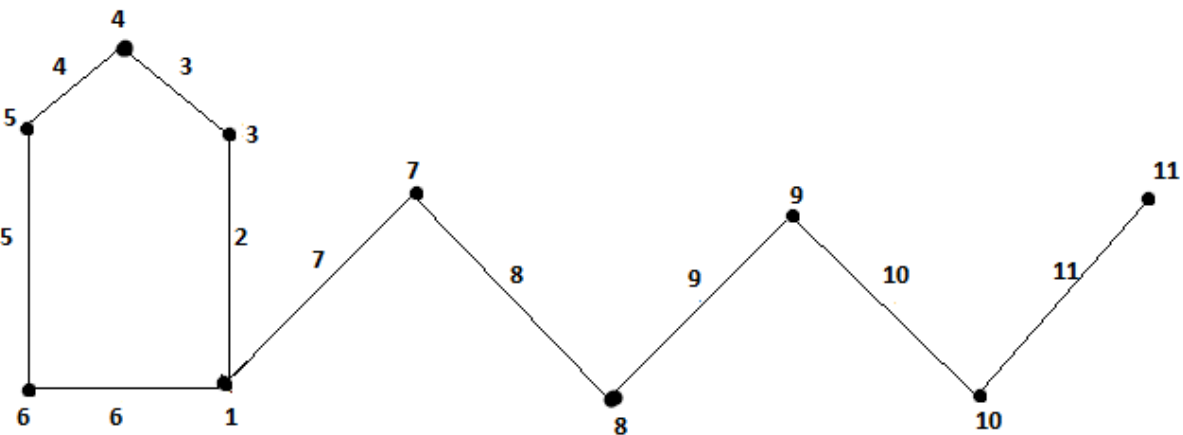


Figure-15

CC. Theorem :2.29

Lehmer -3 mean number of caterpillar is  $3n$

1) *Proof* : Let  $G$  be a graph of  $n$  vertices where each vertex of path is attached to pendant vertex on its both side. Let the vertices be denoted as  $u_1, u_2, \dots, u_n, v_1, v_2, \dots, v_n, w_1, w_2, \dots, w_n$ . Let  $f:V(G) \rightarrow \{1,2,\dots,n\}$  be a function defined by  $f(u_1) = 1, f(u_i) = 3i-1; 2 \leq i \leq n, f(v_i) = 3i; 1 \leq i \leq n, f(w_i) = 3i+1; 1 \leq i \leq n$  and The edges are labeled as  $f(u_i u_{i+1}) = 3i+1; 1 \leq i \leq n-1, f(u_i v_i) = 3i-1; 1 \leq i \leq n, f(u_i w_i) = 3i; 1 \leq i \leq n-1,$  and  $f(u_n w_n) = 3n$ . hence the Lehmer -3 mean number of a caterpillar is  $3n$

DD. Example 2.30

Lehmer -3 mean number of a caterpillar of 5 vertices is  $3n = 3 \times 5$  is given below

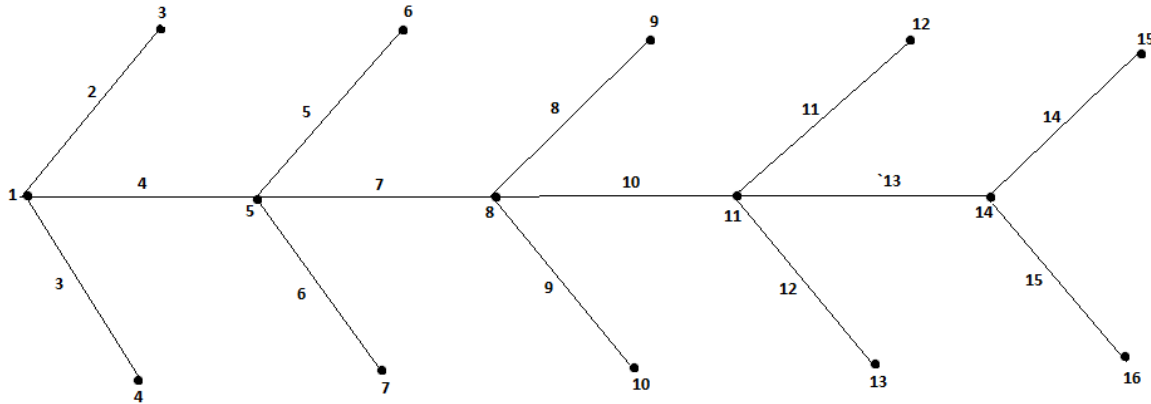


Figure-16

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45.98



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7.129



IMPACT FACTOR:  
7.429



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