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On Ternary Quadratic Diophantine Equation

$$15x^2 + 15y^2 + 24xy = 438z^2$$

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Abstract**: The ternary quadratic Diophantine equation* $15x^2 + 15y^2 + 24xy = 438z^2$ ***is analyzed for its non-trivial distinct integral solutions. Six different patterns of integral solutions are obtained. A few interesting relations among the solutions and special polygonal numbers are presented.

***Keywords**: Ternary quadratic equation, Integral solutions.*

Notations:

$T_{3,n} = \frac{n(n+1)}{2}$ = Triangular number of rank n.

$T_{7,n} = \frac{n(5n-3)}{2}$ = Heptagonal number of rank n.

$T_{10,n} = n(4n-3)$ = Decagonal number of rank n.

$T_{12,n} = n(5n-4)$ = Dodecagonal number of rank n.

$T_{13,n} = \frac{n(11n-9)}{2}$ = Tridecagonal number of rank n.

$T_{17,n} = \frac{n(15n-13)}{2}$ = Heptadecagonal number of rank n.

$T_{18,n} = n(8n-7)$ = Octadecagonal number of rank n.

$Gno_n = (2n-1)$ = Gnomonic number of rank n.

Mathematical Classification: 11D09.

I. INTRODUCTION

Ternary quadratic equations are rich in variety. For more detailed understanding one can see [1-7]. For the non-trivial integral solutions of ternary quadratic Diophantine equations [8-9] has been studied. [10-13] has been referred for various ternary quadratic Diophantine equations. In this communication, we consider yet another interesting ternary quadratic equation $15x^2 + 15y^2 + 24xy = 438z^2$ and obtain different patterns of non-trivial integral solutions. Also, a few interesting relations among the solutions and special polygonal, Centered, Gnomonic, Star numbers are presented.

II. METHOD OF ANALYSIS

The Quadratic Diophantine equation with four unknowns under consideration is

$$15x^2 + 15y^2 + 24xy = 438z^2 \qquad (1)$$

The substitution of the linear transformations

$$x = u + v \quad and \quad y = u - v \qquad (2)$$

in (1) leads to

$$9u^2 + v^2 = 73z^2 \qquad (3)$$

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Four different choices of solutions to (3) are presented below. Once the values of u and v are known, using (2), the corresponding values of X and Y are obtained.

A.Pattern 1

In (3), $9u^2+v^2=73z^2$

Assume that

$$z=9a^2+b^2,\quad a,b\neq 0 \tag{4}$$

We can write $73=(3+8i)(3-8i)$ (5)

Substituting (4) and (5) in (3), we get

$$(3u+iv)(3u-iv)=(3+8i)(3-8i)(3a+ib)^2(3a-ib)^2$$

Equating the positive and negative factors, we get

$$(3u+iv)=(3+8i)(3a+ib)^2 \tag{6}$$

$$(3u-iv)=(3-8i)(3a-ib)^2 \tag{7}$$

Equating the real and imaginary parts in either (6) or (7), we get

$$u=9a^2-b^2-16ab \tag{8}$$

$$v=72a^2-8b^2+18ab \tag{9}$$

Substituting (8) and (9) in (2), we get

$$\left.\begin{aligned} x&=x(a,b)=81a^2-9b^2+2ab\\ y&=y(a,b)=-63a^2+7b^2-34ab\\ z&=z(a,b)=9a^2+b^2 \end{aligned}\right\} \tag{10}$$

Thus (10) represent non-zero distinct integer solution to (1) in two parameters.

B. Observations

1) $x(a,a)-74T_{4,a}=0$
2) $y(a,a)+90T_{4,a}=0$
3) $x(2a,a)+y(2a,a)=6a^2$, a nasty number.
4) $z(a(a+1),b(b+1))-9P_a^2-P_b^2=0$
5) $x(a,a+1)+y(a,a+1)+z(1,5a)+29=(3a-6)^2$, a perfect square.
6) $x(a,a)+y(a,a)+z(a,a)=-6a^2$, a nasty number.

C. Pattern 2

In (3), $9u^2+v^2=73z^2$

Consider the linear transformations

$$\left.\begin{aligned} z&=X+9T\\ u&=X+73T \end{aligned}\right\} \tag{11}$$

or

$$\left.\begin{aligned} z&=X-9T\\ u&=X-73T \end{aligned}\right\} \tag{12}$$

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Substituting (11) or (12) in (3), we get

$$v^2 = 64(X^2 - 657T^2) \tag{13}$$

Write $v = 8V$ (14)

Substituting (14) in (13), we get

$$V^2 = X^2 - 657T^2$$

$$X^2 = 657T^2 + V^2 \tag{15}$$

This is in the standard form $x^2 = Dy^2 + z^2$

The corresponding solutions to (15) are

$$\left.\begin{aligned} T &= 2ab \\ V &= 657a^2 - b^2 \\ X &= 657a^2 + b^2 \end{aligned}\right\} \tag{16}$$

Substituting (16) in (11) and (14), we get

$$\left.\begin{aligned} z &= 657a^2 + b^2 + 18ab \\ u &= 657a^2 + b^2 + 146ab \\ v &= 5256a^2 - 8b^2 \end{aligned}\right\} \tag{17}$$

Substituting (17) in (2), we get

$$\left.\begin{aligned} x &= x(a,b) = 5913a^2 - 7b^2 + 146ab \\ y &= y(a,b) = -4599a^2 + 9b^2 + 146ab \\ z &= z(a,b) = 657a^2 + b^2 + 18ab \end{aligned}\right\} \tag{18}$$

Thus (18) represent non-zero distinct integer solution to (1) in two parameters.

D. Observations

1) $x(a,a) - 6052T_{4,a} = 0$
2) $y(a,a) + 4444T_{4,a} = 0$
3) $z(a,a)$ is a perfect square.
4) $x(a,a+1) - y(a,a+1) \equiv 16 \pmod{32}$
5) $x(a,a+1) - y(a,a+1) - T_{68,a} \equiv -16 \pmod{10463}$
6) $y(1,B+3) = 100Gno_B + 9T_{4,B} - 3980$

E. Pattern 3

The Ternary quadratic equation (3) can be written as

$$9u^2 - 9z^2 = 64z^2 - v^2 \tag{19}$$

Factorizing (19) we have

$$(3u+3z)(3u-3z) = (8z+v)(8z-v)$$

$$\frac{3u+3z}{8z+v} = \frac{8z-v}{3u-3z} = \frac{A}{B}, B \neq 0 \tag{20}$$

This is equivalent to the following two equations.

$$Av-3Bu+z(8A-3B)=0 \tag{21}$$

$$Bv-3uA-z(3A+8B)=0 \tag{22}$$

Applying the method of cross multiplication, we get

$$z=z(A,B)=3A^2+3B^2 \tag{23}$$

$$\left.\begin{aligned} u=u(A,B)=3A^2-3B^2+16AB \\ v=v(A,B)=-24A^2+24B^2+18AB \end{aligned}\right\} \tag{24}$$

Substituting (22) in (2), we get

$$\left.\begin{aligned} x=x(A,B)=-21A^2+21B^2+34AB \\ y=y(A,B)=27A^2-27B^2-2AB \end{aligned}\right\} \tag{25}$$

Thus (23) and (25) represent non-zero distinct integer solution to (1) in two parameters.

F. Observations

1) $x(A,A)-y(A,A)=(6A)^2, a$ perfect square.

2) $z(A,A)$ is a nasty number.

3) $x(A,A+1)+y(A,A+1)+T_{44,A}\equiv -6(\mathrm{mod}\,53)$

4) $z(A,A(A+1))=3T_{4,A}^2+12P_A^5$

5) $x(A,1)+Gno_{13n}+8=y(A,1)+star_A$

6) $x(A,A(4A-3))+y(A,A(4A-3))+96T_{4,A}^2=T_{4,A}\left[Gno_{136A}-143\right]$

G. Pattern 4

(15) can be written as

$$X^2-V^2=657T^2$$

$$(X+V)(X-V)=(657T)T$$

Equating the positive and negative factors we get

$$X+V=657T \tag{26}$$

$$X-V=T \tag{27}$$

Solving (26) and (27), we get

$$\left.\begin{aligned} X=329T \\ V=328T \end{aligned}\right\} \tag{28}$$

Substitute (28) in (14) we get

$$v=2624T$$

$$For\ T=A,\ v=2624A$$

Substitute the value of v in (11), we get

$$u=402A$$

Substituting u and v in (2)

$$\left.\begin{aligned} x &= 3026A \\ y &= -2222A \\ z &= 338A \end{aligned}\right\} \qquad (29)$$

Thus (29) represent non-zero distinct integer solution to (1) in one parameter.

H. Observations

1) $z(A^2)+z(A)-338\,\mathrm{Pr}_A=0$
2) $2x(A^2)+2y(A^2)-4z(A^2)=(16A)^2$, is a perfect square.
3) $y(A^3)+y(A^2)+4444P_A^5=0$
4) $3x(A^2)+3y(A^2)+z(A^2)-59T_{4,A}=(53A)^2$, is a perfect square.
5) $x(A^2)+x(A)-6052P_A^2=0$

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