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# Application of DOE and ANOVA on Nozzle

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***Abstract: Design of experiments (DOE) is a systematic, rigorous approach to engineering problem-solving that applies principles and techniques at the data collection stage so as to ensure the generation of valid, defensible, and supportable engineering conclusions. In addition, all of this is carried out under the constraint of a minimal expenditure of engineering runs, time, and money. This branch of applied statistics deals with planning, conducting, analyzing and interpreting controlled tests to evaluate the factors that control the value of a parameter or group of parameters. Here we apply this method of evaluation of factors in the design of a nozzle. A nozzle is simply known as a tube the control the fluid flow. It plays a major role in the performance. Where (ANOVA) analysis of variance is a collection of statistical models used to analyze the differences between and among groups. After we obtain the values of some specific design parameters, the variation between the alloy nozzle and composite nozzle is determined and confidence interval is established using ANOVA.***
***Keywords: ANOVA, Nozzle, Design of Experiments.***

## I. INTRODUCTION

### *A. Nozzle*

A de Laval nozzle (or convergent-divergent nozzle, CD nozzle or con-di nozzle) is a tube that is pinched in the middle, making an hourglass-shape. It is used as a means of accelerating the flow of a gas passing through it to a supersonic speed.

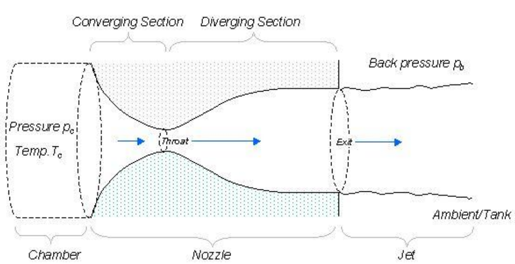



Fig 1.1 Convergent-divergent nozzle configuration

It is widely used in some types of steam turbine and is an essential part of the modern rocket engine and supersonic jet engines. Similar flow properties have been applied to jet streams within astrophysics. In all probability before 1897 no one knew for certain that there are two regimes of flow for gases and vapors and only writers of science fiction thought in terms of travelling at speeds of thousands of miles an hour. The low-speed regime is common in nature but it might be due to steady high-speed flow of gases does not exist in nature. High-speed flow came about as a consequence of man’s ingenuity. It is likely that the first person to produce a steady supersonic flow was Gustaf de Laval (1845-1913) who developed, in 1897, a convergent-divergent nozzle for a small steam turbine for use in a creamery. His invention paved the way steam powered turbo-alternators, turbine powered flight and rocketry. It is important. This principle was used in a rocket engine by Robert Goddard, and very nearly all modern rocket engines that employ hot gas combustion use de Laval nozzles.

### *B. Operation*

Its operation relies on the different properties of gases flowing at subsonic and supersonic speeds. The speed of a subsonic flow of gas will increase if the pipe carrying it narrows because the mass flow rate is constant. The gas flow through a de Laval nozzle is isentropic (gas entropy is nearly constant). At subsonic flow the gas is compressible; sound, a small pressure wave, will propagate through it. At the "throat", where the cross sectional area is a minimum, the gas velocity locally becomes sonic (Mach number = 1.0), a condition called choked flow. As the nozzle cross sectional area increases the gas begins to expand and the gas flow increases to supersonic velocities where a sound wave will not propagate backwards through the gas as viewed in the frame of reference of the nozzle (Mach number > 1.0).

### *C. Advanced designs*

A number of more sophisticated designs have been proposed for altitude compensation and other uses.

Nozzles with an atmospheric boundary include:

*1)* expansion-deflection nozzle,[10]
*2)* plug nozzle,
*3)* aerospike,[10][11]
*4)* Single Expansion Ramp Nozzle (SERN), a linear expansion nozzle, where the gas pressure transfers work only on one side and which could be described as a single-sided aerospike nozzle. Each of these allows the supersonic flow to adapt to the ambient pressure by expanding or contracting, thereby changing the exit ratio so that it is at (or near) optimal exit pressure for the corresponding altitude. The plug and aerospike nozzles are very similar in that they are radial in-flow designs but plug nozzles feature a solid center body (sometimes truncated) and aerospike nozzles have a "base-bleed" of gases to simulate a solid center-body. ED nozzles are radial out-flow nozzles with the flow deflected by a center pintle.
*5)* *Controlled flow-separation nozzles include:*
*a)* expanding nozzle,
*b)* bell nozzles with a removable insert,
*c)* Stepped nozzles, or dual-bell nozzles.[12]

These are generally very similar to bell nozzles but include an insert or mechanism by which the exit area ratio can be increased as ambient pressure is reduced.

*6)* *Dual-mode nozzles include:*
*a)* Dual – expander nozzle,
*b)* Dual – throat nozzle.

These have either two throats or two thrust chambers (with corresponding throats). The central throat is of a standard design and is surrounded by an annular throat, which exhausts gases from the same (dual-throat) or a separate (dual-expander) thrust chamber. Both throats would, in either case, discharge into a bell nozzle. At higher altitudes, where the ambient pressure is lower, the central nozzle would be shut off, reducing the throat area and thereby increasing the nozzle area ratio. These designs require additional complexity, but an advantage of having two thrust chambers is that they can be configured to burn different propellants or different fuel mixture ratios. Similarly, Aerojet has also designed a nozzle called the "Thrust Augmented Nozzle", which injects propellant and oxidiser directly into the nozzle section for combustion, allowing larger area ratio nozzles to be used deeper in an atmosphere than they would without augmentation due to effects of flow separation. They would again allow multiple propellants to be used (such as RP-1), further increasing thrust. Liquid injection thrust vectoring nozzles are another advanced design that allow pitch and yaw control from un-gimbaled nozzles. India's PSLV calls its design "Secondary Injection Thrust Vector Control System"; strontium perchlorate is injected through various fluid paths in the nozzle to achieve the desired control. Some ICBMs and boosters, such as the Titan IIIC and Minuteman II, use similar designs.

### *D. Materials Used*

In order to withstand the higher temperatures of exhaust gases, the nozzle must be manufactured with such type of materials which can tolerate the variation in operating conditions.

Commonly used materials in the manufacturing of nozzles are:

*1)* Nickel-based alloy
*2)* Titanium alloy
*3)* Ceramic matrix composite

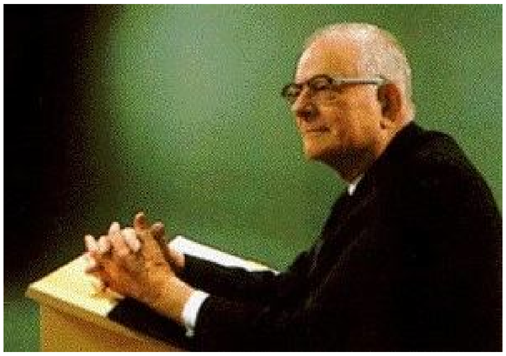
Here we picked out two conditions. One on alloy based nozzle (Ti- 4Al- 6Vi) and one on composite based nozzle. (CMC)

After the survey, we have identified certain design parameters.

## II. LITERATURE SURVEY

The de Laval nozzle was originally developed in the 19th century by Gustaf de Laval for use in steam turbines. It was first used in an early rocket engine developed by Robert Goddard, one of the fathers of modern rocketry. It has since been used in almost all rocket engines, including Walter The ill's implementation, which made possible Germany's V-2 rocket. Sir Ronald Fisher developed in the UK in the first half of the 20th century. He really laid the foundation for statistics and for design of experiments. He and his colleague Frank Yates developed many of the concepts and procedures that we use today. Basic concepts such as orthogonal designs and Latin squares began there in the 20's through the 40's. World War II also had an impact on statistics, inspiring sequential analysis, which arose from World War II as a method to improve the accuracy of long-range artillery guns. Immediately following World War II the first industrial era marked another resurgence in the use of DOE. It was at this time that Box and Wilson (1951) wrote the key paper in response surface designs thinking of the output as a response function and trying to find the optimum conditions for this function. George Box died early in 2013. And, an interesting fact here - he married Fisher's daughter. He worked in the chemical industry in England in his early career and then came to America and worked at the University of Wisconsin for most of his career.

The Second Industrial Era - or the Quality Revolution



*A. W. Edwards Deming*

The importance of statistical quality control was taken to Japan in the 1950's by W Edward Deming. This started what Montgomery calls a second Industrial Era, and sometimes the quality revolution. After the Second World War, Japanese products were of terrible quality. They were cheaply made and not very good. In the 1960s their quality started improving. The Japanese car industry adopted statistical quality control procedures and conducted experiments which started this new era. Total Quality Management (TQM), Continuous Quality Improvement (CQI) are management techniques that have come out of this statistical quality revolution - statistical quality control and design of experiments. Taguchi, a Japanese engineer, discovered and published a lot of the techniques that were later brought to the West, using an independent development of what he referred to as orthogonal arrays. In the West these were referred to as fractional factorial designs. These are both very similar and we will discuss both of these in this course. He came up with the concept of robust parameter design and process robustness.

*B. The Modern Era*

Around 1990 Six Sigma, a new way of representing CQI, became popular. Now it is a company and they employ a technique which has been adopted by many of the large manufacturing companies. This is a technique that uses statistics to make decisions based on quality and feedback loops. It incorporates a lot of the previous statistical and management techniques.

*C. Clinical Trials*

Montgomery omits in this brief history a major part of design of experimentation that evolved - clinical trials. This evolved in the 1960's when medical advances were previously based on anecdotal data; a doctor would examine six patients and from this wrote a paper and published it. The incredible biases resulting from these kinds of anecdotal studies became known. The outcome was a move toward making the randomized double-blind clinical trial the gold standard for approval of any new product, medical device, or procedure. The scientific application of the statistical procedures became very important.

*D. Brief History Behind Anova*

First used by behavioral scientists in the 1930s, use of Analysis of Variance (ANOVA) grew quickly after World War II, and exactly paralleled incorporation of statistical significance testing in general. Detailed consideration of this history suggests several reasons for the pattern of slow growth, followed by rapid institutionalization. In particular, ANOVA stood duty as a warrant of scientific legitimacy among behavioral scientists, a fact that may also be relevant to understanding recent critiques by psychologists of its overuse and misuse.

## III. PARAMETER SELECTION

Before conducting the experiment, the knowledge of the product/process under investigation is of prime importance for identifying the factors likely to influence the outcome. Design specifications of this project are:

*A.* Selection of parameters
*B.* Selection of Orthogonal array
*C.* Establishing the confidence intervals up to 95% using ANOVA
*D.* Design parameters that are selected in this project:
E. Mach number
F. Inlet diameter
G. Throat-to-nozzle diameter
H. Output pressure

Selection of Orthogonal arrayThe array can be selected using a table called Array selector.

| Number of Levels | Number of Parameters (P) | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 31 |
| 2 | L4 | L4 | L8 | L8 | L8 | L8 | L12 | L12 | L12 | L12 | L16 | L16 | L16 | L16 | L32 | L32 | L32 | L32 | L32 | L32 | L32 | L32 | L32 | L32 | L32 | L32 | L32 | L32 | L32 | L32 |
| 3 | L9 | L9 | L9 | L18 | L18 | L18 | L18 | L27 | L27 | L27 | L27 | L27 | L36 | L36 | L36 | L36 | L36 | L36 | L36 | L36 | L36 | L36 | | | | | | | | |
| 4 | L'16 | L'16 | L'16 | L'16 | L'32 | L'32 | L'32 | L'32 | L'32 | | | | | | | | | | | | | | | | | | | | | |
| 5 | L25 | L25 | L25 | L25 | L25 | L50 | L50 | L50 | L50 | L50 | L50 | | | | | | | | | | | | | | | | | | | |

Fig 3.1 Array selector

The concept of orthogonal array is discussed in further chapter. Experimental values recorded for alloy and composite nozzles.

## IV.ORTHOGONAL ARRAY

### *A. Background*

The technique of laying out the conditions of experiments involving multiple factors was first proposed by the Englishman, Sir R. A. Fisher. The method is popularly known as the factorial design of experiments. A full factorial design will identify all possible combinations for a given set of factors. Since most industrial experiments usually involve a significant number of factors, a full factorial design results in a large number of experiments. To reduce the number of experiments to a practical level, only a small set from all the possibilities is selected. The method of selecting a limited number of experiments which produces the most information is known as a partial fraction experiment. Although this method is well known, there are no general guidelines for its application or the analysis of the results obtained by performing the experiments. Taguchi constructed a special set of general design guidelines for factorial experiments that cover many applications.

### *B. Basic Concepts*

1) *Definition:* Taguchi has envisaged a new method of conducting the design of experiments which are based on well defined guidelines. This method uses a special set of arrays called orthogonal arrays. These standard arrays stipulate the way of conducting the minimal number of experiments which could give the full information of all the factors that affect the performance parameter. The crux of the orthogonal arrays method lies in choosing the level combinations of the input design variables for each experiment While there are many standard orthogonal arrays available, each of the arrays is meant for a specific number of independent design variables and levels. For example, if one wants to conduct an experiment to understand the influence of 4 different independent variables with each variable having 3 set values (level values), then an L9 orthogonal array might be the right choice.

The L9 orthogonal array is meant for understanding the effect of 4 independent factors each having 3 factor level values. This array assumes that there is no interaction between any two factors. While in many cases, no interaction model assumption is valid, there are some cases where there is a clear evidence of interaction. A typical case of interaction would be the interaction between the material properties and temperature.

### *C. Properties Of An Orthogonal Array*

The orthogonal array has the following special properties that reduce the number of experiments to be conducted.

1) The vertical column under each independent variables of the above table has a special combination of level settings. All the level settings appear an equal number of times. For L9 array under variable 4, level 1, level 2 and level 3 appears thrice. This is called the balancing property of orthogonal arrays.
2) All the level values of independent variables are used for conducting the experiments.
3) The sequence of level values for conducting the experiments shall not be changed. This means one cannot conduct experiment 1 with variable 1, level 2 setup and experiment 4 with variable 1, level 1 setup. The reason for this is that the array of each factor columns is mutually orthogonal to any other column of level values. The inner product of vectors corresponding to weights is zero. If the above 3 levels are normalized between -1 and 1, then the weighing factors for level 1, level 2 , level 3 are -1 , 0 , 1 respectively. Hence the inner product of weighing factors of independent variable 1 and independent variable 3 would be

$$(-1 * -1+-1*0+-1*1) + (0*0+0*1+0*-1) + (1*0+1*1+1*-1) = 0$$

4) *Minimum Number of Experiments To Be Conducted.:* The design of experiments using the orthogonal array is, in most cases, efficient when compared to many other statistical designs. The minimum number of experiments that are required to conduct the Taguchi method can be calculated based on the degrees of freedom approach.

$$N_{Taguchi} = 1 + \sum_{i=1}^{NV} (L_i - 1)$$

For example, in case of 8 independent variables study having 1 independent variable with 2 levels and remaining 7 independent variables with 3 levels (L18 orthogonal array), the minimum number of experiments required based on the above equation is 16. Because of the balancing property of the orthogonal arrays, the total number of experiments shall be multiple of 2 and 3. Hence the number of experiments for the above case is 18.

### *D. Array*

An L9 array can be used to covers all combinations of two parameters at three levels each. Alternatively, if we are willing to ignore the interactions between parameters (or assume that they are small), an L9 can be used as a 'saturated' array to measure four parameters at three levels each.

| Experiment | P1 | P2 | P3 | P4 |
|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 |
| 2 | 1 | 2 | 2 | 2 |
| 3 | 1 | 3 | 3 | 3 |
| 4 | 2 | 1 | 2 | 3 |
| 5 | 2 | 2 | 3 | 1 |
| 6 | 2 | 3 | 1 | 2 |
| 7 | 3 | 1 | 3 | 2 |
| 8 | 3 | 2 | 1 | 3 |
| 9 | 3 | 3 | 2 | 1 |

Fig 4.1 L9 array

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*E. Purpose*

The main purpose of the L9 tests was to expand our coverage of the parameter space. While the approach was very efficient compared to the full factorial approach (each of which would have required 81 tests), the penalty was a loss of any information on the interactions between factors, and a loss of the 'free' noise estimate.

## V. DESIGN OF EXPERIMENTS

A designed experiment is a test or series of tests in which purposeful changes are made to the input variables of a process or system so that we may observe and identify the reasons for changes in the output response. The process parameters x1, x2, x3,xp are controllable, whereas other variables z1, z2, z3, …,zq.

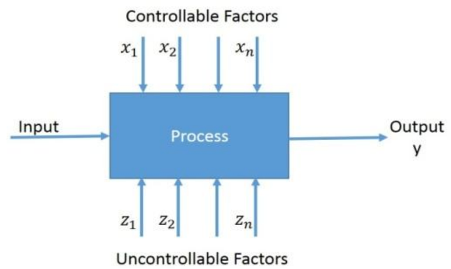


Fig. 2 General model of a process or system

The term y refers to the output variable.

*A. Objectives*

1) Determining which variables are most influential on the response, y.
2) Determining where to set the influential x's so that y is almost always near the desired nominal value.
3) Determining where to set the influential x's so that variability in y is small.
4) Determining where to set the influential x's so that the effects of the uncontrollable z1, z2 …zq are minimized.
5) optimization of manufacturing processes
6) optimization of analytical instrument
7) screening and identification of important factors

*B. Guidelines*

1) Recognition and statement of the problem.
2) Choice of factors and levels.
3) Selection of the response variable.
4) Choice of experimental design.
5) Performing the experiment.
6) Data analysis.
7) Conclusions and recommendations.

*C. Principles and Applications*

1) robustness testing of methods
2) robustness testing of products
3) formulation experiments

*D. The design of an experiment involves the following steps*

1) Selection of independent variables
2) Selection of number of level settings for each independent variable
3) Selection of orthogonal array
4) Assigning the independent variables to each column
5) Conducting the experiments

6) Analyzing the data
7) Inference

*E. The details of the above steps are given below.*

1) Selection of the independent variables
2) Before conducting the experiment, the knowledge of the product/process under investigation is of prime importance for identifying the factors likely to influence the outcome. In order to compile a comprehensive list of factors, the input to the experiment is generally obtained from all the people involved in the project.
3) Deciding the number of levels
4) Once the independent variables are decided, the number of levels for each variable is decided. The selection of number of levels depends on how the performance parameter is affected due to different level settings. If the performance parameter is a linear function of the independent variable, then the number of level setting shall be 2. However, if the independent variable is not linearly related, then one could go for 3, 4 or higher levels depending on whether the relationship is quadratic, cubic or higher order.

   In the absence of exact nature of relationship between the independent variable and the performance parameter, one could choose 2 level settings. After analyzing the experimental data, one can decide whether the assumption of level setting is right or not based on the percent contribution and the error calculations.
5) Selection of an orthogonal array
6) Before selecting the orthogonal array, the minimum number of experiments to be conducted shall be fixed based on the total number of degrees of freedom present in the study. The minimum number of experiments that must be run to study the factors shall be more than the total degrees of freedom available. In counting the total degrees of freedom the investigator commits 1 degree of freedom to the overall mean of the response under study. The number of degrees of freedom associated with each factor under study equals one less than the number of levels available for that factor. Hence the total degrees of freedom without interaction effect is 1 + as already given by equation. For example, in case of 11 independent variables, each having 2 levels, the total degrees of freedom is 12. Hence the selected orthogonal array shall have at least 12 experiments. An L12 orthogonal satisfies this requirement.
7) Assigning the independent variables to columns
8) The order in which the independent variables are assigned to the vertical column is very essential. In case of mixed level variables and interaction between variables, the variables are to be assigned at right columns as stipulated by the orthogonal array.
9) Finally, before conducting the experiment, the actual level values of each design variable shall be decided. It shall be noted that the significance and the percent contribution of the independent variables changes depending on the level values assigned. It is the designer's responsibility to set proper level values.
10) Conducting the experiment
11) Once the orthogonal array is selected, the experiments are conducted as per the level combinations. It is necessary that all the experiments be conducted. The interaction columns and dummy variable columns shall not be considered for conducting the experiment, but are needed while analyzing the data to understand the interaction effect. The performance parameter under study is noted down for each experiment to conduct the sensitivity analysis.
12) Analysis of the data
13) Since each experiment is the combination of different factor levels, it is essential to segregate the individual effect of independent variables. This can be done by summing up the performance parameter values for the corresponding level settings. For example, in order to find out the main effect of level 1 setting of the independent variable 2 (refer Table 2.1), sum the performance parameter values of the experiments 1, 4 and 7. Similarly for level 2, sum the experimental results of 2, 5 and 7 and so on.
14) Once the mean value of each level of a particular independent variable is calculated, the sum of square of deviation of each of the mean value from the grand mean value is calculated. This sum of square deviation of a particular variable indicates whether the performance parameter is sensitive to the change in level setting. If the sum of square deviation is close to zero or insignificant, one may conclude that the design variables is not influencing the performance of the process. In other words, by conducting the sensitivity analysis, and performing analysis of variance (ANOVA), one can decide which independent factor dominates over other and the percentage contribution of that particular independent variable.

## VI. ANALYSIS OF VARIANCE (ANOVA)

Econometricians see it as an uninteresting special case of linear regression. Bayesians see it as an in-flexible classical method. Theoretical statisticians have supplied many mathematical definitions [see, e.g., Speed (1987)]. Instructors see it as one of the hardest topics in classical statistics to teach, especially in its more elaborate forms such as split-plot analysis. We believe, however, that the ideas of ANOVA are useful in many applications of statistics. For the purpose of this paper, we identify ANOVA with the structuring of parameters into batches—that is, with variance components models. There are more general mathematical formulations of the analysis of variance, but this is the aspect that we believe is most relevant in applied statistics, especially for regression modeling.

### *A. General Purpose of ANOVA*

These days, researchers are using ANOVA in many ways. The use of ANOVA depends on the research design. Commonly, researchers are using ANOVA in three ways: one-way ANOVA, two-way ANOVA, and N-way Multivariate ANOVA.

1) *One-Way:* When we compare more than two groups, based on one factor (independent variable), this is called one way ANOVA. For example, it is used if a manufacturing company wants to compare the productivity of three or more employees based on working hours. This is called one way ANOVA.
2) *Two-Way:* When a company wants to compare the employee productivity based on two factors (2 independent variables), then it said to be two way (Factorial) ANOVA. For example, based on the working hours and working conditions, if a company wants to compare employee productivity, it can do that through two way ANOVA. Two-way ANOVA's can be used to see the effect of one of the factors after controlling for the other, or it can be used to see the INTERACTION between the two factors. This is a great way to control for extraneous variables as you are able to add them to the design of the study.Factorial ANOVA can be balanced or unbalanced. This is to say, you can have the same number of subjects in each group (balanced) or not (unbalanced). This can come about, depending on the study, as just a reflection of the population, or an unwanted event such as participants not returning to the study. Not having equal sizes groups can make it appear that there is an effect when this may not be the case. There are several procedures a researcher can do in order to solve this problem:

### *B. Discard Cases (Undesirable)*

Conduct a special kind of ANOVA which can deal with the unbalanced design

There are three types of ANOVA's that can candle an unbalanced design. These are the Classical Experimental design (Type 2 analysis), the Hierarchical Approach (Type 1 analysis), and the Full regression approach (Type 3 analysis). Which approach to use depends on whether the unbalanced data occurred on purpose. If the data is unbalanced because this is a reflection of the population and it was intended, use the Full Regression approach (Type 3).-If the data was not intended to be unbalanced but you can argue some type of hierarchy between the factors, use the Hierarchical approach (Type 1).-If the data was not intended to be unbalanced and you cannot find any hierarchy, use the classical experimental approach (Type 2).

1) *N-Way:* When the factor comparison is taken, then it said to be n-way ANOVA. For example, in productivity measurement if a company takes all the factors for productivity measurement, then it is said to be n-way ANOVA.ANOVA is used very commonly in business, medicine or in psychology research. In business, ANOVA can be used to compare the sales of different designs based on different factors. A psychology researcher can use ANOVA to compare the different attitude or behavior in people and whether or not they are the same depending on certain factors. In medical research, ANOVA is used to test the effectiveness of a drug. One way ANOVA or simply known as Null-hypothesis, is selected in this project. Our null hypothesis is that any difference between the three flavors is due to chance.

$H_0$: μ1 = μ2 = μ3 = μ4

Formulae used in one way ANOVA:

2) *Definition 1:* Suppose we have *k* samples, which we will call groups **(or** treatments); these are the columns in our analysis (corresponding to the 3 flavors in the above example). We will use the index *j* for these. Each group consists of a sample of size $n_j$. The sample elements are the rows in the analysis. We will use the index *i* for these.

Suppose the *j*th group sample is

$\{x_{1j}, \ldots, x_{n_j j}\}$

and so the total sample consists of all the elements

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$\{x_{ij}: 1 \le i \le n_j, 1 \le j \le k\}$

We will use the abbreviation $\bar{x}_j$ for the mean of the *j*th group sample (called the group mean) and $\bar{x}$ for the mean of the total sample (called the total or grand mean).

Let the sum of squares for the *j*th group be

$$SS_j = \sum_i (x_{ij} - \bar{x}_j)^2$$

We now define the following terms:

$$SS_T = \sum_j \sum_i (x_{ij} - \bar{x})^2 \quad SS_W = \sum_j SS_j = \sum_j \sum_i (x_{ij} - \bar{x}_j)^2 \quad SS_B = \sum_j n_j (\bar{x}_j - \bar{x})^2$$

$SS_T$ is the sum of squares for the **total** sample, i.e. the sum of the squared deviations from the grand mean. $SS_W$ is the sum of squares **within** the groups, i.e. the sum of the squared means across all groups. $SS_B$ is the sum of the squares **between** group sample means, i.e. the weighted sum of the squared deviations of the group means from the grand mean.

Where

$$n = \sum_{j=1}^{k} n_j$$

we also define the following degrees of freedom

$$df_T = n - 1 \qquad df_B = k - 1 \qquad df_W = \sum_{j=1}^{k} (n_j - 1) = n - k$$

Finally we define the **mean square** as

$$MS = SS/df$$

and so

$$MS_T = SS_T / df_T \qquad MS_B = SS_B / df_B \qquad MS_W = SS_W / df_W$$

Summarizing:

| | *df* | *SS* | *MS* |
|---|---|---|---|
| ***T*** | $n-1$ | $\sum_j \sum_i (x_{ij} - \bar{x})^2$ | $SS_T/df_T$ |
| ***B*** | $k-1$ | $\sum_j n_j (\bar{x}_j - \bar{x})^2$ | $SS_B/df_B$ |
| ***W*** | $n-k$ | $\sum_j \sum_i (x_{ij} - \bar{x}_j)^2$ | $SS_W/df_W$ |

3) *Observation:* Clearly $MS_T$ is the variance for the total sample. $MS_W$ is the sum of the group sample variances. $MS_B$ is the variance for the "between sample" i.e. the variance of $\{n_1\bar{x}_1, \ldots, n_k\bar{x}_k\}$.
4) *Property 1:* If a sample is made as described in Definition 1, with the $x_{ij}$ independently and normally distributed and with all $\sigma_j^2$ equal, then

$$MS_T \sim \frac{\sigma_T^2}{df_T}\chi^2(df_T) \quad MS_B \sim \frac{\sigma_B^2}{df_B}\chi^2(df_B) \quad MS_W \sim \frac{\sigma_W^2}{df_W}\chi^2(df_W)$$

5) *Property 2:*

$$SS_T = SS_W + SS_B df_T = df_W + df_B$$

6) *Definition 2*: Using the terminology from Definition 1, we define the structural model as follows. First we estimate the group means from the total mean: $\bar{\mu}_j = \mu + a_j$ where$a_j$ denotes the effect of the *j*th group (i.e. the departure of the *j*th group mean from the total mean). We have a similar estimate for the sample of $\bar{x}_j = \bar{x} + a_j$.

The null hypothesis is now equivalent to

$H_0$: $a_j = 0$ for all $j$

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Similarly, we can represent each element in the sample as $x_{ij} = \mu + \alpha_j + \varepsilon_{ij}$ where $\varepsilon_{ij}$ denotes the error for the *i*th element in the *j*th group. As before we have the sample version $x_{ij} = \bar{x} + a_j + eij$, where $e_{ij}$ is the counterpart to $\varepsilon_{ij}$ in the sample.
Also $\varepsilon_{ij} = x_{ij} - (\mu + \alpha_{ij}) = x_{ij} - \mu_j$ and similarly, $e_{ij} = x_{ij} - \bar{x}_j$.
Observation: Since

$$\varepsilon_{ij} = x_{ij} - (\mu + \alpha_j) = x_{ij} - \mu_j$$

## VII. RESULTS AND DISCUSSIONS

In this project with help of design of experiments four parameters were opted. They are as follows:

*A.* Mach number
*B.* Inlet diameter
*C.* Ratio of inlet diameter – to – throat diameter
*D.* Pressure

After running experiments for four times, the values are recorded as shown in figure:

| | A | B | C | D |
|---|---|---|---|---|
| 1 | Di | Dt/Di | Pi (Mpa) | Mach |
| 2 | 32 | 0.1 | 20 | 1.2 |
| 3 | 40 | 0.23 | 40 | 1.4 |
| 4 | 62 | 0.314 | 60 | 1.6 |
| 5 | 80 | 0.412 | 70 | 1.8 |

Fig 7.1 Recorded values

Now we apply analysis of variance (ANOVA) on these values. We performed this experiment using MS Excel. To reduce complexity Single – factor ANOVA was taken. Therefore, the final results are as shown in figure.

Anova: Single Factor

SUMMARY

| Groups | Count | Sum | Average | Variance |
|---|---|---|---|---|
| Column 1 | 4 | 214 | 53.5 | 473 |
| Column 2 | 4 | 1.056 | 0.264 | 0.017485 |
| Column 3 | 4 | 190 | 47.5 | 491.6667 |
| Column 4 | 4 | 6 | 1.5 | 0.066667 |

ANOVA

| ce of Varia | SS | df | MS | F | P-value | F crit |
|---|---|---|---|---|---|---|
| Between G | 9922.839 | 3 | 3307.613 | 13.71385 | 0.000349 | 3.490295 |
| Within Grc | 2894.252 | 12 | 241.1877 | | | |
| Total | 12817.09 | 15 | | | | |

Fig 7.2 ANOVA results

Here, we obtained the value of F = 13. 71385 which is greater than the value of Fcrit= 3. 490295F >FcritTherefore, Null Hypothesis is rejected for this experiment.

## VIII. CONCLUSION

It is observed that the above methods are less time consuming and less economical. The ANOVA procedure performs analysis of variance (ANOVA) for balanced data from a wide variety of experimental designs. In analysis of variance, a continuous response variable, known as a dependent variable, is measured under experimental conditions identified by classification variables, known as independent variables. The variation in the response is assumed to be due to effects in the classification, with random error accounting for the remaining variation.

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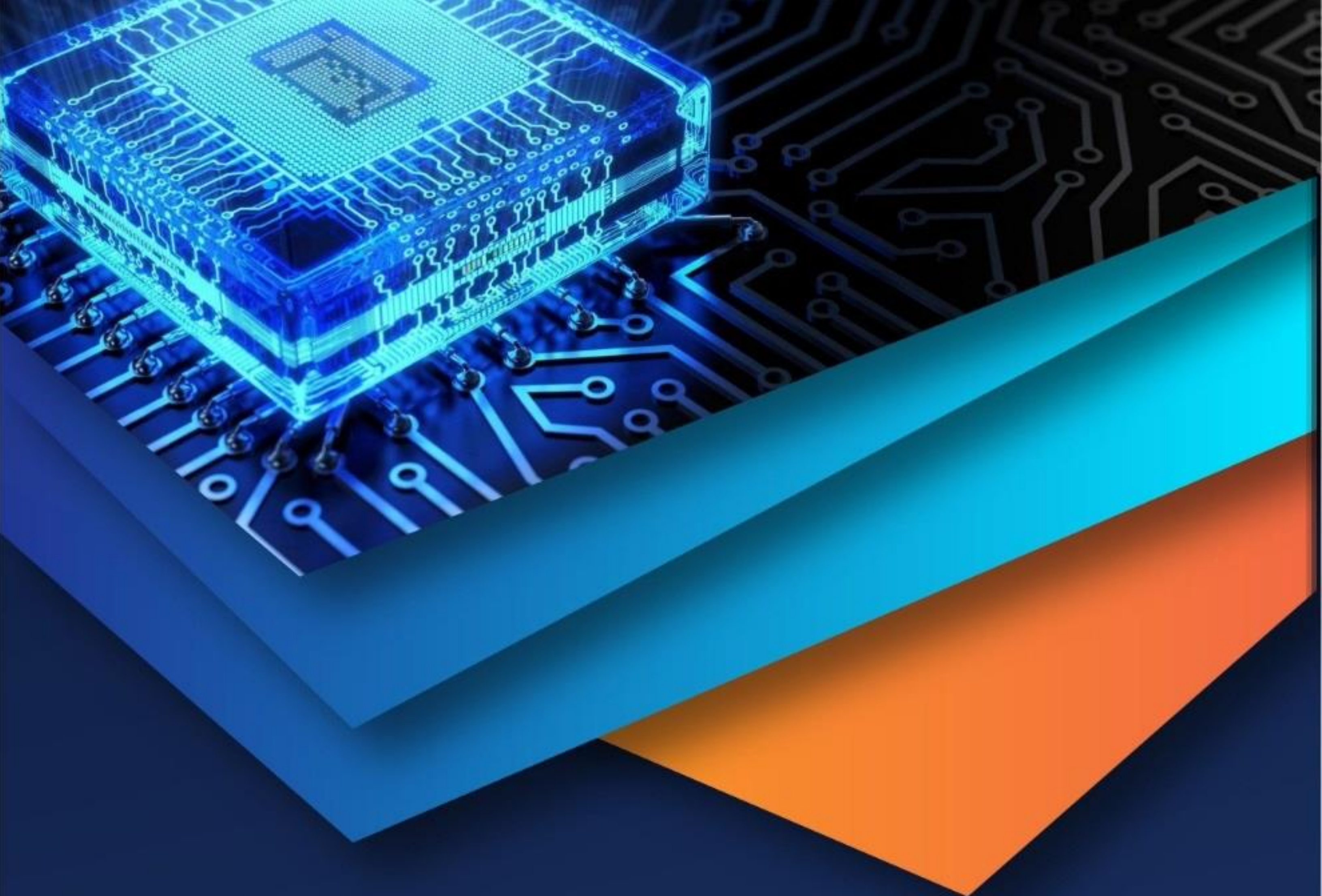


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