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Cototal Bondage Number in Graphs and In Chemical Structures

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Abstract: Let G=(V, E) be a graph with vertex set V and edge set E. A set D ⊆ V is a dominating set of G if every vertex in V-D is adjacent to a vertex in D. We denote by <D> the sub graph induced by D. A dominating set D of a graph G=(V , E) is a Cototal dominating set if every vertex $v \in V - D$ is not an isolated vertex in the induced subgraph <V-D>.The Cototal domination number $\gamma_{cl}(G)$ of G is the minimum cardinality of Cototal dominating set of G. The bondage number b(G) of a nonempty graph G is the minimum cardinality among all sets of edges $E^{'} \subseteq E$ for which $\gamma(G-E') > \gamma(G)$.In this paper ,We define the Cototal bondage number of G, denoted by $b_{cl}(G)$,as the minimum cardinality among all sets of edges $E^{'} \subseteq E$ such that $\gamma_{cl}(G-E') > \gamma(G)$ and we discuss the basic properties of Cototal bondage number of some standard graphs and some interesting results. Also we determine the Cototal domination number and the Cototal bondage number for pyrene torus. Further we compute the Cototal bondage number of Line graphs.

Key words: Cototal bondage number, Cototal domination number ,Line graphs, Pyrene torus.

I. INTRODUCTION

we follow the notations of [2]. Specifically, Let G=(V , E) be a graph with vertex set V and edge set E. A set D ⊆ V is a dominating set of G if every vertex in V-D is adjacent to a vertex in D.We denote by <D> the subgraph induced by D.A dominating set D of a graph G=(V , E) is a Cototal dominating set if every vertex $v \in V - D$ is not an isolated vertex in the induced subgraph <V-D>.The Cototal domination number $\gamma_{cl}(G)$ of G is the minimum cardinality of Cototal dominating set of G. Cototal domination was introduced by V.R.Kulli and b.Janakiram [6].

The bondage number b(G) of a nonempty graph G is the minimum cardinality among all sets of edges $E^{'} \subseteq E$ for which $\gamma(G - E') > \gamma(G)$.Bondage in graphs was introduced by Fink et al.[3] in 1990.

We define the Cototal bondage number of G, denoted by $b_{cl}(G)$, as the minimum cardinality among all sets of edges $E^{'} \subseteq E$ such that $\gamma_{cl}\left(G - E^{'}\right) > \gamma(G)$ .we say that G is a γ_{cl}-strongly stable graph if for all $E^{'} \subseteq E$ either $\gamma_{cl}\left(G - E^{'}\right) = \gamma_{cl}(G)$ and we write $b_{cl}(G) = 0$.We denote by DS_p the double star on p vertices, SP_p the spider on p vertices.

A. Proposition :1.1 For p≥6 , $b_{cl}(C_p) = 2$

B. Proposition:1.2 For p ≥ 4,$b(K_p) = \left\lceil\frac{p}{2}\right\rceil$

In this chapter , we discuss the basic properties of Cototal bondage and some interesting results.

II. RESULTS

*A. Proposition:*2.1 $b_{cl}(P_p) = 0$

*B. Proposition:*2.2 $\gamma_{cl}(K_{1,n}) = \infty \; and \; b_{cl}(K_{1,n}) = \infty$

*C. Proposition:*2.3 $b_{cl}(DS_p) = 0$

D. Theorem:2.4

For p ≥ 4,$b_{cl}(W_p) = 1$

Proof: Let u be the vertex of maximum degree of W_p.

Denote by v the vertex of degree three.

Let X be the graph secured from W_p by deleting edge u v.

Then the vertices {u, v } dominate X. So that $\gamma_{cl}(X) > \gamma_{cl}(W_p)$, for this understanding,$b_{cl}(W_p) = 1$

E. Theorem:2.5

For SP_p ,$b_{cl}(SP_p)$=1

Proof Denote by u the pendant vertex of degree 1.
And mark the vertex ,having degree exactly two and adjacent to u, as v .
By deleting the edge uv from spider tree,then the cototal domination number will be increased by two.
So that $b_{cl}(SP_p)$=1.

F. Theorem:2.6

For p ≥ 4,$b_{cl}\left(K_p\right) = \left\lceil\frac{p}{2}\right\rceil$

*1) Proof:*Let I_E be the collection of all set of independent edges with cardinality $\frac{p}{2}$
Case(i) p is even
Removal of I_E edges from K_p increases the cototal domination number by one.
So that $b_{cl}\left(K_p\right) = \frac{p}{2}$
Case(ii) p is odd
Let $e \notin I_E$ but $e \in E(K_p)$
Removal of $I_E \cup \{e\}$ edges from K_p yields cototal bondage number by $\frac{p}{2}$+1
Hence $b_{cl}\left(K_p\right) = \left\lceil\frac{p}{2}\right\rceil$

G. Theorem:2.7

Let $K_{m,n}$ where 1 < m ≤ n , be a complete bipartite graph.Then $b_{cl}\left(K_{m,n}\right) = n$

1) Proof: Certainly,$\gamma_{cl}\left(K_{m,n}\right) = 2$.
Denote by A_1 and A_2 be the partite sets of V($K_{m,n}$) such that $|A_1| = m$ and $|A_2| = n$.
Let u be the vertex of degree m in B.
Let E_u be the set of all edges which are incident with u.
Now let A be the graph acquired by detaching E_u edges from $K_{m,n}$.
Then each vertex belonging to A_1 is of degree n-1 and for this reason, $\gamma_{cl}\ (A) > 2$
Accordingly,$b_{cl}(K_{m,n}) \leq m$
However, if A is the graph obtained by removing m-1 edges from $K_{m,n}$ then there exists a vertex of degree n in A_1 and there exists a vertex of degree m in A_2.
Henceforth $b_{cl}\left(K_{m,n}\right) > m - 1$
We agree that $b_{cl}\left(K_{m,n}\right) = m$

H. Theorem:2.8

For p ≥ 6 ,$b_{cl}\left(C_p\right) = 2$

1) Proof: Let u be the vertex of degree 2 in C_p where p ≥ 6.
Let w_1 ,w_2 be the vertices which are adjacent to u.
Let X be the graph obtained from C_p, p ≥ 6 by deleting the edges uw_1,uw_2.
Trivially, $\gamma_{cl}(X) > \gamma_{cl}(c_p)$
For this reason, $b_{cl}\left(C_p\right) = 2$, p ≥ 6

I. Remark:2.9

If k is any positive integer, let H_k be the tree obtained from the star $K_{1,k+1}$ by subdividing k edges twice. It can be easily verified that $b_{cl}(H_k) = 1$

J. Theorem :2.10

$$\gamma_{cl}(P_p) = p - 2$$

1) Proof: Let $v_1,v_2,\dots,v_p$ be the vertices of P_p .
Choose the set of all vertices $v_1,v_2,\dots,v_{p-4},v_{p-1},\dots,v_p$ and labeled it by D.
Clearly, D forms a minimal cototal dominating set with cardinality p-2.

K. Remark:2.11
Let A_k be the tree obtained from a star $K_{1,k-1}$ and (k-1) copies of K_2 with a fixed pendant vertex of $K_{1,k-1}$.
It can be easily viewed that $\gamma_{cl} = 0$ and $b_{cl} = \infty$

L Observation:2.12
$\mathfrak{B}$ has the property that every non pendant vertices in T are adjacent to atleast one pendant vertices in T.

L. Theorem:2.13
If $T \in \mathfrak{B}$ then $\gamma_{cl}(T) = l$
1) Proof: Since T ∈ ,Removal of pendant vertices in T yields a path with p- l vertices.
Hence, $\gamma_{cl}(T) = l$

M. Theorem: 2.14
For any tree T, $\gamma_{cl}(T) \geq l$
1) Proof: Let L be the set of pendant vertices of T with $|L| = l$
And S be the set of non pendant vertices of T which are not adjacent to an element of L with $|S|$ =k.
Now, To form a minimal Cototal dominating set , we need atleast l vertices and additionaly $\frac{k}{2}$ vertices.
That is $\gamma_{cl}(T) \geq l + \frac{k}{2} \geq l$

N. Theorem:2.15 $\gamma_{cl}(T) \leq \frac{p+l}{2}$

1) Proof: Let L be the set of all pendant vertices of T with $|L| = l$.

Consider,S is the set of all non pendant vertices of T with cardinality $\frac{p-l}{2}$
Clearly,$S \cup L$ forms a cototal dominating set with cardinality $l + \frac{p-l}{2} = \frac{2l+p-l}{2} = \frac{p+l}{2}$. Hence $\gamma_{cl}(T) \leq \frac{p+l}{2}$

III. COTOTAL DOMINATION AND COTOTAL BONDAGE NUMBER FOR SOME CHEMICAL STRUCTURES

A. Cototal Bondage number of pyrene network
The series of hypothetical benzenoid torus networks are derived by properly joining all the distant pairs of peripheral carbon atoms of 2-dimensional polycyclic aromatic hydrocarbons,namely coronene,pyrene and hexabenzocoronene.This has vast applications in the field of chemistry.Pyrene is an alternative polycyclic aromatic hydrocarbon (PAH) and consists of four fused benzene rings,resulting in a large flat aromatic system.It is a colorless or pale yellow solid which forms during incomplete combustion of organic materials and therefore can be isolated from coal tar together with a broad range of related compounds.
In the last four decades, a number of research works have been reported on both the theoretical and experimental investigation of pyrene concerning its electronic structure,uv absorption and fluorescence emission spectrum.Indeed, this polcyclic aromatic hydrocarbon exhibits a set of many interesting electrochemical and photophysical attributes, which have results in its utilization in a variety of scientific areas.Like most PAHs, pyrene is used to make dyes,plastics and pesticides.Here we introduce the axes for the pyrene network as follows:
Let the centre line perpendicular to the vertical edge direction of hexagon of PY(n) be denoted by α_0 as shown in Figure1.The lines which are parallel and in the anticlockwise direction of α_0 are denoted by α_i,1≤i≤(n-1) and those in the clockwise direction α_0 are denoted by α_{-i} ,$1 \leq i \leq (n-1)$.

Let the outer cycle of PY(n) be denoted by C_n^0. Let the topmost and bottom most vertices in PY(n) be denoted by a and b.

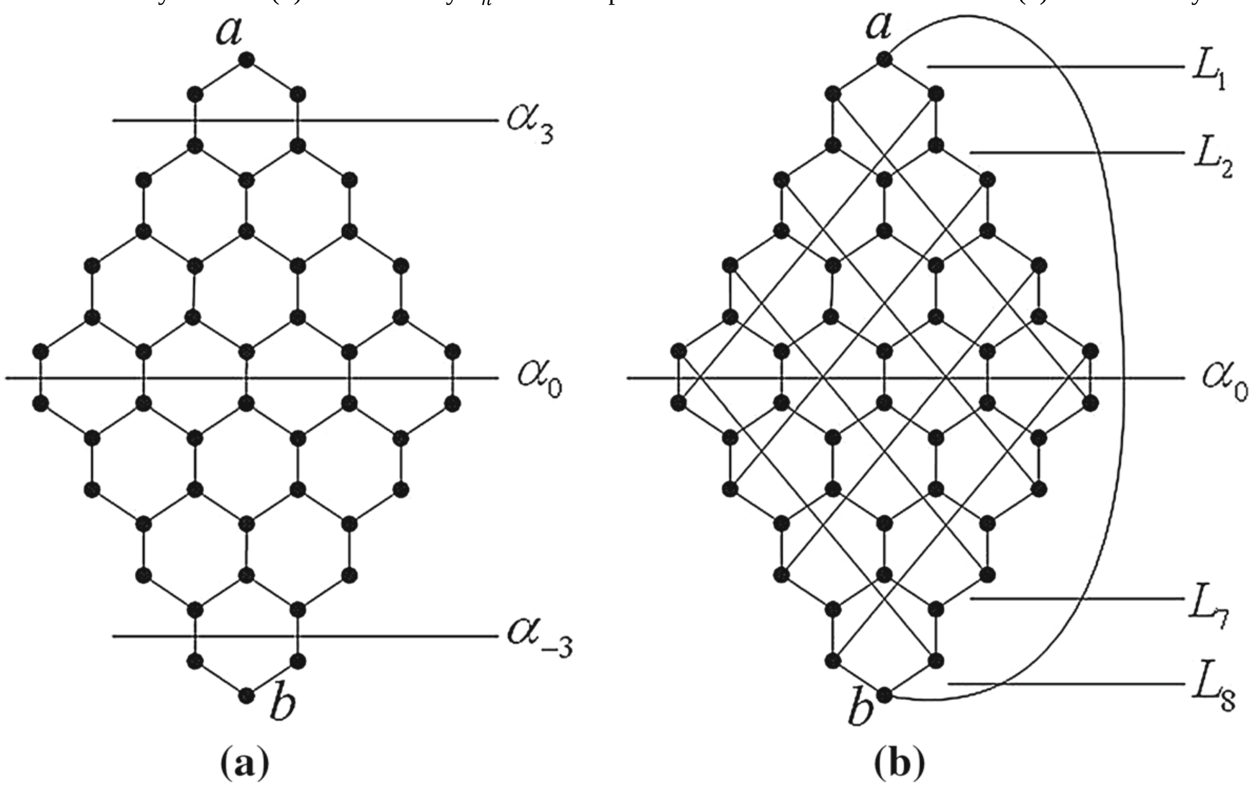

Fig. 1 a Pyrene network *PY(4)*, b pyrene torus network *PT(4)*

B. Theorem:3.1

Let S be a cototal dominating set with the property that if every vertex a∈V(G) is dominated by exactly one vertex of S, then S is a minimum cototal dominating set.

C. Theorem:3.2

Let G be a pyrene network of dimension n.then $\gamma_{cl}(G)=\begin{cases} \left(\frac{2n^2+4n}{4}\right)+1 \; if\ n\ is\ even \\ \left\lceil\frac{2n^2+4n}{4}\right\rceil \; if\ n\ is\ odd \end{cases}$

1) Proof: Start at a vertex a as shown in figure[1] and call it saturated.

Case(i) n is odd

Saturate a sequence of diagonally opposite vertices of hexagons beginning with vertex a and proceed till vertex b is reached.

Case(ii) n is even

The α_0 line divides PY(n) into two subgraphs H_1 and H_2 which are mirror images of each other along α_0.Saturate the vertices as in case(i) till α_{-1} line is reached next saturate a vertex b in H_2 which is the mirror image of a.

Saturate a sequence of diagonally opposite vertices of hexagons beginning with vertex b,reached till α_{-1} line is reached.

By result[3.1],Since all the vertices on graph are dominated exactly once, the saturated vertices form a minimum cototal dominating set. Therefore, $\gamma_{cl}(G) = \left(\frac{2n^2+4n}{4}\right)+1$, $if\ n\ is\ even$ and $\gamma_{cl}(G) = \left\lceil\frac{2n^2+4n}{4}\right\rceil\ if\ n\ is\ odd$

D. Theorem:3.3

If G is a pyrene network, then $b_{cl}(G) = 2$

1) Proof: Clearly all the saturated vertices form a minimum cototal dominating set of G by theorem[3.2] .

Removal of an two edges which are adjacent to any one of saturated vertex, increases the cototal domination number to $\left(\frac{2n^2+4n}{4}\right) + 2\ if\ n\ is\ even$ and $\left\lceil\frac{2n^2+4n}{4}\right\rceil + 1\ if\ n\ is\ odd.$

An $K_{1,\Delta}$ packing of a graph G is a set of vertex disjoint subgraphs of G,each of which is isomorphic to a fixed graph $K_{1,\Delta}$ where Δ is the maximum degree of G.
An $K_{1,\Delta}$ –packing in G is called perfect if it covers all vertices of G.

IV. COTOTAL BONDAGE NUMBER OF PYRENE TORUS

A. Theorem:4.1

In a graph G, if there exists a perfect $K_{1,\Delta}$ –packing then $\gamma_{cl}(G) = t$ where $\Delta(G)$ and t are the maximum degree and the packing number of G respectively

1) Proof: Let S be the set of all vertices are dominated by exactly one vertex in a perfect $K_{1,\Delta}$ packing
Clearly,<V-S > has no isolated vertices and $\gamma(K_{1,\Delta}) = 1$
Hence it is easily noticed that the packing number and the cototal domination number of G are same.
Hence $\gamma_{cl}(G) = t$

B. Theorem:4.2

If G is a pyrene torus with perfect $K_{1,3}$-packing then $t = \left\lfloor\frac{2n^2+4n}{4}\right\rfloor$ where t is the packing number of G.

1) Proof: Start at a vertex a as shown in Figure(2b) and call it saturated.
Saturate the vertices as in Result 1 of the subgraph H≅PY(n-2) obtained by deleting the boundary vertices and the diagonally opposite vertices of a and b in PY(n).
Draw the vertical line β_0 through the vertices a and b.
The β_0 line divides PT(n) into two subgraphs H_1 and H_2 which are mirror images of each other along β_0.
Let the boundary line of line of H_1 and H_2 be C_1 and C_2 respectively.
Label C_1 as follows: Begin with vertex u at distance 3 from b and call it saturated,Traversing in the anticlockwise sense,
Choose the next vertex v at distance 4 from the saturated vertex and saturate it provided the vertex of distance 4 from the saturated vertex and saturate it provided the vertex of distance 4 from u has no neighbour in zigzag horizontal line 4.
If not,Saturate the next vertex on C_n^0 adjacent to b.Proceed till β_0 is reached.
Now we saturate the vertex a and the vertices in C_2 which are mirror images of saturated vertices on C_1.
The subgraph induced by N[a] when a is saturated vertex is isomorphic to $K_{1,3}$.
Now N[a]∩N[u]=Φ for all pairs of saturated vertices.
The wrap around edge incident with a together with the two vertices on C_n^0
adjacent to a, induce $K_{1,3}$.
For n even, C_n^0 contains 2n-1 number of saturated vertices.The closed neighbourhoods of saturated vertices together cover $4\left[\frac{(n-1)^2+1}{2} + (2n-1)\right]$=$2n^2 + 4n$ vertices.
Hence ,the $K_{1,3}$ –packing is perfect and t=$\frac{2n^2+4n}{4}$
For n is odd,ZZC(K) contains k+1 number of saturated vertices, k=1,3,5…..n and 2n+1-k number of saturated vertices,k=n+2,n+4,…..,2n-3.
The Last zigzag horizontal channel contains a saturated vertex.
The closed neighbourhood of these saturated vertices together cover 4[2+4+6+…..+(n+1)+(n-1)+(n-3)+….+6+4]+4=$[2n^2 + 4n] - 2$ vertices
Therefore,$t \geq \left\lfloor\frac{2n^2+4n}{4}\right\rfloor$

Clearly,$t \leq \left\lfloor\frac{2n^2+4n}{4}\right\rfloor$,Hence $t = \left\lfloor\frac{2n^2+4n}{4}\right\rfloor$

C. Theorem:4.3

If G is a pyrene torus, then $\gamma_{cl}(G) = \left\lceil \frac{n^2+2n}{2} \right\rceil$

1) *Proof:* Clearly a pyrene torus PT(n) has a perfect $K_{1,3}$ –packing when n is even and an $K_{1,3}$ –packing of PT(n) with atmost two unsaturated vertices, if n is odd , Therefore $\gamma_{cl}(G) = \left\lceil \frac{2n^2+2n}{2} \right\rceil$.

D. Theorem:4.4

If G is a pyrene torus network , then $b_{cl}(G) = 1$

1) Proof: Clearly a pyrene torus PT(n) has a perfect $K_{1,3}$ packing when n is even and an $K_{1,3}$-packing of PT(n) with atmost two unsaturated vertices, if n is odd by theorem[1]

Removal of an edge which is adjacent to a and b except the wraparound edge e=(a,b),increases the cototal domination number to $\left\lceil \frac{n^2+2n}{2} \right\rceil + 1$ if n is odd.

Hence $b_{cl}(G) = 1$

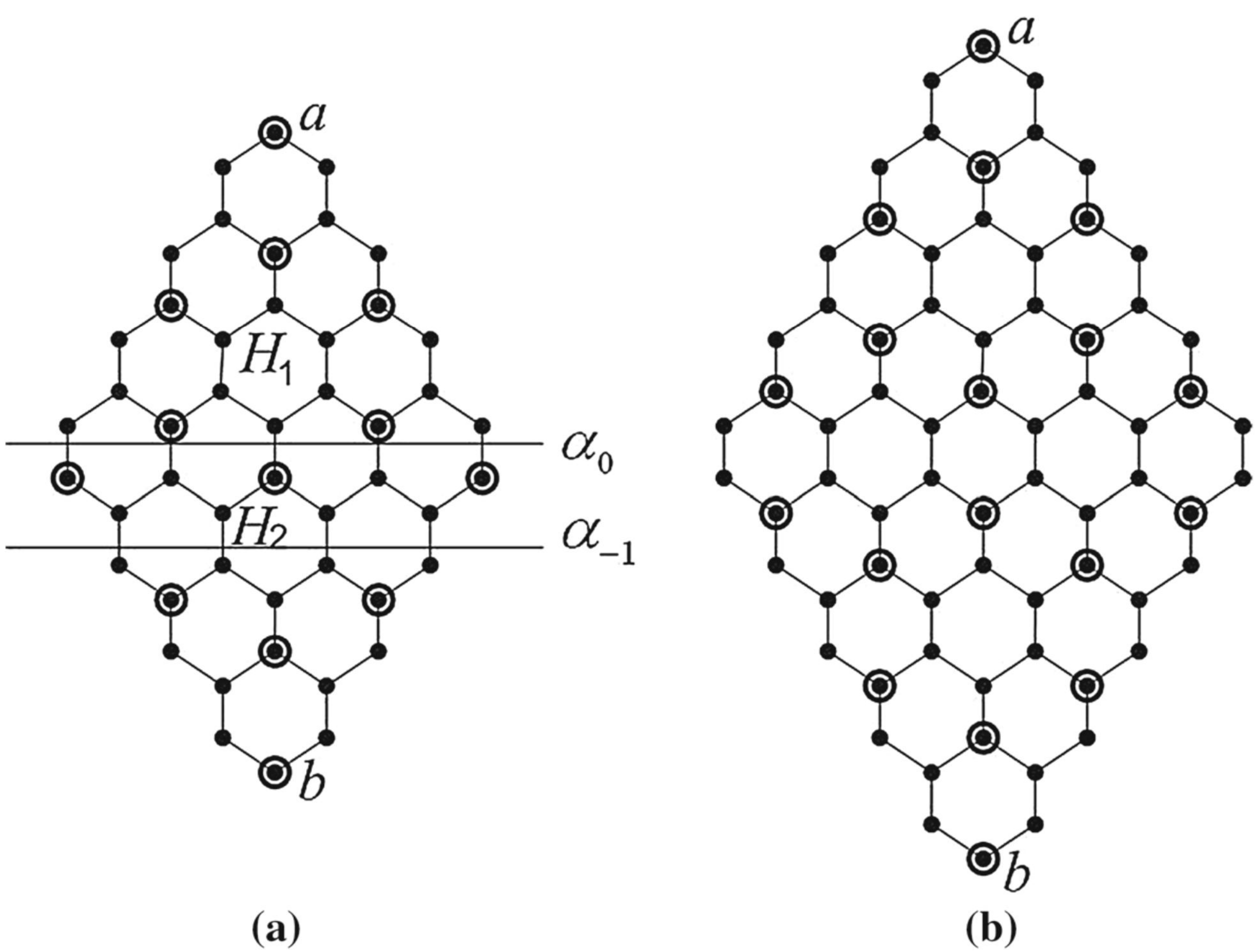

Fig. 2 a Pyrene network *PY(4)*, b pyrene network *PY(5)*

V. COTOTAL BONDAGE NUMBER OF LINE GRAPHS

A. Theorem:5.1

For $p \geq 6$, $b_{cl}\left(L(C_p)\right) = 2$

1) *Proof:* Since the Line graph of cycle is cycle.

By proposition 1.1, $b_{cl}((L(C_p))$ with p≥6 is two.

B. Theorem:5.2

For $K_{1,n}$ ($n \geq 3$) ,$b_{cl}\left(L\left(K_{1,n}\right)\right) = \left\lceil\frac{n+1}{2}\right\rceil$

1) *Proof*: By proposition:1.2, Since the Cototal bondage number of complete graph with $p \geq 4$ vertices is $\left\lceil\frac{p}{2}\right\rceil$

Hence $b_{cl}\left(L\left(K_{1,n}\right)\right) = \left\lceil\frac{n+1}{2}\right\rceil$

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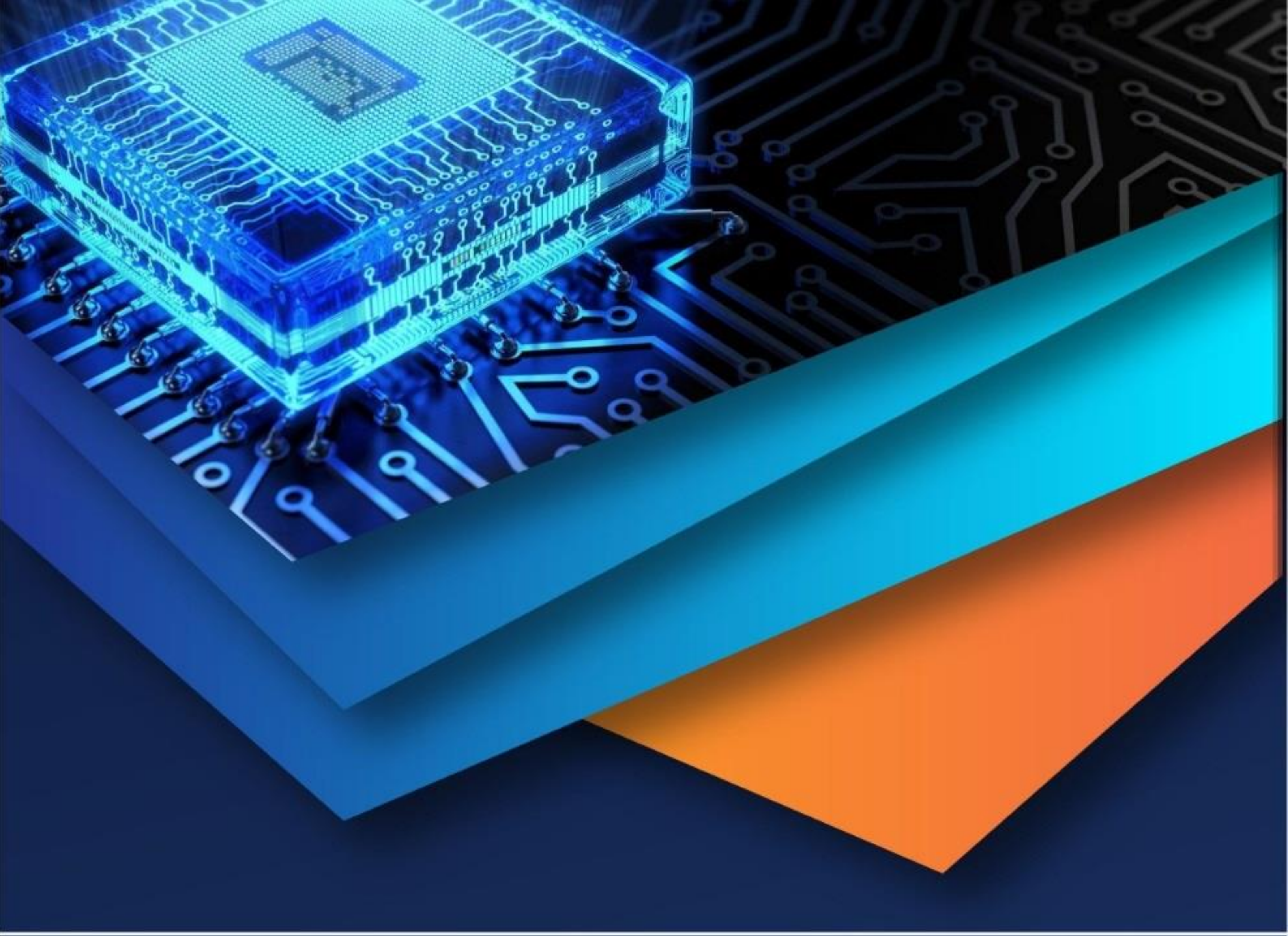

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