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Complementary Domination in Intunionistic Fuzzy Graphs

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Abstract: Let G be an intuitionistic fuzzy graph. Let u and v be two vertices of G. Let D be a minimal dominating set of $G(V,\sigma,\mu)$. If V-D contains a dominating set D', then D' is called a complementary dominating set of G with respect to D. The complementary intuionistic fuzzy domination number $\gamma_{cf}(G)$ of a fuzzy graph G is the minimum cardinality of a complementary dominating set of G. The aim of this study is to define the complementary domination set and complementary domination number $\gamma(G)$. Further discuses the properties and bounds of the complementary domination number of the intunionistic fuzzy graphs.

Key words: Intunionistic Fuzzy Graph, Intunionistic Fuzzy Domination, Intunionistic Complementary Fuzzy Domination and Intunionistic Complementary Fuzzy Domination number

I. INTRODUCTION

The first definition of fuzzy graphs was proposed by Kafmann, from the fuzzy relations introduced by Zadeh. Although Rosenfeld introduced another elaborated definition, including fuzzy vertex and fuzzy edges, and several fuzzy analogs of graph theoretic concepts such as paths, cycles, connectedness and etc. The concept of domination in fuzzy graphs was investigated by A. Somasundaram, S. Somasundaram and A. Somasundaram present the concepts of independent domination, total domination, connected domination of fuzzy graphs . C. Natarajan and S.K. Ayyaswamy introduce the strong (weak) domination in fuzzy graph. Somasundaram, A & Somasundaram,S introduced the concepts of domination and total domination in fuzzy graphs and determined the domination number for several classes of fuzzy graphs and obtained bounds for the same. Somasundaram, A studied several operations on fuzzy graphs such as union, join, composition, Cartesian product and obtained their domination parameters. Nair,PS & Cheng, SC discussed the concepts of clique and fuzzy cliques in fuzzy graphs. Various properties of fuzzy cliques and a characterization of fuzzy cliques were also presented. Monderson, JN & Yao, YY examined the properties of various types of fuzzy cycles, fuzzy trees, fuzzy bridges, and fuzzy cut nodes in fuzzy graphs. Nagoor Gani, A & Basheer Ahmed, M examined the properties of various types of degree, order and size of fuzzy graphs and compared the relationship between degree, order and size of fuzzy graphs. In this paper we develop the complementary efficient domination in fuzzy graphs and investigate the property of domination parameter in fuzzy graph.

II. PRELIMINARIES

An intuitionistic fuzzy graph (IFG) is of the form G=(V,E) , where $V=\{v_1,v_2,...,v_n\}$ such that $\mu_1:V\to[0,1]$, $\gamma_1:V\to[0,1]$ denote the degree of membership and nonmember ship of the element $v_i\in V$ respectively and $0\le\mu_1(v_i)+\gamma_1(v_i)\le 1$ for every $v_i\in V,(i=1,2,...n)$. $E\subseteq V\times V$ where
$\mu_2:V\times V\to[0,1]$, *and* $\gamma_2:V\times V\to[0,1]$ are such that
$\mu_2(v_i,v_j)\le\mu_1(v_i)\wedge\mu_1(v_j)$, $\gamma_2(v_i,v_j)\le\gamma_1(v_i)\vee\gamma_1(v_j)$ and $0\le\mu_2(v_i,v_j)+\gamma_2(v_i,v_j)\le 1$

An arc (v_i,v_j) of an IFG G is called an *strong arc* if $\mu_2(v_i,v_j)=\mu_1(v_i)\wedge\mu_1(v_j)$, $\gamma_2(v_i,v_j)=\gamma_1(v_i)\wedge\gamma_1(v_j)$.

Let G = (V,E) be an IFG. Then the *cardinality of G* is defined to be

$|G| = \left|\sum_{v_i \in V}[\frac{(1+\mu_1(v_i) - \gamma_1(v_i))}{2}]\right| + \left|\sum_{v_i \in V}[\frac{(1+\mu_2(v_i,v_j) - \gamma_1(v_i,v_j))}{2}]\right|$. Let G = (V, E) be an IFG. The *vertex cardinality* of G is defined to be $|G| = \left|\sum_{v_i \in V}[\frac{(1+\mu_1(v_i) - \gamma_1(v_i))}{2}]\right|$ for all $v_i \in V, (i = 1,2,...n)$. Let G = (V, E) be an IFG.

An *edge cardinality* of G is defined to be $|G| = \left|\sum_{v_i \in V}[\frac{(1+\mu_2(v_i,v_j) - \gamma_1(v_i,v_j))}{2}]\right|$ for all $(v_i, v_j) \in V \times V$, Let G = (V, E) be an IFG. A set $D \subseteq V$ is said to be a dominating set of G if every $v \in V - D$ there exist $u \in D$ such that u dominates v.

An intunionistic fuzzy dominating set D of an IFG, G is called minimal dominating set of G if every node $u \in D$, $D - \{u\}$ is not a dominating set in G. An intunionistic fuzzy domination number $\gamma_{if}(G)$ of an IFG, G is the minimum vertex cardinality over all minimal dominating sets in G.

A set $S \subseteq V$ in an IFG, G is said to be an independent if there is no strong between the vertices $v \in S$. An independent set S of IFG ,G is said to be maximal independent set if every node $v \in V - S$ then the set $S \cup \{v\}$ is not an independent set in G. The minimum cardinality among all the maximal independent sets in an IFG, G is called the intunionistic fuzzy independent number.

III. COMPLEMENTRY DOMINATION

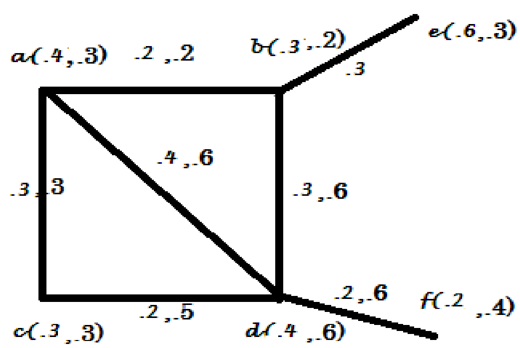
A. Definition 2.1

Let D be a minimal dominating set of an IFG, $G(V,E)$. If V-D contains a dominating set D', then D' is called a complementary dominating set of G with respect to D.

B. Definition 2.2

The complementary intunionistic fuzzy domination number $\gamma_{cif}(G)$ of a fuzzy graph G is the minimum cardinality of a complementary dominating set of G.

1) Example



Minimum complementary dominating set $D' = \{a.e, f\}$ & $\gamma_{cif}(G) = 2.8$

C. Proposition 2.1. For any fuzzy graph G ,

$\gamma_f(G) < \gamma_{cif}(G)$

1) Proof: Clearly every complementary dominating set is a dominating set of G. Therefore $\gamma_{if}(G) < \gamma_{cif}(G)$.

D. Proposition 2.2. For any intunionistic fuzzy graph of order p, $\gamma_{cif}(G) \le p - \Delta_N(G) \le p - \Delta_E(G)$.

1) *Proof:* Let u be a vertex of degree $\Delta_N(G)$ in a fuzzy graph G. Then clearly V-N(u) is a complementary dominating set of G. Therefore $\gamma_{cif}(G) \le |V - N(u)| = p - \Delta_N(G)$. Since $\Delta_N(G) > \Delta_E(G)$, therefore $\gamma_{cif}(G) \le p - \Delta_N(G) \le p - \Delta_E(G)$.

E. *Proposition 2.3.* For any intunionstic fuzzy graph G , $\gamma_{cif}(G) \le \beta_{if}(G)$.

1) *Proof :* Let D be the minimum dominating set of a fuzzy graph G and I be a maximum independent set in V-D. Every vertex in $V-(D \cup I)$ must be adjacent to exactly one vertex in I. If every vertex in D is adjacent to at least one vertex in I. then I is the complementary dominating set of G.

Otherwise $D' \subset D$ be such that no vertex in D' is adjacent to an edge in I. then every vertex in D' is adjacent to at least one vertex in $V-(D \cup I)$ so far each vertex in D' pick in $V-(D \cup I)$ a vertex adjacent to it and let R be the set of all vertex. Then $|R|_{if} \le |D'|_{if}$ and $\gamma_{cif}(G) \le |I \cup R| \le |I| + |D'| \le \beta_{if}(G)$.

F. *Theorem 2.1.* Let G be an intunionistic fuzzy graph, then

$$\gamma_f(G) \le \gamma_{cif}(G).$$

1) *Proof:* D' is a minimum intunionistic fuzzy dominating set with respect to D.D' is also a dominating set but not minimum dominating set , therefore $|D|_{if} \le |D'|_{if} \Rightarrow \gamma_{if}(G) \le \gamma_{cif}(G)$.

G. *Theorem 2.2.* For any fuzzy graph G, then

$$\gamma_{cif}(G) \le \frac{O(G)}{2}.$$

1) *Proof:* Let D' be the minimum complementary dominating set with respect to D. $D' \subseteq V - D$

$$\Rightarrow |D'|_{if} \le |V|_{if} - |D|_{if}$$

$$\Rightarrow \gamma_{cif}(G) \le O(G) - \gamma_{if}(G).\ [\text{Since } \gamma_{if}(G) \le \frac{O(G)}{2}]$$

$$\Rightarrow \gamma_{cif}(G) \le O(G) - \frac{O(G)}{2}$$

$$\Rightarrow \gamma_{cif}(G) \le \frac{O(G)}{2}.$$

Hence proved.

2) *Remark:* $\gamma_{if}(G) + \gamma_{cif}(G) \le O(G)$

H. *Theorem 2.3.* Let G be a fuzzy graph. D is the minimum dominating set of G. If ⟨V-D⟩ does not contain any strong arc, then

$$\gamma_{if}(G) + \gamma_{cif}(G) = O(G).$$

1) *Proof:* Let G be an intunionistic fuzzy graph. D is the minimum dominating set of G. Then $|D|_f = \gamma_f(G)$. If $<V-D>$ does not contain any strong arc, therefore every vertex in $<V-D>$ is dominate itself.

$$\Rightarrow |D|_{if} = |V - D'|_{if}$$

$$\Rightarrow \gamma_{cif}(G) = |V|_{if} - |D|_{if}$$

$$\Rightarrow \gamma_{cif}(G) + \gamma_{if}(G) = O(G).$$

Hence proved.

I. *Theorem 2.4.* Let G be a strong intunionistic fuzzy graph. Let D, be the minimum complementary dominating set with respect to D, then

$$\Rightarrow \gamma_{cif}(G) + \gamma_{if}(G) < O(G).$$

1) *Proof:* Let G be a strong fuzzy graph. Let D, be the minimum complementary dominating set with respect to D, therefore $D' \subseteq V - D$. Assume G is strong fuzzy graph then $<V-D>$ is also a strong fuzzy graph. Therefore a vertex in $<V-D>$ has the strong arc between some other vertices. So we have D' ⊂ V-D.

$\Rightarrow |D'|_{if} < |V-D|_{if}$

$\Rightarrow \gamma_{cif}(G) < |V|_{if} - |D|_{if}$

$\Rightarrow \gamma_{cif}(G) + \gamma_{if}(G) < O(G).$

Hence proved.

J. *Theorem 2.5.* Let G be a complete intunionistic fuzzy graph D' be the inverse fuzzy dominating set with respect to D of G. Then

$\gamma_{cif}(G) + \gamma_{if}(G) = \sigma_1 + \sigma_2.$

Here σ_1, σ_2 be the least two membership values of the vertices in G.

1) *Proof:* Let $\sigma_1(G) = u$, $\sigma_2(G) = v$ (say), be the least two membership values of vertices in G. Let G be the complete intunionistic fuzzy graph. D' is the minimum complementary dominating set with respect to D. Therefore $|D|_{if} = \gamma_{if}(G)$.

$\Rightarrow \gamma_{if}(G) = \sigma_1$ [since G is complete]

G is a complete fuzzy graph. Therefore ⟨V-D⟩ is also a complete graph. $D' \in V - D$

$\Rightarrow |D'|_{if} = \gamma_{cif}(G) = \sigma_2$

$\Rightarrow \gamma_{cif}(G) + \gamma_{if}(G) = \sigma_1 + \sigma_2.$

K. *Theorem 2.6.* Let G be an intunionistic fuzzy graph, G does not contain any strong arc, then $\gamma_{cif}(G) = 0$.

1) *Proof:* Let G be an intunionistic fuzzy graph, G does not contain any strong arc. Let D' be the minimum complementary dominating set with respect t to D in G.

$\Rightarrow |D|_{if} = |V|_{if} = O(G)$

There is no strong arc in G. Therefore every vertex dominates itself. $D' \in V - D$

$\Rightarrow |D'|_{if} = |V-D|_{if}$

$\Rightarrow \gamma_{cif}(G) \le |V|_{if} - |D|_{if}$

$\Rightarrow \gamma_{cif}(G) = 0$. Hence proved.

L. *Theorem 2.7* Let G_1 and G_2 are an intunionistic fuzzy graph and D_1' and D_2' be a complementary dominating set with respect to D_1 and D_2 respectively. Then $G_1 \times G_2$ is a fuzzy graph and we have $D_1 \times (V_2\text{-}D_2)$ or $D_2 \times (V_1\text{-} D_1)$ is a complementary dominating set of $G_1 \times G_2$.

1) *Proof:* Let D_1^{-1} and D_2^{-1} be a complementary intunionistic fuzzy dominating set with respect to D_1 and D_2 of G_1 and G_2 respectively. We know that $\gamma_f(G_1 \times G_2) = \min\{|D_1 \times V_2|_f, |V_1 \times D_2|_f\}$. Therefore we note that the complementary dominating set $D_1 \times D_2' \subseteq V_1 \times V_2 - (D_1 \times V_2)$ or $D_1 \times D_2' \subseteq V_1 \times V_2 - (D_2 \times V_1)$. D_1, D_2 are minimum dominating set of G_1 and G_2 respectively.

The sub graph $<V_1 \times V_2 - (D_1 \times D_2)>$ of a fuzzy graph $G_1 \times G_2$. $D_1 \times V_2$ and $D_2 \times V_1$ are minimum dominating set of $G_1 \times G_2$. Therefore $D_1 \times V_2$ dominates $D_2 \times V_1$ and vise versa.

The sub graph $<V_1 \times V_2 - (D_1 \times D_2)>$ in $G_1 \times G_2$. $D_2 \times V_1$ is a dominating set of $G_1 \times G_2$. Therefore sub graph $<(V_1 - D_1) \times V_2)>$ in $G_1 \times G_2$. D_2 dominates V_2. Therefore $D_2 \times (V_1\text{-}D_1)$ is a dominating set of $<(V_1\text{-}D_1) \times V_2>$. Therefore $D_2 \times (V_1\text{-}D_1)$ is a complementary fuzzy dominating set of $G_1 \times G_2$. Similarly $D_1 \times (V_2\text{-}D_2)$ is a complementary fuzzy dominating set of $G_1 \times G_2$.

IV. CONCLUSION

In this paper, deals complementary domination and complementary domination number for intunionistic fuzzy graphs. Some interesting properties of these new concepts are proved. Further, discuss the bounds in operations on fuzzy graphs.

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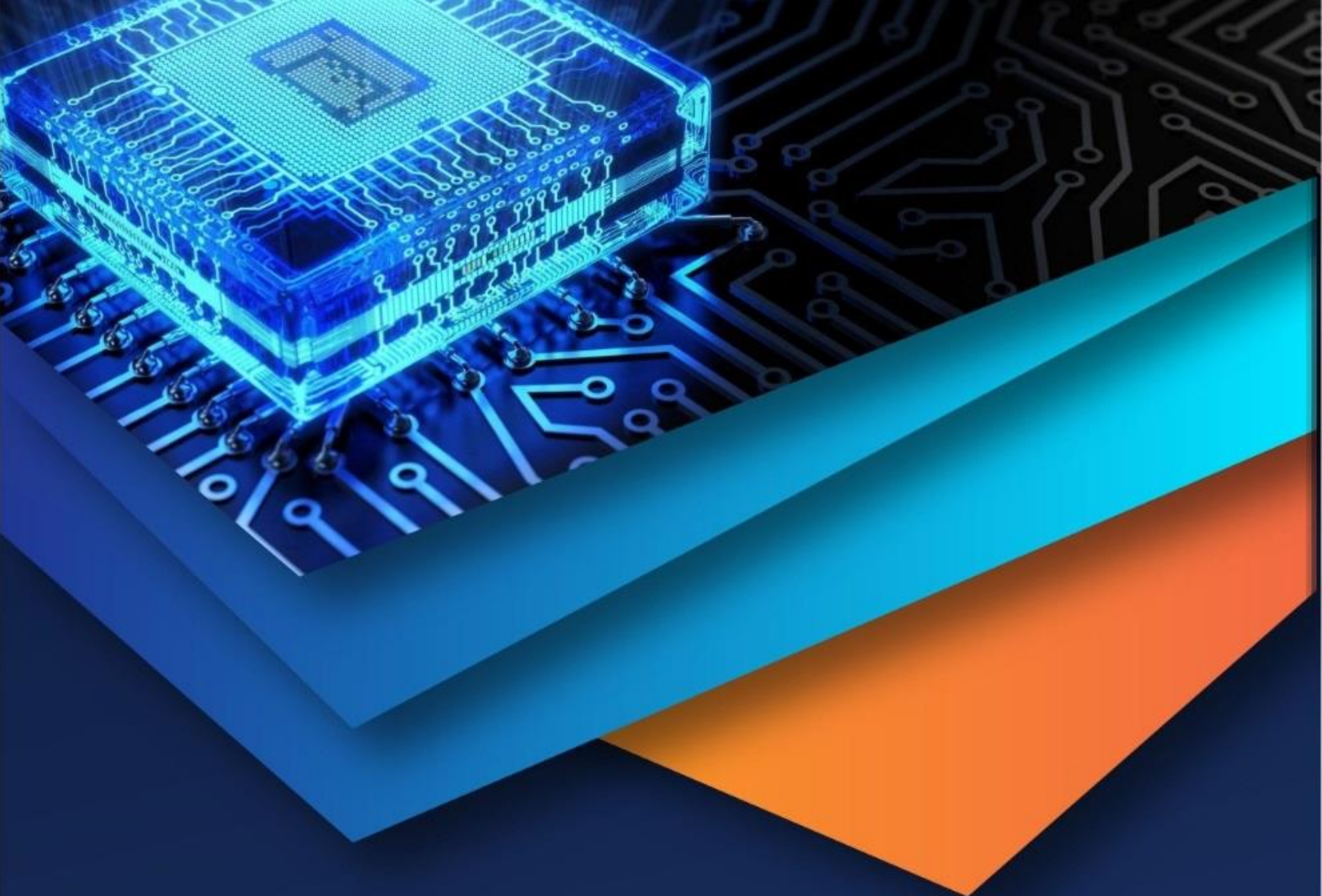

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