



iJRASET

International Journal For Research in
Applied Science and Engineering Technology



INTERNATIONAL JOURNAL FOR RESEARCH

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY

Volume: 2017 **Issue:** conference **Month of publication:** December 2017

DOI:

www.ijraset.com

Call:  08813907089

E-mail ID: ijraset@gmail.com

Lower level subsets of an anti-fuzzy HX ideal of a HX ring

R. Muthuraj¹, N. Ramila Gandhi²

¹ PG & Research Department of Mathematics, H.H.The Rajah's College, Pudukkottai – 622 001, Tamilnadu, India.

²Department of Mathematics, PSNA College of Engineering and Technology, Dindigul-624 622, Tamilnadu, India. E-mail: ¹

Abstract: In this paper, we introduce the concept of lower level subsets of an anti-fuzzy HX ideal of a HX ring. We also discuss the relation between a given anti-fuzzy HX ideals of a HX ring and its lower level HX ideals and investigate the conditions under which a given HX ring has a properly inclusive chain of HX ideals. We introduce the concept of homomorphism and anti homomorphism of lower level subsets of an anti-fuzzy HX ideal and discuss some of its properties.

Keywords: HX ring, anti-fuzzy HX ideal, homomorphism, lower level subset.

AMS Subject Classification (2000): 20N25, 03E72, 03F055, 06F35, 03G25.

I. INTRODUCTION

In 1965, Lotfi.A.Zadeh [9] introduced the concept of fuzzy set. Fuzzy sets attracted many mathematicians and grew enormously by finding applications in many areas. We introduce a notion of anti fuzzy HX ideal of a HX ring and some of its properties are discussed. We prove that a fuzzy subset of a HX ring is an anti fuzzy HX ideal if and only if the lower level subsets are HX ideals of a HX ring. In 1982 Wang-jin Liu [6] introduced the concept of fuzzy subring and fuzzy ideal. In 1988, Professor Li Hong Xing [5] proposed the concept of HX ring and derived some of its properties, then Professor Zhong [10] gave the structures of HX ring on a class of ring.

II. PRELIMINARIES

In this section, we site the fundamental definitions that will be used in the sequel. Throughout this paper, $R = (R, +, \cdot)$ is a Ring, e is the additive identity element of R and xy , we mean $x \cdot y$.

III. LOWER LEVEL SUBSETS OF AN ANTI-FUZZY HX IDEAL

A. Definition

Let λ_μ be an anti-fuzzy HX ideal of a HX ring \mathfrak{R} . For any $t \in [0,1]$, we define the set $L(\lambda_\mu; t) = \{ A \in \mathfrak{R} / \lambda_\mu(A) \leq t \}$ is called a lower level subset of λ_μ .

B. Theorem

Let λ_μ be an anti-fuzzy HX right ideal of a HX ring \mathfrak{R} and $L(\lambda_\mu; t)$ is non-empty, then for $t \in [0,1]$, $L(\lambda_\mu; t)$ is a HX right ideal of \mathfrak{R} .

1) *Proof:* Let λ_μ be an anti-fuzzy HX right ideal of a HX ring \mathfrak{R} .

For any $A, B \in L(\lambda_\mu; t)$, we have, $\lambda_\mu(A) \leq t$ and $\lambda_\mu(B) \leq t$.

$$\begin{aligned} \text{Now, } \lambda_\mu(A - B) &\leq \max \{ \lambda_\mu(A), \lambda_\mu(B) \} \\ &\leq \max \{ t, t \} = t, \text{ for some } t \in [0,1]. \\ \lambda_\mu(A - B) &\leq t. \end{aligned}$$

For any $A \in L(\lambda_\mu; t)$ and $B \in \mathfrak{R}$, we have, $\lambda_\mu(A) \leq t$.

$$\begin{aligned} \text{Now, } \lambda_\mu(AB) &\leq \lambda_\mu(A) \leq t. \\ \lambda_\mu(AB) &\leq t. \end{aligned}$$

Hence, $A - B, AB \in L(\lambda_\mu; t)$.

Hence, $L(\lambda_\mu; t)$ is a HX right ideal of a HX ring \mathfrak{R} .

C. Theorem

Let λ_μ be an anti-fuzzy HX left ideal of a HX ring \mathfrak{R} and $L(\lambda_\mu; t)$ is non-empty, then for any $t \in [0,1]$, $L(\lambda_\mu; t)$ is a HX left ideal of \mathfrak{R} .

1) *Proof:* Let λ_μ be an anti-fuzzy HX left ideal of a HX ring \mathfrak{R} .

For any $A, B \in L(\lambda_\mu; t)$, we have, $\lambda_\mu(A) \leq t$ and $\lambda_\mu(B) \leq t$.

Now,
$$\lambda_\mu(A - B) \leq \max\{\lambda_\mu(A), \lambda_\mu(B)\} \leq \max\{t, t\} = t, \text{ for some } t \in [0,1].$$

$$\lambda_\mu(A - B) \leq t.$$

For any $A \in L(\lambda_\mu; t)$ and $B \in \mathfrak{R}$, we have, $\lambda_\mu(A) \leq t$.

Now, $\lambda_\mu(BA) \leq \lambda_\mu(A) \leq t$.

$$\lambda_\mu(BA) \leq t$$

Hence, $A - B, BA \in L(\lambda_\mu; t)$.

Hence, $L(\lambda_\mu; t)$ is a HX left ideal of a HX ring \mathfrak{R} .

D. Theorem

Let λ_μ be an anti-fuzzy HX ideal of a HX ring \mathfrak{R} and $L(\lambda_\mu; t)$ is non-empty, then for $t \in [0,1]$, $L(\lambda_\mu; t)$ is a HX ideal of \mathfrak{R} .

1) *Proof:* It is clear.

E. Theorem

Let \mathfrak{R} be a HX ring and λ_μ be a fuzzy subset of \mathfrak{R} such that $L(\lambda_\mu; t)$ is a HX right ideal of \mathfrak{R} for all $t \in [0,1]$ then λ_μ is an anti-fuzzy HX right ideal of \mathfrak{R} .

1) *Proof:* It is clear.

F. Theorem

Let \mathfrak{R} be a HX ring and λ_μ be a fuzzy subset of \mathfrak{R} such that $L(\lambda_\mu; t)$ is a HX left ideal of \mathfrak{R} for all $t \in [0,1]$ then λ_μ is an anti-fuzzy HX left ideal of \mathfrak{R} .

1) *Proof:* It is clear.

G. Theorem

Let \mathfrak{R} be a HX ring and λ_μ be a fuzzy subset of \mathfrak{R} such that $L(\lambda_\mu; t)$ is a HX ideal of \mathfrak{R} for all $t \in [0,1]$ then λ_μ is an anti-fuzzy HX ideal of \mathfrak{R} .

1) *Proof:* It is clear.

H. Theorem

A fuzzy subset λ_μ of \mathfrak{R} is a fuzzy HX ideal of a HX ring \mathfrak{R} if and only if the level HX subsets $L(\lambda_\mu; t)$, $t \in \text{Image } \lambda_\mu$, are HX ideals of \mathfrak{R} .

1) *Proof:* It is clear.

I. Theorem

Let λ_μ be an anti-fuzzy HX ideal of a HX ring \mathfrak{R} . If two lower level HX ideals, $L(\lambda_\mu; t_1), L(\lambda_\mu; t_2)$ with $t_1 < t_2$ of λ_μ are equal if and only if there is no A in \mathfrak{R} such that

$$t_1 \leq \lambda_\mu(A) < t_2.$$

1) *Proof:* It is clear.

J. Theorem

Any HX ideal H of a HX ring \mathfrak{R} can be realized as a lower level HX ideal of some anti-fuzzy HX ideal of \mathfrak{R} .

1) *Proof:* It is clear.

K. Remark

As a consequence of the Theorem 3.9 and 3.10, the lower level HX ideals of an anti-fuzzy HX ideal λ^μ of a HX ring \mathfrak{R} form a chain. Since $\lambda^\mu(Q) \leq \lambda^\mu(A)$ for all A in \mathfrak{R} and therefore $L(\lambda^\mu; t_0)$, where $\lambda^\mu(Q) = t_0$ is the smallest and we have the chain : $\{Q\} = L(\lambda_\mu; t_0) \subset L(\lambda_\mu; t_1) \subset L(\lambda_\mu; t_2) \subset \dots \subset L(\lambda_\mu; t_n) = \mathfrak{R}$, where $t_0 < t_1 < t_2 < \dots < t_n$.

III. HOMOMORPHISM AND ANTI HOMOMORPHISM OF A LOWER LEVEL SUBSETS OF AN ANTI-FUZZY HX IDEAL OF A HX RING

In this section, we introduce the concept of homomorphism and anti homomorphism of lower level subsets of an anti-fuzzy HX ideal and discuss some of its properties. Throughout this section, $t \in [0,1]$.

A. Theorem

Let R_1 and R_2 be any two rings, \mathfrak{R}_1 and \mathfrak{R}_2 be HX rings on R_1 and R_2 respectively. Let λ_μ be an anti-fuzzy HX right ideal on \mathfrak{R}_1 . If $f : \mathfrak{R}_1 \rightarrow \mathfrak{R}_2$ is a homomorphism and onto, then the anti-image of a lower level HX right ideal $L(\lambda_\mu; t)$ of an anti-fuzzy HX right ideal λ_μ of a HX ring \mathfrak{R}_1 is a lower level HX right ideal $L(f(\lambda_\mu); t)$ of an anti-fuzzy HX right ideal $f(\lambda_\mu)$ of a HX ring \mathfrak{R}_2 .

1) *Proof:* Let R_1 and R_2 be any two rings and $f : \mathfrak{R}_1 \rightarrow \mathfrak{R}_2$ be a homomorphism.

Let λ_μ be an anti-fuzzy HX right ideal of a HX ring \mathfrak{R}_1 . Clearly, $f(\lambda_\mu)$ is an anti-fuzzy HX right ideal of a HX ring \mathfrak{R}_2 . Let X and Y in \mathfrak{R}_1 , implies $f(X)$ and $f(Y)$ in \mathfrak{R}_2 .

Let $L(\lambda_\mu; t)$ is a lower level HX right ideal of an anti-fuzzy HX right ideal λ_μ of a HX ring \mathfrak{R}_1 .

Choose $t \in [0,1]$ in such a way that $X, Y \in L(\lambda_\mu; t)$ and hence $X-Y \in L(\lambda_\mu; t)$.

Then, $\lambda_\mu(X) \leq t$ and $\lambda_\mu(Y) \leq t$ and $\lambda_\mu(X-Y) \leq t$.

For this $t \in [0,1]$, let $X \in L(\lambda_\mu; t)$ and $Y \in \mathfrak{R}_1$ then $XY \in L(\lambda_\mu; t)$, as $L(\lambda_\mu; t)$ is a lower level HX right ideal of an anti-fuzzy HX right ideal λ_μ of a HX ring \mathfrak{R}_1 .

Then, $\lambda_\mu(X) \leq t$ and $\lambda_\mu(XY) \leq t$.

We have to prove that $L(f(\lambda_\mu); t)$ is a lower level HX right ideal of an anti-fuzzy HX right ideal $f(\lambda_\mu)$ of a HX ring \mathfrak{R}_2 .

Let $X, Y \in L(\lambda_\mu; t)$ and hence $X-Y \in L(\lambda_\mu; t)$.

For $f(X), f(Y) \in L(f(\lambda_\mu); t)$,

$$\begin{aligned} (f(\lambda_\mu))(f(X) - f(Y)) &= (f(\lambda_\mu))(f(X-Y)), \\ &= \lambda_\mu(X-Y) \\ &\leq t \\ (f(\lambda_\mu))(f(X) - f(Y)) &\leq t. \\ (f(X) - f(Y)) &\in L(f(\lambda_\mu); t). \end{aligned}$$

let $X \in L(\lambda_\mu; t)$ and $Y \in \mathfrak{R}_1$ then $XY \in L(\lambda_\mu; t)$, as $L(\lambda_\mu; t)$ is a lower level HX right ideal of an anti-fuzzy HX right ideal λ_μ of a HX ring \mathfrak{R}_1 .

For, $f(X) \in L(f(\lambda_\mu); t)$ and $f(Y) \in \mathfrak{R}_2$,

$$\begin{aligned} (f(\lambda_\mu))(f(X) f(Y)) &\leq (f(\lambda_\mu))f(X) \\ &\leq t \\ (f(\lambda_\mu))(f(X)f(Y)) &\leq t. \\ (f(X) f(Y)) &\in L(f(\lambda_\mu); t). \end{aligned}$$

Hence, $L(f(\lambda_\mu); t)$ is a lower level HX right ideal of an anti-fuzzy HX right ideal $f(\lambda_\mu)$ of a HX ring \mathfrak{R}_2 .

B. Theorem

Let R_1 and R_2 be any two rings, \mathfrak{R}_1 and \mathfrak{R}_2 be HX rings on R_1 and R_2 respectively. Let λ_μ be an anti-fuzzy HX left ideal on \mathfrak{R}_1 . If $f : \mathfrak{R}_1 \rightarrow \mathfrak{R}_2$ is a homomorphism and onto, then the anti-image of a lower level HX left ideal $L(\lambda_\mu; t)$ of an anti-fuzzy HX left ideal λ_μ of a HX ring \mathfrak{R}_1 is a lower level HX left ideal $L(f(\lambda_\mu); t)$ of an anti-fuzzy HX left ideal $f(\lambda_\mu)$ of a HX ring \mathfrak{R}_2 .

1) *Proof:* Let R_1 and R_2 be any two rings and $f : \mathfrak{R}_1 \rightarrow \mathfrak{R}_2$ be a homomorphism.

Let λ_μ be an anti-fuzzy HX left ideal of a HX ring \mathfrak{R}_1 . Clearly, $f(\lambda_\mu)$ is an anti-fuzzy HX left ideal of a HX ring \mathfrak{R}_2 . Let X and Y in \mathfrak{R}_1 , implies $f(X)$ and $f(Y)$ in \mathfrak{R}_2 .

Let $L(\lambda_\mu; t)$ is a lower level HX left ideal of an anti-fuzzy HX left ideal λ_μ of a HX ring \mathfrak{R}_1 .

Choose $t \in [0,1]$ in such a way that $X, Y \in L(\lambda_\mu; t)$ and hence $X-Y \in L(\lambda_\mu; t)$.

Then, $\lambda_\mu(X) \leq t$ and $\lambda_\mu(Y) \leq t$ and $\lambda_\mu(X-Y) \leq t$.

For this $t \in [0,1]$, let $X \in L(\lambda_\mu; t)$ and $Y \in \mathfrak{R}_1$ then $XY \in L(\lambda_\mu; t)$, as $L(\lambda_\mu; t)$ is a lower level HX left ideal of an anti-fuzzy HX left ideal λ_μ of a HX ring \mathfrak{R}_1 .

Then, $\lambda_\mu(X) \leq t$ and $\lambda_\mu(XY) \leq t$.

We have to prove that $L(f(\lambda_\mu); t)$ is a lower level HX left ideal of an anti-fuzzy HX left ideal $f(\lambda_\mu)$ of a HX ring \mathfrak{R}_2 .

Let $X, Y \in L(\lambda_\mu; t)$ and hence $X-Y \in L(\lambda_\mu; t)$.

For $f(X), f(Y) \in L(f(\lambda_\mu); t)$,

$$\begin{aligned} (f(\lambda_\mu))(f(X) - f(Y)) &= (f(\lambda_\mu))(f(X-Y)), \\ &= \lambda_\mu(X-Y) \\ &\leq t \\ (f(\lambda_\mu))(f(X) - f(Y)) &\leq t. \\ (f(X) - f(Y)) &\in L(f(\lambda_\mu); t). \end{aligned}$$

let $X \in L(\lambda_\mu; t)$ and $Y \in \mathfrak{R}_1$ then $YX \in L(\lambda_\mu; t)$, as $L(\lambda_\mu; t)$ is a lower level HX left ideal of an anti-fuzzy HX left ideal λ_μ of a HX ring \mathfrak{R}_1 .

For, $f(X) \in L(f(\lambda_\mu); t)$ and $f(Y) \in \mathfrak{R}_2$,

$$\begin{aligned} (f(\lambda_\mu))(f(Y) f(X)) &\leq (f(\lambda_\mu))(f(X)) \\ &\leq t \\ (f(\lambda_\mu))(f(Y)f(X)) &\leq t. \\ (f(Y) f(X)) &\in L(f(\lambda_\mu); t). \end{aligned}$$

Hence, $L(f(\lambda_\mu); t)$ is a lower level HX left ideal of an anti-fuzzy HX left ideal $f(\lambda_\mu)$ of a HX ring \mathfrak{R}_2 .

C. Theorem

Let R_1 and R_2 be any two rings, \mathfrak{R}_1 and \mathfrak{R}_2 be HX rings on R_1 and R_2 respectively. Let λ_μ be an anti-fuzzy HX ideal on \mathfrak{R}_1 . If $f : \mathfrak{R}_1 \rightarrow \mathfrak{R}_2$ is a homomorphism and onto, then the anti-image of a lower level HX ideal $L(\lambda_\mu; t)$ of an anti-fuzzy HX ideal λ_μ of a HX ring \mathfrak{R}_1 is a lower level HX ideal $L(f(\lambda_\mu); t)$ of an anti-fuzzy HX ideal $f(\lambda_\mu)$ of a HX ring \mathfrak{R}_2 .

1) *Proof:* It is clear.

D. Theorem

Let R_1 and R_2 be any two rings, \mathfrak{R}_1 and \mathfrak{R}_2 be HX rings on R_1 and R_2 respectively. Let η_α be an anti-fuzzy HX right ideal on \mathfrak{R}_2 . If $f : \mathfrak{R}_1 \rightarrow \mathfrak{R}_2$ is a homomorphism on HX rings. Let $L(\eta_\alpha; t)$ be a lower level HX right ideal of an anti-fuzzy HX right ideal η_α of a HX ring \mathfrak{R}_2 then $L(f^{-1}(\eta_\alpha); t)$ is a lower level HX right ideal of an anti-fuzzy HX right ideal $f^{-1}(\eta_\alpha)$ of a HX ring \mathfrak{R}_1 .

1) *Proof:* Let R_1 and R_2 be any two rings and $f : \mathfrak{R}_1 \rightarrow \mathfrak{R}_2$ be a homomorphism.

Let η_α be an anti-fuzzy HX right ideal of a HX ring \mathfrak{R}_2 . Clearly, $f^{-1}(\eta_\alpha)$ is an anti-fuzzy HX right ideal of a HX ring \mathfrak{R}_1 . Let X and Y in \mathfrak{R}_1 , implies $f(X)$ and $f(Y)$ in \mathfrak{R}_2 .

Let $L(\eta_\alpha; t)$ be a lower level HX right ideal of an anti-fuzzy HX right ideal η_α of the HX ring \mathfrak{R}_2 . Let $X, Y \in \mathfrak{R}_1$ then $f(X), f(Y) \in \mathfrak{R}_2$.

Choose $t \in [0, 1]$ in such a way that $f(X), f(Y) \in L(\eta_\alpha; t)$ and hence $f(X) - f(Y) \in L(\eta_\alpha; t)$.

Then, $\eta_\alpha(f(X)) \leq t$, $\eta_\alpha(f(Y)) \leq t$ and $\eta_\alpha(f(X) - f(Y)) \leq t$.

For this $t \in [0, 1]$, $f(X) \in L(\eta_\alpha; t)$ and $f(Y) \in \mathfrak{R}_2$ then $f(X)f(Y) \in L(\eta_\alpha; t)$, as $L(\eta_\alpha; t)$ be a lower level HX right ideal of an anti-fuzzy HX right ideal η_α of the HX ring \mathfrak{R}_2 .

Then, $\eta_\alpha(f(X)) \leq t$, $\eta_\alpha(f(X)f(Y)) \leq t$.

We have to prove that $L(f^{-1}(\eta_\alpha); t)$ is a lower level HX right ideal of an anti-fuzzy HX right ideal $f^{-1}(\eta_\alpha)$ of a HX ring \mathfrak{R}_1 . Now, Let $X, Y \in L(f^{-1}(\eta_\alpha); t)$.

$$\begin{aligned} (f^{-1}(\eta_\alpha))(X - Y) &= \eta_\alpha(f(X - Y)) \\ &= \eta_\alpha(f(X) - f(Y)) \\ &\leq t \\ (f^{-1}(\eta_\alpha))(X - Y) &\leq t \\ X - Y &\in L(f^{-1}(\eta_\alpha); t). \end{aligned}$$

Let $f(X) \in L(\eta_\alpha; t)$ and $f(Y) \in \mathfrak{R}_2$ then $f(X)f(Y) \in L(\eta_\alpha; t)$, as $L(\eta_\alpha; t)$ be a lower level HX right ideal of an anti-fuzzy HX right ideal η_α of the HX ring \mathfrak{R}_2 .

$$\begin{aligned} (f^{-1}(\eta_\alpha))(XY) &\leq (f^{-1}(\eta_\alpha))(X) \\ &= \eta_\alpha(f(X)) \\ &\leq t \\ (f^{-1}(\eta_\alpha))(XY) &\leq t \\ XY &\in L(f^{-1}(\eta_\alpha); t). \end{aligned}$$

Hence, $L(f^{-1}(\eta_\alpha); t)$ is a lower level HX right ideal of an anti-fuzzy HX right ideal of a HX ring \mathfrak{R}_1 .

E. Theorem

Let R_1 and R_2 be any two rings, \mathfrak{R}_1 and \mathfrak{R}_2 be HX rings on R_1 and R_2 respectively. Let η_α be an anti-fuzzy HX left ideal on \mathfrak{R}_2 . If $f : \mathfrak{R}_1 \rightarrow \mathfrak{R}_2$ is a homomorphism on HX rings. Let $L(\eta_\alpha; t)$ be a lower level HX left ideal of an anti-fuzzy HX left ideal η_α of a HX ring \mathfrak{R}_2 then $L(f^{-1}(\eta_\alpha); t)$ is a lower level HX left ideal of an anti-fuzzy HX left ideal $f^{-1}(\eta_\alpha)$ of a HX ring \mathfrak{R}_1 .

1) *Proof:* Let R_1 and R_2 be any two rings and $f : \mathfrak{R}_1 \rightarrow \mathfrak{R}_2$ be a homomorphism.

Let η_α be an anti-fuzzy HX left ideal of a HX ring \mathfrak{R}_2 . Clearly, $f^{-1}(\eta_\alpha)$ is an anti-fuzzy HX left ideal of a HX ring \mathfrak{R}_1 . Let X and Y in \mathfrak{R}_1 , implies $f(X)$ and $f(Y)$ in \mathfrak{R}_2 .

Let $L(\eta_\alpha; t)$ be a lower level HX left ideal of an anti-fuzzy HX left ideal η_α of the HX ring \mathfrak{R}_2 . Let $X, Y \in \mathfrak{R}_1$ then $f(X), f(Y) \in \mathfrak{R}_2$.

Choose $t \in [0, 1]$ in such a way that $f(X), f(Y) \in L(\eta_\alpha; t)$ and hence $f(X)-f(Y) \in L(\eta_\alpha; t)$.

Then, $\eta_\alpha(f(X)) \leq t$, $\eta_\alpha(f(Y)) \leq t$ and $\eta_\alpha(f(X)-f(Y)) \leq t$.

For this $t \in [0, 1]$, $f(X) \in L(\eta_\alpha; t)$ and $f(Y) \in \mathfrak{R}_2$ then $f(X)f(Y) \in L(\eta_\alpha; t)$, as $L(\eta_\alpha; t)$ be a lower level HX left ideal of an anti-fuzzy HX left ideal η_α of the HX ring \mathfrak{R}_2 .

Then, $\eta_\alpha(f(X)) \leq t$, $\eta_\alpha(f(X)f(Y)) \leq t$.

We have to prove that $L(f^{-1}(\eta_\alpha); t)$ is a lower level HX left ideal of an anti-fuzzy HX left ideal $f^{-1}(\eta_\alpha)$ of a HX ring \mathfrak{R}_1 . Now, Let $X, Y \in L(f^{-1}(\eta_\alpha); t)$.

$$\begin{aligned} (f^{-1}(\eta_\alpha))(X-Y) &= \eta_\alpha(f(X-Y)) \\ &= \eta_\alpha(f(X) - f(Y)) \\ &\leq t \\ (f^{-1}(\eta_\alpha))(X-Y) &\leq t \\ X-Y &\in L(f^{-1}(\eta_\alpha); t). \end{aligned}$$

Let $f(X) \in L(\eta_\alpha; t)$ and $f(Y) \in \mathfrak{R}_2$ then $f(X)f(Y) \in L(\eta_\alpha; t)$, as $L(\eta_\alpha; t)$ be a lower level HX left ideal of an anti-fuzzy HX left ideal η_α of the HX ring \mathfrak{R}_2 .

$$\begin{aligned} (f^{-1}(\eta_\alpha))(XY) &\leq (f^{-1}(\eta_\alpha))(Y) \\ &= \eta_\alpha(f(Y)) \\ &\leq t \\ (f^{-1}(\eta_\alpha))(XY) &\leq t \\ XY &\in L(f^{-1}(\eta_\alpha); t). \end{aligned}$$

Hence, $L(f^{-1}(\eta_\alpha); t)$ is a lower level HX left ideal of an anti-fuzzy HX left ideal of a HX ring \mathfrak{R}_1 .

F. Theorem

Let R_1 and R_2 be any two rings, \mathfrak{R}_1 and \mathfrak{R}_2 be HX rings on R_1 and R_2 respectively. Let η_α be an anti-fuzzy HX ideal on \mathfrak{R}_2 . If $f : \mathfrak{R}_1 \rightarrow \mathfrak{R}_2$ is a homomorphism on HX rings. Let $L(\eta_\alpha; t)$ be a lower level HX ideal of an anti-fuzzy HX ideal η_α of a HX ring \mathfrak{R}_2 then $L(f^{-1}(\eta_\alpha); t)$ is a lower level HX ideal of an anti-fuzzy HX ideal $f^{-1}(\eta_\alpha)$ of a HX ring \mathfrak{R}_1 .

1) *Proof:* It is clear.

G. Theorem

Let R_1 and R_2 be any two rings, \mathfrak{R}_1 and \mathfrak{R}_2 be HX rings on R_1 and R_2 respectively. Let λ_μ be an anti-fuzzy HX right ideal on \mathfrak{R}_1 . If $f : \mathfrak{R}_1 \rightarrow \mathfrak{R}_2$ is an anti homomorphism and onto, then the anti-image of a lower level HX right ideal $L(\lambda_\mu; t)$ of an anti-fuzzy HX right ideal λ_μ of a HX ring \mathfrak{R}_1 is a lower level HX left ideal $L(f(\lambda_\mu); t)$ of an anti-fuzzy HX left ideal $f(\lambda_\mu)$ of a HX ring \mathfrak{R}_2 .

1) *Proof:* Let R_1 and R_2 be any two rings and $f: \mathfrak{R}_1 \rightarrow \mathfrak{R}_2$ be an anti homomorphism.

Let λ_μ be an anti-fuzzy HX right ideal of a HX ring \mathfrak{R}_1 . Clearly, $f(\lambda_\mu)$ is an anti-fuzzy HX left ideal of a HX ring \mathfrak{R}_2 . Let X and Y in \mathfrak{R}_1 , implies $f(X)$ and $f(Y)$ in \mathfrak{R}_2 .

Let $L(\lambda_\mu; t)$ is a lower level HX right ideal of an anti-fuzzy HX right ideal λ_μ of a HX ring \mathfrak{R}_1 .

Choose $t \in [0,1]$ in such a way that $X, Y \in L(\lambda_\mu; t)$ and hence $Y-X \in L(\lambda_\mu; t)$.

Then, $\lambda_\mu(X) \leq t$ and $\lambda_\mu(Y) \leq t$ and $\lambda_\mu(Y-X) \leq t$.

For this $t \in [0,1]$, let $X \in L(\lambda_\mu; t)$ and $Y \in \mathfrak{R}_1$ then $XY \in L(\lambda_\mu; t)$, as $L(\lambda_\mu; t)$ is a lower level HX right ideal of an anti-fuzzy HX right ideal λ_μ of a HX ring \mathfrak{R}_1 .

Then, $\lambda_\mu(X) \leq t$ and $\lambda_\mu(XY) \leq t$.

We have to prove that $L(f(\lambda_\mu); t)$ is a lower level HX left ideal of an anti-fuzzy HX left ideal $f(\lambda_\mu)$ of a HX ring \mathfrak{R}_2 .

Let $X, Y \in L(\lambda_\mu; t)$ and hence $Y-X \in L(\lambda_\mu; t)$.

For $f(X), f(Y) \in L(f(\lambda_\mu); t)$,

$$\begin{aligned} (f(\lambda_\mu))(f(X)-f(Y)) &= (f(\lambda_\mu))(f(Y-X)), \\ &= \lambda_\mu(Y-X) \\ &\leq t \\ (f(\lambda_\mu))(f(X)-f(Y)) &\leq t. \\ (f(X)-f(Y)) &\in L(f(\lambda_\mu); t). \end{aligned}$$

let $X \in L(\lambda_\mu; t)$ and $Y \in \mathfrak{R}_1$ then $XY \in L(\lambda_\mu; t)$, as $L(\lambda_\mu; t)$ is a lower level HX right ideal of an anti-fuzzy HX right ideal λ_μ of a HX ring \mathfrak{R}_1 .

For, $f(X) \in L(f(\lambda_\mu); t)$ and $f(Y) \in \mathfrak{R}_2$,

$$\begin{aligned} (f(\lambda_\mu))(f(Y)f(X)) &\leq (f(\lambda_\mu))(f(X)) \\ &\leq t \\ (f(\lambda_\mu))(f(Y)f(X)) &\leq t. \\ (f(Y)f(X)) &\in L(f(\lambda_\mu); t). \end{aligned}$$

Hence, $L(f(\lambda_\mu); t)$ is a lower level HX right ideal of an anti-fuzzy HX right ideal $f(\lambda_\mu)$ of a HX ring \mathfrak{R}_2 .

H. Theorem

Let R_1 and R_2 be any two rings, \mathfrak{R}_1 and \mathfrak{R}_2 be HX rings on R_1 and R_2 respectively. Let λ_μ be an anti-fuzzy HX left ideal on \mathfrak{R}_1 . If $f: \mathfrak{R}_1 \rightarrow \mathfrak{R}_2$ is an anti homomorphism and onto, then the anti-image of a lower level HX left ideal $L(\lambda_\mu; t)$ of an anti-fuzzy HX left ideal λ_μ of a HX ring \mathfrak{R}_1 is a lower level HX right ideal $L(f(\lambda_\mu); t)$ of an anti-fuzzy HX right ideal $f(\lambda_\mu)$ of a HX ring \mathfrak{R}_2 .

1) *Proof:* Let R_1 and R_2 be any two rings and $f: \mathfrak{R}_1 \rightarrow \mathfrak{R}_2$ be an anti homomorphism.

Let λ_μ be an anti-fuzzy HX left ideal of a HX ring \mathfrak{R}_1 . Clearly, $f(\lambda_\mu)$ is an anti-fuzzy HX right ideal of a HX ring \mathfrak{R}_2 . Let X and Y in \mathfrak{R}_1 , implies $f(X)$ and $f(Y)$ in \mathfrak{R}_2 .

Let $L(\lambda_\mu; t)$ is a lower level HX left ideal of an anti-fuzzy HX left ideal λ_μ of a HX ring \mathfrak{R}_1 .

Choose $t \in [0,1]$ in such a way that $X, Y \in L(\lambda_\mu; t)$ and hence $Y-X \in L(\lambda_\mu; t)$.

Then, $\lambda_\mu(X) \leq t$ and $\lambda_\mu(Y) \leq t$ and $\lambda_\mu(Y-X) \leq t$.

For this $t \in [0,1]$, let $X \in L(\lambda_\mu; t)$ and $Y \in \mathfrak{R}_1$ then $XY \in L(\lambda_\mu; t)$, as $L(\lambda_\mu; t)$ is a lower level HX left ideal of an anti-fuzzy HX left ideal λ_μ of a HX ring \mathfrak{R}_1 .

Then, $\lambda_\mu(X) \leq t$ and $\lambda_\mu(XY) \leq t$.

We have to prove that $L(f(\lambda_\mu); t)$ is a lower level HX right ideal of an anti-fuzzy HX right ideal $f(\lambda_\mu)$ of a HX ring \mathfrak{R}_2 .

Let $X, Y \in L(\lambda_\mu; t)$ and hence $Y-X \in L(\lambda_\mu; t)$.

For $f(X), f(Y) \in L(f(\lambda_\mu); t)$,

$$\begin{aligned} (f(\lambda_\mu))(f(X)-f(Y)) &= (f(\lambda_\mu))(f(Y-X)), \\ &= \lambda_\mu(Y-X) \\ &\leq t \\ (f(\lambda_\mu))(f(X)-f(Y)) &\leq t. \\ (f(X)-f(Y)) &\in L(f(\lambda_\mu); t). \end{aligned}$$

let $X \in L(\lambda_\mu; t)$ and $Y \in \mathfrak{R}_1$ then $YX \in L(\lambda_\mu; t)$, as $L(\lambda_\mu; t)$ is a lower level HX left ideal of an anti-fuzzy HX left ideal λ_μ of a HX ring \mathfrak{R}_1 .

For, $f(X) \in L(f(\lambda_\mu); t)$ and $f(Y) \in \mathfrak{R}_2$,

$$\begin{aligned} (f(\lambda_\mu))(f(X) f(Y)) &\leq (f(\lambda_\mu))f(X) \\ &\leq t \\ (f(\lambda_\mu))(f(X)f(Y)) &\leq t. \\ (f(X) f(Y)) &\in L(f(\lambda_\mu); t). \end{aligned}$$

Hence, $L(f(\lambda_\mu); t)$ is a lower level HX right ideal of an anti-fuzzy HX right ideal $f(\lambda_\mu)$ of a HX ring \mathfrak{R}_2 .

I. Theorem

Let R_1 and R_2 be any two rings, \mathfrak{R}_1 and \mathfrak{R}_2 be HX rings on R_1 and R_2 respectively. Let λ_μ be an anti-fuzzy HX ideal on \mathfrak{R}_1 . If $f : \mathfrak{R}_1 \rightarrow \mathfrak{R}_2$ is an anti homomorphism and onto, then the anti-image of a lower level HX ideal $L(\lambda_\mu; t)$ of an anti-fuzzy HX ideal λ_μ of a HX ring \mathfrak{R}_1 is a lower level HX ideal $L(f(\lambda_\mu); t)$ of an anti-fuzzy HX ideal $f(\lambda_\mu)$ of a HX ring \mathfrak{R}_2 .

1) *Proof:* It is clear.

J. Theorem

Let R_1 and R_2 be any two rings, \mathfrak{R}_1 and \mathfrak{R}_2 be HX rings on R_1 and R_2 respectively. Let η_α be an anti-fuzzy HX right ideal on \mathfrak{R}_2 . If $f : \mathfrak{R}_1 \rightarrow \mathfrak{R}_2$ is an anti homomorphism on HX rings. Let $L(\eta_\alpha; t)$ be a lower level HX right ideal of an anti-fuzzy HX right ideal η_α of a HX ring \mathfrak{R}_2 then $L(f^{-1}(\eta_\alpha); t)$ is a lower level HX left ideal of an anti-fuzzy HX left ideal $f^{-1}(\eta_\alpha)$ of a HX ring \mathfrak{R}_1 .

1) *Proof:* Let R_1 and R_2 be any two rings and $f : \mathfrak{R}_1 \rightarrow \mathfrak{R}_2$ be an anti homomorphism.

Let η_α be an anti-fuzzy HX right ideal of a HX ring \mathfrak{R}_2 . Clearly, $f^{-1}(\eta_\alpha)$ is an anti-fuzzy HX left ideal of a HX ring \mathfrak{R}_1 . Let X and Y in \mathfrak{R}_1 , implies $f(X)$ and $f(Y)$ in \mathfrak{R}_2 .

Let $L(\eta_\alpha; t)$ be a lower level HX right ideal of an anti-fuzzy HX right ideal η_α of the HX ring \mathfrak{R}_2 . Let $X, Y \in \mathfrak{R}_1$ then $f(X), f(Y) \in \mathfrak{R}_2$.

Choose $t \in [0, 1]$ in such a way that $f(X), f(Y) \in L(\eta_\alpha; t)$ and hence, $f(Y) - f(X) \in L(\eta_\alpha; t)$.

Then, $\eta_\alpha(f(X)) \leq t$, $\eta_\alpha(f(Y)) \leq t$ and $\eta_\alpha(f(Y) - f(X)) \leq t$.

For this $t \in [0, 1]$, $f(X) \in L(\eta_\alpha; t)$ and $f(Y) \in \mathfrak{R}_2$ then $f(X)f(Y) \in L(\eta_\alpha; t)$, as $L(\eta_\alpha; t)$ be a lower level HX right ideal of an anti-fuzzy HX right ideal η_α of the HX ring \mathfrak{R}_2 .

Then, $\eta_\alpha(f(X)) \leq t$, $\eta_\alpha(f(X)f(Y)) \leq t$.

We have to prove that $L(f^{-1}(\eta_\alpha); t)$ is a lower level HX left ideal of an anti-fuzzy HX left ideal $f^{-1}(\eta_\alpha)$ of a HX ring \mathfrak{R}_1 . Now, Let $X, Y \in L(f^{-1}(\eta_\alpha); t)$.

$$\begin{aligned} (f^{-1}(\eta_\alpha))(X - Y) &= \eta_\alpha(f(X - Y)) \\ &= \eta_\alpha(f(Y) - f(X)) \\ &\leq t \\ (f^{-1}(\eta_\alpha))(X - Y) &\leq t \\ X - Y &\in L(f^{-1}(\eta_\alpha); t). \end{aligned}$$

Let $f(X) \in L(\eta_\alpha; t)$ and $f(Y) \in \mathfrak{R}_2$ then $f(X)f(Y) \in L(\eta_\alpha; t)$, as $L(\eta_\alpha; t)$ be a lower level HX right ideal of an anti-fuzzy HX right ideal η_α of the HX ring \mathfrak{R}_2 .

$$\begin{aligned} (f^{-1}(\eta_\alpha))(YX) &\leq f^{-1}(\eta_\alpha)(X) \\ &= \eta_\alpha(f(X)) \\ &\leq t \\ (f^{-1}(\eta_\alpha))(YX) &\leq t \\ YX &\in L(f^{-1}(\eta_\alpha); t). \end{aligned}$$

Hence, $L(f^{-1}(\eta_\alpha); t)$ is a lower level HX left ideal of an anti-fuzzy HX left ideal of a HX ring \mathfrak{R}_1 .

K. Theorem

Let R_1 and R_2 be any two rings, \mathfrak{R}_1 and \mathfrak{R}_2 be HX rings on R_1 and R_2 respectively. Let η_α be an anti-fuzzy HX left ideal on \mathfrak{R}_2 . If $f : \mathfrak{R}_1 \rightarrow \mathfrak{R}_2$ is a homomorphism on HX rings. Let $L(\eta_\alpha; t)$ be a lower level HX left ideal of an anti-fuzzy HX left ideal η_α of a HX ring \mathfrak{R}_2 then $L(f^{-1}(\eta_\alpha); t)$ is a lower level HX right ideal of an anti-fuzzy HX right ideal $f^{-1}(\eta_\alpha)$ of a HX ring \mathfrak{R}_1 .

1) *Proof:* Let R_1 and R_2 be any two rings and $f : \mathfrak{R}_1 \rightarrow \mathfrak{R}_2$ be an anti homomorphism.

Let η_α be an anti-fuzzy HX left ideal of a HX ring \mathfrak{R}_2 . Clearly, $f^{-1}(\eta_\alpha)$ is an anti-fuzzy HX right ideal of a HX ring \mathfrak{R}_1 . Let X and Y in \mathfrak{R}_1 , implies $f(X)$ and $f(Y)$ in \mathfrak{R}_2 .

Let $L(\eta_\alpha; t)$ be a lower level HX left ideal of an anti-fuzzy HX left ideal η_α of the HX ring \mathfrak{R}_2 . Let $X, Y \in \mathfrak{R}_1$ then $f(X), f(Y) \in \mathfrak{R}_2$.

Choose $t \in [0, 1]$ in such a way that $f(X), f(Y) \in L(\eta_\alpha; t)$ and hence $f(Y) - f(X) \in L(\eta_\alpha; t)$.

Then, $\eta_\alpha(f(X)) \leq t$, $\eta_\alpha(f(Y)) \leq t$ and $\eta_\alpha(f(Y) - f(X)) \leq t$.

For this $t \in [0, 1]$, $f(X) \in L(\eta_\alpha; t)$ and $f(Y) \in \mathfrak{R}_2$ then $f(X)f(Y) \in L(\eta_\alpha; t)$, as $L(\eta_\alpha; t)$ be a lower level HX left ideal of an anti-fuzzy HX left ideal η_α of the HX ring \mathfrak{R}_2 .

Then, $\eta_\alpha(f(X)) \leq t$, $\eta_\alpha(f(X)f(Y)) \leq t$.

We have to prove that $L(f^{-1}(\eta_\alpha); t)$ is a lower level HX right ideal of an anti-fuzzy HX right ideal $f^{-1}(\eta_\alpha)$ of a HX ring \mathfrak{R}_1 . Now, Let $X, Y \in L(f^{-1}(\eta_\alpha); t)$.

$$\begin{aligned} (f^{-1}(\eta_\alpha))(X - Y) &= \eta_\alpha(f(X - Y)) \\ &= \eta_\alpha(f(Y) - f(X)) \\ &\leq t \\ (f^{-1}(\eta_\alpha))(X - Y) &\leq t \\ X - Y &\in L(f^{-1}(\eta_\alpha); t). \end{aligned}$$

Let $f(X) \in L(\eta_\alpha; t)$ and $f(Y) \in \mathfrak{R}_2$ then $f(X)f(Y) \in L(\eta_\alpha; t)$, as $L(\eta_\alpha; t)$ be a lower level HX left ideal of an anti-fuzzy HX left ideal η_α of the HX ring \mathfrak{R}_2 .

$$\begin{aligned} (f^{-1}(\eta_\alpha))(XY) &\leq (f^{-1}(\eta_\alpha))(X) \\ &= \eta_\alpha(f(X)) \\ &\leq t \\ (f^{-1}(\eta_\alpha))(XY) &\leq t \\ X Y &\in L(f^{-1}(\eta_\alpha); t). \end{aligned}$$

Hence, $L(f^{-1}(\eta_\alpha); t)$ is a lower level HX right ideal of an anti-fuzzy HX right ideal of a HX ring \mathfrak{R}_1 .

L. Theorem

Let R_1 and R_2 be any two rings, \mathfrak{R}_1 and \mathfrak{R}_2 be HX rings on R_1 and R_2 respectively. Let η_α be an anti-fuzzy HX ideal on \mathfrak{R}_2 . If $f : \mathfrak{R}_1 \rightarrow \mathfrak{R}_2$ is an anti homomorphism on HX rings. Let $L(\eta_\alpha; t)$ be a lower level HX ideal of an anti-fuzzy HX ideal η_α of a HX ring \mathfrak{R}_2 then $L(f^{-1}(\eta_\alpha); t)$ is a lower level HX ideal of an anti-fuzzy HX ideal $f^{-1}(\eta_\alpha)$ of a HX ring \mathfrak{R}_1 .

1) *Proof:* It is clear.

REFERENCES

- [1] Azam.F. A., Mamun.A.A., Nasrin. F., Anti fuzzy ideal of a ring, Annals of fuzzy Mathematics and informatics, volume 5, No.2, 349-360, (2013).
- [2] Bing-xue Yao and Yubin-Zhong, The construction of power ring, Fuzzy information and Engineering (ICFIE), ASC 40, 181-187, (2007).
- [3] Chandrasekara Rao.K and Swaminathan.V, Anti-homomorphism in fuzzy ideals, World Academy of Science, Engineering and Technology, Vol:44, 08-29, (2010).
- [4] Chang Bum, Kim, A note on some fuzzy ideals of a ring, Journal of Science Institute, Kookmin University, Volume 9, 29-37, (1991).
- [5] Li Hong Xing, HX ring, BUSEFAL, 34(1), 3-8, (1988).
- [6] Liu, W.J., Operations on fuzzy ideals, Fuzzy Sets and Systems, 11, 31-41, (1983).
- [7] Malik, Davender S. and Mordeson, John N., Fuzzy homomorphism of rings, Fuzzy Sets and Systems, 46, 139-146, (1992).
- [8] Rajesh Kumar, Certain fuzzy ideals of rings redefined, Fuzzy sets and systems, 46, 251-260, (1992).
- [9] Zadeh.L.A., Fuzzy sets, Information and control, 8, 338-353, (1965).
- [10] Zhong YB, The structure of HX-ring on a class of ring. Fuzzy Systems and Mathematics, 9(4) 73-77, (1995).



10.22214/IJRASET



45.98



IMPACT FACTOR:
7.129



IMPACT FACTOR:
7.429



INTERNATIONAL JOURNAL FOR RESEARCH

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY

Call : 08813907089  (24*7 Support on Whatsapp)