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Border and Exterior of Soft Semi #gα Closed Sets in Soft Bi-Topological Spaces

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Abstract: ***In this paper, we introduce a new class of closed sets via, soft semi #gα-closed sets in bi-topological spaces. And also we study the concepts of border and exterior of soft semi #gα-closed sets in bi-topological spaces which are denoted by $(1,2)^*$ soft semi #gα-bd (F,A) and $(1,2)^*$ soft semi #gα-ext (F,A), where (F,A)is any soft set of (X,E) and also investigate their basic properties.***

Keywords: ***$(1,2)^*$soft semi #gα--closed set, $(1,2)^*$soft semi #gα--open set, $(1,2)^*$soft semi #gα—interior, $(1,2)^*$soft semi #gα—closure, $(1,2)^*$soft semi #gα—border and $(1,2)^*$soft semi #gα—exterior.***

I. INTRODUCTION

In 1963, the concepts of bi-topological spaces was originally initiated by J.C. Kelly[3]. The theory of generalized closed sets in topological spaces which was found by Levine[8] in 1970. The concepts of generalized and semi generalized closed sets was introduced and studied by Lellis[7] in classical topology. He defined a bi-topological space (X,τ_1,τ_2) to be a set X with two topologies τ_1 *and* τ_2 on X and initiated the systematic study of bi-topological spaces. The soft theory is rapidly processing in different field of mathematics. It was first proposed by Russian researcher Molodtsov[9] in 1999.Muhammad Shabir and Manazza Naz [10] introduced soft topological spaces in 2011. It was defined over an initial universe with a fixed set of parameters. N. Cagman and S. Karatas[2] introduced topology on a set called "soft topology" and initiated the theory of soft topological spaces in 2013.In this paper we defined and examined the basic properties of $(1,2)^*$ soft semi #gα-border and $(1,2)^*$ soft semi #gα-exterior in soft bi-topological spaces and study their properties.

II. PRELIMINARIES

In this section we have presented some of the basic definitions and results of soft set, soft topological space, bi-topological space to use in the sequel. Throughout this paper, X is an initial universe, E is the set of parameters, P(X) is the power set of X, and A⊆ X.

A. Definition 2.1.

Let $\tilde{\tau}$ be the collection of soft sets over X, then $\tilde{\tau}$ is called a soft topology on X if$\tilde{\tau}$ satisfies the following axioms:
$\emptyset$, $\tilde{X}$ belongs to $\tilde{\tau}$.
The union of any number of soft sets in $\tilde{\tau}$ belongs to $\tilde{\tau}$.
The intersection of any two soft sets in $\tilde{\tau}$ belongs to $\tilde{\tau}$.
The triplet (X,$\tilde{\tau}$,E) is called a soft topological space over X. For simplicity, we can take the soft topological space (X,$\tilde{\tau}$,E) as X throughout the work.

B. Definition 2.2.

A set X together with two different topologies is called bi-topological space. It is denoted by (X,τ_1,τ_2).

C. Definition 2.3.

A soft set (F,A) of a soft topological space (X,$\tilde{\tau}$,E) is called

1) soft $\alpha -$ closed [4] if $\tilde{s}cl(\tilde{s}\,int\,(\tilde{s}cl(F,A)))$ $\tilde{\subseteq}$ (F,A). The complement of soft α-closed set is called soft α-open.
2) soft semi – closed [2] if $\tilde{s}int(\tilde{s}cl(F,A))$ $\tilde{\subseteq}$ (F,A). The complement of soft semi – closed set is called soft semi-open.
3) soft g-closed [5] if $\tilde{s}cl($F,A$)$ $\tilde{\subseteq}$ (U,E), whenever (F,A) $\tilde{\subseteq}$ (U,E) and (U,E) is soft open in (X , $\tilde{\tau}$, E). The complement of soft g-closed set is called soft g-open.

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4) soft $g^{\#}$ α- closed [6] if $\tilde{s}\,\alpha cl$(F,A) $\tilde{\subseteq}$ (U,E), whenever (F,A) $\tilde{\subseteq}$ (U,E) and (U,E) is soft g-open in (X , $\tilde{\tau}$, E). The complement of soft $g^{\#}$ α -closed set is called soft $g^{\#}$ α -open.soft #gα- closed [8] if $\tilde{s}\,\alpha cl$(F,A) $\tilde{\subseteq}$ (U,E), whenever (F,A) $\tilde{\subseteq}$ (U,E) and (U,E) is soft $g^{\#}$α- open in (X , $\tilde{\tau}$, E). The complement of soft #gα -closed set is called soft #gα -open.
5) soft semi #gα-closed [9] if $\tilde{s}\,scl$(F,A) $\tilde{\subseteq}$ (U,E), whenever (F,A) $\tilde{\subseteq}$ (U,E) and (U,E) is soft #gα- open in (X , $\tilde{\tau}$, E). The complement of soft semi #gα -closed set is called soft semi #gα –open
6) The union of all soft semi #gα open sets [10] each contained in a set (F,A) of (X , $\tilde{\tau}$, E) is called soft semi #gα interior of (F,A) which is denoted by $\tilde{s}$ semi #gα-int(F,A)
7) The intersection of all soft semi #gα- closed sets [10], each containing a set (F,A) of (X , $\tilde{\tau}$, E) is called soft semi #gα-closure of (F,A), which is denoted by $\tilde{s}$ semi #gα-closure of (F,A).

D. Definition 2.4.

Let X be a non-empty soft set on the universe X ,$\tilde{\tau}_1$,$\tilde{\tau}_2$ are different soft topologies on $\tilde{X}$. Then ($\tilde{X}$, $\tilde{\tau}_1$, $\tilde{\tau}_2$)is called a soft bi-topological space.

E. Definition 2.5.

Let F_A ∈S(U). Power soft set of F_A is defined by ,$\tilde{P}$ (F_A) = {F_{Ai} $\tilde{\subseteq}$ F_A: i ∈ I}
And its cardinality is defined by$\left|\tilde{P}\,(F_A)\right|$ = 2 $\sum_{x\in E}|f_A(x)|$ where $|f_A(X)|$ is cardinality of $f_A(X)$.

F. Example 2.6.

Let U = $\{u_1,u_2,u_3\}$, E = $\{x_1,x_2\}$ and F_E = X = {(x_1 ,{u_1,u_2,u_3}), (x_2 ,{u_1,u_2,u_3})}. And let ($\tilde{X}$, $\tilde{\tau}_1$, $\tilde{\tau}_2$) be a soft bi-topological space , where $\tilde{\tau}_1$ = {∅, $_{2}$, $_{3}$, $_{5}$, X } , $\tilde{}_{2}$ = { , $_{2}$, $_{8}$, $_{14}$, X } , then $\widetilde{}_{1,2}$ soft open sets are { , $_{2}$ $_{3}$, $_{5}$, $_{8}$, $_{14}$, $_{17}$, $_{32}$, X }and $\widetilde{}_{1,2}$ soft closed sets are { , $_{4}$ $_{6}$, $_{7}$, $_{12}$, $_{31}$, $_{44}$, $_{46}$,X }.
Then ,

$_{1}$=
$_{2}$ = {($_{1}$,{ $_{1}$})}
$_{3}$= {($_{1}$,{ $_{2}$})}
$_{4}$ = {($_{1}$,{ $_{3}$})}
$_{5}$ = {($_{1}$,{ $_{1}$, $_{2}$})}
$_{6}$ = {($_{1}$,{ $_{2}$, $_{3}$})}
$_{7}$= {($_{1}$,{ $_{3}$, $_{1}$})}
$_{8}$= {($_{2}$,{ $_{1}$})}
$_{9}$ = {($_{2}$,{ $_{2}$})}
$_{10}$= {($_{2}$,{ $_{3}$})}
$_{11}$= {($_{2}$,{ $_{1}$, $_{2}$})}
$_{12}$ = {($_{2}$,{ $_{2}$, $_{3}$})}
$_{13}$= {($_{1}$,{ $_{3}$, $_{1}$})}
$_{14}$= {($_{1}$,{ $_{1}$} ,($_{2}$,{ $_{1}$})}
$_{15}$= {($_{1}$,{ $_{1}$} ,($_{2}$,{ $_{2}$})}
$_{16}$= {($_{1}$,{ $_{1}$} ,($_{2}$,{ $_{1}$, $_{2}$ })}
$_{17}$= {($_{1}$,{ $_{2}$} ,($_{2}$,{ $_{1}$})}
$_{18}$ = {($_{1}$,{ $_{2}$} ,($_{2}$,{ $_{2}$})}
$_{19}$= {($_{1}$,{ $_{2}$} ,($_{2}$,{ $_{1}$, $_{2}$})}
$_{20}$= {($_{1}$,{ $_{3}$} ,($_{2}$,{ $_{1}$})}
$_{21}$= {($_{1}$,{ $_{3}$} ,($_{2}$,{ $_{2}$})}
$_{22}$= {($_{1}$,{ $_{3}$} ,($_{2}$,{ $_{1}$, $_{2}$})}

${}_{23}= \{({}_{1}, \{ {}_{3}\}, ({}_{2}, \{ {}_{3}, {}_{1}\})\}$

${}_{24}= \{({}_{1}, \{ {}_{1}\}, ({}_{2}, \{ {}_{3}\})\}$

${}_{25}= \{({}_{1}, \{ {}_{1}\}, ({}_{2}, \{ {}_{2}, {}_{3}\})\}$

${}_{26}= \{({}_{1}, \{ {}_{2}\}, ({}_{2}, \{ {}_{3}, {}_{1}\})\}$

${}_{27}= \{({}_{1}, \{ {}_{2}\}, ({}_{2}, \{ {}_{3}\})\}$

${}_{28}= \{({}_{1}, \{ {}_{2}\}, ({}_{2}, \{ {}_{2}, {}_{3}\})\}$

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${}_{33}= \{({}_{1}, \{ {}_{1}, {}_{2}\}, ({}_{2}, \{ {}_{2}\})\}$

${}_{34}= \{({}_{1}, \{ {}_{1}, {}_{2}\}, ({}_{2}, \{ {}_{1}, {}_{2}\})\}$

${}_{35}= \{({}_{1}, \{ {}_{2}, {}_{3}\}, ({}_{2}, \{ {}_{1}\})\}$

${}_{36}= \{({}_{1}, \{ {}_{2}, {}_{3}\}, ({}_{2}, \{ {}_{2}\})\}$

${}_{37}= \{({}_{1}, \{ {}_{2}, {}_{3}\}, ({}_{2}, \{ {}_{1}, {}_{2}\})\}$

${}_{38}= \{({}_{1}, \{ {}_{3}, {}_{1}\}, ({}_{2}, \{ {}_{1}\})\}$

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${}_{40}= \{({}_{1}, \{ {}_{3}, {}_{1}\}, ({}_{2}, \{ {}_{1}, {}_{2}\})\}$

${}_{41}= \{({}_{1}, \{ {}_{1}, {}_{2}\}, ({}_{2}, \{ {}_{3}\})\}$

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${}_{43}= \{({}_{1}, \{ {}_{2}, {}_{3}\}, ({}_{2}, \{ {}_{3}\})\}$

${}_{44}= \{({}_{1}, \{ {}_{2}, {}_{3}\}, ({}_{2}, \{ {}_{2}, {}_{3}\})\}$

${}_{45}= \{({}_{1}, \{ {}_{3}, {}_{1}\}, ({}_{2}, \{ {}_{3}\})\}$

${}_{46}=\{({}_{1}, \{ {}_{3}, {}_{1}\}, ({}_{2}, \{ {}_{2}, {}_{3}\}$

${}_{47}= \{({}_{1}, \{ {}_{1}, {}_{2}, {}_{3}\})\}$

${}_{48}= \{({}_{1}, \{ {}_{1}, {}_{2}, {}_{3}\}, ({}_{2}, \{ {}_{1}\})\}$

${}_{49}= \{({}_{1}, \{ {}_{1}, {}_{2}, {}_{3}\}, ({}_{2}, \{ {}_{2}\})\}$

${}_{50}= \{({}_{1}, \{ {}_{1}, {}_{2}, {}_{3}\}, ({}_{2}, \{ {}_{1}, {}_{2}\})\}$

${}_{51}= \{({}_{1}, \{ {}_{1}, {}_{2}, {}_{3}\}, ({}_{2}, \{ {}_{3}\})\}$

${}_{52}= \{({}_{1}, \{ {}_{1}, {}_{2}, {}_{3}\}, ({}_{2}, \{ {}_{2}, {}_{3}\})\}$

${}_{53}= \{({}_{1}, \{ {}_{1}, {}_{2}, {}_{3}\}, ({}_{2}, \{ {}_{3}, {}_{1}\})\}$

${}_{54}= \{({}_{1}, \{ {}_{1}\}, ({}_{2}, \{ {}_{1}, {}_{2}, {}_{3}\})\}$

${}_{55}= \{({}_{1}, \{ {}_{2}\}, ({}_{2}, \{ {}_{1}, {}_{2}, {}_{3}\})\}$

${}_{56}= \{({}_{1}, \{ {}_{1}, {}_{2}\}, ({}_{2}, \{ {}_{1}, {}_{2}, u_{3}\})\}$

${}_{57}= \{({}_{1}, \{ {}_{3}\}, ({}_{2}, \{ {}_{1}, {}_{2}, {}_{3}\})\}$

${}_{58}= \{({}_{1}, \{ {}_{2}, {}_{3}\}, ({}_{2}, \{ {}_{1}, {}_{2}, {}_{3}\})\}$

${}_{59}= \{({}_{1}, {}_{1}, {}_{3}\}, ({}_{2}, \{ {}_{1}, {}_{2}, {}_{3}\})\}$

${}_{60}= \{({}_{1} \{ {}_{1}, {}_{3}\}, ({}_{2}, \{ {}_{1}, {}_{3}\})\}$

${}_{61}=\{({}_{1} \{ {}_{1}, {}_{2}\}, ({}_{2}, \{ {}_{1}, {}_{3}\})\}$

${}_{62}= \{({}_{1} \{ {}_{2}, {}_{3}\}, ({}_{2}, \{ {}_{1}, {}_{3}\})\}$

${}_{63}= \{ ({}_{2}, \{ {}_{1}, {}_{2}, {}_{3}\})\}$

$_{64}$= {($_1$,{ $_1$, $_2$, $_3$},($_2$,{ $_1$, $_2$, $_3$})} = X. Are all soft subsets of . So |~ ( ) | = 2^6 = 64

G. Definition 2.7.

A soft set (F,A) of a soft bi-topological space (~, ~$_1$, ~$_2$,E) is called (1,2)* soft semi #gα-closed if ~scl(F,A)⊆̃ (U,E), whenever (F,A) ⊆̃ (U,E) and (U,E) is (1,2)* soft #gα-open in (~, ~$_1$, ~$_2$,E). The complement of (1,2)* soft semi #gα-closed set is called (1,2)* soft semi #gα-open.

H. Theorem 2.8.

If (F,A) and ~$_{1,2}$(G,B) are soft subset of (X,E), then

1) (F,A) is (1,2)* soft semi #gα-open iff (1,2)* soft semi #gα-int(F,A) ≅ (F,A).
2) (1,2)* soft semi #gα-int (F,A) is (1,2)* soft semi #gα − open
3) (F,A) is (1,2)* soft semi #gα-closed iff(1,2)* soft semi #gα-cl(F,A) ≅ (F,A)
4) (1,2)*soft semi #gα-cl (F,A) is (1,2)* soft semi #gα-closed.
5) (1,2)* soft semi #gα-cl ((X,E)\(F,A)) ≅ (X,E)\ (1,2)* soft semi #gα-int(F,A).
6) (1,2)* soft semi #gα-int((X,E)\(F,A)) ≅ (X,E)\ (1,2)* soft semi #gα-cl(F,A).
7) If (F,A) is (1,2)* soft semi #gα-open in (~, ~$_1$, ~$_2$,E) and ~$_{1,2}$(G,B) is (1,2)* soft semi #gα-open in (~, ~$_1$, ~$_2$,E), then (F,A)∩̃ ~$_{1,2}$(G,B) is (1,2)* soft semi #gα-open in (~, ~$_1$, ~$_2$,E)
8) A point x ∈̃ (1,2)* soft semi #gα-cl (F,A) iff every (1,2)* soft semi #gα-open set in (X,E) containing x intersects (F,A).
9) Arbitary intersection of (1,2)* soft semi #gα-closed sets in (~, ~$_1$, ~$_2$,E) is also (1,2)* soft semi #gα-closed in (~, ~$_1$, ~$_2$,E).

Proof We know that ,(1,2)* soft union of all (1,2)* soft open sets contained in (F,A) is called ~ int(F,A). So (1,2)* soft semi #gα-open sets also is in (F,A).

Since (1,2)* soft semi #gα-int (F,A) are(1,2)* soft open sets. Therefore (1,2)* soft semi #gα-int(F,A) is (1,2)* soft semi #gα-open sets.

Let the (1,2)* soft intersection of all (1,2)* soft closed sets containing (F,A) is called ~ cl(F,A). So (1,2)* soft semi #gα-closed sets is in (F,A). (iv) is similar to (iii).

(1,2)* soft complement of (X,E) and (F,A) is equal to (1,2)* soft complement of (X,E) and(1,2)* soft semi #gα-int (F,A).

Similarly (vi) can be proved. (vii) and (viii) are follow from the definition of (1,2)* soft interior. (ix) obivious from the definition of (1,2)* soft closure.

I. Definition 2.9.

For any soft subset (F,A) of ~$_{1,2}$(X,E),

The soft border of (F,A) is defined by soft bd (F,A) ≅ (F,A)\ (1,2)* soft int(F,A).

The soft exterior of (F,A) is defined by soft ext (F,A) ≅ (1,2)* soft int((X,E) \ (F,A)).

III.SOFT SEMI #G-ALPHA BORDER AND EXTERIOR OF A SET IN BI-TOPOLOGICAL SPACES

In this section, we introduce and study the concepts of border and exterior of soft semi #gα-closed sets in soft bi-topological spaces.

A. Definition 3.1.

For any soft subset (F,A) of ~$_{1,2}$(X,E) , (1,2)* soft semi #gα-border of (F,A) is defined by

(1,2)*soft semi #gα-bd (F,A) ≅ (F,A)\ (1,2)* soft semi #gα- int(F,A).

B. Theorem 3.2.

In a soft bi-topological space (X, ~$_{1,2}$,E), for any soft subset (F,A) of (X,E), the following statements hold.

(1,2)*soft semi #gα-bd (□)≅ (1,2)* soft semi #gα-bd (X,E) ≅□.

(1,2)*soft semi #gα-bd (F,A) ⊆̃ (F,A).

(F,A)≅ (1,2)* soft semi #gα- int (F,A) ∪̃ (1,2)* soft semi #gα-bd (F,A).

(1,2)*soft semi #gα-int (F,A)∩̃ (1,2)* soft semi #gα-bd (F,A) ≅□.

soft semi #gα-int (F,A) ≅ (F,A) \(1,2)* soft semi #gα-bd (F,A).

Proof Let us take (1,2)*soft semi #gα-bd (□) is in (1,2)* soft semi #gα-bd (X,E) and is an empty set.

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$(1,2)^*$soft semi #gα-bd (F,A) it should be any subset of (F,A).
Since soft union of $(1,2)^*$ soft semi #gα-bd (F,A) and $(1,2)^*$ soft semi #gα-int(F,A) are for any subset of (F,A).
The soft intersection of $(1,2)^*$soft semi #gα-bd (F,A) and $(1,2)^*$ soft semi #gα-int (F,A) are empty set.
The soft complement of $(1,2)^*$soft semi #gα-bd (F,A) is $(1,2)^*$ soft semi #gα-int (F,A).

C. Theorem 3.3.

$(1,2)^*$ soft semi #gα-int($(1,2)^*$ soft semi #gα-bd (F,A)) ≅□.(F,A) is $(1,2)^*$ soft semi #gα-open if and only if $(1,2)^*$ soft semi #gα-bd (F,A) ≅□.
soft semi #gα-bd ($(1,2)^*$ soft semi #gα-int(F,A))≅□.
$(1,2)^*$ soft semi #gα-bd ($(1,2)^*$ soft semi #gα-bd(F,A)) ≅ $(1,2)^*$ soft semi #gα-bd (F,A).
soft semi #gα-bd (F,A) ≅ (F,A) ⋒ $(1,2)^*$ soft semi #gα-cl((X,E) \ (F,A)).
Proof:Let x ∈̃ $(1,2)^*$soft semi #gα-int ($(1,2)^*$ soft semi #gα-bd (F,A)). Then x ∈̃ $(1,2)^*$ soft semi #gα-bd(F,A), since $(1,2)^*$ soft semi #gα-bd (F,A) ⊆̃ (F,A), x ∈̃ $(1,2)^*$ soft semi #gα-int ($(1,2)^*$ soft semi #gα-bd (F,A)) ⊆̃ $(1,2)^*$ soft semi #gα-int (F,A). Therefore x ∈̃ $(1,2)^*$ soft semi #gα-int (F,A) ⋒ $(1,2)^*$ soft semi #gα-bd(F,A) which is contradiction to the above theorem (iv). Thus (i) is proved.
(F,A) is $(1,2)^*$ soft semi #gα-open iff $(1,2)^*$ soft semi #gα-int(F,A) ≅ (F,A) [Theorem 2.8(i)]. But $(1,2)^*$ soft semi #gα-bd (F,A) ≅ (F,A) \ $(1,2)^*$ soft semi #gα-int (F,A) implies $(1,2)^*$ soft semi #gα-bd (F,A)≅□. This proves (ii) and (iii).
And when (F,A) ≅ $(1,2)^*$ soft semi #gα-bd (F,A) Definition 3.1 becomes $(1,2)^*$ soft semi #gα-bd ($(1,2)^*$ soft semi #gα-bd (F,A)) ≅ $(1,2)^*$ soft semi #gα-bd (F,A) \ $(1,2)^*$ soft semi #gα-int ($(1,2)^*$ soft semi #gα-bd(F,A)). Using (iii), we will get (iv).
(v) $(1,2)^*$ soft semi #gα-bd (F,A) ≅ (F,A) \ $(1,2)^*$ soft semi #gα-int (F,A) ≅ (F,A) ⋒ ($\widetilde{}_{1,2}$(X,E) \ $(1,2)^*$ soft semi #gα-int (F,A)) ≅ (F,A) ⋒ $(1,2)^*$ soft semi #gα-cl((X,E) \ (F,A)) [Theorem 2.8 (v)]. Hence (v) is also proved.

D. Definition.3.4.

For any soft subset (F,A) of $\widetilde{}_{1,2}$(X,E) , its $(1,2)^*$ soft semi #gα-exterior is defined by, $(1,2)^*$ soft semi #gα-ext (F,A) ≅ $(1,2)^*$ soft semi #gα-int((X,E)\(F,A)).

E. Theorem 3.5.

For any ≅ $(1,2)^*$soft subets (F,A) and $\widetilde{}_{1,2}$(G,B) of $\widetilde{}_{1,2}$(X,E), in soft bi-topological space (X, $\widetilde{}_{1,2}$,E), the following statements hold.
$(1,2)^*$soft semi #gα-ext (□) ≅ $(1,2)^*$ soft semi #gα-ext (X,E) ≅ ~.
If (F,A) ⊆̃ $\widetilde{}_{1,2}$(G,B), then $(1,2)^*$ soft semi #gα-ext (G,B) ⊆̃ $(1,2)^*$ soft semi #gα-ext (F,A).
$(1,2)^*$soft semi #gα-ext (F,A) is $(1,2)^*$ soft semi #gα-open.
(F,A) is $(1,2)^*$ soft semi #gα-closed if and only if $(1,2)^*$ soft semi #gα-ext (F,A) ≅ $\widetilde{}_{1,2}$(X,E) \ (F,A).
$(1,2)^*$ soft semi #gα-ext (F,A) ≅(X,E)\ $(1,2)^*$ soft semi #gα-cl(F,A).
Proof:Let us take (i) $(1,2)^*$soft semi #gα-ext (□) is in $(1,2)^*$ soft semi #gα-ext (X,E) and is an empty set.
And (ii) if any soft subset of (F,A) is contained in $\widetilde{}_{1,2}$(G,B) then, $(1,2)^*$ soft semi #gα-ext (F,A) is always contained in $(1,2)^*$ soft semi #gα-ext (G,B).
Since $(1,2)^*$soft semi #gα-int (F,A) is $(1,2)^*$ soft semi #gα-open, proof of (iii) is follow from the definition 3.4.
is$(1,2)^*$ soft semi #gα-cl (F,A) is $(1,2)^*$ soft semi #gα-closed.
Since $(1,2)^*$ soft semi #gα-int((X,E) \ (F,A))≅ (X,E) \ $(1,2)^*$ soft semi #gα-cl(F,A), (v) follows from definition 3.4.

F. Theorem 3.6.

$(1,2)^*$ soft semi #gα-ext ($(1,2)^*$soft semi #gα-ext (F,A)) ≅ $(1,2)^*$ soft semi #gα-int($(1,2)^*$soft semi #gα-cl(F,A)).
(F,A) is $(1,2)^*$ soft semi #gα-regular, then $(1,2)^*$ soft semi #gα-ext($(1,2)^*$soft semi #gα-ext (F,A))≅ (F,A).
$(1,2)^*$soft semi #gα-ext (F,A) ≅ $(1,2)^*$soft semi #gα-ext ((X,E) \($(1,2)^*$soft semi #gα-ext (F,A)).
$(1,2)^*$soft semi #gα-int (F,A) ⊆̃ $(1,2)^*$ soft semi #gα-ext ($(1,2)^*$ soft semi #gα-ext (F,A)).
Proof:Since $(1,2)^*$ soft semi #gα- int (X,E)\(F,A)) ≅ (X,E)\ $(1,2)^*$ soft semi #gα-ext(F,A), (i) follows from definition 3.4.Similarly (ii) can be proved. If (F,A) is $(1,2)^*$ soft semi #gα-regular, from the above theorem (iv), we have $(1,2)^*$ soft semi #gα-ext (F,A) ≅(X,E)\(F,A) which is also $(1,2)^*$ soft semi #gα-regular.

Thus $(1,2)^*$soft semi #gα-ext(soft semi #gα-ext (F,A)) $\cong$(F,A), (ii) is proved. (iii) $(1,2)^*$ soft semi #gα-ext((X,E) \ $(1,2)^*$ soft semi #gα-ext (F,A)) $\cong$ $(1,2)^*$ soft semi #gα-ext ((X,E)\ $(1,2)^*$ soft semi #gα-int ((X,E)\(F,A))) $\cong$ $(1,2)^*$ soft semi #gα-int ((X,E)\((X,E)\ $(1,2)^*$ soft semi #gα-int((X,E)\(F,A)))) $\cong$ $(1,2)^*$ soft semi #gα-int($(1,2)^*$ soft semi #gα-int(X,E)\(F,A)$\cong$ ($(1,2)^*$ soft semi #gα-int (X,E)\(F,A)) $\cong$ $(1,2)^*$ soft semi #gα-ext(F,A). Hence (iii) is proved.
Since (F,A) $\tilde{\subseteq}$ $(1,2)^*$ soft semi #gα-cl(F,A), using (i) (iv) can be proved.

G. Theorem 3.7.

(X,E) $\cong$ $(1,2)^*$ soft semi #gα-int(F,A) $\tilde{\cup}$ $(1,2)^*$ soft semi #gα-ext (F,A) $\tilde{\cup}$ $(1,2)^*$ soft semi #gα-fr (B,E).
$(1,2)^*$ soft semi #gα-ext ((F,A) $\tilde{\cup}$ $\tilde{}_{1,2}$(G,B)) $\tilde{\subseteq}$ $(1,2)^*$ soft semi #gα-ext(F,A) $\tilde{\cap}$ $(1,2)^*$ soft semi #gα-ext (B,E).
$(1,2)^*$ soft semi #gα-ext ((F,A) $\tilde{\cap}$ $\tilde{}_{1,2}$(G,B) $\tilde{\subseteq}$ $(1,2)^*$ soft semi #gα-ext(F,A) $\tilde{\cup}$ $(1,2)^*$ soft semi #gα-ext (B,E).

Proof If we know that complement of $(1,2)^*$soft semi #gα-int (X,E) is in (F,A) and union of $(1,2)^*$ soft semi #gα-int(F,A) is $\tilde{}_{1,2}$(X,E).
And then any subset of (F,A) of $\tilde{}_{1,2}$(,) , its exterior is complement of $\tilde{}_{1,2}$(X,E) , its exterior is complement of $\tilde{}_{1,2}$(X,E) and (F,A). So union of all$(1,2)^*$ soft semi #gα-interior , exterior and frontier is in $\tilde{}_{1,2}$(X,E). Hrence (i) is proved.
Proof of (ii) union of all exterior of (F,A) and $\tilde{}_{1,2}$(G,B) is contained in intersection of $(1,2)^*$ soft semi #gα-ext(F,A) and $(1,2)^*$ soft semi #gα-ext (B,E). Hence (ii) is proved.
And next , proof of (iii) intersection of all exterior of (F,A) and $\tilde{}_{1,2}$(G,B) is contained in union of all $(1,2)^*$ soft semi #gα-ext(F,A) and $(1,2)^*$ soft semi #gα-ext (B,E). Hence (ii) is proved.
Example 3.8.
Let U = { $_1$, $_2$, $_3$} , E = { $_1$, $_2$} and = X = {($_1$,{ $_1$, $_2$, $_3$}), ($_2$,{ $_1$, $_2$, $_3$})}. And let (X , $\tilde{}_1$, $\tilde{}_2$) be a soft bi-topological space , where $\tilde{}_1$ = { , $_2$, $_3$, $_5$, X } , $\tilde{}_2$ = { , $_2$, $_8$, $_{14}$, X } , then $\tilde{}_{1,2}$ soft open sets are{ , $_2$ $_3$, $_5$, $_8$, $_{14}$, $_{17}$, $_{32}$,X} and $\tilde{}_{1,2}$ soft closed sets are { , $_4$ $_6$, $_7$, $_{12}$, $_{31}$, $_{44}$, $_{46}$, X }. Consider a $(1,2)^*$soft semi #gα-closed set ~ = {($_1$,{ $_1$, $_2$, $_3$}), ($_2$,{ $_1$})}
$(1,2)^*$soft semi #gα-border :
{ {($_1$,{ $_1$, $_2$, $_3$}), ($_2$,{ $_1$})} }
$(1,2)^*$soft semi #gα-exterior :
{ {($_1$,{ $_1$, $_2$, $_3$}), ($_2$,{ $_1$})}

IV. CONCLUSIONS

In this paper ,Border and Exterior of soft semi gα-closed sets in soft bi-topological spaces were introduced and studied with already existing sets in soft bi-topological spaces. The scope for further research can be focused on the applications of soft bi-topological spaces.

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