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Lower level subsets of an anti-fuzzy HX ideal of a HX ring

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Abstract: ***In this paper, we introduce the concept of lower level subsets of an anti-fuzzy HX ideal of a HX ring. We also discuss the relation between a given anti-fuzzy HX ideals of a HX ring and its lower level HX ideals and investigate the conditions under which a given HX ring has a properly inclusive chain of HX ideals. We introduce the concept of homomorphism and anti homomorphism of lower level subsets of an anti-fuzzy HX ideal and discuss some of its properties.***

Keywords: HX ring, anti-fuzzy HX ideal, homomorphism, lower level subset.

AMS Subject Classification (2000): 20N25, 03E72, 03F055 , 06F35, 03G25.

I. INTRODUCTION

In 1965,Lotfi.A.Zadeh [9] introduced the concept of fuzzy set. Fuzzy sets attracted many mathematicians and grew enormously by finding applications in many areas. We introduce a notion of anti fuzzy HX ideal of a HX ring and some of its properties are discussed. We prove that a fuzzy subset of a HX ring is an anti fuzzy HX ideal if and only if the lower level subsets are HX ideals of a HX ring. In 1982 Wang-jin Liu [6] introduced the concept of fuzzy subring and fuzzy ideal. In 1988, Professor Li Hong Xing [5] proposed the concept of HX ring and derived some of its properties, then Professor Zhong [10] gave the structures of HX ring on a class of ring.

II. PRELIMINARIES

In this section, we site the fundamental definitions that will be used in the sequel. Throughout this paper, $R = (R, +, \cdot)$ is a Ring, e is the additive identity element of R and xy, we mean $x \cdot y$.

III. LOWER LEVEL SUBSETS OF AN ANTI-FUZZY HX IDEAL

A. Definition

Let λ_μ be an anti-fuzzy HX ideal of a HX ring $\Re$. For any $t \in [0,1]$, we define the set $L(\lambda_\mu ; t) = \{ A \in \Re / \lambda_\mu (A) \leq t \}$ is called a lower level subset of λ_μ.

B. Theorem

Let λ_μ be an anti-fuzzy HX right ideal of a HX ring $\Re$ and $L(\lambda_\mu ; t)$ is non-empty, then for $t \in [0,1]$, $L(\lambda_\mu ; t)$ is a HX right ideal of $\Re$.

1) Proof: Let λ_μ be an anti-fuzzy HX right ideal of a HX ring $\Re$.

For any $A , B \in L(\lambda_\mu; t)$, we have , $\lambda_\mu (A) \leq t$ and $\lambda_\mu (B) \leq t$.

Now, $\lambda_\mu (A – B) \leq \max \{ \lambda_\mu (A) , \lambda_\mu (B) \}$

$\leq \max \{ t , t \} = t$, for some $t \in [0,1]$.

$\lambda_\mu (A – B) \leq t$.

For any $A \in L(\lambda_\mu; t)$ and $B \in \Re$, we have , $\lambda_\mu (A) \leq t$.

Now, $\lambda_\mu (AB) \leq \lambda_\mu (A) \leq t$.

$\lambda_\mu (AB) \leq t$.

Hence, $A – B , AB \in L(\lambda_\mu; t)$.

Hence, $L(\lambda_\mu; t)$ is a HX right ideal of a HX ring $\Re$.

C. Theorem

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Let λ_μ be an anti-fuzzy HX left ideal of a HX ring $\Re$ and L (λ_μ ; t) is non-empty, then for any $t \in [0,1]$, L(λ_μ ; t) is a HX left ideal of $\Re$.

1) Proof: Let λ_μ be an anti-fuzzy HX left ideal of a HX ring $\Re$.

For any A , B $\in$ L(λ_μ; t), we have , λ_μ (A) $\leq$ t and λ_μ (B) $\leq$ t.

Now, λ_μ (A – B) $\leq$ max { λ_μ (A) , λ_μ (B) }

$\leq$ max { t , t } = t , for some t$\in$ [0,1].

λ_μ (A – B) $\leq$ t.

For any A $\in$ L(λ_μ; t) and B $\in \Re$, we have , λ_μ (A) $\leq$ t.

Now, λ_μ (BA) $\leq \lambda_\mu$ (A) $\leq$ t.

λ_μ (BA) $\leq$ t

Hence, A – B , BA $\in$ L(λ_μ; t) .

Hence , L(λ_μ; t) is a HX left ideal of a HX ring $\Re$.

D. Theorem

Let λ_μ be an anti-fuzzy HX ideal of a HX ring $\Re$ and L(λ_μ ; t) is non-empty, then for $t \in [0,1]$, L(λ_μ ; t) is a HX ideal of $\Re$.

1) Proof: It is clear.

E. Theorem

Let $\Re$ be a HX ring and λ_μ be a fuzzy subset of $\Re$ such that L(λ_μ ; t) is a HX right ideal of $\Re$ for all t$\in$ [0,1] then λ_μ is an anti-fuzzy HX right ideal of $\Re$.

1) Proof: It is clear.

F. Theorem

Let $\Re$ be a HX ring and λ_μ be a fuzzy subset of $\Re$ such that L(λ_μ ; t) is a HX left ideal of $\Re$ for all t$\in$ [0,1] then λ_μ is an anti-fuzzy HX left ideal of $\Re$.

1) Proof: It is clear.

G. Theorem

Let $\Re$ be a HX ring and λ_μ be a fuzzy subset of $\Re$ such that L(λ_μ ; t) is a HX ideal of $\Re$ for all t$\in$ [0,1] then λ_μ is an anti-fuzzy HX ideal of $\Re$.

1) Proof: It is clear.

H. Theorem

A fuzzy subset λ_μ of $\Re$ is a fuzzy HX ideal of a HX ring $\Re$ if and only if the level HX subsets L(λ_μ ; t), $t \in$ Image λ_μ, are HX ideals of $\Re$.

1) Proof: It is clear.

I. Theorem

Let λ_μ be an anti-fuzzy HX ideal of a HX ring $\Re$. If two lower level HX ideals, L(λ_μ ; t_1), L(λ_μ ; t_2) with $t_1 < t_2$ of λ_μ are equal if and only if there is no A in $\Re$ such that

$t_1 \leq \lambda_\mu (A) < t_2$.

1) Proof: It is clear.

J. Theorem

Any HX ideal H of a HX ring $\Re$ can be realized as a lower level HX ideal of some anti-fuzzy HX ideal of $\Re$.

1) Proof: It is clear.

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K. Remark

As a consequence of the Theorem 3.9 and 3.10, the lower level HX ideals of an anti-fuzzy HX ideal λ^{μ} of a HX ring $\Re$ form a chain. Since $\lambda^{\mu}(Q) \leq \lambda^{\mu}(A)$ for all A in $\Re$ and therefore $L(\lambda^{\mu}; t_0)$, where $\lambda^{\mu}(Q) = t_0$ is the smallest and we have the chain :

$\{Q\} = L(\lambda_{\mu}; t_0) \subset L(\lambda_{\mu}; t_1) \subset L(\lambda_{\mu}; t_2) \subset \ldots \subset L(\lambda_{\mu}; t_n) = \Re$,

where $t_0 < t_1 < t_2 < \ldots\ldots < t_n$.

III. HOMOMORPHISM AND ANTI HOMOMORPHISM OF A LOWER LEVEL SUBSETS OF AN ANTI-FUZZY HX IDEAL OF A HX RING

In this section, we introduce the concept of homomorphism and anti homomorphism of lower level subsets of an anti-fuzzy HX ideal and discuss some of its properties. Throughout this section, $t \in [0,1]$.

A. Theorem

Let R_1 and R_2 be any two rings, $\Re_1$ and $\Re_2$ be HX rings on R_1 and R_2 respectively. Let λ_{μ} be an anti-fuzzy HX right ideal on $\Re_1$. If $f : \Re_1 \rightarrow \Re_2$ is a homomorphism and onto, then the anti-image of a lower level HX right ideal $L(\lambda_{\mu}; t)$ of an anti-fuzzy HX right ideal λ_{μ} of a HX ring $\Re_1$ is a lower level HX right ideal $L(f(\lambda_{\mu}); t)$ of an anti-fuzzy HX right ideal $f(\lambda_{\mu})$ of a HX ring $\Re_2$.

1) Proof: Let R_1 and R_2 be any two rings and $f : \Re_1 \rightarrow \Re_2$ be a homomorphism.

Let λ_{μ} be an anti-fuzzy HX right ideal of a HX ring $\Re_1$. Clearly, $f(\lambda_{\mu})$ is an anti-fuzzy HX right ideal of a HX ring $\Re_2$. Let X and Y in $\Re_1$, implies f (X) and f (Y) in $\Re_2$.

Let $L(\lambda_{\mu}; t)$ is a lower level HX right ideal of an anti-fuzzy HX right ideal λ_{μ} of a HX ring $\Re_1$.

Choose $t \in [0,1]$ in such a way that $X, Y \in L(\lambda_{\mu}; t)$ and hence $X-Y \in L(\lambda_{\mu}; t)$.

Then, $\lambda_{\mu}(X) \leq t$ and $\lambda_{\mu}(Y) \leq t$ and $\lambda_{\mu}(X-Y) \leq t$.

For this $t \in [0,1]$, let $X \in L(\lambda_{\mu}; t)$ and $Y \in \Re_1$ then $XY \in L(\lambda_{\mu}; t)$, as $L(\lambda_{\mu}; t)$ is a lower level HX right ideal of an anti-fuzzy HX right ideal λ_{μ} of a HX ring $\Re_1$.

Then, $\lambda_{\mu}(X) \leq t$ and $\lambda_{\mu}(XY) \leq t$.

We have to prove that $L(f(\lambda_{\mu}); t)$ is a lower level HX right ideal of an anti-fuzzy HX right ideal $f(\lambda_{\mu})$ of a HX ring $\Re_2$.

Let $X, Y \in L(\lambda_{\mu}; t)$ and hence $X-Y \in L(\lambda_{\mu}; t)$.

For $f(X), f(Y) \in L(f(\lambda^{\mu}); t)$,

$$\begin{aligned} (f(\lambda_{\mu}))(f(X) - f(Y)) &= (f(\lambda_{\mu}))(f(X-Y)), \\ &= \lambda_{\mu}(X-Y) \\ &\leq t \\ (f(\lambda_{\mu}))(f(X) - f(Y)) &\leq t . \\ (f(X) - f(Y)) &\in L(f(\lambda_{\mu}); t). \end{aligned}$$

let $X \in L(\lambda_{\mu}; t)$ and $Y \in \Re_1$ then $XY \in L(\lambda_{\mu}; t)$, as $L(\lambda_{\mu}; t)$ is a lower level HX right ideal of an anti-fuzzy HX right ideal λ_{μ} of a HX ring $\Re_1$.

For, $f(X) \in L(f(\lambda_{\mu}); t)$ and $f(Y) \in \Re_2$,

$$\begin{aligned} (f(\lambda_{\mu}))(f(X)\, f(Y)) &\leq (f(\lambda_{\mu}))f(X) \\ &\leq t \\ (f(\lambda_{\mu}))(f(X)(f(Y)) &\leq t. \\ (f(X)\, f(Y)) &\in L(f(\lambda_{\mu}); t). \end{aligned}$$

Hence, $L(f(\lambda_{\mu}); t)$ is a lower level HX right ideal of an anti-fuzzy HX right ideal $f(\lambda_{\mu})$ of a HX ring $\Re_2$.

B. Theorem

Let R_1 and R_2 be any two rings, $\Re_1$ and $\Re_2$ be HX rings on R_1 and R_2 respectively. Let λ_{μ} be an anti-fuzzy HX left ideal on $\Re_1$. If $f : \Re_1 \rightarrow \Re_2$ is a homomorphism and onto, then the anti-image of a lower level HX left ideal $L(\lambda_{\mu}; t)$ of an anti-fuzzy HX left ideal λ_{μ} of a HX ring $\Re_1$ is a lower level HX left ideal $L(f(\lambda_{\mu}); t)$ of an anti-fuzzy HX left ideal $f(\lambda_{\mu})$ of a HX ring $\Re_2$.

1) Proof: Let R_1 and R_2 be any two rings and $f : \Re_1 \rightarrow \Re_2$ be a homomorphism.

Let λ_{μ} be an anti-fuzzy HX left ideal of a HX ring $\Re_1$. Clearly, $f(\lambda_{\mu})$ is an anti-fuzzy HX left ideal of a HX ring $\Re_2$. Let X and Y in $\Re_1$, implies f (X) and f (Y) in $\Re_2$.

Let $L(\lambda_{\mu}; t)$ is a lower level HX left ideal of an anti-fuzzy HX left ideal λ_{μ} of a HX ring $\Re_1$.

Choose $t \in [0,1]$ in such a way that $X, Y \in L(\lambda_{\mu}; t)$ and hence $X-Y \in L(\lambda_{\mu}; t)$.

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Then, $\lambda_\mu(X) \le t$ and $\lambda_\mu(Y) \le t$ and $\lambda_\mu(X-Y) \le t$.

For this $t \in [0,1]$, let $X \in L(\lambda_\mu ; t)$ and $Y \in \Re_1$ then $XY \in L(\lambda_\mu ; t)$, as $L(\lambda_\mu ; t)$ is a lower level HX left ideal of an anti-fuzzy HX left ideal λ_μ of a HX ring $\Re_1$.

Then, $\lambda_\mu(X) \le t$ and $\lambda_\mu(XY) \le t$.

We have to prove that $L(f(\lambda_\mu) ; t)$ is a lower level HX left ideal of an anti-fuzzy HX left ideal $f(\lambda_\mu)$ of a HX ring $\Re_2$.

Let $X, Y \in L(\lambda_\mu ; t)$ and hence $X-Y \in L(\lambda_\mu ; t)$.

For $f(X), f(Y) \in L(f(\lambda^\mu) ; t)$,

$(f(\lambda_\mu))(f(X) - f(Y)) = (f(\lambda_\mu))(f(X-Y))$,

$= \lambda_\mu(X-Y)$

$\le t$

$(f(\lambda_\mu))(f(X) - f(Y)) \le t$.

$(f(X) - f(Y)) \in L(f(\lambda_\mu); t)$.

let $X \in L(\lambda_\mu ; t)$ and $Y \in \Re_1$ then $YX \in L(\lambda_\mu ; t)$, as $L(\lambda_\mu ; t)$ is a lower level HX left ideal of an anti-fuzzy HX left ideal λ_μ of a HX ring $\Re_1$.

For, $f(X) \in L(f(\lambda_\mu) ; t)$ and $f(Y) \in \Re_2$,

$(f(\lambda_\mu))(f(Y)\, f(X)) \le (f(\lambda_\mu))(f(X))$

$\le t$

$(f(\lambda_\mu))(f(Y)(f(X)) \le t.$

$(f(Y)\, f(X)) \in L(f(\lambda_\mu); t)$.

Hence, $L(f(\lambda_\mu) ; t)$ is a lower level HX left ideal of an anti-fuzzy HX left ideal $f(\lambda_\mu)$ of a HX ring $\Re_2$.

C. Theorem

Let R_1 and R_2 be any two rings, $\Re_1$ and $\Re_2$ be HX rings on R_1 and R_2 respectively. Let λ_μ be an anti-fuzzy HX ideal on $\Re_1$. If $f : \Re_1 \to \Re_2$ is a homomorphism and onto, then the anti-image of a lower level HX ideal $L(\lambda_\mu ; t)$ of an anti-fuzzy HX ideal λ_μ of a HX ring $\Re_1$ is a lower level HX ideal $L(f(\lambda_\mu); t)$ of an anti-fuzzy HX ideal $f(\lambda_\mu)$ of a HX ring $\Re_2$.

1) Prof: It is clear.

D. Theorem

Let R_1 and R_2 be any two rings , $\Re_1$ and $\Re_2$ be HX rings on R_1 and R_2 respectively. Let η_α be an anti-fuzzy HX right ideal on $\Re_2$. If $f : \Re_1 \to \Re_2$ is a homomorphism on HX rings. Let $L(\eta_\alpha ; t)$ be a lower level HX right ideal of an anti-fuzzy HX right ideal η_α of a HX ring $\Re_2$ then $L(f^{-1}(\eta_\alpha); t)$ is a lower level HX right ideal of an anti-fuzzy HX right ideal $f^{-1}(\eta_\alpha)$ of a HX ring $\Re_1$.

1) Proof: Let R_1 and R_2 be any two rings and $f : \Re_1 \to \Re_2$ be a homomorphism.

Let η_α be an anti-fuzzy HX right ideal of a HX ring $\Re_2$. Clearly, $f^{-1}(\eta_\alpha)$ is an anti-fuzzy HX right ideal of a HX ring $\Re_1$. Let X and Y in $\Re_1$, implies f(X) and f(Y) in $\Re_2$.

Let $L(\eta_\alpha ; t)$ be a lower level HX right ideal of an anti-fuzzy HX right ideal η_α of the HX ring $\Re_2$.Let $X, Y \in \Re_1$ then f(X) ,f(Y) $\in \Re_2$.

Choose $t \in [0, 1]$ in such a way that $f(X), f(Y) \in L(\eta_\alpha; t)$ and hence ,$f(X)-f(Y) \in L(\eta_\alpha ; t)$.

Then, $\eta_\alpha(f(X)) \le t$, $\eta_\alpha(f(Y)) \le t$ and $\eta_\alpha(f(X)-f(Y)) \le t$.

For this $t \in [0, 1]$, $f(X) \in L(\eta_\alpha; t)$ and $f(Y) \in \Re_2$ then $f(X)f(Y) \in L(\eta_\alpha; t)$, as $L(\eta_\alpha ; t)$ be a lower level HX right ideal of an anti-fuzzy HX right ideal η_α of the HX ring $\Re_2$.

Then, $\eta_\alpha(f(X)) \le t$, $\eta_\alpha(f(X)f(Y)) \le t$.

We have to prove that $L(f^{-1}(\eta_\alpha); t)$ is a lower level HX right ideal of an anti-fuzzy HX right ideal $f^{-1}(\eta_\alpha)$ of a HX ring $\Re_1$. Now, Let $X, Y \in L(f^{-1}(\eta_\alpha) ; t)$.

$(f^{-1}(\eta_\alpha))(X-Y) = \eta_\alpha(f(X-Y))$

$= \eta_\alpha(f(X) - f(Y))$

$\le t$

$(f^{-1}(\eta_\alpha))(X-Y) \le t$

$X-Y \in L(f^{-1}(\eta_\alpha); t)$.

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Let f(X) $\in$ L(η_α ; t) and f(Y)$\in\Re_2$ then f(X)f(Y)$\in$ L(η_α ; t), as L(η_α ; t) be a lower level HX right ideal of an anti-fuzzy HX right ideal η_α of the HX ring $\Re_2$.

$$\begin{aligned} (f^{-1}(\eta_\alpha))(XY) &\leq (f^{-1}(\eta_\alpha))(X) \\ &= \eta_\alpha(f(X)) \\ &\leq t \\ (f^{-1}(\eta_\alpha))(XY) &\leq t \\ XY &\in L(f^{-1}(\eta_\alpha);\ t). \end{aligned}$$

Hence, L($f^{-1}(\eta_\alpha)$; t) is a lower level HX right ideal of an anti-fuzzy HX right ideal of a HX ring $\Re_1$.

E. Theorem

Let R_1 and R_2 be any two rings , $\Re_1$ and $\Re_2$ be HX rings on R_1 and R_2 respectively. Let η_α be an anti-fuzzy HX left ideal on $\Re_2$. If f : $\Re_1\rightarrow \Re_2$ is a homomorphism on HX rings. Let L(η_α ; t) be a lower level HX left ideal of an anti-fuzzy HX left ideal η_α of a HX ring $\Re_2$ then L($f^{-1}(\eta_\alpha)$; t) is a lower level HX left ideal of an anti-fuzzy HX left ideal $f^{-1}(\eta_\alpha)$ of a HX ring $\Re_1$.

1) Proof: Let R_1 and R_2 be any two rings and f : $\Re_1\rightarrow \Re_2$ be a homomorphism.

Let η_α be an anti-fuzzy HX left ideal of a HX ring $\Re_2$. Clearly, $f^{-1}(\eta_\alpha)$ is an anti-fuzzy HX left ideal of a HX ring $\Re_1$. Let X and Y in $\Re_1$, implies f(X) and f(Y) in $\Re_2$.

Let L(η_α ; t) be a lower level HX left ideal of an anti-fuzzy HX left ideal η_α of the HX ring $\Re_2$. Let X ,Y $\in\Re_1$ then f(X) ,f(Y) $\in\Re_2$.

Choose t $\in$ [0 , 1] in such a way that f(X), f(Y)$\in$ L(η_α; t) and hence ,f(X)–f(Y) $\in$ L(η_α ; t).

Then, $\eta_\alpha(f(X)) \leq t$, $\eta_\alpha(f(Y)) \leq t$ and $\eta_\alpha(f(X)–f(Y)) \leq t$.

For this t $\in$ [0 , 1], f(X) $\in$ L(η_α; t) and f(Y)$\in\Re_2$ then f(X)f(Y) $\in$ L(η_α; t) , as L(η_α ; t) be a lower level HX left ideal of an anti-fuzzy HX left ideal η_α of the HX ring $\Re_2$.

Then, $\eta_\alpha(f(X)) \leq t$, $\eta_\alpha(f(X)f(Y)) \leq t$.

We have to prove that L($f^{-1}(\eta_\alpha)$; t) is a lower level HX left ideal of an anti-fuzzy HX left ideal $f^{-1}(\eta_\alpha)$ of a HX ring $\Re_1$. Now, Let X , Y $\in$ L($f^{-1}(\eta_\alpha)$; t).

$$\begin{aligned} (f^{-1}(\eta_\alpha))\ (X-Y) &= \eta_\alpha(f(X–Y)) \\ &= \eta_\alpha(f(X) - f(Y)) \\ &\leq t \\ (f^{-1}(\eta_\alpha))(X–Y) &\leq t \\ X–Y &\in L(f^{-1}(\eta_\alpha);\ t). \end{aligned}$$

Let f(X) $\in$ L(η_α ; t) and f(Y)$\in\Re_2$ then f(X)f(Y)$\in$ L(η_α ; t), as L(η_α ; t) be a lower level HX left ideal of an anti-fuzzy HX left ideal η_α of the HX ring $\Re_2$.

$$\begin{aligned} (f^{-1}(\eta_\alpha))(XY) &\leq (f^{-1}(\eta_\alpha))(Y) \\ &= \eta_\alpha(f(Y)) \\ &\leq t \\ (f^{-1}(\eta_\alpha))(XY) &\leq t \\ XY &\in L(f^{-1}(\eta_\alpha);\ t). \end{aligned}$$

Hence, L($f^{-1}(\eta_\alpha)$; t) is a lower level HX left ideal of an anti-fuzzy HX left ideal of a HX ring $\Re_1$.

F. Theorem

Let R_1 and R_2 be any two rings, $\Re_1$ and $\Re_2$ be HX rings on R_1 and R_2 respectively. Let η_α be an anti-fuzzy HX ideal on $\Re_2$. If f : $\Re_1\rightarrow \Re_2$ is a homomorphism on HX rings. Let L(η_α ; t) be a lower level HX ideal of an anti-fuzzy HX ideal η_α of a HX ring $\Re_2$ then L($f^{-1}(\eta_\alpha)$; t) is a lower level HX ideal of an anti-fuzzy HX ideal $f^{-1}(\eta_\alpha)$ of a HX ring $\Re_1$.

1) Proof: It is clear.

G. Theorem

Let R_1 and R_2 be any two rings, $\Re_1$ and $\Re_2$ be HX rings on R_1 and R_2 respectively. Let λ_μ be an anti-fuzzy HX right ideal on $\Re_1$. If f : $\Re_1\rightarrow \Re_2$ is an anti homomorphism and onto, then the anti-image of a lower level HX right ideal L(λ_μ ; t) of an anti-fuzzy HX right ideal λ_μ of a HX ring $\Re_1$ is a lower level HX left ideal L(f(λ_μ); t) of an anti-fuzzy HX left ideal f (λ_μ) of a HX ring $\Re_2$.

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1) Proof: Let R_1 and R_2 be any two rings and $f : \Re_1 \rightarrow \Re_2$ be an anti homomorphism.

Let λ_μ be an anti-fuzzy HX right ideal of a HX ring $\Re_1$. Clearly, $f(\lambda_\mu)$ is an anti-fuzzy HX left ideal of a HX ring $\Re_2$. Let X and Y in $\Re_1$, implies f (X) and f (Y) in $\Re_2$.

Let $L(\lambda_\mu ; t)$ is a lower level HX right ideal of an anti-fuzzy HX right ideal λ_μ of a HX ring $\Re_1$.

Choose $t \in [0,1]$ in such a way that $X, Y \in L(\lambda_\mu ; t)$ and hence $Y-X \in L(\lambda_\mu ; t)$.

Then, $\lambda_\mu(X) \leq t$ and $\lambda_\mu(Y) \leq t$ and $\lambda_\mu(Y-X) \leq t$.

For this $t \in [0,1]$, let $X \in L(\lambda_\mu ; t)$ and $Y \in \Re_1$ then $XY \in L(\lambda_\mu ; t)$, as $L(\lambda_\mu ; t)$ is a lower level HX right ideal of an anti-fuzzy HX right ideal λ_μ of a HX ring $\Re_1$.

Then, $\lambda_\mu(X) \leq t$ and $\lambda_\mu(XY) \leq t$.

We have to prove that $L(f(\lambda_\mu) ; t)$ is a lower level HX left ideal of an anti-fuzzy HX left ideal $f(\lambda_\mu)$ of a HX ring $\Re_2$.

Let $X, Y \in L(\lambda_\mu ; t)$ and hence $Y-X \in L(\lambda_\mu ; t)$.

For $f(X), f(Y) \in L(f(\lambda^\mu) ; t)$,

$$\begin{aligned} (f(\lambda_\mu))(f(X)-f(Y)) &= (f(\lambda_\mu))(f(Y-X)), \\ &= \lambda_\mu(Y-X) \\ &\leq t \\ (f(\lambda_\mu))(f(X)-f(Y)) &\leq t . \\ (f(X)-f(Y)) &\in L(f(\lambda_\mu); t). \end{aligned}$$

let $X \in L(\lambda_\mu ; t)$ and $Y \in \Re_1$ then $XY \in L(\lambda_\mu ; t)$, as $L(\lambda_\mu ; t)$ is a lower level HX right ideal of an anti-fuzzy HX right ideal λ_μ of a HX ring $\Re_1$.

For, $f(X) \in L(f(\lambda_\mu) ; t)$ and $f(Y) \in \Re_2$,

$$\begin{aligned} (f(\lambda_\mu))(f(Y)\, f(X)) &\leq (f(\lambda_\mu))(f(X)) \\ &\leq t \\ (f(\lambda_\mu))(f(Y)(f(X)) &\leq t. \\ (f(Y)\, f(X)) &\in L(f(\lambda_\mu); t). \end{aligned}$$

Hence, $L(f(\lambda_\mu) ; t)$ is a lower level HX right ideal of an anti-fuzzy HX right ideal $f(\lambda_\mu)$ of a HX ring $\Re_2$.

H. Theorem

Let R_1 and R_2 be any two rings, $\Re_1$ and $\Re_2$ be HX rings on R_1 and R_2 respectively. Let λ_μ be an anti-fuzzy HX left ideal on $\Re_1$. If $f : \Re_1 \rightarrow \Re_2$ is an anti homomorphism and onto, then the anti-image of a lower level HX left ideal $L(\lambda_\mu ; t)$ of an anti-fuzzy HX left ideal λ_μ of a HX ring $\Re_1$ is a lower level HX right ideal $L(f(\lambda_\mu); t)$ of an anti-fuzzy HX right ideal $f(\lambda_\mu)$ of a HX ring $\Re_2$.

1) Proof: Let R_1 and R_2 be any two rings and $f : \Re_1 \rightarrow \Re_2$ be an anti homomorphism.

Let λ_μ be an anti-fuzzy HX left ideal of a HX ring $\Re_1$. Clearly, $f(\lambda_\mu)$ is an anti-fuzzy HX right ideal of a HX ring $\Re_2$. Let X and Y in $\Re_1$, implies f (X) and f (Y) in $\Re_2$.

Let $L(\lambda_\mu ; t)$ is a lower level HX left ideal of an anti-fuzzy HX left ideal λ_μ of a HX ring $\Re_1$.

Choose $t \in [0,1]$ in such a way that $X, Y \in L(\lambda_\mu ; t)$ and hence $Y-X \in L(\lambda_\mu ; t)$.

Then, $\lambda_\mu(X) \leq t$ and $\lambda_\mu(Y) \leq t$ and $\lambda_\mu(Y-X) \leq t$.

For this $t \in [0,1]$, let $X \in L(\lambda_\mu ; t)$ and $Y \in \Re_1$ then $XY \in L(\lambda_\mu ; t)$, as $L(\lambda_\mu ; t)$ is a lower level HX left ideal of an anti-fuzzy HX left ideal λ_μ of a HX ring $\Re_1$.

Then, $\lambda_\mu(X) \leq t$ and $\lambda_\mu(XY) \leq t$.

We have to prove that $L(f(\lambda_\mu) ; t)$ is a lower level HX right ideal of an anti-fuzzy HX right ideal $f(\lambda_\mu)$ of a HX ring $\Re_2$.

Let $X, Y \in L(\lambda_\mu ; t)$ and hence $Y-X \in L(\lambda_\mu ; t)$.

For $f(X), f(Y) \in L(f(\lambda^\mu) ; t)$,

$$\begin{aligned} (f(\lambda_\mu))(f(X) - f(Y)) &= (f(\lambda_\mu))(f(Y-X)), \\ &= \lambda_\mu(Y-X) \\ &\leq t \\ (f(\lambda_\mu))(f(X) - f(Y)) &\leq t . \\ (f(X) - f(Y)) &\in L(f(\lambda_\mu); t). \end{aligned}$$

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let $X \in L(\lambda_\mu ; t)$ and $Y \in \Re_1$ then $YX \in L(\lambda_\mu ; t)$, as $L(\lambda_\mu ; t)$ is a lower level HX left ideal of an anti-fuzzy HX left ideal λ_μ of a HX ring $\Re_1$.

For, $f(X) \in L(f(\lambda_\mu) ; t)$ and $f(Y) \in \Re_2$,

$$\begin{aligned} (f(\lambda_\mu))(f(X)\, f(Y)) &\leq (f(\lambda_\mu))f(X) \\ &\leq t \\ (f(\lambda_\mu))(f(X)(f(Y)) &\leq t. \\ (f(X)\, f(Y)) &\in L(f(\lambda_\mu); t). \end{aligned}$$

Hence, $L(f(\lambda_\mu) ; t)$ is a lower level HX right ideal of an anti-fuzzy HX right ideal $f(\lambda_\mu)$ of a HX ring $\Re_2$.

I. Theorem

Let R_1 and R_2 be any two rings, $\Re_1$ and $\Re_2$ be HX rings on R_1 and R_2 respectively. Let λ_μ be an anti-fuzzy HX ideal on $\Re_1$. If $f : \Re_1 \rightarrow \Re_2$ is an anti homomorphism and onto, then the anti-image of a lower level HX ideal $L(\lambda_\mu ; t)$ of an anti-fuzzy HX ideal λ_μ of a HX ring $\Re_1$ is a lower level HX ideal $L(f(\lambda_\mu); t)$ of an anti-fuzzy HX ideal $f(\lambda_\mu)$ of a HX ring $\Re_2$.

1) Proof: It is clear.

J. Theorem

Let R_1 and R_2 be any two rings , $\Re_1$ and $\Re_2$ be HX rings on R_1 and R_2 respectively. Let η_α be an anti-fuzzy HX right ideal on $\Re_2$. If $f: \Re_1 \rightarrow \Re_2$ is an anti homomorphism on HX rings. Let $L(\eta_\alpha ; t)$ be a lower level HX right ideal of an anti-fuzzy HX right ideal η_α of a HX ring $\Re_2$ then $L(f^{-1}(\eta_\alpha); t)$ is a lower level HX left ideal of an anti-fuzzy HX left ideal $f^{-1}(\eta_\alpha)$ of a HX ring $\Re_1$.

1) Proof: Let R_1 and R_2 be any two rings and $f : \Re_1 \rightarrow \Re_2$ be an anti homomorphism.

Let η_α be an anti-fuzzy HX right ideal of a HX ring $\Re_2$. Clearly, $f^{-1}(\eta_\alpha)$ is an anti-fuzzy HX left ideal of a HX ring $\Re_1$. Let X and Y in $\Re_1$, implies f(X) and f(Y) in $\Re_2$.

Let $L(\eta_\alpha ; t)$ be a lower level HX right ideal of an anti-fuzzy HX right ideal η_α of the HX ring $\Re_2$.Let $X , Y \in \Re_1$ then $f(X) , f(Y) \in \Re_2$.

Choose $t \in [0 , 1]$ in such a way that $f(X), f(Y) \in L(\eta_\alpha; t)$ and hence ,$f(Y)–f(X) \in L(\eta_\alpha ; t)$.

Then, $\eta_\alpha(f(X)) \leq t$, $\eta_\alpha(f(Y)) \leq t$ and $\eta_\alpha(f(Y)–f(X)) \leq t$.

For this $t \in [0 , 1]$, $f(X) \in L(\eta_\alpha; t)$ and $f(Y) \in \Re_2$ then $f(X)f(Y) \in L(\eta_\alpha; t)$, as $L(\eta_\alpha ; t)$ be a lower level HX right ideal of an anti-fuzzy HX right ideal η_α of the HX ring $\Re_2$.

Then, $\eta_\alpha(f(X)) \leq t$, $\eta_\alpha(f(X)f(Y)) \leq t$.

We have to prove that $L(f^{-1}(\eta_\alpha); t)$ is a lower level HX left ideal of an anti-fuzzy HX left ideal $f^{-1}(\eta_\alpha)$ of a HX ring $\Re_1$. Now, Let $X , Y \in L(f^{-1}(\eta_\alpha) ; t)$.

$$\begin{aligned} (f^{-1}(\eta_\alpha))\, (X - Y) &= \eta_\alpha(f(X–Y)) \\ &= \eta_\alpha(f(Y)–f(X)) \\ &\leq t \\ (f^{-1}(\eta_\alpha))(X–Y) &\leq t \\ X–Y &\in L(f^{-1}(\eta_\alpha); t). \end{aligned}$$

Let $f(X) \in L(\eta_\alpha ; t)$ and $f(Y) \in \Re_2$ then $f(X)f(Y) \in L(\eta_\alpha ; t)$, as $L(\eta_\alpha ; t)$ be a lower level HX right ideal of an anti-fuzzy HX right ideal η_α of the HX ring $\Re_2$.

$$\begin{aligned} (f^{-1}(\eta_\alpha))\, (YX) &\leq f^{-1}(\eta_\alpha)(X) \\ &= \eta_\alpha(f(X)) \\ &\leq t \\ (f^{-1}(\eta_\alpha))\, (YX) &\leq t \\ YX &\in L(f^{-1}(\eta_\alpha); t). \end{aligned}$$

Hence, $L(f^{-1}(\eta_\alpha); t)$ is a lower level HX left ideal of an anti-fuzzy HX left ideal of a HX ring $\Re_1$.

K. Theorem

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Let R_1 and R_2 be any two rings , $\Re_1$ and $\Re_2$ be HX rings on R_1 and R_2 respectively. Let η_α be an anti-fuzzy HX left ideal on $\Re_2$. If $f : \Re_1 \rightarrow \Re_2$ is a homomorphism on HX rings. Let $L(\eta_\alpha ; t)$ be a lower level HX left ideal of an anti-fuzzy HX left ideal η_α of a HX ring $\Re_2$ then $L(f^{-1}(\eta_\alpha); t)$ is a lower level HX right ideal of an anti-fuzzy HX right ideal $f^{-1}(\eta_\alpha)$ of a HX ring $\Re_1$.

1) Proof: Let R_1 and R_2 be any two rings and $f : \Re_1 \rightarrow \Re_2$ be an anti homomorphism.

Let η_α be an anti-fuzzy HX left ideal of a HX ring $\Re_2$. Clearly, $f^{-1}(\eta_\alpha)$ is an anti-fuzzy HX right ideal of a HX ring $\Re_1$. Let X and Y in $\Re_1$, implies f(X) and f(Y) in $\Re_2$.

Let $L(\eta_\alpha ; t)$ be a lower level HX left ideal of an anti-fuzzy HX left ideal η_α of the HX ring $\Re_2$. Let $X, Y \in \Re_1$ then $f(X), f(Y) \in \Re_2$.

Choose $t \in [0, 1]$ in such a way that $f(X), f(Y) \in L(\eta_\alpha; t)$ and hence, $f(Y) - f(X) \in L(\eta_\alpha ; t)$.

Then, $\eta_\alpha(f(X)) \leq t$, $\eta_\alpha(f(Y)) \leq t$ and $\eta_\alpha(f(Y) - f(X)) \leq t$.

For this $t \in [0, 1]$, $f(X) \in L(\eta_\alpha; t)$ and $f(Y) \in \Re_2$ then $f(X)f(Y) \in L(\eta_\alpha; t)$, as $L(\eta_\alpha ; t)$ be a lower level HX left ideal of an anti-fuzzy HX left ideal η_α of the HX ring $\Re_2$.

Then, $\eta_\alpha(f(X)) \leq t$, $\eta_\alpha(f(X)f(Y)) \leq t$.

We have to prove that $L(f^{-1}(\eta_\alpha); t)$ is a lower level HX right ideal of an anti-fuzzy HX right ideal $f^{-1}(\eta_\alpha)$ of a HX ring $\Re_1$. Now, Let $X , Y \in L(f^{-1}(\eta_\alpha) ; t)$.

$$\begin{aligned} (f^{-1}(\eta_\alpha))(X - Y) &= \eta_\alpha(f(X-Y)) \\ &= \eta_\alpha(f(Y) - f(X)) \\ &\leq t \\ (f^{-1}(\eta_\alpha))(X-Y) &\leq t \\ X-Y &\in L(f^{-1}(\eta_\alpha); t). \end{aligned}$$

Let $f(X) \in L(\eta_\alpha ; t)$ and $f(Y) \in \Re_2$ then $f(X)f(Y) \in L(\eta_\alpha ; t)$, as $L(\eta_\alpha ; t)$ be a lower level HX left ideal of an anti-fuzzy HX left ideal η_α of the HX ring $\Re_2$.

$$\begin{aligned} (f^{-1}(\eta_\alpha))(XY) &\leq (f^{-1}(\eta_\alpha))(X) \\ &= \eta_\alpha(f(X)) \\ &\leq t \\ (f^{-1}(\eta_\alpha))(XY) &\leq t \\ XY &\in L(f^{-1}(\eta_\alpha); t). \end{aligned}$$

Hence, $L(f^{-1}(\eta_\alpha); t)$ is a lower level HX right ideal of an anti-fuzzy HX right ideal of a HX ring $\Re_1$.

L. Theorem

Let R_1 and R_2 be any two rings, $\Re_1$ and $\Re_2$ be HX rings on R_1 and R_2 respectively. Let η_α be an anti-fuzzy HX ideal on $\Re_2$. If $f : \Re_1 \rightarrow \Re_2$ is an anti homomorphism on HX rings. Let $L(\eta_\alpha ; t)$ be a lower level HX ideal of an anti-fuzzy HX ideal η_α of a HX ring $\Re_2$ then $L(f^{-1}(\eta_\alpha); t)$ is a lower level HX ideal of an anti-fuzzy HX ideal $f^{-1}(\eta_\alpha)$ of a HX ring $\Re_1$.

1) Proof: It is clear.

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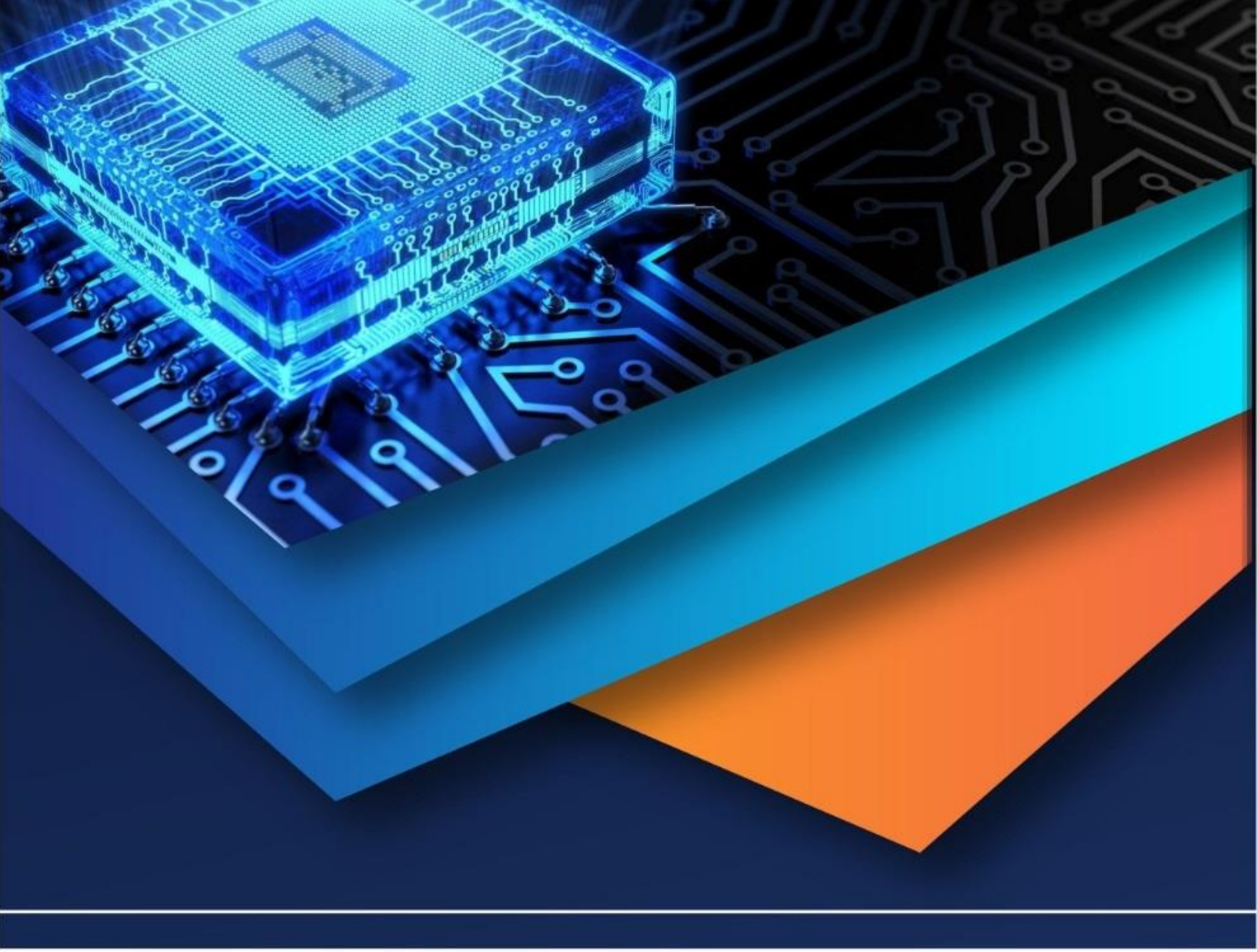
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