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## Simulation of Quadruple Tank Process for Liquid Level Control

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Abstract: A Quadruple Tank Process (QTP) is a two double tank process in which the tanks are arranged in such a way that the upper two tanks have an effect on the liquid level of the lower two tanks. A PID controller is used to control the liquid level in the lower two tanks. This system is divided into two Single Input Single Output systems to make the system simpler and easy to operate. The inputs and outputs are grouped for the pair that yields a good Relative Gain Array. The Niederlinski Index (NI) and Condition Number (CN) are calculated to check that pairing chosen would lead to a stable system. A control technique known as decoupling is used in the decentralised model of the system. The PID controllers are applied in each SISO system along with a decoupler. The system is simulated in MATLAB environment and the results are obtained.

Keyword: QTP-Quadruple Tank Process, MIMO, SISO, Decoupling, RGA, Niederlinski Index, Condition Number.

#### I. INTRODUCTION

Control is to regulate the value of a quantity under consideration in order to maintain a process at the desired operating conditions, safely and efficiently [1]. Multiple Input and Multiple Output (MIMO) [2] are little difficult to control that a system with single input and single output. Quadruple Tank Process (QTP) [3] are such MIMO systems that has four interconnected tank and two pumps. It is used in many industries such as paper making and water treatment plants.

In this paper a transfer function matrix of the Quadruple Tank Process (QTP) is considered after linearization of the state space equations, in order to remove the square root which leads to non-linearity term. The system thus considered is then simulated using MATLAB environment. A decentralised model is constructed using 1-2/2-1 pairing of the inputs and outputs. Then decouplers are applied to remove the effect caused due to interactions between the tanks. This technique is known as Decoupling [4]. The outline of the paper is as follows: the implementation of QTP, i.e., decoupling is described in section 3. The results and discussion of the level control of quadruple tank system is described in section 4 and conclusion is presented in section 5.

#### **II. METHODOLOGY**

The schematic diagram of the physical model is shown in fig 1. The Quadruple tank process is a system comprising of four tank and two pumps. The objective is to control the level of the liquid in the lower tanks, tank2 and tank 4 in fig 1. The system has two water basins from which their respective pumps draw water. The pump voltages are V1 volts and V2 volts for pump 1 and pump 2, respectively. Consider the operation in tank 2; the input comes from pump 1 via Out 2 and from pump 2 via output of tank 1. Similarly, for tank 4 the input comes from pump 1 via Out 2 and from pump 1 via the output of tank 3. Since the change in pump voltages V1 and V2 cause a change in the flow of liquid in the tanks. Thus, these voltages are the manipulated variables whereas the level of the tanks L2 and L4 are the controlled output of the system. The arrangement of the system causes more flow of liquid in tank 2 and tank 4 [5] from the tanks just above them. The schematic diagram of the QTP is shown in Figure 1.

The transfer function matrix obtained after linearization is given in equation (1):

$$\mathbf{G}(\mathbf{s}) = \begin{bmatrix} \frac{3}{179s^2 + 27s + 1} & \frac{1.7}{19s + 1} \\ \frac{1.3}{16s + 1} & \frac{2.56}{172s^2 + 28s + 1} \end{bmatrix}$$
(1)

where  $G_{11}$  denotes interaction between input 1 and output 1,  $G_{12}$  denotes the interaction between input 1 and output 2,  $G_{21}$  denotes the interaction between the input 2 and output 1, and  $G_{22}$  denotes the interaction between the input 2 and output 2.



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Water Basins

Figure 1. Schematic diagram of Quadruple Tank Process

#### **III.IMPLEMENTATION OF QTP**

The system is divided into two SISO systems i.e. the inputs and outputs are paired into two systems that are interconnected to each other. The interactions between the tanks cause an effect on the liquid level of another tank. The pairing should be done with utmost care as all the pairs may not lead to a stable system. This pairing is done because it makes the system simple and easy to design. Relative Gain Array (RGA) is found for all the possible combinations of the inputs and outputs. After finding RGA, Niederlinski Index (NI) is calculated. Finally, Condition Number (CN) would be calculated.

#### A. Relative Gain Array

It is an n x n matrix which determines the degree of interactions between various variables [6]. It is given by equation (2):

$$RGA_{A} = \begin{bmatrix} \lambda_{11} & \lambda_{12} \\ \lambda_{21} & \lambda_{22} \end{bmatrix}$$
(2)

where  $\lambda_{ij}$  is the relative gain between i<sup>th</sup> input and j<sup>th</sup> output. These relative gains or elements  $\lambda_{ij}$  are obtained by equation (3):

$$\lambda_{ij} = \frac{\binom{(d_{2ij})}{(d_{2ij})} \text{all loops open}}{\binom{(d_{2ij})}{(d_{2ij})} \text{all loops open except loop uj}}$$
(3)

RGA can also be found from transfer function matrix by doing Hadamard or Schur product which is element by element multiplication of the transfer function matrix G(s) and the transpose of the matrix,  $G^{-T}$  in MATLAB environment and is as given in equation (4):

$$\mathbf{RGA}(\Lambda) \triangleq \mathbf{G} \times (\mathbf{G}^{-1})^{\mathrm{T}}$$
(4)

#### B. Niederlinski Index

The RGA is found for all the possible pairing. The most suitable pairing 1-2/2-1 is selected. Niederlinski Index(NI) is used to avoid any pairings of variables which would be unstable [7]. It is given in the equation (5):

$$\mathbf{NI} = \frac{|\mathbf{G}(\mathbf{0})|}{\sum_{n=1}^{n} \mathbf{Gn}(\mathbf{0})}$$
(5)

where |G| denotes the determinant of transfer function matrix (taking only the gains) and  $G_{ii}$  is the transfer function relating to the input *i* and output *i*.



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A negative value of the Niederlinski Index indicates instability for the pairing chosen. Niederlinski Index should be large but very large positive value would make it impossible to conclude any result. So, a non-negative NI value is good enough to ensure the stability.

#### C. Condition Number

It is the ratio of the strong interactions to the week interactions. It is also the ratio of the square roots of the maximum to the minimum Eigen values of the product matrix  $G^TG$  [8]. It is given in equation (6):

$$CN = \frac{SV(max)}{SV(min)}$$
(6)

where  $SV_{\mbox{min/or}\mbox{max}}$  is the singular values.

The CN is calculated using only the gains of the transfer function matrix. The Condition Number describes how much the output value changes for even a small change in input. A low value of the condition number indicates that the system is well conditioned whereas if the condition number is a large value then the system is said to be ill conditioned.

Thus the rules for Condition Number are given as below:

- 1) If CN < 0, the system can be decoupled.
- 2) If det = non- negative value, the matrix is well conditioned.
- 3) If CN is not too large then the matrix is ill conditioned

#### D. Decoupling

The interactions between the tanks are high then, decouplers are used to reduce the effect on the tanks. The decouplers are additional controllers used in addition to the PID controllers. The overshoots that occur during the decentralized approach is overcome here. The schematic diagram is shown in Figure 2.



Figure 2. Decoupling control scheme

The figure 2 shows the decoupling control scheme for the Quadruple Tank Process. The decouplers are applied before the PID controllers in each SISO system. The scheme is shown for 1-2/2-1 pairing which is already decided by RGA, NI and CN. Figure 2 shows the input to the decouplers is the output from the feedback controllers.

The decoupling transfer function matrix is given by equation (7):

$$D = \begin{bmatrix} 1 & -\frac{G_{11}}{G_{12}} \\ -\frac{G_{22}}{G_{21}} & 1 \end{bmatrix} = \begin{bmatrix} 1 & -\frac{G_{12}}{G_{11}} \\ -\frac{G_{21}}{G_{22}} & 1 \end{bmatrix}$$
(7)



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#### IV. RESULTS AND DISCUSSION

The relative gain array output for equation (1) is obtained and given in equation (8). The negative off diagonal elements indicate a presence of the interaction. The main diagonal elements are positive which indicate strong flow from input 1 to output 2, and input 2 to output 1. According to the RGA rules the sum of elements in any row and column should be equal to 1.

$$rga = \begin{bmatrix} 1.4040 & -0.4040 \\ -0.4040 & 1.4040 \end{bmatrix}$$
(8)

The NI index obtained for equation (1) is 0.5364 which is a non negative number which is desirable as discussed in section 3.2. Non-negative NI indicates that the pairing done in equation (1) is a stable choice.

The condition number for equation (1) is 3.727 and the system G is non singular as its determinant is 4.2752. So, the system is well conditioned according to the rules discussed in 3.3.

The decoupled model is designed and tested step input w1 is 10-14 and w2 is 10-11. The figure 3 shows the response of the decoupled system. There are no overshoots.



Figure 3. Decoupled system response without any external disturbance

Once the systems are designed and tested it is very important to test the system against any disturbance. Internal disturbance due to the physical components is something that cannot be tested beforehand. However, the response for the external disturbance can be tested by giving a disturbance at the inputs and at the outputs. The external disturbance is added to the QTP system designed and its response is observed.

Three cases are considered to test the response to the external disturbance and they are as follows:

- A. Disturbance at output 1 only
- B. Disturbance at output 2 onl
- C. Disturbance at both output 1 and 2



Figure 4. Decoupled system response with external disturbance at output Y<sub>1</sub> only





Figure 5. Decoupled system response with external disturbance at output Y2 only



Figure 6. Decoupled system response with external disturbance at output  $Y_1$  and  $Y_2$ 

The decoupled system response is shown in figure 4 with external disturbance at only output  $Y_1$ ; figure 5 shows the system response with disturbance at output  $Y_2$ ; and figure 6 shows the system response with disturbance at output  $Y_1$  and output  $Y_2$ .

#### **V. CONCLUSIONS**

The QTP is modelled successfully using SIMULINK in MATLAB 2016a using decoupling technique. The pairing of the system matrix is done using the results of relative gain array, NI and condition number. The system is simulated for decoupled technique without any disturbance first. Then, disturbance is added at output Y1, Y2 and then at both output Y1 and Y2. The results for both the techniques are obtained. The offset is very less and the overshoot is less than 5%. So, decoupling is very easy and effective to control the liquid level in QTP. The future scope would be to implement Model Adaptive Reference Control technique for the QTP system.

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