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Mixed Convection in MHD Slip Flow of Alumina Water Nanofluid Over a Flat Plate

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Abstract*— Heat transfer in magnetohydrodynamic (MHD) slip flow of an incompressible, viscous, electrically conducting, mixed convective and steady alumina-water nanofluid over a flat plate has been analyzed. The governing equations are transformed into a set of simultaneous ordinary differential equations by using similarity transformation. The set of equations thus obtained has been solved using Adaptive Runge-Kutta method with shooting technique. The effects of magnetic parameter and heat source parameter on velocity and temperature distribution, shear stress and temperature gradient were depicted graphically and analyzed. Significant changes were observed in the heat transfer rate.

***Keywords*— Magnetohydrodynamic, Heat source, Boundary layer slip, Volume fraction and Mixed convection.**

2010 Mathematics Subject Classification: 74F10, 76W05, 76N20, 65M06, 76R99.

I. INTRODUCTION

Wang et al. [4] were studied the mixed convective boundary layer flow of non- Newtonian fluids along vertical wavy plates. The authors found that Prandtl number and buoyancy parameters were seen to enhance the influence of plate surfaces on the local Nusselt number in Newtonian fluids or non-Newtonian fluids. Moreover, the irregular surfaces have higher total heat flux than that of corresponding flat plate for any fluid. Vadasz et al. [5] investigated the heat transfer enhancement in nanofluid suspensions. The results were shown excessive improvement in the thermal conductivity of the suspension. Molla and Yao [7] investigated mixed convective heat transfer of non-Newtonian fluid over a flat plate using a modified power law viscosity model. The results were obtained for a shear thinning fluid in terms of the velocity and temperature distribution, and for wall shear stress and heat transfer rates. Ahmad and Pop [8] studied the steady mixed convection boundary layer flow past a vertical flat plate embedded in a porous medium filled with nanofluids. The effects of various parameters on velocity distribution were analyzed. Bachok et al. [9] were analyzed heat transfer characteristics of mixed convective flow over a permeable vertical flat plate embedded in an anisotropic fluid. They were found that dual solutions exist for both assisting and opposing flows. Mohammad et al. [10] studied the heat transfer of an alumina-water nanofluid flow inside a wide rectangular micro channel. Results show that the velocity and temperature difference between the phases is very small and negligible. The average Nusselt number increases as the Reynolds number and volume concentration increase and also with the decay in the nanoparticles size. Aladag et al. [11] made experimental investigation of the viscosity of nanofluids at low temperature. It has been found that carbon nano tube water based nanofluid behaves as Newtonian fluid at high shear rate whereas Alumina water based nanofluid is non-Newtonian. Hamed and Kasera [12] studied the variation iteration method solution for mixed convection over horizontal flat plate and analyzed various parameters numerically and graphically.

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The objective of present paper is to study the mixed convective heat transfer in MHD slip flow of alumina water nanofluid over a flat plate.

II. MATHEMATICAL DESCRIPTION

The physical model of the problem is given here along with flow configuration and coordinate system. Present problem deals with analysis of two dimensional MHD boundary layer slip flow of alumina-water nanofluid. The magnetic field B is imposed in transverse direction to the flow. The plate length is considered infinite and the uniform velocity at infinity is u_∞. The temperature on the surface of the plate is T_w, and far from the surface it is T_∞. The continuity, momentum and energy equations representing flow are as following:

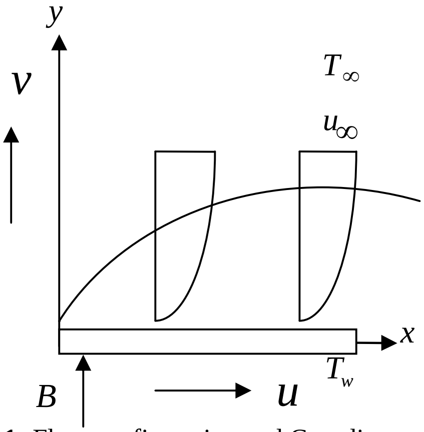

Figure 1: Flow configuration and Coordinate system

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad (1)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\upsilon_{nf}\frac{\partial^2 u}{\partial y^2}-\frac{\sigma_{nf}B^2}{\rho_{nf}}(u-u_\infty)+g\beta_T(T-T_\infty) \qquad (2)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k_{nf}}{\rho_{nf}C_{pnf}}\frac{\partial^2 T}{\partial y^2}+\frac{H_0}{\rho C_{pnf}}(T-T_\infty) \qquad (3)$$

With the following boundary conditions:

$u=V_{sf}\frac{\partial u}{\partial y}$, $v=0$, and $T=T_w+T_{sf}\frac{\partial T}{\partial y}$ at y = 0

$u\to u_\infty$, and $T\to T_\infty$ when $y\to\infty$ (4)

Here $v_{sf}=v_0\sqrt{\frac{u_\infty x}{\upsilon_{nf}}}$, $t_{sf}=t_0\sqrt{\frac{u_\infty x}{\upsilon_{nf}}}$ are velocity slip factor and thermal slip factor with initial values v_0, t_0 respectively.

TABLE 1
NANOFLUID PROPERTIES

Dynamic Viscosity [1]	$\mu_{nf}=\mu_f/(1-\emptyset)^{2.5}$
Density [2]	$\rho_{nf}=(1-\emptyset)\rho_f+\emptyset\rho_s$
Specific heat [3]	$C_{pnf}=(1-\emptyset)C_{pf}+\emptyset(C_{ps})$
Thermal conductivity [4]	$(k_s/k_f)+2-\emptyset(1-(k_s/k_f))k_{nf}=$
Kinematic Viscosity	$\upsilon_{nf}=\mu_{nf}/\rho_{nf}$
Thermal Diffusivity	$\alpha_{nf}=k_{nf}/\rho_{nf}C_{pnf}$

TABLE 2
THE PHYSICAL PROPERTIES OF ALUMINA AND WATER AT ROOM TEMPERATURE ARE

	Density (Kg/m^3)	Thermal Conductivity (W/m.K)	Specific Heat (J/Kg.K)
Alumina	3970	36	769
Water	1000.52	0.597	4181.8

III. METHOD OF SOLUTION

To solve the governing equations (2) and (3) with the boundary conditions (4) following similarity transformation has been introduced:

$\psi=\sqrt{u_\infty\upsilon_{nf}x}\quad f(\eta)$, $\eta=\frac{y}{x}\sqrt{\frac{xu_\infty}{\upsilon_{nf}}}$, $\theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty}$, $M=\frac{\sigma xB^2}{u_\infty\rho_{nf}}$, $G_r=\frac{g\beta_T(T-T_\infty)x}{u_\infty^2}$ and $S=\frac{xH}{\rho_{nf}C_{pnf}u_\infty}$ where $B=\frac{B_0}{\sqrt{x}}$, $\beta_T=\frac{\beta_{T0}}{x}$, $H=\frac{H_0}{x}$, $Pr=\frac{\upsilon_{nf}}{\alpha_{nf}}$.

Here B_0, β_{T0}, H_0 are constants and ψ is the stream function which satisfies equation (1) with $u=\frac{\partial\psi}{\partial y}$ and $v=-\frac{\partial\psi}{\partial x}$

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After using above transformation, the equations (2) and (3) reduce to the nonlinear differential equations as follows:

$$f'''(\eta)+\frac{1}{2}f(\eta)f''(\eta)+M\left(1-f'(\eta)\right)+G_r\theta(\eta)=0 \qquad (5)$$

$$\theta''(\eta)+\frac{1}{2}Prf(\eta)\theta'(\eta)+PrS\theta(\eta)=0 \qquad (6)$$

and the boundary conditions (4) reduce as follows:

$$f(0)=0,\quad f'(0)=\gamma f''(0),\quad \theta(0)=1+\varepsilon\theta'(0),$$

$$\lim_{\eta\to\infty}f'(\eta)=1,\ \lim_{\eta\to\infty}\theta(\eta)=0 \qquad (7)$$

where $\gamma=\frac{V_{sf}^{0}u_{\infty}}{v_{nf}}$ is the velocity slip parameter and $\varepsilon=\frac{T_{sf}^{0}u_{\infty}}{v_{nf}}$ is the thermal slip parameter with initial values V_{sf}^{0} and T_{sf}^{0}.

To solve the set of non-linear differential equations (5) and (6) subject to the boundary conditions (7) Adaptive Runge-Kutta method with shooting technique has been applied. This method is based on the discretization of the problem domain and the calculation of unknown boundary conditions. The domain of the problem is discretized and the boundary conditions for $\eta=\infty$ are replaced by $f'(\eta_{max})=1$, and $\theta(\eta_{max})=0$ where η_{max}; is sufficiently large value of η corresponding to step size at which the boundary conditions (7) for $f(\eta)$ is satisfied.

Onto account of the consistency and to fulfill stability criteria $\eta_{max}=8$ and step size $\Delta\eta=0.01$ have been taken. To solve the problem the nonlinear equations (5) and (6) are first converted into first order ordinary linear differential equations as follows:

$f'(\eta)=y(2)$;
$f''(\eta)=y(3)$;
$f'''(\eta)=[-\frac{1}{2}y(1)y(3)-M\{1-y(2)\}-Gr\,y(4)]$;
$\theta'=y(5)$;
$\theta''(\eta)=[-\frac{1}{2}Pr\,y(1)y(5)-SPry(4)]$

There are three conditions on the boundary $\eta=0$ and two conditions at $\eta=\infty$ as given in equation (7). Shooting technique has been used to find required missing initial conditions. The value of unknowns $f''(0)$ and $\theta'(0)$ has been tabulated in table 3

TABLE 3

VALUES OF $f''(0)$ AND $\theta'(0)$ FOR $Pr=6.6556406$ AND $\phi=0.05$ OBTAINED BY SHOOTING METHOD

Parameters		$f''(0)$	$\theta'(0)$
	0	0.4161	-0.2803
Magnetic Parameter (M)	0.1	0.5121	-0.3483
$[\varepsilon=0.1,\ \gamma=0.1,$	0.3	0.6581	-0.4363
$Gr=0.1,\ S=0.1]$	0.5	0.7731	-0.4865
	0.7	0.8693	-0.5287
	1.0	0.9923	-0.5689
	0	0.7658	-0.7868
Heat Source Parameter (S)	0.1	0.7728	-0.4907
$[\varepsilon=0.1,\ \gamma=0.1,$	0.3	0.8054	-0.6509
$Gr=0.1,\ M=0.5]$	0.7	0.6937	-1.7999
	1.0	0.7189	-0.7799
	1.5	0.7203	-0.7745

IV. RESULT AND DISCUSSION

The computations have been made for velocity, temperature, temperature gradient, Shear stress profile and other physical parameters involved in the flow. The results were depicted graphically to analyze them and their physical explanation is also given corresponding to different values of magnetic parameter and heat source parameter. The physical and thermal properties of alumina water nanofluid corresponding to different volume fraction are also tabulated below in table 4.

Figures 2-3 exhibit the velocity and shear stress profiles obtained by the numerical simulations for various values of magnetic parameter M. It is noticed that the velocity increases with an increase in the magnetic parameter while shear stress profile increases upto $\eta=1.4586667$ and then decreases asymptotically. The temperature and temperature gradient profile decrease with an increase in the magnetic parameter M as shown in figures 4 - 5. Figures 6-7 show the effect of heat source parameter on velocity and shear stress profile respectively. It has been observed that the velocity decreases

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and shear stress increases with an increase in the heat source parameter. The effects of heat source parameter on temperature and temperature gradient have also been studied through figures 8 and 9. It is noticed that the temperature decreases upto $\eta = 1.67466667$ and then increases. Similarly temperature gradient decreases upto $\eta = 1.17688889$ and then increases.

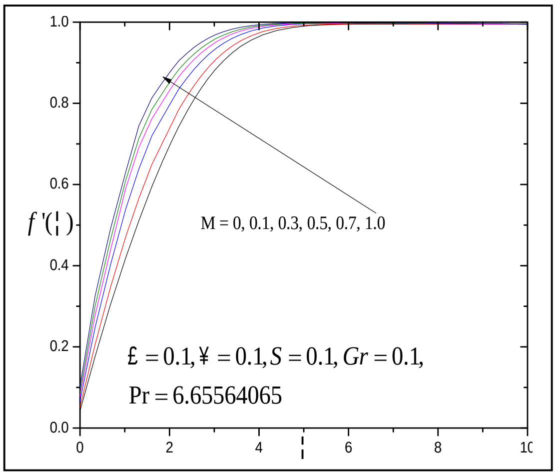

Fig.2 Effect of magnetic parameter on velocity

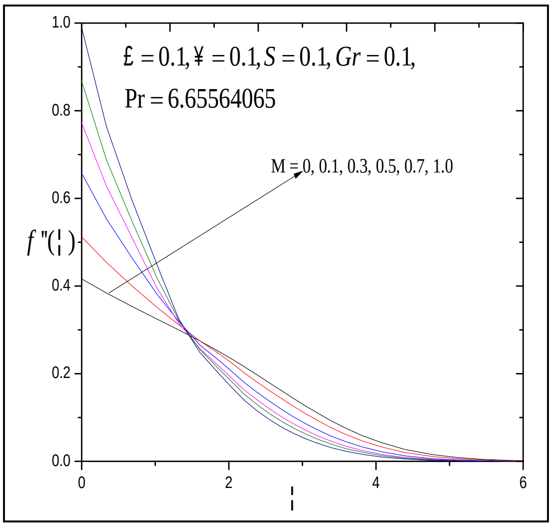

Fig. 3 Effect of magnetic parameter on Shear stress

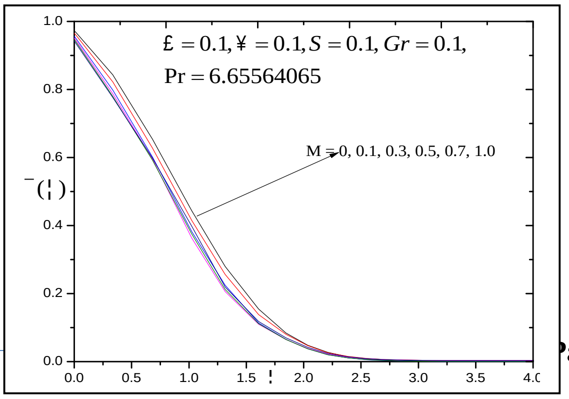

Fig. 4 Effect of magnetic parameter on temperature

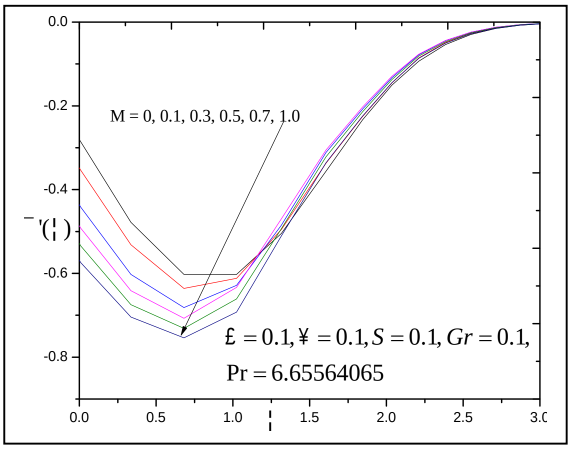

Fig.5 Effect of magnetic parameter on temperature gradient

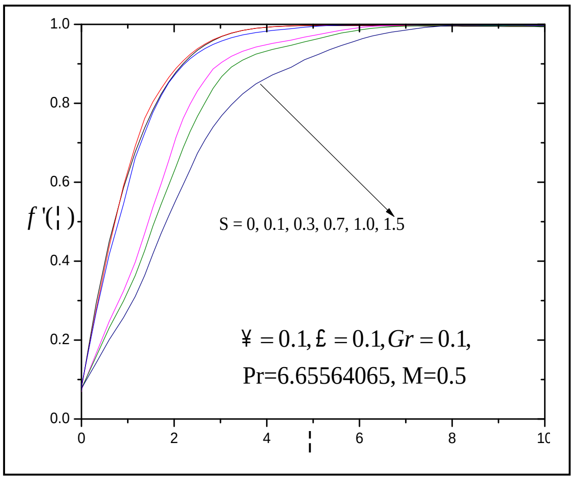

Fig.6 Effect of heat source on velocity.

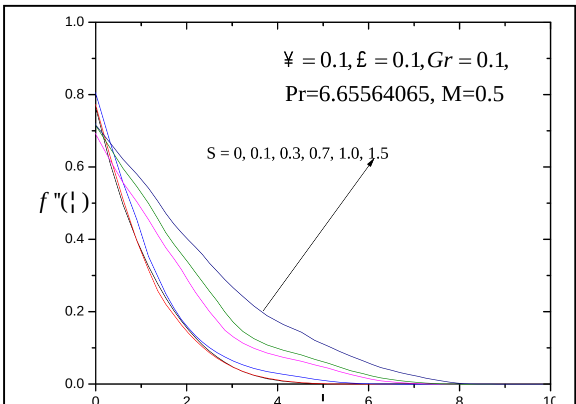

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Fig.7 Effect of heat source on shear stress

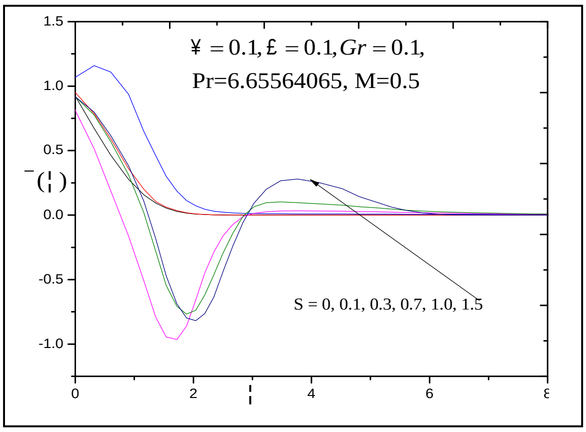

Fig.8 Effect of heat source on temperature

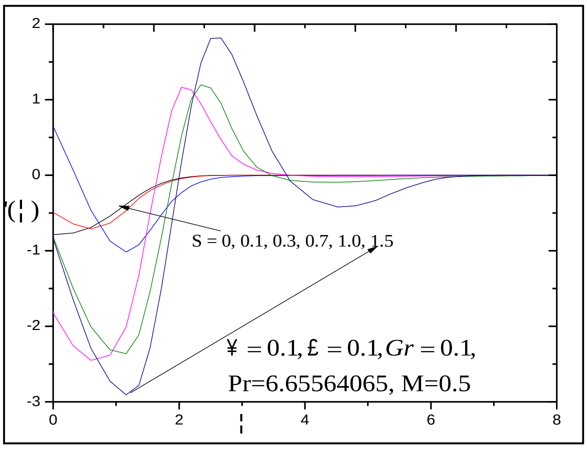

Fig.9 Effect of heat source on temperature gradient

V. NOMENCLATURE

B : Magnetic field

C_{pnf} : Specific heat capacity

G_r : Thermal Grashof number

g : Gravitational acceleration

H : Heat source

k_f :Thermal conductivity of water

k_{nf} : Thermal conductivity of nanofluid

k_s : Thermal conductivity of alumina particles

L : Reference length

M : Magnetic parameter

n : Empirical shape factor

Pr : Prandtal number

S : Heat source parameter

T_w : Temperature at the wall

t_{sf} : Thermal slip factor

T_∞ :Temperature at infinity

u_∞ : Uniform velocity at infinity

u, v : Velocity components in x and y direction

ϵ : Thermal slip parameter

β_T :Thermal expansion coefficient

γ : Velocity slip parameter

σ_{nf} : Electrical conductivity of nanofluid

ω : Spherecity

ϕ : Volume fraction of nanoparticles

ρ_f : Density of water

ρ_s : Density of alumina particles

ρ_{nf} : Nanofluid density

μ_{nf} : Dynamic viscosity of nanofluid

υ_{nf} : Kinematic viscosity of nanofluid

v_{sf} : Velocity slip factor.

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TABLE 4
PHYSICAL PARAMETERS FOR AL_2O_3 NANOFLUID AT 20^0C

Vol. Fr. ϕ	Thermal Conductivity K_{nf}	Dynamic Viscosity μ_{nf}	Prandtal Number Pr	Thermal Diffusivity α_{nf}	Kinematic Viscosity ϑ_{nf}	Density ρ_{nf}	Heat Capacity C_{pnf}
0.00	0.5970000	1.0020000	7.0186994	0.1426872	1.0014792	1000.5200	4181.800
0.01	0.6142114	1.0274950	6.9385106	0.1437426	0.9973600	1030.2148	4147.672
0.02	0.6317568	1.0539075	6.8622847	0.1448988	0.9943372	1059.9096	4113.544
0.03	0.6496460	1.0812805	6.7898403	0.1461537	0.9923606	1089.6044	4079.416
0.04	0.6678893	1.1096592	6.7210103	0.1475057	0.9913875	1119.2992	4045.288
0.05	0.6864972	1.1390917	6.6556406	0.1489536	0.9913818	1148.9940	4011.160
0.06	0.7054809	1.1696288	6.5935894	0.1504967	0.9923135	1178.6888	3977.032
0.07	0.7248517	1.2013245	6.5347257	0.1521346	0.9941582	1208.3836	3942.904
0.08	0.7446217	1.2342358	6.4789292	0.1538674	0.9968963	1238.0784	3908.776
0.09	0.7648033	1.2684233	6.4260890	0.1556954	1.0014792	1000.5200	4181.800
0.10	0.7854095	1.3039515	6.3761029	0.1576193	0.9973600	1030.2148	4147.672

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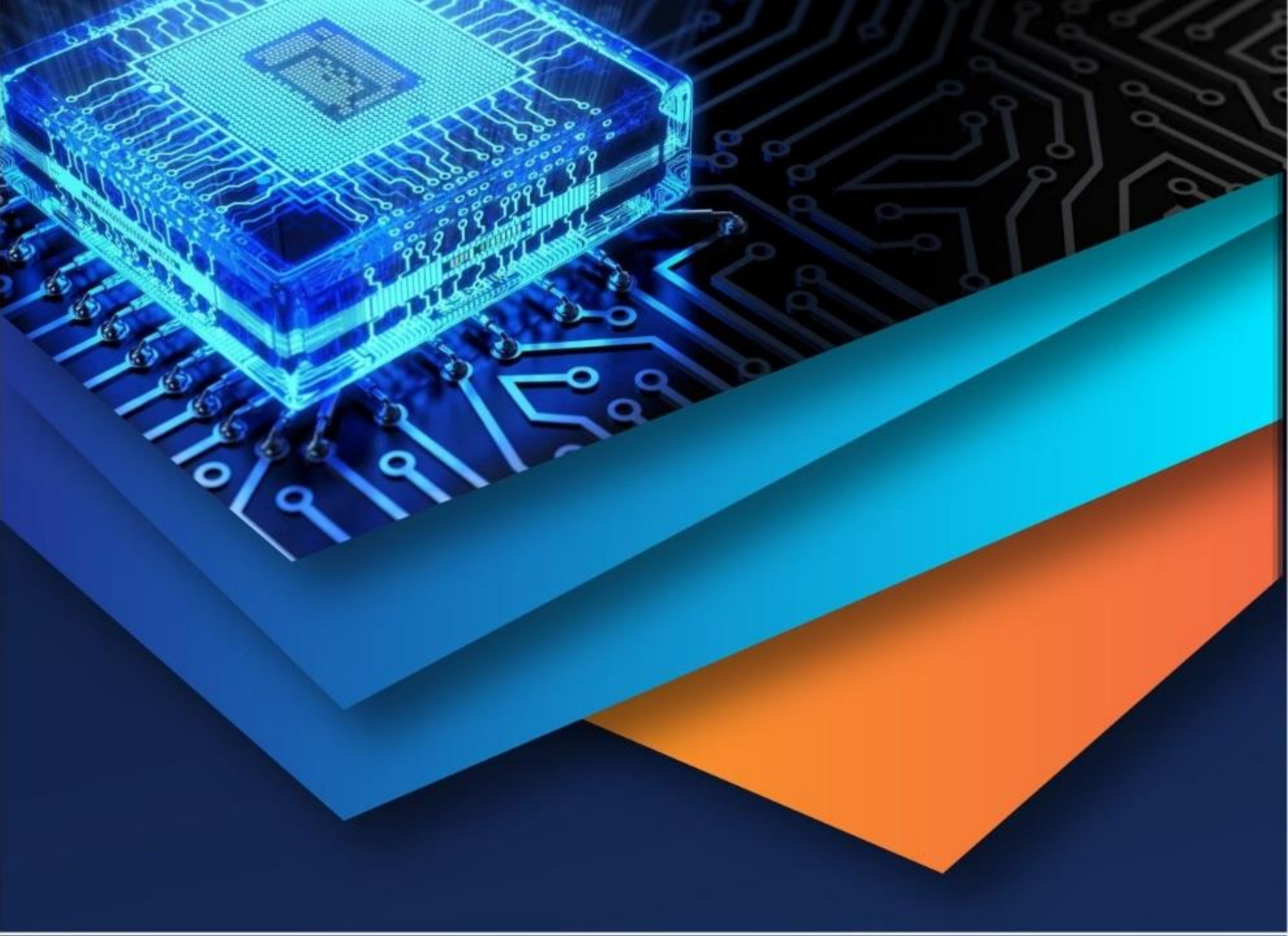

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