Vertex Polynomial of Path related graphs

A. M. Anto[1]
[1]Assistant Professor in Mathematics, MalanKara Catholic College, Kaliyakavilai, Tamil Nadu, India.

Abstract: ***The vertex polynomial of the graph $G = (V, E)$ is defined as $V(G, x) = \sum_{k=0}^{\Delta(G)} v_k x^k$, where $\Delta(G) = max\{d(v)/v\epsilon V\}$ and v_k is the number of vertices of degree k. In this paper I find the Vertex Polynomial of some Path related graphs.***
Keywords: Vertex Polynomial, Splitting graph, Degree splitting graph, Path, corona.

I. INTRODUCTION

Here I consider simple undirected graphs only. The terms not defined here we can refer Frank Harary [2]. The vertex set is denoted by V and the edge set by E. For $v\epsilon V$, $d(v)$ is the number of edges incident with v, the maximum degree of the graph G is defined as $\Delta(G) = \max\{d(v)/v\epsilon V\}$. Let $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ be two graphs, the union $G_1 \cup G_2$ is defined to be $G = (V, E)$ where $V = V_1 \cup V_2$ and $E = E_1 \cup E_2$, the sum $G_1 + G_2$ is defined as $G_1 \cup G_2$ together with all the lines joining points of V_1 to V_2. The Cartesian product of two graphs G_1 and G_2 denoted by $G = G_1 \times G_2$ is the graph G such that $V(G) = V(G_1) \times V(G_2)$, that is every vertex of $G_1 \times G_2$ is an ordered pair (u, v), where $u \in V(G_1)$ and $v \in V(G_2)$ and two distinct vertices (u, v) and (x, y) are adjacent in $G_1 \times G_2$ if either $u = x$ and $vy \in E(G_2)$ or $v = y$ and $ux \in E(G_1)$. If G is of order n, the corona of G with H, $G \odot H$ is the graph obtained by taking one copy of G and n copies of H and joining the i^{th} vertex of G with an every vertex in the i^{th} copy of H. The graph G with $V = S_1 \cup S_2 \cup \ldots \cup S_i \cup T$, where each S_i is a set of vertices having at least two vertices and having the same degree and $T = V \backslash \cup S_i$. The degree splitting graph of G denoted by $DS(G)$ and is obtained from G by adding the vertices $w_1, w_2, \ldots, w_t$ and joining w_i to each vertex of S_i, $1 \le i \le t$ [5]. For each vertex v of a graph G, take a new vertex v′, join v′ to all the vertices of G which are adjacent to v. The graph $S(G)$ thus obtained is called splitting graph of G [1]. The Path consisting of length n is denoted by P_n. The graph $G = (V, E)$ is simply denoted by G.

II. MAIN RESULTS

A. *Theorem: 2.1*

The graph $P_m \cup P_n$ has the vertex polynomial $V(P_m \cup P_n , x) = (m + n - 4)x^2 + 4x$.

1) *Proof:* The graphs P_m and P_n have degree m and n respectively. Then $P_m \cup P_n$ has order $m + n$. Among this $m + n$ vertices, $m - 2$, $n - 2$ vertices have degree 2 and 4 vertices have degree 1. Therefore, $V(P_m \cup P_n , x) = (m + n - 4)x^2 + 4x$.

B. *Theorem: 2.2*

The graph $S(P_m \cup P_n)$ has the vertex polynomial $V(S(P_m \cup P_n) , x) = (m + n - 4)x^4 + (m + n - 4)x^2 + 4x^2 + 4x$.

1) *Proof:* The graphs P_m and P_n have degree m and n respectively. Then, $S(P_m \cup P_n)$ has order $2(m + n)$. Among this $2(m + n)$ vertices, $m + n - 4$ vertices have degree 4, $m + n - 4$ vertices have degree 2, 4 vertices have degree 2 and 4 vertices have degree 1. Hence, $V(S(P_m \cup P_n) , x) = (m + n - 4)x^4 + (m + n - 4)x^2 + 4x^2 + 4x$.

C. *Theorem: 2.3*

The graph $DS(P_m \cup P_n)$ has the vertex polynomial $V(DS(P_m \cup P_n) , x) = x^{m+n-4} + x^4 + (m + n - 4)x^3 + 4x^2$.

1) *Proof:* The graphs P_m and P_n have degree m and n respectively. Then the graph $DS(P_m \cup P_n)$ has order $m + n + 2$. Among this $m + n + 2$ vertices, one vertex has degree $m + n - 4$, one vertex has degree 4, $m + n - 4$ vertices have degree 3 and 4 vertices have degree 2. Hence, $V(DS(P_m \cup P_n) , x) = x^{m+n-4} + x^4 + (m + n - 4)x^3 + 4x^2$.

D. *Theorem: 2.4*

The graph $P_m + P_n$ has the vertex polynomial $V(P_m + P_n , x) = (m - 2)x^{n+2} + (n - 2)x^{m+2} + 2x^{n+1} + 2x^{m+1}$.

1) *Proof:* The graph $P_m + P_n$ has order $m + n$. Among this $m + n$ vertices, $m - 2$ vertices have degree $n + 2$, $n - 2$ vertices have degree $m + 2$, 2 vertices have degree $n + 1$ and 2 vertices have degree $n + 1$. Therefore, $V(P_m + P_n , x) = (m - 2)x^{n+2} + (n - 2)x^{m+2} + 2x^{n+1} + 2x^{m+1}$.

E. *Theorem: 2.5*

The graph $S(P_m + P_n)$ has the vertex polynomial $V(\mathrm{S}(P_m + P_n), x) = (m-2)x^{2(n+2)} + (n-2)x^{2(m+2)} + 2x^{2(n+1)} + 2x^{2(m+1)} + (m-2)x^{n+2} + (n-2)x^{m+2} + 2x^{n+1} + 2x^{m+1}$.

1) *Proof:* The graph $S(P_m + P_n)$ has order $2(m+n)$. Among this $2(m+n)$ vertices, $m-2$ vertices have degree $2(n+2)$, $n-2$ vertices have degree $2(m+2)$, 2 vertices have degree $2(n+1)$, 2 vertices have degree $2(m+1)$, $m-2$ vertices have degree $n+2$, $n-2$ vertices have degree $m+2$, 2 vertices have degree $n+1$ and 2 vertices have degree $m+1$. Hence, $V(\mathrm{S}(P_m + P_n), x) = (m-2)x^{2(n+2)} + (n-2)x^{2(m+2)} + 2x^{2(n+1)} + 2x^{2(m+1)} + (m-2)x^{n+2} + (n-2)x^{m+2} + 2x^{n+1} + 2x^{m+1}$.

F. Theorem: 2.6

The graph $DS(P_m + P_n)$ has the vertex polynomial $V(\mathrm{DS}(P_m + P_n), x) = x^{n-2} + x^{m-2} + (n-2)x^{m+3} + (m-2)x^{n+3} + 2x^{m+2} + 2x^{n+2} + 2x^2$.

1) *Proof*: The graph $DS(P_m + P_n)$ has order $m+n+4$. Among this $m+n+4$ vertices, one vertex has degree $n-2$, one vertex has degree $m-2$, $n-2$ vertices have degree $m+3$, $m-2$ vertices have degree $n+3$, 2 vertices have degree $m+2$, 2 vertices have degree $n+2$ and 2 vertices have degree 2. Therefore, $V(\mathrm{DS}(P_m + P_n), x) = x^{n-2} + x^{m-2} + (n-2)x^{m+3} + (m-2)x^{n+3} + 2x^{m+2} + 2x^{n+2} + 2x^2$.

G. Theorem: 2.7

The graph $P_m \odot P_n$ has the vertex polynomial $V(P_m \odot P_n, x) = (m-2)x^{n+2} + 2x^{n+1} + (mn-2m)x^3 + 2mx^2$.

1) *Proof:* The graph $P_m \odot P_n$ has order $m(n+1)$. Among this $m(n+1)$ vertices, $m-2$ vertices have degree $n+2$, 2 vertices have degree $n+1$, $mn-2m$ vertices have degree 3 and $2m$ vertices have degree 2. Therefore, $V(P_m \odot P_n, x) = (m-2)x^{n+2} + 2x^{n+1} + (mn-2m)x^3 + 2mx^2$.

H. Theorem: 2.8

The graph $S(P_m \odot P_n)$ has the vertex polynomial $V(S(P_m \odot P_n), x) = (m-2)x^{2(n+2)} + (m-2)x^{n+2} + 2x^{2(n+1)} + 2x^{n+1} + (mn-2m)x^6 + (mn-2m)x^3 + 2mx^4 + 2mx^2$.

1) *Proof:* The graph $S(P_m \odot P_n)$ has order $2m(n+1)$. Among this $2m(n+1)$ vertices, $m-2$ vertices have degree $2(n+2)$, $m-2$ vertices have degree $n+2$, 2 vertices have degree $2(n+1)$, 2 vertices have degree $n+1$, $mn-2m$ vertices have degree 6, $mn-2m$ vertices have degree 3, $2m$ vertices have degree 4 and $2m$ vertices have degree 2. Hence, $V(P_m \odot P_n, x) = (m-2)x^{2(n+2)} + (m-2)x^{n+2} + 2x^{2(n+1)} + 2x^{n+1} + (mn-2m)x^6 + (mn-2m)x^3 + 2mx^4 + 2mx^2$.

I. Theorem: 2.9

The graph $DS(P_m \odot P_n)$ has the vertex polynomial $V(DS(P_m \odot P_n), x) = (m-2)x^{n+3} + 2x^{n+2} + x^{mn-2m} + x^{2m} + x^{m-2} + (mn-2m)x^4 + 2mx^3 + x^2$.

1) *Proof:* The graph $S(P_m \odot P_n)$ has order $2m(n+1)$. Among this $2m(n+1)$ vertices, $m-2$ vertices have degree $n+3$, 2 vertices have degree $n+2$, one vertex has degree $mn-2m$, one vertex has degree $2m$, one vertex has degree $m-2$, $mn-2m$ vertices have degree 4, $2m$ vertices have degree 3 and one vertex have degree 2. Therefore, $V(DS(P_m \odot P_n), x) = (m-2)x^{n+3} + 2x^{n+2} + x^{mn-2m} + x^{2m} + x^{m-2} + (mn-2m)x^4 + 2mx^3 + x^2$.

REFERENCES

[1] E.Sampathkumar and H.B.Walikar, On splitting graph of a graph, J. Karnatak Univ. Sci., (25-26) (1980-81), 13-16.

[2] Frank Harary, 1872,"Graph Theory", Addition – Wesley Publishing Company.

[3] Gary Chartrant and Ping Znank, "Introduction to Graph Theory", Tata McGraw-Hill Edition.

[4] J.Devaraj, E.Sukumaran "On Vertex Polynomial", International J. of Math.sci & Engg Appls(IJMESA) Vol. 6 No. 1 (January, 2012), pp. 371-380.

[5] S. S. Sandhya, C. Jeyasekaran, C. D. Raj (2013), "Harmonic Mean Labelling Of Degree Splitting Graphs" Bulletin of Pure and Applied Sciences, 32E, 99-112.