iJRASET
International Journal For Research in
Applied Science and Engineering Technology

INTERNATIONAL JOURNAL FOR RESEARCH

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY

Volume: 6 **Issue:** I **Month of publication:** January 2018

DOI: http://doi.org/10.22214/ijraset.2018.1364

International Journal for Research in Applied Science & Engineering Technology (IJRASET)
ISSN: 2321-9653; IC Value: 45.98; SJ Impact Factor :6.887
Volume 6 Issue I, January 2018- Available at www.ijraset.com

On Ternary Quadratic Diophantine Equation $15x^2+15y^2+24xy=438z^2$

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***Abstract*: The ternary quadratic Diophantine equation $15x^2+15y^2+24xy=438z^2$ is analyzed for its non-trivial distinct integral solutions. Six different patterns of integral solutions are obtained. A few interesting relations among the solutions and special polygonal numbers are presented.**

***Keywords*: Ternary quadratic equation, Integral solutions.**

Notations:

$T_{3,n}=\frac{n(n+1)}{2}$ = Triangular number of rank n.

$T_{7,n}=\frac{n(5n-3)}{2}$ = Heptagonal number of rank n.

$T_{10,n}=n(4n-3)$ = Decagonal number of rank n.

$T_{12,n}=n(5n-4)$ = Dodecagonal number of rank n.

$T_{13,n}=\frac{n(11n-9)}{2}$ = Tridecagonal number of rank n.

$T_{17,n}=\frac{n(15n-13)}{2}$ = Heptadecagonal number of rank n.

$T_{18,n}=n(8n-7)$ = Octadecagonal number of rank n.

$Gno_n=(2n-1)$ = Gnomonic number of rank n.

Mathematical Classification: 11D09.

I. INTRODUCTION

Ternary quadratic equations are rich in variety. For more detailed understanding one can see [1-7]. For the non-trivial integral solutions of ternary quadratic Diophantine equations [8-9] has been studied. [10-13] has been referred for various ternary quadratic Diophantine equations. In this communication, we consider yet another interesting ternary quadratic equation $15x^2+15y^2+24xy=438z^2$ and obtain different patterns of non-trivial integral solutions. Also, a few interesting relations among the solutions and special polygonal, Centered, Gnomonic, Star numbers are presented.

II. METHOD OF ANALYSIS

The Quadratic Diophantine equation with four unknowns under consideration is

$$15x^2+15y^2+24xy=438z^2 \quad (1)$$

The substitution of the linear transformations

$$x=u+v \ \ and \ \ y=u-v \quad (2)$$

in (1) leads to

$$9u^2+v^2=73z^2 \quad (3)$$

Four different choices of solutions to (3) are presented below. Once the values of u and v are known, using (2), the corresponding values of X and Y are obtained.

A. Pattern 1

In (3), $9u^2 + v^2 = 73z^2$

Assume that

$$z = 9a^2 + b^2, \quad a, b \neq 0 \qquad (4)$$

We can write $73 = (3+8i)(3-8i)$ (5)

Substituting (4) and (5) in (3), we get

$$(3u+iv)(3u-iv) = (3+8i)(3-8i)(3a+ib)^2(3a-ib)^2$$

Equating the positive and negative factors, we get

$$(3u+iv) = (3+8i)(3a+ib)^2 \qquad (6)$$

$$(3u-iv) = (3-8i)(3a-ib)^2 \qquad (7)$$

Equating the real and imaginary parts in either (6) or (7), we get

$$u = 9a^2 - b^2 - 16ab \qquad (8)$$

$$v = 72a^2 - 8b^2 + 18ab \qquad (9)$$

Substituting (8) and (9) in (2), we get

$$\left.\begin{aligned} x &= x(a,b) = 81a^2 - 9b^2 + 2ab \\ y &= y(a,b) = -63a^2 + 7b^2 - 34ab \\ z &= z(a,b) = 9a^2 + b^2 \end{aligned}\right\} \qquad (10)$$

Thus (10) represent non-zero distinct integer solution to (1) in two parameters.

B. Observations

1) $x(a,a) - 74T_{4,a} = 0$
2) $y(a,a) + 90T_{4,a} = 0$
3) $x(2a,a) + y(2a,a) = 6a^2$, a nasty number.
4) $z(a(a+1), b(b+1)) - 9P_a^2 - P_b^2 = 0$
5) $x(a,a+1) + y(a,a+1) + z(1,5a) + 29 = (3a-6)^2$, a perfect square.
6) $x(a,a) + y(a,a) + z(a,a) = -6a^2$, a nasty number.

C. Pattern 2

In (3), $9u^2 + v^2 = 73z^2$

Consider the linear transformations

$$\left.\begin{aligned} z &= X + 9T \\ u &= X + 73T \end{aligned}\right\} \qquad (11)$$

or

$$\left.\begin{aligned} z &= X - 9T \\ u &= X - 73T \end{aligned}\right\} \qquad (12)$$

Substituting (11) or (12) in (3), we get

$$v^2 = 64(X^2 - 657T^2) \tag{13}$$

Write $v = 8V$ (14)

Substituting (14) in (13), we get

$$V^2 = X^2 - 657T^2$$

$$X^2 = 657T^2 + V^2 \tag{15}$$

This is in the standard form $x^2 = Dy^2 + z^2$

The corresponding solutions to (15) are

$$\left.\begin{aligned} T &= 2ab \\ V &= 657a^2 - b^2 \\ X &= 657a^2 + b^2 \end{aligned}\right\} \tag{16}$$

Substituting (16) in (11) and (14), we get

$$\left.\begin{aligned} z &= 657a^2 + b^2 + 18ab \\ u &= 657a^2 + b^2 + 146ab \\ v &= 5256a^2 - 8b^2 \end{aligned}\right\} \tag{17}$$

Substituting (17) in (2), we get

$$\left.\begin{aligned} x = x(a,b) &= 5913a^2 - 7b^2 + 146ab \\ y = y(a,b) &= -4599a^2 + 9b^2 + 146ab \\ z = z(a,b) &= 657a^2 + b^2 + 18ab \end{aligned}\right\} \tag{18}$$

Thus (18) represent non-zero distinct integer solution to (1) in two parameters.

D. Observations

1) $x(a,a) - 6052T_{4,a} = 0$
2) $y(a,a) + 4444T_{4,a} = 0$
3) $z(a,a)$ is a perfect square.
4) $x(a,a+1) - y(a,a+1) \equiv 16 \pmod{32}$
5) $x(a,a+1) - y(a,a+1) - T_{68,a} \equiv -16 \pmod{10463}$
6) $y(1,B+3) = 100Gno_B + 9T_{4,B} - 3980$

E. Pattern 3

The Ternary quadratic equation (3) can be written as

$$9u^2 - 9z^2 = 64z^2 - v^2 \tag{19}$$

Factorizing (19) we have

$$(3u+3z)(3u-3z) = (8z+v)(8z-v)$$

$$\frac{3u+3z}{8z+v} = \frac{8z-v}{3u-3z} = \frac{A}{B}, B \neq 0 \tag{20}$$

This is equivalent to the following two equations.

$$Av - 3Bu + z(8A - 3B) = 0 \quad (21)$$

$$Bv - 3uA - z(3A + 8B) = 0 \quad (22)$$

Applying the method of cross multiplication, we get

$$z = z(A, B) = 3A^2 + 3B^2 \quad (23)$$

$$\left.\begin{array}{l} u = u(A, B) = 3A^2 - 3B^2 + 16AB \\ v = v(A, B) = -24A^2 + 24B^2 + 18AB \end{array}\right\} \quad (24)$$

Substituting (22) in (2), we get

$$\left.\begin{array}{l} x = x(A, B) = -21A^2 + 21B^2 + 34AB \\ y = y(A, B) = 27A^2 - 27B^2 - 2AB \end{array}\right\} \quad (25)$$

Thus (23) and (25) represent non-zero distinct integer solution to (1) in two parameters.

F. Observations

1) $x(A, A) - y(A, A) = (6A)^2, a$ perfect square.

2) $z(A, A)$ is a nasty number.

3) $x(A, A+1) + y(A, A+1) + T_{44,A} \equiv -6 \pmod{53}$

4) $z(A, A(A+1)) = 3T_{4,A}^2 + 12P_A^5$

5) $x(A,1) + Gno_{13n} + 8 = y(A,1) + star_A$

6) $x(A, A(4A-3)) + y(A, A(4A-3)) + 96T_{4,A}^2 = T_{4,A}\left[Gno_{136A} - 143\right]$

G. Pattern 4

(15) can be written as

$$X^2 - V^2 = 657T^2$$

$$(X + V)(X - V) = (657T)T$$

Equating the positive and negative factors we get

$$X + V = 657T \quad (26)$$

$$X - V = T \quad (27)$$

Solving (26) and (27), we get

$$\left.\begin{array}{l} X = 329T \\ V = 328T \end{array}\right\} \quad (28)$$

Substitute (28) in (14) we get

$$v = 2624T$$

$$For\ T = A,\ v = 2624A$$

Substitute the value of v in (11), we get

$$u = 402A$$

Substituting u and v in (2)

$$\left.\begin{aligned} x &= 3026A \\ y &= -2222A \\ z &= 338A \end{aligned}\right\} \qquad (29)$$

Thus (29) represent non-zero distinct integer solution to (1) in one parameter.

H. Observations

1) $z(A^2) + z(A) - 338\,\mathrm{Pr}_A = 0$
2) $2x(A^2) + 2y(A^2) - 4z(A^2) = (16A)^2$, is a perfect square.
3) $y(A^3) + y(A^2) + 4444P_A^5 = 0$
4) $3x(A^2) + 3y(A^2) + z(A^2) - 59T_{4,A} = (53A)^2$, is a perfect square.
5) $x(A^2) + x(A) - 6052P_A^2 = 0$

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