iJRASET
International Journal For Research in
Applied Science and Engineering Technology

INTERNATIONAL JOURNAL FOR RESEARCH

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY

Volume: 6 **Issue:** I **Month of publication:** January 2018

DOI: http://doi.org/10.22214/ijraset.2018.1391

International Journal for Research in Applied Science & Engineering Technology (IJRASET)
ISSN: 2321-9653; IC Value: 45.98; SJ Impact Factor :6.887
Volume 6 Issue I, January 2018- Available at www.ijraset.com

Pseudo-Complete Color Critical Graphs

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Abstract: ***A pseudo-complete coloring of a graph G is an assignment of colors to the vertices of G such that for any two distinct colors, there exist adjacent vertices having those colors. The maximum number of colors used in a pseudo-complete coloring of G is called the pseudo-achromatic number of G and is denoted by*** $\psi_s(G)$***. A graph G is called edge critical if*** $\psi_s(G-e)<\psi_s(G)$ ***for any edge e of G. A graph G is called vertex critical if*** $\psi_s(G-v)<.\ \psi_s(G)$ ***for every vertex v of G. These graphs are generally called as pseudo-achromatic number critical graphs (called shortly as PAN Critical graphs). In this paper, we investigate the properties of these critical graphs. We also investigate the locally critical elements of graphs.***
Keywords: Pseudo-complete coloring, Pseudo-achromatic number, critical graph

I. INTRODUCTION

By a graph we mean a finite undirected graph without loops, multiple edges or isolated vertices.

An assignment of colors to the vertices of a graph is called a proper coloring, if any two adjacent vertices receive distinct colors. An assignment of colors to the vertices of a graph G is called a pseudo-complete coloring, if for any two distinct colors; there exist adjacent vertices having those colors. A proper and pseudo-complete coloring of G is called a complete coloring of G.

The minimum number of colors used in a proper coloring of G is called the chromatic number of G and is denoted by $\chi(G)$. The maximum number of colors used in a complete coloring of Gis called the achromatic number of G and is denoted by $\psi(G)$ [3]. The maximum number of colors used in a pseudo-complete coloring of Gis called the pseudo-achromatic number of G and is denoted by $\psi_s(G)$ [5]. A graph which admits a pseudo-complete coloring by k colors is called a k- pseudo-complete colorable graph. Several bounds for these coloring parameters were obtained in [3. 4, 5, 6] and a detailed study of this parameter and critical graphs with respect to it were studied by Suresh Kumar in his doctoral dissertation [7].

The concept of critical graphs with respect to chromatic number was introduced and studied by Dirac [1, 2]. In this paper, we introduce the concept of critical graphs with respect to pseudo-achromatic number, obtain characterizations of them and determine the pseudo-achromatic number of several classes of graphs.

Let n be any positive real number. Then, [n] denote the greatest integer less than or equal to n and {n} denotes the smallest integer greater than or equal to n.

For terms not defined explicitly here, reader can refer Harary [8].

II. CRITICAL PATHS AND CRITICAL CYCLES

A. *Definition2.1.* A graph G is called k-edge critical if$\psi_s(G)$= k and $\psi_s(G-e)$ <k, for every edge e E E(G). A graph G is called k-vertex critical if $\psi_s(G) = k$ and $\psi_s(G-v) < k$ for every vertex v of G. A graph G is called k-contraction critical (shortly, k-con-critical) if$\psi_s(G) = k$ and $\psi_s(G||e) < k$ for every edge e of G, where G||e denotes the graph obtained from G by contracting the edge e. Following observations are quite useful later.

B. *Proposition 2.2.* A graph G is k-edge critical if f G is k-pseudo-complete colorable and $|E(G)| = \binom{k}{2}$

C. *Proposition 2.3.* A k-edge critical graph is k--con-critical and a k-con-critical graph is k-vertex-critical.

D. *Remark 2.4.*None of the statements in Proposition 2.3can be reversed. For example, C8is 4-con-critical but not edge critical. Also, C4is 3-vertex critical, but not con-critical.

E. *Theorem2.5.* Let G be a k-pseudo-complete colorable graph. Then, $|V(G)| \geq k\left\{\frac{k-1}{\Delta}\right\}$ where Δ is the maximum degree of a vertex of G and {n} is the smallest integer less than or equal to n. Proof. Consider any k-pseudo-complete coloring of G. Then for any color c, there exist k—1 edges in G such that one end vertex of each of these edges receive the color c. Hence there must be at least $k\left\{\frac{k-1}{\Delta}\right\}$vertices with color c, so that$|V(G)| \geq k\left\{\frac{k-1}{\Delta}\right\}$.

F. *Corollary 2.6.*$\psi_s(G) = max\left\{k: k\left\{\frac{k-1}{\Delta}\right\} \leq |V(G)|\right\}$for any graph G with the maximum degree, Δ.

G. *Corollary 2.7.* If G is the Petersen Graph, $\psi_s(G) = 5$.

1) *Proof*. Clearly, E(G) can be partitioned into a 2-factor, say { (v1v2v3v4v5v1), (v6v7v8v9viov6)} and a 1-factor, {v1v6, v2v9, v3v7, v4v10, v5v8}. Now the function $f: V(G) \to \{1,2,3,4.5\}$definedby$f(v_I) = i, 1 \le i \le 5$ and$f(v_I) = 2i - 11(mod\ 5), 6 \le i \le 10$assigns a 5-pseudo-complete coloring for G Hence,$\psi_s(G) \ge 5$. Also, by Corollary 2.6, $\psi_s(G) \le 5$.

H. *Corollary 2.8.*IfG is the 3-cube, Q_3,$\psi_s(G) = 4$

1) *Proof*. By Corollary 2.6,$\psi_s(G) \le 4$. The function $f: V(G) \to \{1,2,3,4\}$defined below assigns a 4-pseudo-complete coloring to Q_3 so that the corollary follows.

$$f(0,0,0) = f(1,1,0) = 1\ ,\ f(0,0,1) = f(1,1,1) = 2$$
$$f(0,1,1) = f(1,0,0) = 3\ ,\ f(0,1,0) = f(1,0,1) = 4$$

I. *Corollary 2.9.For the 4-cube Q_4,* $\psi_s(Q_4) = 8$

1) *Proof*. By Corollary 2.6, $\psi_s(Q_4) \le 8$. The function $f : V(Q_4) \to \{1,2,3,4, 5,6, 7, 8\}$ defined below assigns a 8-pseudo-complete coloring to Q_4 so that the corollary follows.

$$f(0,1,1,0) = f(1,0,0,1) = 1\ ,\ f(1,1,1,0) = f(0,0,0,1) = 2$$
$$f(0,1,0,0) = f(1,0,1,0) = 3\ ,\ f(0,0,1,0) = f(1,1,1,1) = 4$$
$$f(1,1,0,0) = f(0,1,1,1) = 5\ ,\ f(1,0,0,0) = f(0,1,1,1) = 6$$
$$f(0,1,0,1) = f(1,0,1,1) = 7\ ,\ f(0,0,0,0) = f(1,1,0,1) = 8$$

J. *Corollary 2.10.For the 5-cube* Q_5,$\psi_s(Q_5) = 11$.

1) *Proof*. By Corollary 2.6, $\psi_s(Q_5) \le 11$. An 11-pseudo complete coloring of Q_5 is shown below, so that the corollary follows.

f(0,1,1,0,0) = f(1, 0, 0,1,0) =1, f(1,1,1,0,0) =f(1,1,1,0,1) = f(0,0,0,1,1) =2
f(0,1,0,0,0) = f(1,0,1,0,0) = 3, $f(0,0,1,0,0) = f(1,1,1,1,0) = f(0,0,1,0,1) = 4$
$f(1,1,0,0,0) = f(0,1,1,1,0) = f(0,1,1,1,1) = 5$, $f(1,0,0,0,0) = f(0,0,1,1,0) = f(0,0,1,1,1) = 6$, $f(1,0,1,1,0) = f(0,1,0,1,1) = 7$, $f(0,0,0,0,0) = f(1,1,0,1,0) = f(0,0,0,0,1) = f(1,1,0,1,1,) = 8$
$f(1,0,0,0,1) = f(0,1,0,1,0) = f(0,1,1,0,1) = 9$
$f(1,1,0,0,1) = f(1,0,0,1,1) = f(1,0,1,0,1) = f(1,0,1,1,1) = 10$
$f(0,0,0,1,0) = f(0,1,0,0,1) = f(1,1,1,1,1) = 11$

Thesecorollaries motivate the following conjecture:

K. *Conjecture 2.11.*$\psi_s(Q_n) = max\left\{k: k\left\{\frac{k-1}{\Delta}\right\} \le 2^n\right\}$

L. *Corollary 2.12.* For $n \ge 2, \psi_s(K_{n,n}) = n + 1$ n + 1.

1) Proof. By Corollary 2.6, $\psi_s(K_{n,n}) \le n + 1$. Also if $X = \{x_1, x_2, \dots . x_n\}, Y = \{y_1, y_2, \dots . y_n\}$is a bipartition of $K_{n,n}$ then the function$f: V(K_{n,n}) \to \{1,2, \dots . n + 1\}$defined by $f(x_1) = 1, f(y_1) = 2,)\ and\ f(x_i) = f(y_i) = i + 1, 2 \le i \le n$gives a pseudo complete coloring of G so that $\psi_s(K_{n,n}) = n + 1$ 1

Now we proceed to characterize critical cycles and critical paths.

M. *Theorem 2.13.* Let n(k) denote the integer$\binom{k}{2}$ or $\binom{k}{2} + 1$, according as*k*is odd or even. Then a cycle C_n is k-con-critical if and only if n = n(k).

1) *Proof*. Let $C_n = \langle v_1, v_2, \dots . v_{n(k)}. v_1\rangle$ and $Z_n = \{1,2 \dots . n\}$

We first prove that C_n is k-pseudo-complete colorable.

Case 1.kis even.

Define a function $f: V(C_{n(k)}) \to Z_n$ as follows:

$$f(v_i) = \begin{cases} (1 + (-1)^{n+1}\{i/2\})(mod\ k) & if\ 1 \le i \le k \\ (\{i/k\} + (-1)^{1+g_i}\{g_i/2\})(mod\ k) & otherwise \end{cases}$$

Where $g_i = (i - \{i/k\}(k - 2) + k - 3)(mod\ k)$

Let $j_1, j_2 \in Z_{k,} j_1 < j_2$ Suppose $j_2 - j_1 = k/2$ let

International Journal for Research in Applied Science & Engineering Technology (IJRASET)
ISSN: 2321-9653; IC Value: 45.98; SJ Impact Factor :6.887
Volume 6 Issue I, January 2018- Available at www.ijraset.com

$$If\ j_1 = 1, f(\{v_k, v_{k+1}\}) = \{j_1, j_2\}$$
$$If\ j_1 \geq 2, f(\{v_{j_1-2j_1+3}, v_{j_1-2j_1+4}\}) = \{j_1, j_2\}$$

Now, suppose $j_2 - j_1 \neq k/2$.Put $n = \left\{\frac{(j_1+j_2)(mod\ k)}{2}\right\}$

Choose j to be the least positive integer such that $f(v_{(n-1)k+j})$=j1 or j2. Then it can be easily verified that $f(\{v_{(n-1)k+j}, v_{(n-1)k+j+1}\}) = \{j_1, j_2\}$. Thus in all cases, f assigns a k-pseudo-complete coloring to Cn(k).

Case 2.kis odd.

Define a function $f: V(C_{n(k)}) \to Z_k$as follows:

$$f(v_i) = \begin{cases} k & if\ i \equiv 1(mod\ k) \\ \left(\left\{\frac{i}{k}\right\} + (-1)^{g_i}\left[\frac{g_i}{2}\right]\right)(mod\ k-1) & otherwise \end{cases}$$

where$g_i = \left(i - \left\{\frac{i}{k}\right\}k + k - 1\right)(mod\ k)$

Let $j_1, j_2 \in Z_k\ and\ j_1 < j_2$. Suppose $j_2 = k$.

If $j_1 \leq \left\{\frac{k}{2}\right\}, then\ f\left(v_{(j_1-1)k+1}, v_{(j_1-k)k+2}\right) = \{j_1, j_2\}$

If $j_1 > \left\{\frac{k}{2}\right\}, then\ f\left(v_{(j_1-\{k/2\})k}, v_{(j_1-\{k/2\})k+1}\right) = \{j_1, j_2\}$

Now, suppose $j_1 < k$. Put $n = \left\{\frac{(j_1+j_2)(mod\ k-1)}{2}\right\}$. Choose j to be the least positive integer such that $f\left(v_{(n-1)k+j}\right) = j_1\ or\ j_2$

Then,$f\left(v_{(n-1)k+j}, v_{(n-1)k+j+1}\right) = \{j_1, j_2\}$

Thus in all cases, f assigns a k-pseudo-complete coloring to $C_{n(k)}$. Hence it follows from Theorem 2.5 that $C_{n(k)}$ is k-con-critical. Conversely, suppose Cn is k-con-critical. Since $C_{n(k)}$is k-pseudo-complete colorable, n<n(k) and by Theorem 2.5, n>n(k).

N. *Corollary 2.14.* Letn(k) denote the integer$\binom{k}{2}$ or $\binom{k}{2} + \frac{k}{2}$, according as k is odd or even. Then $\psi_s(C_n) = max\{k: n(k) \leq n\}$

O. *Corollary 2.15.* Letm(k) denote the integer $\binom{k}{2} + 1\ or\ \binom{k}{2} + \frac{k}{2}$, according as k is odd or even. Then a path P_n is k-con-critical if and only if n = m(k).

If P_n is k-pseudo-complete colorable, then n >m(k), by from Theorem 2.5. But since $C_{n(k)}$is k-pseudo-complete colorable, so is $P_{m(k)}$. Hence P_n is k-con-critical if and only if n = m(k).

Corollary 2.16.Let m(k) denote the integer $\binom{k}{2} + 1\ or\ \binom{k}{2} + \frac{k}{2}$, according as k is odd or even. Then $\psi_s(P_n) = max\{k: m(k) \leq n\}$

The following propositions can easily be deduced from Theorem 2.5, Theorem 2.13 and Corollary 2.15.

P. *Proposition 2.17.* A cycle, C_n is k-vertex critical iff

$$n = \begin{cases} \binom{k}{2} + \frac{k}{2} & if\ k\ is\ even \\ \binom{k}{2}\ or\ \binom{k}{2} + 1 & if\ k\ is\ odd \end{cases}$$

Q. *Proposition 2.18.* A Path, P_n is k-vertex critical iff

$$n = \begin{cases} \binom{k}{2} + \frac{k}{2} & if\ k\ is\ even \\ \binom{k}{2} + 1 & if\ k\ is\ odd \end{cases}$$

R. *Theorem 2.19.* There is no k-edge critical cycle, if k is even

Suppose$C_n$is a k-edge critical cycle and k is even. By Theorem 2.13, $n = \binom{k}{2} + \frac{k}{2}$. Now, for any edge e of

$C_n, C_n \sim e$(a path on $\binom{k}{2} + \frac{k}{2}$vertices) is not k-pseudo-complete colorable, a contradiction.

S. *Corollary 2.20.* A cycle,C_n, is k-edge critical if and only if k is odd and$n = \binom{k}{2}$.

T. *Theorem 2.21.* There is no k-edge critical path, if $k \geq 4$is even.

Suppose P_n is k-edge critical and $k \geq 4$is even. By Corollary 2.15, $n = \binom{k}{2} + \frac{k}{2}$ so that, $|E(P_n)| > \binom{k}{2}$, a contradiction toProposition.2.2.

U. *Corollary. 2.22.*A path P_n is k-edge critical if and only if either k is odd and $n = \binom{k}{2} + 1$or n=k=2

V. *Remark 2.23.* Corollaries 2.14, 2.7, 2.8, 2.9, 2.10 and 2.12 show that the regular graphs such as cycles, Petersen graph, n-cubes for n = 3, 4, 5 and $K_{n,n}$are solutions of the equation:

$$\psi_s(G) = max\{k: k\{(k-1)/\Delta\}\} \leq |V(G)|$$

$O_s(G)$ = max{k : *k* [(k _ i)Ai C iv(G)I}.

This motivates us to propose the following conjecture for which Conjecture 2.11 is a special case.

W. *Conjecture 2.24.* For any n-regular graph *G*,

$$\psi_s(G) = max\{k: k\{(k-1)/n\}\} \leq |V(G)|$$

III. CHARACTERIZATION OF PAN CRITICAL GRAPHS

Let v be a vertex of G having degree d and let n be an integer such that1 ≤ ≤ . Then an n-splitting of v is the replacement of v by a set S of n independent vertices such that N(S) = N({v}), degree of ≥ 1 for all ∈ and $\sum_{u \in S} \deg u = d$, where N(S) denotes the set of all neighbors of the vertices in S.

Clearly, the complete graph on k vertices and the graph consisting of $\binom{}{2}$disjoint copies of $_2$are k-edge critical. The following theorem shows that any k-edge critical graph can be obtained from these graphs by simple operations, which preserve edge criticality.

A. *Theorem 3.1. The following statements are equivalent:*

G is a k-edge critical graph.

G can be obtained from Kkby a sequence of n-splitting operations.

Let H be the graph consisting of disjoint copies of K2 and let C bea k-pseudo-complete coloring of H. Then G can be obtained from H by a sequence of identifications of vertices of same color.

1) *Proof.* Let G be a k-edge critical graph and let $c_1, c_2 \dots, c_k$be the colors used in a k-pseudo-complete coloring of G. Let $d_{i1}, d_{i2}, \dots d_{im_i}$denote the degreesof the vertices of G with colors $c_i, 1 \leq i \leq k$. Clearly $\sum_{j=1}^{m_i} d_{ij=1}$. Now, let$V(K_k) = \{v_1, v_2, \dots v_k\}$. Then G can be obtained from Kk by performing an mi-splitting operation on vi, for each i, $1 \leq i \leq k$, such that the mj vertices which replace vi have degrees $d_{i1}, d_{i2}, \dots d_{im_i}$. Hence (1) implies (2). Also, since an n-splitting operation preserves the edge criticality and Kkis k-edge critical, (2) implies (1).

If G is k-edge critical, G is k-pseudo-complete colorable and G has exactly $\binom{k}{2}$edges. Now H can be obtained from G by recursively performing 2-splitting operations and each of these operations can be reversed by an identification of a pair of vertices of same color. Hence (1) implies (3). Also, since an identification of vertices of same color preserves the edge criticality and H is k-edge critical, (3) implies (1).

B. Corollary 3.2. A graph G is 2-edge critical if and only if G = K2.

C. Corollary 3.3. A graph G is 3-edge critical if and only if G = K3, P4, 3K2 or K2UK1,2

D. Corollary 3.4. A necessary condition for a cycle,$C_n$with a chord, to be kedge critical is that k is odd and $n = \binom{k}{2} - 1$

If a cycle C_n with a chord is k-edge critical, then the path P_{n+2} is also k-edge critical, because P_{n+2}can be obtained from $C_n$with a

International Journal for Research in Applied Science & Engineering Technology (IJRASET)
ISSN: 2321-9653; IC Value: 45.98; SJ Impact Factor :6.887
Volume 6 Issue I, January 2018- Available at www.ijraset.com

chord by a 2-splitting operation at each of the end vertices of the chord. So, by Corollary 2.22, k is odd and $n = \binom{k}{2} - 1$

E. Corollary 3.5. A necessary condition for a θ-graph θ(a,b,c) consisting of 3 internally disjoint paths of lengths a, b and c where $2 \le a \le b \le c$, to be
k-edge critical is that k is odd and a + b + c =$\binom{k}{2}$
The following theorem shows that there is no forbidden-subgraph characterization of any of the three types of critical graphs.

F. *Theorem 3.6***.** Any graph G is an induced subgraph of some connected edge critical graph.

1) *Proof. Let* V(G) = {v1, v2, , vn}. For each i, $1 \le i \le n$, such that degreeof vi is not equal to n—1, add a new vertex ui and join ui to all the vertices $v_j$with j >i and is not adjacent to vi. Let G' denote the resulting graph. Clearly G' is n-edge critical. Also G' is connected, unless G is the disjoint union of two complete graphs. If G' is disconnected, we add a new vertex u0 to G' and join u0 to all the vertices of G. The resulting graph is connected and (n+1)-edge critical.

Now, we investigate the local criticality concepts such as critical vertices, critical edges and non-contractible edges.

2) *Local Criticality*

Now, we investigate the local criticality concepts such as critical vertices, critical edges and non-contractible edges of a graph.

G. *Definition 3.7.* Let G be a graph. A vertex v E V(G) is called a critical vertex of G. if 7/),(C — v) <V),(G). An edge e EE(G) is called a critical edge of G, if O_s(G e) < i./),(G). An edge e eE(G) is called a non-contractible edge of G, if 0_5(G1ie) < 0,(G).

H. *Remark 3.8.* We observe that e = uv is a critical edge of a graph *G* with 1/),(G) = *k* if and only if in any k-pseudo-complete coloring of *G*, there exist two colors c1 and c2 such that u and v are the only adjacent vertices, having the colors c_1 and c2 respectively. Hence the end vertices of a critical edge are critical. However, critical vertices need not be the end vertices of critical edges. For example, in the graph P11 ± K1, the unique vertex of degree 11 is critical, but none of the edges incident at it is critical.

I. *Remark 3.9.* Since *G—e* can be obtained from G||e, by a 2-splitting operation, any critical edge is non-contractible. Again, a non-contractible edge need not be critical. For example in the graph P_H any edge incident at the unique vertex of degree 11 is non-contractible, but is not critical.

J. *Remark 3.10.* We observe that an edge e ={u, v} of Gis non-contractible if and only if in any k-pseudo-complete coloring of *G*, u and v have distinct colors where $k = \psi_s(G)$. Hence, the end vertices of a non-contractible edge arecritical. Again, a critical vertex of *G* need not be an end vertex of some non-contractible edge of *G*. For example, every vertex of the cycle C_4 is critical, but none of its edges is non-contractible. The next proposition gives a condition for a critical vertex to be an end vertex of some non-contractible edge.

K. *Proposition 3.11.* Let G be a graph on p vertices and let v be a vertex of G with degree p —1. Then, any vertex $u \ne v$ is critical if and only if the edge e = uvis non-contractible.

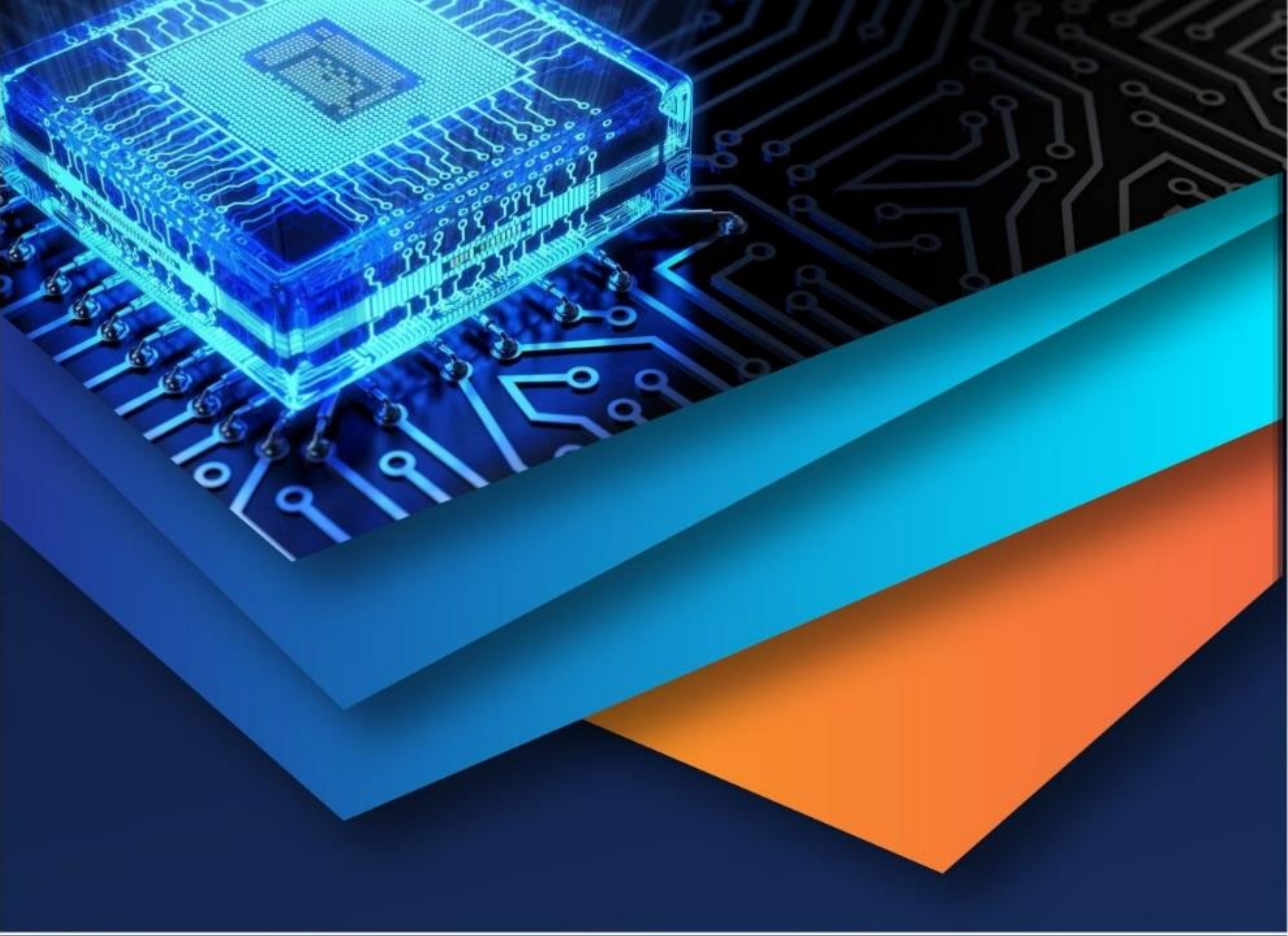
1) *Proof.*If a graph G has p vertices and a vertex v of G has degree p-1, then clearly v is a critical vertex of G. Suppose that a vertex u of G is critical, $u \ne v$ and e = uv is not non-contractible. Then, $\psi_s(G) - 1 \le \psi_s(G - \{u, v\}) \le \psi_s(G - v) \le \psi_s(G) - 1$so that $\psi_s(G - \{u, v\}) = \psi_s(G - v)$ Thus, u is not a critical vertex of G—v and hence u is not a critical vertex of G, a contradiction. Converse follows from Remark 3.10.

L. *Corollary 3.12.* If G is k-con-critical, then G + is (k + n)-con-critical.

*M. Remark 3.13.*If G is a k-vertex critical graph, then $G+K_{ri}$is (k+n)-vertex critical. However, if G is k-edge critical, then $G+K_{ri}$ need not be edge critical.

REFERENCES

[1] G. A. Dirac, Some theorems on abstract graphs, Proc. London Math. Soc. 2 (3) (1952), 69-81.
[2] G. A. Dirac, A property of 4-chromatic graphs and some remarks on critical graphs, J. London Math. Soc. 27 (1952), 85-92.
[3] F. Harary, S. Hedetniemi and G. Prins, An Interpolation theorem for graphical homomorphisms, Portugal Math. 26 (1967), 453-462.
[4] F. Harary and S. Hedetniemi, Achromatic number of a graph, J. Comb. Theory 8 (1970), 154--161.
[5] R. P. Gupta, Bounds on the chromatic and achromatic numbers of complementary graphs, Recent Progress in Combinatorics, Ed. W. T. Tutte, Academic Press, New York (1969).
[6] E. Sampathkumar and V. N. Bhave, Partition graphs and coloring numbers of a graph, Discrete Math. 16 (1976), 57-60.
[7] J.Suresh Kumar, On Pseudo-complete colorings and Graphoidal colorings of a graph, Ph.D Thesis, Submitted to M.S. University, Thirunelveli, Tamilnadu, India (June, 2002)
[8] Frank Harary, Graph Theory, Reading Mass, 1969.



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