iJRASET
International Journal For Research in Applied Science and Engineering Technology



INTERNATIONAL JOURNAL FOR RESEARCH

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY

Volume: 6 **Issue:** I **Month of publication:** January 2018

DOI: http://doi.org/10.22214/ijraset.2018.1201

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International Journal for Research in Applied Science & Engineering Technology (IJRASET)
ISSN: 2321-9653; IC Value: 45.98; SJ Impact Factor :6.887
Volume 6 Issue I, January 2018- Available at www.ijraset.com

Border and Exterior of Soft Semi #gα Closed Sets in Soft Bi-Topological Spaces

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Abstract*: In this paper, we introduce a new class of closed sets via, soft semi #gα-closed sets in bi-topological spaces. And also we study the concepts of border and exterior of soft semi #gα-closed sets in bi-topological spaces which are denoted by* $(1,2)^*$ *soft semi #gα-bd (F,A) and* $(1,2)^*$ *soft semi #gα-ext (F,A), where (F,A)is any soft set of (X,E) and also investigate their basic properties.*

Keywords: $(1,2)^*$*soft semi #gα--closed set,* $(1,2)^*$*soft semi #gα--open set,* $(1,2)^*$*soft semi #gα—interior,* $(1,2)^*$*soft semi #gα—closure,* $(1,2)^*$*soft semi #gα—border and* $(1,2)^*$*soft semi #gα—exterior.*

I. INTRODUCTION

In 1963, the concepts of bi-topological spaces was originally initiated by J.C. Kelly[3]. The theory of generalized closed sets in topological spaces which was found by Levine[8] in 1970. The concepts of generalized and semi generalized closed sets was introduced and studied by Lellis[7] in classical topology. He defined a bi-topological space (X,τ_1,τ_2) to be a set X with two topologies τ_1 *and* τ_2 on X and initiated the systematic study of bi-topological spaces. The soft theory is rapidly processing in different field of mathematics. It was first proposed by Russian researcher Molodtsov[9] in 1999.Muhammad Shabir and Manazza Naz [10] introduced soft topological spaces in 2011. It was defined over an initial universe with a fixed set of parameters. N. Cagman and S. Karatas[2] introduced topology on a set called "soft topology" and initiated the theory of soft topological spaces in 2013.In this paper we defined and examined the basic properties of $(1,2)^*$ soft semi #gα-border and $(1,2)^*$ soft semi #gα-exterior in soft bi-topological spaces and study their properties.

II. PRELIMINARIES

In this section we have presented some of the basic definitions and results of soft set, soft topological space, bi-topological space to use in the sequel. Throughout this paper, X is an initial universe, E is the set of parameters, P(X) is the power set of X, and A⊆ X.

A. Definition 2.1.

Let $\tilde{\tau}$ be the collection of soft sets over X, then $\tilde{\tau}$ is called a soft topology on X if$\tilde{\tau}$ satisfies the following axioms:
$\emptyset$, $\tilde{X}$ belongs to $\tilde{\tau}$.
The union of any number of soft sets in $\tilde{\tau}$ belongs to $\tilde{\tau}$.
The intersection of any two soft sets in $\tilde{\tau}$ belongs to $\tilde{\tau}$.
The triplet (X,$\tilde{\tau}$,E) is called a soft topological space over X. For simplicity, we can take the soft topological space (X,$\tilde{\tau}$,E) as X throughout the work.

B. Definition 2.2.

A set X together with two different topologies is called bi-topological space. It is denoted by (X,τ_1,τ_2).

C. Definition 2.3.

A soft set (F,A) of a soft topological space (X,$\tilde{\tau}$,E) is called

1) soft α − closed [4] if $\tilde{s}cl(\tilde{s}int\ (\tilde{s}cl(F,A)))\ \tilde{\subseteq}$ (F,A). The complement of soft α-closed set is called soft α-open.
2) soft semi – closed [2] if $\tilde{s}int(\tilde{s}cl(F,A))\ \tilde{\subseteq}$ (F,A). The complement of soft semi – closed set is called soft semi-open.
3) soft g-closed [5] if $\tilde{s}cl(F,A)\ \tilde{\subseteq}$ (U,E), whenever (F,A) $\tilde{\subseteq}$ (U,E) and (U,E) is soft open in (X , $\tilde{\tau}$, E). The complement of soft g-closed set is called soft g-open.

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4) soft $g^{\#}$ α- closed [6] if $\tilde{s}\alpha cl(F,A) \tilde{\subseteq} (U,E)$, whenever $(F,A) \tilde{\subseteq} (U,E)$ and (U,E) is soft g-open in $(X, \tilde{\tau}, E)$. The complement of soft $g^{\#}$ α -closed set is called soft $g^{\#}$ α -open.soft #gα- closed [8] if $\tilde{s}\alpha cl(F,A) \tilde{\subseteq} (U,E)$, whenever $(F,A) \tilde{\subseteq} (U,E)$ and (U,E) is soft $g^{\#}$α- open in $(X, \tilde{\tau}, E)$. The complement of soft #gα -closed set is called soft #gα -open.

5) soft semi #gα-closed [9] if $\tilde{s}scl(F,A) \tilde{\subseteq} (U,E)$, whenever $(F,A) \tilde{\subseteq} (U,E)$ and (U,E) is soft #gα- open in $(X, \tilde{\tau}, E)$. The complement of soft semi #gα -closed set is called soft semi #gα –open

6) The union of all soft semi #gα open sets [10] each contained in a set (F,A) of $(X, \tilde{\tau}, E)$ is called soft semi #gα interior of (F,A) which is denoted by $\tilde{s}$ semi #gα-int(F,A)

7) The intersection of all soft semi #gα- closed sets [10], each containing a set (F,A) of $(X, \tilde{\tau}, E)$ is called soft semi #gα-closure of (F,A), which is denoted by $\tilde{s}$ semi #gα-closure of (F,A).

D. Definition 2.4.

Let X be a non-empty soft set on the universe X ,$\tilde{\tau}_1, \tilde{\tau}_2$ are different soft topologies on $\tilde{X}$. Then $(\tilde{X}, \tilde{\tau}_1, \tilde{\tau}_2)$is called a soft bi-topological space.

E. Definition 2.5.

Let $F_A \in S(U)$. Power soft set of F_A is defined by ,$\tilde{P}(F_A) = \{F_{Ai} \tilde{\subseteq} F_A : i \in I\}$

And its cardinality is defined by $|\tilde{P}(F_A)| = 2^{\sum_{x\in E}|f_A(x)|}$ where $|f_A(X)|$ is cardinality of $f_A(X)$.

F. Example 2.6.

Let U = $\{u_1, u_2, u_3\}$, E = $\{x_1, x_2\}$ and F_E = X = $\{(x_1, \{u_1, u_2, u_3\}), (x_2, \{u_1, u_2, u_3\})\}$. And let $(\tilde{X}, \tilde{\tau}_1, \tilde{\tau}_2)$ be a soft bi-topological space , where $\tilde{\tau}_1$ = {Ø, $_2$, $_3$, $_5$, X } , $\widetilde{}_2$ = { , $_2$, $_8$, $_{14}$, X } , then $\widetilde{}_{1,2}$ soft open sets are { , $_2$ $_3$, $_5$, $_8$, $_{14}$, $_{17}$, $_{32}$, X }and $\widetilde{}_{1,2}$ soft closed sets are { , $_4$ $_6$, $_7$, $_{12}$, $_{31}$, $_{44}$, $_{46}$,X }. Then ,

$_1$ =

$_2$ = {($_1$,{ $_1$})}

$_3$= {($_1$,{ $_2$})}

$_4$ = {($_1$,{ $_3$})}

$_5$ = {($_1$,{ $_1$, $_2$})}

$_6$ = {($_1$,{ $_2$, $_3$})}

$_7$= {($_1$,{ $_3$, $_1$})}

$_8$= {($_2$,{ $_1$})}

$_9$ = {($_2$,{ $_2$})}

$_{10}$= {($_2$,{ $_3$})}

$_{11}$= {($_2$,{ $_1$, $_2$})}

$_{12}$ = {($_2$,{ $_2$, $_3$})}

$_{13}$= {($_1$,{ $_3$, $_1$})}

$_{14}$= {($_1$,{ $_1$} ,($_2$,{ $_1$})}

$_{15}$= {($_1$,{ $_1$} ,($_2$,{ $_2$})}

$_{16}$= {($_1$,{ $_1$} ,($_2$,{ $_1$, $_2$ })}

$_{17}$= {($_1$,{ $_2$} ,($_2$,{ $_1$})}

$_{18}$ = {($_1$,{ $_2$} ,($_2$,{ $_2$})}

$_{19}$= {($_1$,{ $_2$} ,($_2$,{ $_1$, $_2$})}

$_{20}$= {($_1$,{ $_3$} ,($_2$,{ $_1$})}

$_{21}$= {($_1$,{ $_3$} ,($_2$,{ $_2$})}

$_{22}$= {($_1$,{ $_3$} ,($_2$,{ $_1$, $_2$})}

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$_{23}= \{(\ _1 ,\{ \ _3\} ,(\ _2,\{ \ _3, \ _1\})\}$

$_{24}= \{(\ _1 ,\{ \ _1\} ,(\ _2,\{ \ _3\})\}$

$_{25}= \{(\ _1 ,\{ \ _1\} ,(\ _2,\{ \ _2, \ _3\})\}$

$_{26}= \{(\ _1 ,\{ \ _2\} ,(\ _2,\{ \ _3, \ _1\})\}$

$_{27}= \{(\ _1 ,\{ \ _2\} ,(\ _2,\{ \ _3\})\}$

$_{28}= \{(\ _1 ,\{ \ _2\} ,(\ _2,\{ \ _2, \ _3\})\}$

$_{29}= \{(\ _1 ,\{ \ _1\} ,(\ _2,\{ \ _3, \ _1\})\}$

$_{30}= \{(\ _1 ,\{ \ _3\} ,(\ _2, \ _3\})\}$

$_{31}= \{(\ _1 ,\{ \ _3\} ,(\ _2,\{ \ _2, \ _3\})\}$

$_{32}= \{(\ _1 ,\{ \ _1, \ _2\} ,(\ _2,\{ \ _1\})\}$

$_{33}= \{(\ _1 ,\{ \ _1, \ _2\} ,(\ _2,\{ \ _2\})\}$

$_{34}= \{(\ _1 ,\{ \ _1, \ _2\} ,(\ _2,\{ \ _1, \ _2\})\}$

$_{35}= \{(\ _1 ,\{ \ _2, \ _3\} ,(\ _2,\{ \ _1\})\}$

$_{36}= \{(\ _1 ,\{ \ _2, \ _3\} ,(\ _2,\{ \ _2\})\}$

$_{37}= \{(\ _1 ,\{ \ _2, \ _3\} ,(\ _2,\{ \ _1, \ _2\})\}$

$_{38}= \{(\ _1 ,\{ \ _3, \ _1\} ,(\ _2,\{ \ _1\})\}$

$_{39}= \{(\ _1 ,\{ \ _3, \ _1\} ,(\ _2,\{ \ _2\})\}$

$_{40}= \{(\ _1 ,\{ \ _3, \ _1\} ,(\ _2,\{ \ _1, \ _2\})\}$

$_{41}= \{(\ _1 ,\{ \ _1, \ _2\} ,(\ _2,\{ \ _3\})\}$

$_{42}= \{(\ _1 ,\{ \ _1, \ _2\} ,(\ _2,\{ \ _2 , \ _3\})\}$

$_{43}= \{(\ _1 ,\{ \ _2, \ _3\} ,(\ _2,\{ \ _3\})\}$

$_{44}= \{(\ _1 ,\{ \ _2, \ _3\} ,(\ _2,\{ \ _2, \ _3\})\}$

$_{45}= \{(\ _1 ,\{ \ _3, \ _1\} ,(\ _2,\{ \ _3\})\}$

$_{46}=\{(\ _1 ,\{ \ _3, \ _1\} ,(\ _2,\{ \ _2, \ _3\}$

$_{47}= \{(\ _1 ,\{ \ _1, \ _2, \ _3\})\}$

$_{48}= \{(\ _1 ,\{ \ _1, \ _2, \ _3\},(\ _2,\{ \ _1\})\}$

$_{49}= \{(\ _1 ,\{ \ _1, \ _2, \ _3\},(\ _2,\{ \ _2\})\}$

$_{50}= \{(\ _1 ,\{ \ _1, \ _2, \ _3\},(\ _2,\{ \ _1, \ _2\})\}$

$_{51}= \{(\ _1 ,\{ \ _1, \ _2, \ _3\},(\ _2,\{ \ _3\})\}$

$_{52}= \{(\ _1 ,\{ \ _1, \ _2, \ _3\},(\ _2,\{ \ _2, \ _3\})\}$

$_{53}= \{(\ _1 ,\{ \ _1, \ _2, \ _3\},(\ _2,\{ \ _3, \ _1\})\}$

$_{54}= \{(\ _1 ,\{ \ _1\},(\ _2,\{ \ _1, \ _2, \ _3\})\}$

$_{55}= \{(\ _1 ,\{ \ _2\},(\ _2,\{ \ _1, \ _2, \ _3\})\}$

$_{56}= \{(\ _1 ,\{ \ _1 , \ _2\},(\ _2,\{ \ _1, \ _2, u_3\})\}$

$_{57}= \{(\ _1 ,\{ \ _3\},(\ _2,\{ \ _1, \ _2, \ _3\})\}$

$_{58}= \{(\ _1 ,\{ \ _2, \ _3\},(\ _2,\{ \ _1, \ _2, \ _3\})\}$

$_{59}= \{(\ _1 , \ _1 , \ _3\},(\ _2,\{ \ _1, \ _2, \ _3\})\}$

$_{60}= \{(\ _1 \{ \ _1, \ _3\},(\ _2,\{ \ _1, \ _3\})\}$

$_{61}=\{(\ _1 \{ \ _1, \ _2\},(\ _2,\{ \ _1, \ _3\})\}$

$_{62}= \{(\ _1 \{ \ _2, \ _3\},(\ _2,\{ \ _1, \ _3\})\}$

$_{63}= \{ (\ _2,\{ \ _1, \ _2, \ _3\})\}$

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$_{64}$= {($_1$,{ $_1$, $_2$, $_3$},($_2$,{ $_1$, $_2$, $_3$})} = X. Are all soft subsets of . So |˜ ( ) | = 2^6 = 64

G. Definition 2.7.

A soft set (F,A) of a soft bi-topological space (˜, ˜$_1$, ˜$_2$,E) is called (*1,2*)*soft semi #gα-closed if ˜scl(F,A)⊑̃ (U,E), whenever (F,A) ⊑̃ (U,E) and (U,E) is (*1,2*)* soft #gα-open in (˜, ˜$_1$, ˜$_2$,E). The complement of (*1,2*)* soft semi #gα-closed set is called (*1,2*)* soft semi #gα-open.

H. Theorem 2.8.

If (F,A) and ˜$_{1,2}$(G,B) are soft subset of (X,E), then

1) (F,A) is (*1,2*)* soft semi #gα-open iff (*1,2*)* soft semi #gα-int(F,A) ≅̃ (F,A).

2) (*1,2*)* soft semi #gα-int (F,A) is (*1,2*)* soft semi #gα − open

3) (F,A) is (*1,2*)* soft semi #gα-closed iff(*1,2*)* soft semi #gα-cl(F,A) ≅̃ (F,A)

4) (*1,2*)*soft semi #gα-cl (F,A) is (*1,2*)* soft semi #gα-closed.

5) (*1,2*)* soft semi #gα-cl ((X,E)\(F,A)) ≅̃ (X,E)\ (*1,2*)* soft semi #gα-int(F,A).

6) (*1,2*)* soft semi #gα-int((X,E)\(F,A)) ≅̃ (X,E)\ (*1,2*)* soft semi #gα-cl(F,A).

7) If (F,A) is (*1,2*)* soft semi #gα-open in (˜, ˜$_1$, ˜$_2$,E) and ˜$_{1,2}$(G,B) is (*1,2*)* soft semi #gα-open in (˜, ˜$_1$, ˜$_2$,E), then (F,A)∩̃ ˜$_{1,2}$(G,B) is (*1,2*)* soft semi #gα-open in (˜, ˜$_1$, ˜$_2$,E)

8) A point x ∈̃ (*1,2*)* soft semi #gα-cl (F,A) iff every (*1,2*)* soft semi #gα-open set in (X,E) containing x intersects (F,A).

9) Arbitary intersection of (*1,2*)* soft semi #gα-closed sets in (˜, ˜$_1$, ˜$_2$,E) is also (*1,2*)* soft semi #gα-closed in (˜, ˜$_1$, ˜$_2$,E).

Proof We know that ,(*1,2*)* soft union of all (*1,2*)* soft open sets contained in (F,A) is called ˜ int(F,A). So (*1,2*)* soft semi #gα-open sets also is in (F,A).

Since (*1,2*)* soft semi #gα-int (F,A) are(*1,2*)* soft open sets. Therefore (*1,2*)* soft semi #gα-int(F,A) is (*1,2*)* soft semi #gα-open sets.

Let the (*1,2*)* soft intersection of all (*1,2*)* soft closed sets containing (F,A) is called ˜ cl(F,A). So (*1,2*)* soft semi #gα-closed sets is in (F,A). (iv) is similar to (iii).

(*1,2*)* soft complement of (X,E) and (F,A) is equal to (*1,2*)* soft complement of (X,E) and(*1,2*)* soft semi #gα-int (F,A).

Similarly (vi) can be proved. (vii) and (viii) are follow from the definition of (*1,2*)* soft interior. (ix) obivious from the definition of (*1,2*)* soft closure.

I. Definition 2.9.

For any soft subset (F,A) of ˜$_{1,2}$(X,E),

The soft border of (F,A) is defined by soft bd (F,A) ≅̃ (F,A)\ (*1,2*)* soft int(F,A).

The soft exterior of (F,A) is defined by soft ext (F,A) ≅̃ (*1,2*)* soft int((X,E) \ (F,A)).

III.SOFT SEMI #G-ALPHA BORDER AND EXTERIOR OF A SET IN BI-TOPOLOGICAL SPACES

In this section, we introduce and study the concepts of border and exterior of soft semi #gα-closed sets in soft bi-topological spaces.

A. Definition 3.1.

For any soft subset (F,A) of ˜$_{1,2}$(X,E) , (*1,2*)* soft semi #gα-border of (F,A) is defined by

(*1,2*)*soft semi #gα-bd (F,A) ≅̃ (F,A)\ (*1,2*)* soft semi #gα- int(F,A).

B. Theorem 3.2.

In a soft bi-topological space (X, ˜$_{1,2}$,E), for any soft subset (F,A) of (X,E), the following statements hold.

(*1,2*)*soft semi #gα-bd (□)≅̃ (*1,2*)* soft semi #gα-bd (X,E) ≅̃□.

(*1,2*)*soft semi #gα-bd (F,A) ⊑̃ (F,A).

(F,A)≅̃ (*1,2*)* soft semi #gα- int (F,A) ∪̃ (*1,2*)* soft semi #gα-bd (F,A).

(*1,2*)*soft semi #gα-int (F,A)∩̃ (*1,2*)* soft semi #gα-bd (F,A) ≅̃□.

soft semi #gα-int (F,A) ≅̃ (F,A) \(*1,2*)* soft semi #gα-bd (F,A).

Proof Let us take (*1,2*)*soft semi #gα-bd (□) is in (*1,2*)* soft semi #gα-bd (X,E) and is an empty set.

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$(1,2)^*$soft semi #gα-bd (F,A) it should be any subset of (F,A).
Since soft union of $(1,2)^*$ soft semi #gα-bd (F,A) and $(1,2)^*$ soft semi #gα-int(F,A) are for any subset of (F,A).
The soft intersection of $(1,2)^*$soft semi #gα-bd (F,A) and $(1,2)^*$ soft semi #gα-int (F,A) are empty set.
The soft complement of $(1,2)^*$soft semi #gα-bd (F,A) is $(1,2)^*$ soft semi #gα-int (F,A).

C. Theorem 3.3.
$(1,2)^*$ soft semi #gα-int($(1,2)^*$ soft semi #gα-bd (F,A)) ≅□.(F,A) is $(1,2)^*$ soft semi #gα-open if and only if $(1,2)^*$ soft semi #gα-bd (F,A) ≅□.
soft semi #gα-bd ($(1,2)^*$ soft semi #gα-int(F,A))≅□.
$(1,2)^*$ soft semi #gα-bd ($(1,2)^*$ soft semi #gα-bd(F,A)) ≅ $(1,2)^*$ soft semi #gα-bd (F,A).
soft semi #gα-bd (F,A) ≅ (F,A) ∩̃ $(1,2)^*$ soft semi #gα-cl((X,E) \ (F,A)).
Proof:Let x ∈̃ $(1,2)^*$soft semi #gα-int ($(1,2)^*$ soft semi #gα-bd (F,A)). Then x ∈̃ $(1,2)^*$ soft semi #gα-bd(F,A), since $(1,2)^*$ soft semi #gα-bd (F,A) ⊆̃ (F,A), x ∈̃ $(1,2)^*$ soft semi #gα-int ($(1,2)^*$ soft semi #gα-bd (F,A)) ⊆̃ $(1,2)^*$ soft semi #gα-int (F,A). Therefore x ∈̃ $(1,2)^*$ soft semi #gα-int (F,A) ∩̃ $(1,2)^*$ soft semi #gα-bd(F,A) which is contradiction to the above theorem (iv). Thus (i) is proved.
(F,A) is $(1,2)^*$ soft semi #gα-open iff $(1,2)^*$ soft semi #gα-int(F,A) ≅ (F,A) [Theorem 2.8(i)]. But $(1,2)^*$ soft semi #gα-bd (F,A) ≅ (F,A) \ $(1,2)^*$ soft semi #gα-int (F,A) implies $(1,2)^*$ soft semi #gα-bd (F,A)≅□. This proves (ii) and (iii).
And when (F,A) ≅ $(1,2)^*$ soft semi #gα-bd (F,A) Definition 3.1 becomes $(1,2)^*$ soft semi #gα-bd ($(1,2)^*$ soft semi #gα-bd (F,A)) ≅ $(1,2)^*$ soft semi #gα-bd (F,A) \ $(1,2)^*$ soft semi #gα-int ($(1,2)^*$ soft semi #gα-bd(F,A)). Using (iii), we will get (iv).
(v) $(1,2)^*$ soft semi #gα-bd (F,A) ≅ (F,A) \ $(1,2)^*$ soft semi #gα-int (F,A) ≅ (F,A) ∩̃ ($\tilde{}_{1,2}$(X,E) \ $(1,2)^*$ soft semi #gα-int (F,A)) ≅ (F,A) ∩̃ $(1,2)^*$ soft semi #gα-cl((X,E) \ (F,A)) [Theorem 2.8 (v)]. Hence (v) is also proved.

D. Definition.3.4.
For any soft subset (F,A) of $\tilde{}_{1,2}$(X,E) , its $(1,2)^*$ soft semi #gα-exterior is defined by, $(1,2)^*$ soft semi #gα-ext (F,A) ≅ $(1,2)^*$ soft semi #gα-int((X,E)\(F,A)).

E. Theorem 3.5.
For any ≅ $(1,2)^*$soft subets (F,A) and $\tilde{}_{1,2}$(G,B) of $\tilde{}_{1,2}$(X,E), in soft bi-topological space (X, $\tilde{}_{1,2}$,E), the following statements hold.
$(1,2)^*$soft semi #gα-ext (□) ≅ $(1,2)^*$ soft semi #gα-ext (X,E) ≅ $\tilde{}$.
If (F,A) ⊆̃ $\tilde{}_{1,2}$(G,B), then $(1,2)^*$ soft semi #gα-ext (G,B) ⊆̃ $(1,2)^*$ soft semi #gα-ext (F,A).
$(1,2)^*$soft semi #gα-ext (F,A) is $(1,2)^*$ soft semi #gα-open.
(F,A) is $(1,2)^*$ soft semi #gα-closed if and only if $(1,2)^*$ soft semi #gα-ext (F,A) ≅ $\tilde{}_{1,2}$(X,E) \ (F,A).
$(1,2)^*$ soft semi #gα-ext (F,A) ≅(X,E)\ $(1,2)^*$ soft semi #gα-cl(F,A).
Proof:Let us take (i) $(1,2)^*$soft semi #gα-ext (□) is in $(1,2)^*$ soft semi #gα-ext (X,E) and is an empty set.
And (ii) if any soft subset of (F,A) is contained in $\tilde{}_{1,2}$(G,B) then, $(1,2)^*$ soft semi #gα-ext (F,A) is always contained in $(1,2)^*$ soft semi #gα-ext (G,B).
Since $(1,2)^*$soft semi #gα-int (F,A) is $(1,2)^*$ soft semi #gα-open, proof of (iii) is follow from the definition 3.4.
is$(1,2)^*$ soft semi #gα-cl (F,A) is $(1,2)^*$ soft semi #gα-closed.
Since $(1,2)^*$ soft semi #gα-int((X,E) \ (F,A))≅ (X,E) \ $(1,2)^*$ soft semi #gα-cl(F,A), (v) follows from definition 3.4.

F. Theorem 3.6.
$(1,2)^*$ soft semi #gα-ext ($(1,2)^*$soft semi #gα-ext (F,A)) ≅ $(1,2)^*$ soft semi #gα-int($(1,2)^*$soft semi #gα-cl(F,A)).
(F,A) is $(1,2)^*$ soft semi #gα-regular, then $(1,2)^*$ soft semi #gα-ext($(1,2)^*$soft semi #gα-ext (F,A))≅ (F,A).
$(1,2)^*$soft semi #gα-ext (F,A) ≅ $(1,2)^*$soft semi #gα-ext ((X,E) \($(1,2)^*$soft semi #gα-ext (F,A)).
$(1,2)^*$soft semi #gα-int (F,A) ⊆̃ $(1,2)^*$ soft semi #gα-ext ($(1,2)^*$ soft semi #gα-ext (F,A)).
Proof:Since $(1,2)^*$ soft semi #gα- int (X,E)\(F,A)) ≅ (X,E)\ $(1,2)^*$ soft semi #gα-ext(F,A), (i) follows from definition 3.4.Similarly (ii) can be proved. If (F,A) is $(1,2)^*$ soft semi #gα-regular, from the above theorem (iv), we have $(1,2)^*$ soft semi #gα-ext (F,A) ≅(X,E)\(F,A) which is also $(1,2)^*$ soft semi #gα-regular.

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Thus $(1,2)^*$soft semi #gα-ext(soft semi #gα-ext (F,A)) ≅(F,A), (ii) is proved. (iii) $(1,2)^*$ soft semi #gα-ext((X,E) \ $(1,2)^*$ soft semi #gα-ext (F,A)) ≅ $(1,2)^*$ soft semi #gα-ext ((X,E)\ $(1,2)^*$ soft semi #gα-int ((X,E)\(F,A))) ≅ $(1,2)^*$ soft semi #gα-int ((X,E)\((X,E)\ $(1,2)^*$ soft semi #gα-int((X,E)\(F,A)))) ≅ $(1,2)^*$ soft semi #gα-int($(1,2)^*$ soft semi #gα-int(X,E)\(F,A)≅ ($(1,2)^*$ soft semi #gα-int (X,E)\(F,A)) ≅ $(1,2)^*$ soft semi #gα-ext(F,A). Hence (iii) is proved.
Since (F,A) ⊑ $(1,2)^*$ soft semi #gα-cl(F,A), using (i) (iv) can be proved.

G. Theorem 3.7.
(X,E) ≅ $(1,2)^*$ soft semi #gα-int(F,A) Ũ $(1,2)^*$ soft semi #gα-ext (F,A) Ũ $(1,2)^*$ soft semi #gα-fr (B,E).
$(1,2)^*$ soft semi #gα-ext ((F,A) Ũ $\sim_{1,2}$(G,B)) ⊑ $(1,2)^*$ soft semi #gα-ext(F,A) Ñ $(1,2)^*$ soft semi #gα-ext (B,E).
$(1,2)^*$ soft semi #gα-ext ((F,A) Ñ $\sim_{1,2}$(G,B) ⊑ $(1,2)^*$ soft semi #gα-ext(F,A) Ũ $(1,2)^*$ soft semi #gα-ext (B,E).
Proof If we know that complement of $(1,2)^*$soft semi #gα-int (X,E) is in (F,A) and union of $(1,2)^*$ soft semi #gα-int(F,A) is $\sim_{1,2}$(X,E).
And then any subset of (F,A) of $\sim_{1,2}$(,) , its exterior is complement of $\sim_{1,2}$(X,E) , its exterior is complement of $\sim_{1,2}$(X,E) and (F,A). So union of all$(1,2)^*$ soft semi #gα-interior , exterior and frontier is in $\sim_{1,2}$(X,E). Hrence (i) is proved.
Proof of (ii) union of all exterior of (F,A) and $\sim_{1,2}$(G,B) is contained in intersection of $(1,2)^*$ soft semi #gα-ext(F,A) and $(1,2)^*$ soft semi #gα-ext (B,E). Hence (ii) is proved.
And next , proof of (iii) intersection of all exterior of (F,A) and $\sim_{1,2}$(G,B) is contained in union of all $(1,2)^*$ soft semi #gα-ext(F,A) and $(1,2)^*$ soft semi #gα-ext (B,E). Hence (ii) is proved.
Example 3.8.
Let U = { $_1$, $_2$, $_3$} , E = { $_1$, $_2$} and = X = {($_1$,{ $_1$, $_2$, $_3$}), ($_2$,{ $_1$, $_2$, $_3$})}. And let (X , $\sim_1$, $\sim_2$) be a soft bi-topological space , where $\sim_1$ = { , $_2$, $_3$, $_5$, X } , $\sim_2$ = { , $_2$, $_8$, $_{14}$, X } , then $\sim_{1,2}$ soft open sets are { , $_2$ $_3$, $_5$, $_8$, $_{14}$, $_{17}$, $_{32}$, X } and $\sim_{1,2}$ soft closed sets are { , $_4$ $_6$, $_7$, $_{12}$, $_{31}$, $_{44}$, $_{46}$, X }. Consider a $(1,2)^*$soft semi #gα-closed set ~ = {($_1$,{ $_1$, $_2$, $_3$}), ($_2$,{ $_1$})}
$(1,2)^*$soft semi #gα-border :
{ {($_1$,{ $_1$, $_2$, $_3$}), ($_2$,{ $_1$})} }
$(1,2)^*$soft semi #gα-exterior :
{ {($_1$,{ $_1$, $_2$, $_3$}), ($_2$,{ $_1$})}

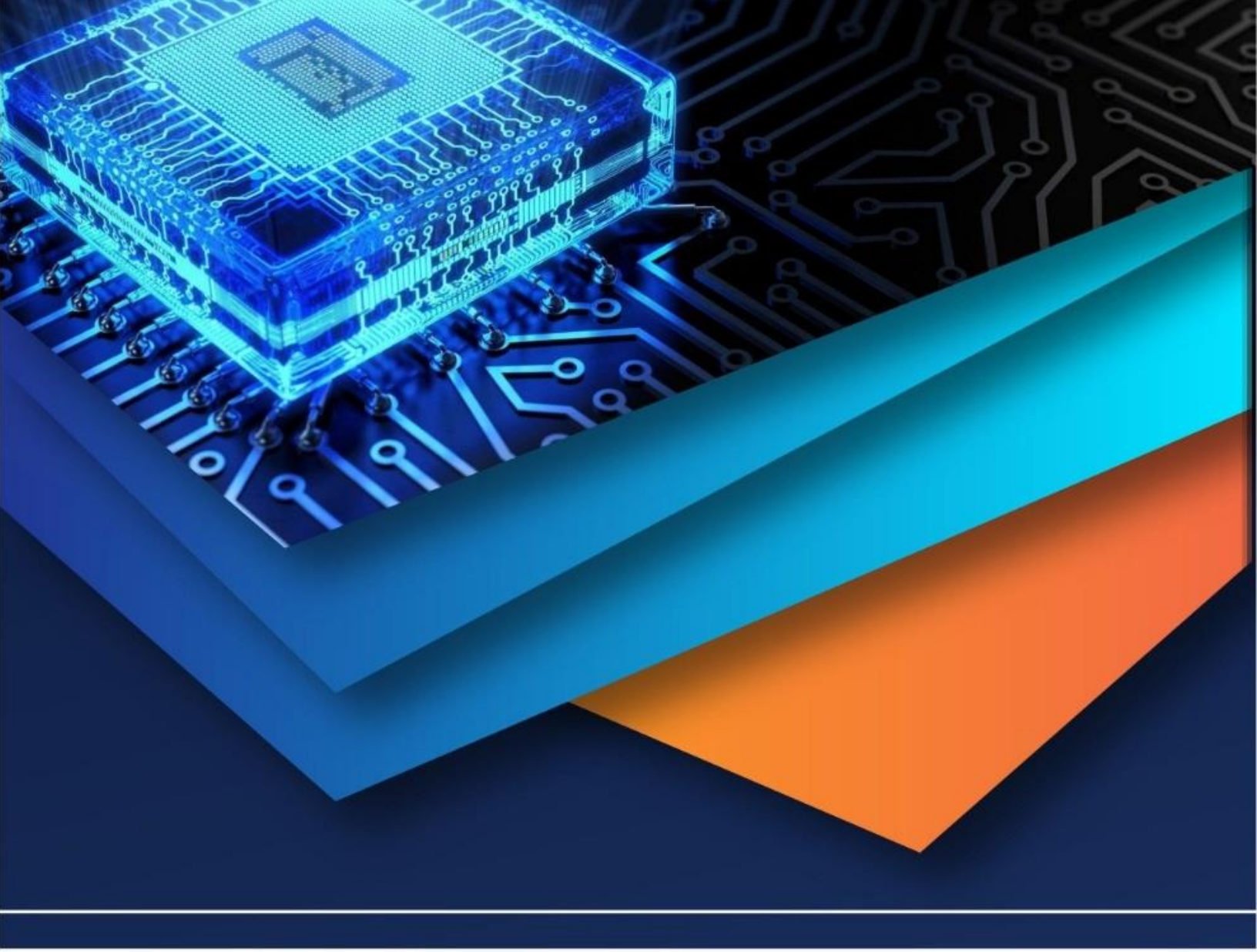
IV. CONCLUSIONS

In this paper ,Border and Exterior of soft semi gα-closed sets in soft bi-topological spaces were introduced and studied with already existing sets in soft bi-topological spaces. The scope for further research can be focused on the applications of soft bi-topological spaces.

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