iJRASET
International Journal For Research in
Applied Science and Engineering Technology

INTERNATIONAL JOURNAL FOR RESEARCH

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY

Volume: 6 Issue: II Month of publication: February 2018

DOI: http://doi.org/10.22214/ijraset.2018.2096

International Journal for Research in Applied Science & Engineering Technology (IJRASET)
ISSN: 2321-9653; IC Value: 45.98; SJ Impact Factor: 6.887
Volume 6 Issue II, February 2018- Available at www.ijraset.com

Weak and Strong forms of ğ-semi-irresolute Functions

R. Nagendran[1], R. Latha[2] S.Karpagam[3]
[1, 2, 3] Department of Mathematics, Sree Sastha Institute of Engineering and Technology, Chennai-123. Tamil Nadu, India

Abstract: The purpose of this paper is to give two new types of irresolute functions called completely ğ-semi-irresolute functions and weakly ğ-semi- irresolute functions. We obtain their characterizations and basic properties. 2010 Mathematics Subject Classification: 54C10, 54C08, 54C05.
Keywords and phrases: ğ-semi-open set, ğ-semi-irresolute function, completely ğ-semi-irresolute function and weakly ğ-semi-irresolute function.

I. INTRODUCTION AND PRELIMINARIES

Functions and of course irresolute functions stand among the most important and most researched points in the whole of mathematical science. In 1972, Crossley and Hildebrand [2] introduced the notion of irresoluteness. Many different forms of irresolute functions have been introduced over the years. Various interesting problems arise when one considers irresoluteness. Its importance is significant in various areas of mathematics and related sciences. Recently, as generalization of closed sets, the notion of ğ-semi-closed sets were introduced and studied by Veerakumar [16]. In this paper, we introduce and characterize the concepts of completely ğ-semi-irresolute and weakly ğ-semi-irresolute functions.

A. Definition 1.1 A subset A of a space X is called
1) regular open if $A = \text{int}(\text{cl}A)$;
2) semi-open if $A \subseteq \text{cl}(\text{int}A)$.
The complement of regular open (resp. semi-open) is called regular closed (resp. semi-closed).

B. Definition 1.2
1) A subset A of a space X is called:
2) ĝ-closed if $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is semi-open in X;
3) *g-closed if $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is ĝ-open in X
4) #g-semi-closed if $\text{scl}(A) \subseteq U$ whenever $A \subseteq U$ and U is *g-open in X
5) ğ-semi-closed if $\text{scl}(A) \subseteq U$ whenever $A \subseteq U$ and U is #g-semi-open in X. The complement of ğ-semi-closed (resp. ĝ-closed, *g-closed and #g-semi-closed) is called ğ-semi-open(resp. ĝ-open, *g-open and #g-semi-open).

B. Definition 1.3 A function $f: X \to Y$ is called:
1) strongly continuous if $f^{-1}(V)$ is both open and closed in X for each subset V of Y;
2) completely continuous if $f^{-1}(V)$ is regular open in X for each open subset V of Y;
3) ğ-semi-irresolute if $f^{-1}(V)$ is ğ-semi-closed in X for each ğ-semi-closed subset V of Y
4) pre ğ-semi-closed if f(V) is ğ-semi-closed in Y for each ğ-semi-closed subset V of X.

II. COMPLETELY Ğ-SEMI-IRRESOLUTE FUNCTIONS

A. Definition 2.1 A function $f: X \to Y$ is called completely ğ-semi-irresolute if the inverse image of each ğ-semi-open subset of Y is regular open in X.
Clearly, every strongly continuous function is completely ğ-semi-irresolute and every completely ğ-semi-irresolute function is ğ-semi-irresolute.

B. *Remark 2.2* The converses of the above implications are not true in general as seen from the following examples.

C. *Example 2.3* Let X = Y = { a, b, c}, $\tau = \sigma = \{\phi, X = Y, \{a\}, \{b, c\}\}$. Then the identity function f: $(X,\tau) \to (Y,\sigma)$ is completely ğ-semi-irresolute but not strongly continuous.

D. *Example 2.4* Let X = Y = { a, b, c}, $\tau = \{\phi, X, \{a\}, \{a, b\}\}$ and $\sigma = \{\phi, Y, \{a\}\}$. Then the identity function f: $(X,\tau) \to (Y,\sigma)$ is ğ-semi-irresolute but not completely ğ-semi-irresolute.

E. *Theorem 2.5* The following statements are equivalent for a function f: X → Y.
1) f is completely ğ-semi-irresolute.
2) f: $(X,\tau) \to$ (Y, ĞSO(X)) is completely continuous.
3) $f^{-1}(F)$ is regular closed in X for each ğ-semi-closed set F of Y.
4) Proof⇔ (ii) : It follows from the definitions.

⇒ (iii) : Let F be any ğ-semi-closed set of Y. Then Y\F ∈ ĞSO(Y). By (i), $f^{-1}(Y\backslash F) = X\backslash f^{-1}(F)$ ∈ RO(X). We have $f^{-1}(F)$ ∈ RC(X). The converse is similar.

F. *Definition 2.6* A space X is said to be almost connected (resp. ğ-semi-connected) if there does not exist disjoint regular open (resp. ğ-semi-open) sets A and B such that A U B = X.

G. *Theorem 2.7* If f: X → Y is completely ğ-semi-irresolute surjective function and X is almost connected, then Y is ğ-semi-connected.
1) *Proof* Suppose that Y is not ğ-semi-connected. Then there exist disjoint ğ-semi-open sets A and B of Y such that A U B = X. Since f is completely ğ-semi-irresolute surjective, $f^{-1}(A)$ and $f^{-1}(B)$ are regular open sets in X. Moreover, $f^{-1}(A)$ U $f^{-1}(B)$ = X, $f^{-1}(A) \neq \phi$ and $f^{-1}(B) \neq \phi$. This shows that X is not almost connected, which is a contradiction to the assumption that X is almost connected. By contradiction, Y is ğ-semi-connected.

H. *Definition 2.8* A space (X,τ) is said to be ğ-semi-T_1 (resp. r-T_1) if for each pair of distinct points x and y of X, there exist ğ-semi-open (resp. regular open) sets U1 and U2 such that $x \in U_1$ and $y \in U2$, $x \notin U2$ and $y \notin U1$.

I. *Theorem 2.9* If f: $(X,\tau) \to (Y,\sigma)$ is completely ğ-semi-irresolute injective function and Y is ğ-semi-T_1, then X is r-T_1.
1) *Proof* Suppose that Y is ğ-semi-T_1. For any two distinct points x and y of X, there exist ğ-semi-open sets F_1 and F2 in Y such that $f(x) \in F_1$, $f(y) \in F2$, $f(x) \notin F2$ and $f(y) \notin F_1$. Since f is injective completely ğ-semi-irresolute function, we have X is r-T_1.

J. *Definition 2.10* space (X,τ) is said to be ğ-semi-T_2 if for each pair of distinct points x and y in X, there exist disjoint ğ-semi-open sets A and B in X such that $x \in A$ and $y \in B$ and $A \cap B = \phi$.

III. WEAKLY Ğ-SEMI-IRRESOLUTE FUNCTIONS

A. *Definition 3.1* A function f: X → Y is said to be weakly ğ-semi-irresolute if for each point $x \in X$ and each V ∈ ĞSO(Y, f(x)), there exists a U ∈ ĞSO(X, x) such that $f(U) \subset$ ğ-semi-cl(V).

It is evident that every ğ-semi-irresolute function is weakly ğ-semi-irresolute but the converse is not true.

B. *Example 3.2* Let X = Y = { a, b, c}, $\tau = \{\phi, X, \{a, b\}\}$ and $\sigma = \{\phi, Y, \{a\}, \{a, b\}\}$. Then the identity function f: $(X,\tau) \to (Y,\sigma)$ is weakly ğ-semi-irresolute but not ğ-semi-irresolute.

C. *Theorem 3.3* For a function f: X → Y, the following statements are equivalent:
1) f is weakly ğ-semi-irresolute.
2) $f^{-1}(V) \subset$ ğ-semi-int (f^{-1}(ğ-semi-cl(V))) for every V ∈ ĞSO(Y).

3) ğ-semi-cl(f^{-1}(V)) $\subset$ f^{-1}(ğ-semi-cl(V)) for every V $\in$ ĞSO(Y).

(i) $\Rightarrow$ (ii) : Suppose that V $\in$ ĞSO(Y) and let x $\in$ f^{-1}(V). It follows from (i) that f(U) $\subset$ ğ-semi-cl(V) for some U $\in$ ĞSO(X, x). Therefore, we have U $\subset$ f^{-1}(ğ-semi-cl(V)) and x $\in$ U $\subset$ ğ-semi-int (f^{-1}(ğ-semi-cl(V))). This shows that f^{-1}(V) $\subset$ ğ-semi-int (f^{-1}(ğ-semi-cl(V))).

(ii) $\Rightarrow$ (iii) : Suppose that V $\in$ ĞSO(Y) and x $\notin$ f^{-1}(ğ-semi-cl(V)). Then f(x) $\notin$ ğ-semi-cl(V). There exists G $\in$ ĞSO(Y, f(x)) such that G $\cap$ V= ϕ. Since V $\in$ ĞSO(Y) , we have ğ-semi-cl(G) $\cap$ V = ϕ and hence ğ-semi-int (f^{-1}(ğ-semi-cl(G))) $\cap$ f^{-1}(V) = ϕ. By (ii), we have x $\in$ f^{-1}(G) $\subset$ ğ-semi-int (f^{-1}(ğ-semi-cl(G))) $\in$ ĞSO(X). Therefore, we obtain x $\notin$ ğ-semi-cl(f^{-1}(V)). This shows that ğ-semi-cl(f^{-1}(V)) $\subset$ f^{-1}(ğ-semi-cl(V)).

4) $\Rightarrow$ (i): Let x $\in$ X and V $\in$ ĞSO(Y, f(x)). Then x $\notin$ f^{-1}(ğ-semi-cl(Y \ ğ-semi-cl(V))). Since

5) Y \ ğ-semi-cl(V) $\in$ ĞSO(Y), by (iii), we have x $\notin$ ğ-semi-cl(f^{-1}(Y \ ğ-semi-cl(V))). Hence there exists U $\in$ ĞSO(X, x) such that U $\cap$ f^{-1}(Y \ ğ-semi-cl(V)) = ϕ. Therefore, we obtain f(U) $\cap$ (Y \ ğ-semi-cl(V)) = ϕ and hence f(U) $\subset$ ğ-semi-cl(V).This shows that f is weakly ğ-semi-irresolute.

D. *Theorem 3.4* A space X is ğ-semi-T_2 if and only if for any pair of distinct points x, y of X there exist ğ-semi-open sets U and V such that x $\in$ U and y $\in$ V and ğ-semi-cl(U) $\cap$ ğ-semi-cl(V) = ϕ.

E. *Theorem 3.5* If Y is a ğ-semi-T_2 space and f: X $\rightarrow$ Y is a weakly ğ-semi-irresolute injection, then X is ğ-semi-T_2.

1) *Proof* Let x and y be any two distinct points of X. Since f is injective , we have f(x) $\neq$ f(y). Since Y is ğ-semi-T_2, by Theorem 3.4 there exist V $\in$ ĞSO(Y, f(x)) and W $\in$ ĞSO(Y, f(y)) such that ğ-semi-cl(V) $\cap$ ğ-semi-cl(W) = ϕ. Since f is weakly ğ-semi-irresolute, there exist G $\in$ ĞSO(X, x) and H $\in$ ĞSO(X, y) such that f(G) $\subset$ ğ-semi-cl(V) and f(H) $\subset$ ğ-semi-cl(W). Hence we obtain G $\cap$ H = ϕ. This shows that X is ğ-semi-T_2.

F. *Definition 3.6* A function f: X $\rightarrow$ Y is said to have a strongly ğ-semi-closed graph if for each (x, y) $\in$ (X x Y)\G(f), there exist U $\in$ ĞSO(X, x) and V $\in$ĞSO(Y, y) such that (ğ-semi-cl(U) x ğ- semi-cl(V)) $\cap$ G(f) = ϕ.

G. *Theorem 3.7* If Y is a ğ-semi-T_2 space and f: X $\rightarrow$ Y is weakly ğ-semi-irresolute, then G(f) is strongly ğ-semi-closed.Let (x, y) $\in$ (X x Y)\G(f). Then y $\neq$ f(x) and by Theorem 3.4 there exist V $\in$ ĞSO(Y, f(x)) and W $\in$ ĞSO(Y, y) such that ğ-semi-cl(V) $\cap$ ğ-semi-cl(W) = ϕ. Since f is weakly ğ-semi-irresolute, there exists U $\in$ ĞSO(X, x) such that f(ğ-semi-cl(U)) $\subset$ ğ-semi-cl(V). Therefore, we obtain f(ğ-semi-cl(U)) $\cap$ ğ-semi-cl(W) = ϕ and hence (ğ-semi-cl(U) x ğ-semi-cl(W)) $\cap$ G(f) = ϕ. This shows that G(f) is strongly ğ-semi-closed in X x Y.

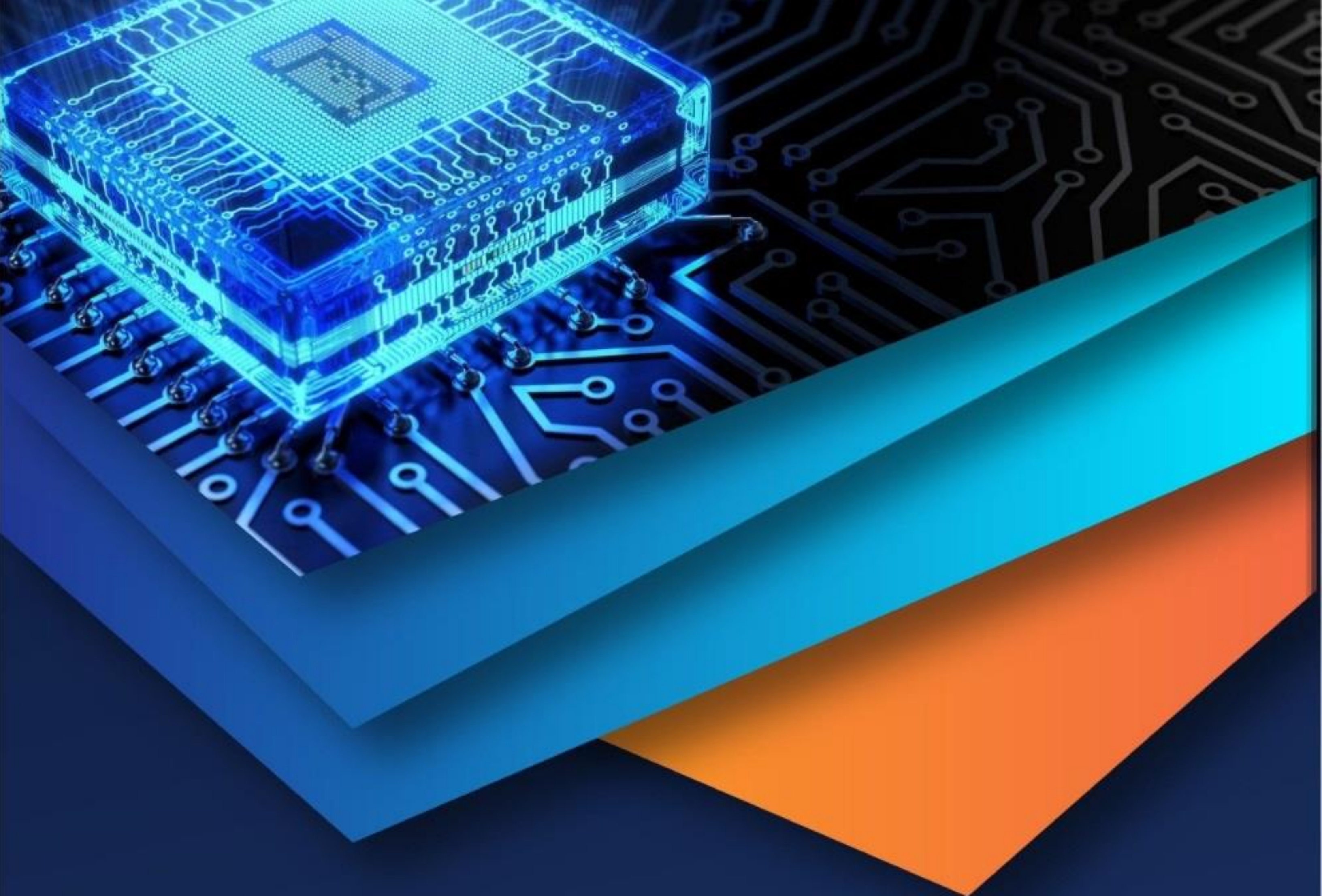
H. *Theorem 3.8* If a function f: X $\rightarrow$ Y is weakly ğ-semi-irresolute, injective and G(f) is strongly ğ-semi-closed, then X is ğ-semi-T_2.

1) *Proof* Let x and y be a pair of distinct points of X. Since f is injective, f(x) $\neq$ f(y) and (x, f(y)) $\notin$ G(f). Since G(f) is strongly ğ-semi-closed, there exist G $\in$ ĞSO(X, x) and V $\in$ ĞSO(Y, f(y)) such that f(ğ-semi-cl(G)) $\cap$ ğ-semi-cl(V) = ϕ. Since f is weakly ğ-semi-irresolute, there exists H $\in$ ĞSO(X, y) such that f(H) $\subset$ ğ-semi-cl(V). Hence we have f(ğ-semi-cl(G)) $\cap$ f(H) = ϕ; hence G $\cap$ H = ϕ. This shows that X is ğ-semi-T_2.

REFERENCES

[1] S.P. Arya and R. Gupta, On strongly continuous maps, Kyungpook Math. J. 14 (1974), 131-143.
[2] S.G. Crossley and S.K. Hildebrand, Semi-Topological properties, Fund. Math. 74 (1972), 233-254.
[3] K. Dlaska, N. Ergun and M. Ganster, Countable S-closed spaces, Math. Slovaca, 44 (1994), 337-348.
[4] E. Ekici, Generalization of perfectly continuous, Regular set- connected and clopen functions, Acta. Math. Hungar., 107(3
[5] E. Ekici, Properties of Regular set-connected Functions, Kyungpook Math. J. 44 (2004), 395-403.
[6] N. Ergun, On nearly paracompact spaces, Intanbul. Univ. Fen. Mec. Ser. A, 45 (1980), 65-87.
[7] N. Levine, Semi-open sets and semi-continuity in topological spaces, Amer. Math. Monthly, 70 (1963), 36-41.
[8] N. Levine, Strong continuity in topological spaces, Amer. Math. Monthly, 67 (1960), 269.
[9] N. Levine, Generalized closed sets in topology, Rend. Circ. Math. Palermo, 19 (2) (1970), 89-96.

[10] P.E. Long and L.L. Herrington, Basic properties of regular closed functions, Rend. Circ. Mat. Palermo, 27 (1978), 20-28.
[11] G.D. Maio, S-closed spaces, S-sets and S-continuous functions, Acad. Sci. Torino, 118 (1984), 125-134.
[12] T. Noiri and G.D. Maio, Properties of α-compact spaces, Rend. Circ. Mat. Palermo. Ser II, (1998), 359-369.
[13] M.K. Singal, A.R. Singal and A. Mathur, On nearly compact spaces, Boll. UMI, 4 (1969), -710.
[14] M.H. Stone , Applications of the theory of Boolean rings to general topology, Trans. Amer. Math. Soc. 41(1937), 375-381.
[15] P. Sundaram, N.Rajesh, M.Lellis Thivagar and Zbigniew Duszyski, ğ-semi-closed sets intopological spaces,
[16] T. Thomson, S-closed spaces, Proc. Amer. Math. Soc. 60 (1976), 335-338.
[17] M.K.R.S. Veerakumar, #g-semi-closed sets in topological spaces, Antarctica J. Math 2(2) (2005), 201-222.
19. M.K.R.S. Veerakumar, ĝ-closed sets in toplogical spaces, Bull.Allahabad Math. Soc., 18 (2003), 99-112.



10.22214/IJRASET

INDEX COPERNICUS 45.98

IMPACT FACTOR: 7.129

IMPACT FACTOR: 7.429

TOGETHER WE REACH THE GOAL

INTERNATIONAL JOURNAL FOR RESEARCH

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY