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Development of an EOQ Inventory Model for Perishable Items with Exponentially Decaying Demand, Shortages and Partial Backlogging

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Abstract: In this paper an EOQ inventory mathematical model is studied for perishable items with exponential decaying demand, partial backlogging and shortages. Shortages are partially backordered and allowed. The rare of backlogging rate is dependent on the waiting time for the next replenishment. The final results are solved with the help of numerical example associated with the model is discussed.

I. INTRODUCTION

Deterioration of goods is a general problem of our daily life. Fruits, Flowers, foods, vegetables, medicines, fashionable items, electronic items, foods, vegetables, etc. are some examples of such type of items. The loss because of deterioration cannot be avoided. The study of inventory models for perishable items was started 1960s. Many research papers in this area have been published already, the study was not remarkable. In 1964, van Zyl proposed one of the first inventory models for perishable items with two periods. Because at that time there are so many literature reviews on inventory system. In 1963, Ghare and Schrader were the two most famous researchers, who proposed many important inventory models for deteriorating inventory items for a constant demand. In 1973, Covert and Philip were formulated a model with assumptions of a constant rate of demand and without shortages by using a variable rate of deterioration of two-parameter Weibull distribution. After that in 1974, Philip extended the inventory model for perishable items by taking a variable rate of deterioration of three-parameter Weibull distribution. But, all the above models are considered only constant demand later, in 1977, Shah and Jaiswal, proposed an order-level inventory model for perishable items with a constant rate of deterioration. After that in 1978, Aggarwal corrected the model presented by Shah and Jaiswal's analysis (1977) and gave an order-level inventory model by which the error in calculating the average inventory holding cost is reduced. In1977 Su et al. studied a production inventory model for perishable items with an exponentially decreasing demand over a fix period. The first valuable review on inventory problems for obtaining required ordering policies for both. inventory with continuous exponential decay and perishable inventory with fixed lifetime, After that in 1983, the researchers Mak, Hollier and Park also studied some models with backlogging of constant rates in their study. In few inventory items, such as fashionable goods, the main problem was to determine whether the backlogging will be accepted or not. This problem was solved by some researchers bytaking the factor of length of the waiting time for the next replenishment In 1981, Dave and Patel presented an inventory model for perishable items where demand was proportional to time, instantaneous replenishment and without shortage. In 1991, Raafat gives the literature survey on regularly perishable inventory models studied until 1990 and continuing to classify the study according to Silver (1981), who observed a gap between practical and theoretical study of inventory control in his research. In 1991, Moon, Gallego and Simchi-Levi presented a study on the effects of decreasing production due to the reason of a manufacturing equipment of a family of items, taking same cycle for all the items. In 1994, Hariga and Benkherouf considered both case of exponentially decaying and growing markets and generalized Hollier and Mak's model (1983). In 1995, Wee (1995) studied an inventory model for perishable items where the demand decreases exponentially with time as well as cost of items.In this study during the shortage period, the rate of backlogging was taken as a fixed fraction of rate of demand. After that Wilson extended Harris' model and presented a result to determine economic order quantity. In 1999, Chang and Dye presented an inventory model where the proportion of customers who were ready to allow back logging is the reciprocal of a waiting time linear function. A complete review perishable inventory models was given by Goyal and Giri in 2001. They suggested that the consideration of constant demand rate is not always effective for several inventory items such as, fashionable clothes electronic goods, computer chipsetc. as they face the problem of fluctuations in the rate of demand. Several perishable items experience a cycle of increasing demand during the growing stage of their product life cycle. On the other side, the demand of few perishable items may decrease because of the choice of customers ofmany new better products present in the market. A loss of consumer confidence due to the negative impact on demand of such products creates a big loss in market. In the study of inventory models, two types of time dependent demands have been assumed one is continuous time dependent demand and other is discrete time dependent demand. Most of the continuous time inventory models have been presented by taking thr pattern of linearly increasing/decreasing demand or exponentially increasing/decreasing demand. In 1998, Benkherouf extended the study of Wee (1995) for the optimal procedure by considering independent rate of demand. In 1999, Chung and Tsai also stated that the Newton's method by Wee (1995) was not satisfactory condition of first order of the total cost. They also stated it to separate the nonzero quantity, and then the Newton's method was applied by them. During the study of inventory, shortages are assumed completely backlogged by many researchers. In common study it is observed that during the shortage period, some customers like to wait for but the others not. There due to the lack of sales the cost of opportunity must be considered in the modeling. .In 2005, Yang, G., Ronald, R.J. and Chu, P. proposed Inventory models for variable lead time . In 2007, Alamri and Balkhi studied some inventory models on the effects of learning and forgetting on the optimal production lot size for deteriorating items with time-varying demand and deterioration. In 2007, Dye et al also proposed an optimal selling price lot size model with a variable deterioration and exponential, shortages and partial backlogging. In their study they consider that a part of customers who backlog their orders increases exponentially as the waiting time for the next replenishment decreases. In 2008, Roy presented an inventory model where the rate of deterioration rate was proportional to time. In 2009, Pareek et al. proposed an inventory model for perishable with shortages and re-sale value. In 2013, Mishra and Singh studied an inventory model for quadratic demand, time-dependent perishable items with recover value and shortages. And perishable time dependent inventory model with variable (time) holding cost wand partial backlogging. An inventory model with common type of demand, deterioration and backlogging proposed by Hung in 2013. In 2013, Mishra et al. also proposed an inventory model for perishable items with time-dependent demand, time-varying holding cost and partial backlogging. In 1990, Datta and Pal was studied the models on same pattern. In 1997, Sarker, Mukherjee and Balan considered inventory-level dependent demand and introducing a totally new concept of decrease in demand. In 1973, Montgomery, Bazarra and Keswani proposed both deterministic and probabilistic models considering the situation in which, during the stock-out period a part of demand is backordered and rest is lost forever.

A. Notation

The mathematical model in this paper is developed on the basis of the following notations.

A	=	The opportunity cost when per unit sales is lost.
t_1	=	The time for which the shortages started.
T	=	The ordering cycle length
U	=	The inventory level, which is maximum for each ordering cycle
B	=	The amount of demand, which is maximum and backlogged for each ordering cycle
Q	=	The order quantity for each ordering cycle
I (t)	=	The inventory level at time t
H	=	The holding cost/ unit time
C	=	The inventory per unit cost.
P	=	The per orderinventory ordering cost.
S	=	The per unit time shortage.

B. Assumptions And Limitations

The mathematical model in this paper is developed on the basis of the following assumptions and limitations.

1) At an infinite rate, the replenishment occurs instantaneously.
2) Singe item inventory system.
3) The planning horizon is infinite
4) Shortages are allowed.
5) The deteriorating rate, μ $(0 < \mu < 1)$, which is constant and during the given period the replacement of deteriorated units is not allowed.
6) The demand rate, which decreases exponentially is D(t) and given as

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$$D(t)=\begin{cases} Me^{-\alpha t}, & I(t)\geq 0 \\ E, & I(t)\leq 0 \end{cases}$$

where α ($0<\alpha<\mu$) is a constant due to the decreased demand rate and M (>0) is initial demand.

In case of period of shortage the rate of backlogging isa variable and depends on the waiting time length for the next replenishment. If the waiting time is longer, the rate of backlogging will be smaller. Hence, the proportion of customers who are ready to accept the condition of backlogging at time t is decreasing with the waiting time (T-t), which is the waiting for the next replenishment. To handle this conditional backlogging rate $\frac{1}{1+\beta(T-t)}$ is defined in case of negative inventory i. The backlogging parameter β is a positive constant, $t_1\leq t\leq T$.

C. Model Formulation

In present study an EOQ inventory mathematical model is proposed for perishable items with exponential decaying demand, partial backlogging and shortages. Shortages are partially backordered and allowed. The rare of backlogging depends on the waiting time for the next replenishment. The aim of the present study is to obtain the optimal order quantity and the ordering cycle length to minimize the total inventory cost as low as possible. Here U is the maximum inventory level and replenishment is considered at time t = 0. When the market demand and deterioration of the item are increasing then the inventory level decreases during the period [0,t] , and ultimately falls to zero at $t=t_1$. Therefore during the time interval [t,T], shortages are allowed to consider and during the period [t,T] .all of the demand is partially backlogged

Now the differential equation generated by our inventory system is given as follows.

$$\frac{dI(t)}{dt}+\mu I(t)=Me^{-\alpha t}, \quad 0\leq t\leq t_1 \tag{1}$$

using the boundary condition I (0) =U. The solution of equation (1) is

$$I(t)=\frac{Me^{-\alpha t}}{\mu-\alpha}\left[e^{(\mu-\alpha)(t-t_1)}-1\right], \quad 0\leq t\leq t_1 \tag{2}$$

Therefore, the maximum inventory level for each cycle is given as

$$U=I(0)=\frac{M}{\mu-\alpha}\left[e^{(\mu-\alpha)t_1}-1\right] \tag{3}$$

Now during the cycle of shortage [t_1 ,T], the demand at time t partially backlogged at the fraction $\frac{1}{1+\beta(T-t)}$.

Thus, the differential equation for the amount of demand backlogged is given as

$$\frac{dI(t)}{dt}=-\frac{E}{1+\beta(T-t)}, \quad t_1\leq t\leq T \tag{4}$$

Using the boundary condition I(t_1). The solution of equation (4) is given by

$$I(t)=\frac{E}{\beta}\{\ln[1+\beta(T-t)]-\ln[1+\beta(T-t_1)]\}, \quad t_1\leq t\leq T \tag{5}$$

By putting t □□T in (5), we have the maximum amount of demand backlogged per cycle as follows

$$B=-I(T)=\frac{E}{\beta}\ln[1+\beta(T-t_1)] \tag{6}$$

Hence, the order quantity per cycle is given by

$$Q=U+B=\frac{M}{\mu-\alpha}\left[e^{(\mu-\alpha)t_1}-1\right]+\frac{E}{\beta}\ln[1+\beta(T-t_1)] \tag{7}$$

Now, we determine Holding Cost per cycle

$$HC=\int_0^{t_1} hI(t)dt=\frac{hM}{\mu(\mu-\alpha)}e^{-\alpha t}[e^{\alpha t_1}-1-\frac{\mu}{\alpha}(e^{\alpha t_1}-1)] \quad (8)$$

Now, we determine Deterioration Cost per cycle

$$DC=C\left[U-\int_0^{t_1} D(t)dt\right]=C\left[U-\int_0^{t_1} Me^{-\alpha t}dt\right]$$
$$=CM\left\{\frac{1}{\mu-\alpha}[e^{(\mu-\alpha)t_1}-1]-\frac{1}{\alpha}(e^{\alpha t_1}-1)\right\} \quad (9)$$

Now, we determine Shortage Cost per cycle

$$SC=S\left[-\int_{t_1}^{T} I(t)dt\right]=SE\left\{\frac{T-t_1}{\beta}-\frac{1}{\beta^2}\ln[1+\beta(T-t_1)]\right\} \quad (10)$$

Now, we determine the Opportunity Cost due to lost sales per cycle

$$OC=A\left[\int_{t_1}^{T}\left[1-\frac{1}{1+\beta(T-t)}\right]Edt\right]=AE\left\{(T-t_1)-\frac{1}{\beta}\ln[1+\beta(T-t_1)]\right\} \quad (11)$$

Therefore, the average total cost per unit time per cycle is

TVC((t_1 ,T)= holding cost + deterioration cost + ordering cost + shortage cost + opportunity cost due to lost sales)/ length of ordering cycle

$$=\frac{1}{T}\left\{\frac{hM}{\mu(\mu-\alpha)}e^{-\alpha t}[e^{\alpha t_1}-1-\frac{\mu}{\alpha}(e^{\alpha t_1}-1)]+CM\left[\frac{1}{\mu-\alpha}[e^{(\mu-\alpha)t_1}-1]-\frac{1}{\alpha}(e^{\alpha t_1}-1)\right]\right.$$
$$\left.+SE\left[\frac{T-t_1}{\beta}-\frac{1}{\beta^2}\ln[1+\beta(T-t_1)]\right]+AE\left[(T-t_1)-\frac{1}{\beta}\ln[1+\beta(T-t_1)]\right]\right\} \quad (12)$$

Our aim is to find the optimal values of t_1 and T so that the average total cost (TVC)per unit time is minimized. The optimal solutions t_1 *and T* required to satisfy the equations given below.

$$\frac{\partial TVC}{\partial t_1}=0 \text{ and } \frac{\partial TVC}{\partial T}=0$$

$$\frac{\partial TVC}{\partial t_1}=\frac{1}{T}\left\{\frac{M(h+\mu C)}{\mu}\left[e^{(\mu-\alpha)t_1}-e^{-\alpha t_1}\right]-\frac{E(S+\beta L)}{\beta}\left[1-\frac{1}{1+\beta(T-t_1)}\right]\right\}=0 \quad (13)$$

And

$$\frac{\partial TVC}{\partial T}=\frac{1}{T^2}\left\{\frac{E(S+\beta L)}{\beta}\left[\frac{(T-t_1)(\beta t_1-1)}{1+\beta(T-t_1)}+\frac{1}{\beta}\ln[1+\beta(T-t_1)]\right]\right.$$
$$\left.+\frac{M(h+\mu C)}{\mu}[1-e^{-\alpha t_1}]-P\right\}=0 \quad (14)$$

Let $\frac{M(h+\mu C)}{\mu}=X$ and $\frac{E(S+\beta L)}{\beta}=Y$

Substitute these values in (13) and (14), we have

$$T = t_1 + \frac{\left\{\frac{1}{\beta}\frac{X}{Y}[e^{(\mu-\alpha)t_1} - e^{-\alpha t_1}]\right\}}{\left\{1 - \frac{X}{Y}[e^{(\mu-\alpha)t_1} - e^{-\alpha t_1}]\right\}} \tag{15}$$

and

$$Y\left\{\frac{(T-t_1)(\beta t_1 - 1)}{1+\beta(T-t_1)} + \frac{1}{\beta}\ln[1+\beta(T-t_1)]\right\} - \frac{X}{\mu-\alpha}\left[e^{(\mu-\alpha)t_1} - 1\right] + \frac{X}{\alpha}\left[1 - e^{-\alpha t_1}\right] - P = 0 \tag{16}$$

Substituting the value of T from (15) into (16), we have

$$\frac{Y}{\beta}[e^{(\mu-\alpha)t_1} - e^{-\alpha t_1}](\beta t_1 - 1) - \frac{Y}{\beta}\ln\left[1 - \frac{X}{Y}[e^{(\mu-\alpha)t_1} - e^{-\alpha t_1}]\right] - \frac{X}{\mu-\alpha}[e^{(\mu-\alpha)t_1} - 1] + \frac{X}{\alpha}[1 - e^{-\alpha t_1}] - P = 0 \tag{17}$$

If we let $W = \left(1 + \frac{X}{Y}\right) - \left(1 + \frac{X}{Y}\right)^{\frac{-\alpha}{\mu-\alpha}}$, then we have the following results.

1) Result-1

If $\mu > \alpha$ and $\frac{XW}{\theta-\alpha}\ln\left(1+\frac{Y}{X}\right) - \frac{Y}{\beta}\ln\left(1 - \frac{X}{Y}W\right) - \frac{XW(\alpha-\beta)}{\alpha\beta}Y - P > 0$, then the solution of (13) and (14) exists and is unique.

2) Result-2

If $\mu > \alpha$ and $\frac{XW}{\theta-\alpha}\ln\left(1+\frac{Y}{X}\right) - \frac{Y}{\beta}\ln\left(1 - \frac{X}{Y}W\right) - \frac{XW(\alpha-\beta)}{\alpha\beta}Y - P > 0$, then the total cost per unit time $TVC(t_1, T)$ is convex ad reaches its global minimum at point $[t_1*, T*]$.

Now by using the values of t_1* and T* in equations (3) and (12), respectively, the optimal maximum inventory level U* and the minimum average total cost TVC* per unit time can be obtained. We can also obtain the optimal order quantity Q* (from equation (7).

D. Numerical Example

Assume that there is a product with an exponentially decreasing function ofdemand

$D(t) = Me^{-\alpha t}$ with M>0, α>0, where M and α are constants.

The parameters of the inventory model are M = 12, μ = 0.075, β = 2.05, α = 0.032, h=

= 0.55, C = 1.5, P = 10, S = 2.5, L = 2, and E = 8. With the help of these values of Parameters, we check the condition

$$\frac{XW}{\theta-\alpha}\ln\left(1+\frac{Y}{X}\right) - \frac{Y}{\beta}\ln\left(1 - \frac{X}{Y}W\right) - \frac{XW(\alpha-\beta)}{\alpha\beta}Y - P = 281.2122 > 0$$

Now the optimal shortage at point t_1* can be determined which is 1.4675per unit time and the optimal length of ordering cycle T* = 1.8543 unit time. Now, calculate the optimal maximum inventory level U*, which is 18.421 units, the optimal order quantity Q* = 20.1253 units and the per unit time minimum average total cost TVC* = 11.1710.

II. CONCLUSION

In the present study an EOQ inventory mathematical model is proposed for perishable items with exponential decaying demand, partial backlogging and shortages. In practical situations not only demand depends on time, the costs also affected by shortages. Here we also considered constant rate of deterioration and backlogging rate is taken inversely proportional to the waiting time for the next replenishment. It is also showing that the minimized objective cost function is derive the optimal solution and is jointly convex. Also the numerical example is given to explain and illustrate the model. The future hope of the study is to extend the proposal model in many situations in present business environment such as for perishable items with linear and quadratic increasing demand, price dependent demand, power demand and stock-in/ stock-out dependent demand,. Also, it may extend the deterministic demand function to probabilistic varying demand patterns. Also the model can be generalized for the lot size economic production model.

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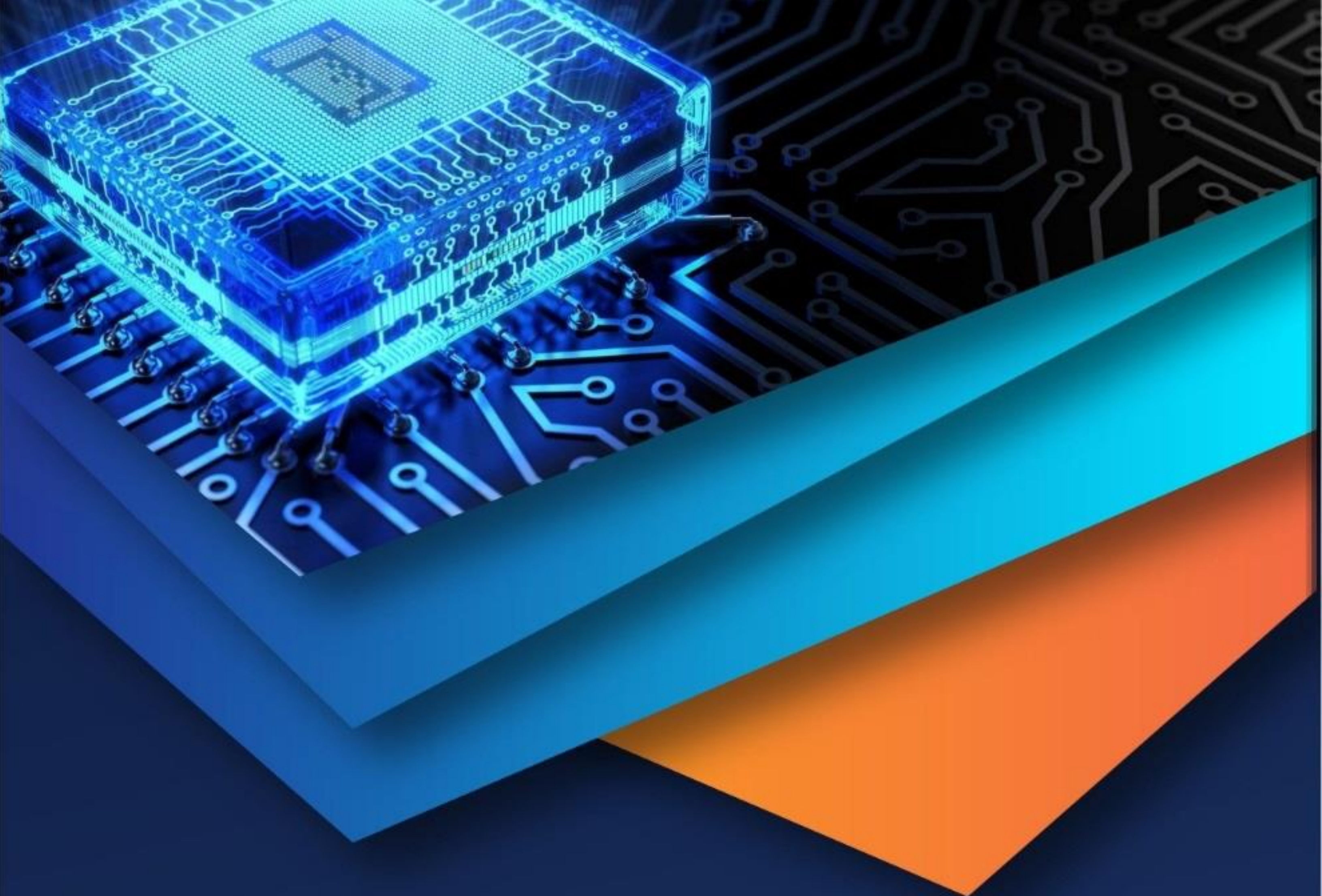

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